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THE

PRACTICAL DRAUGHTSMAN'S
BOOK OF INDUSTRIAL DESIGN:

FORMING A COMPLETE COURSE OF

Mechanical, Engineering, and Architectural Drawing.

FOUNDED UPON THE "NOUVEAU COURS RAISONNÉ DE DESSIN INDUSTRIEL" OF
MM. ARMENGAUD, AINÉ, ARMENGAUD, JEUNE, AND AMOUROUX,
CIVIL ENGINEERS, PARIS.

CONTAINING ADDITIONAL PLATES AS EXAMPLES OF THE MOST USEFUL AND
GENERALLY EMPLOYED MECHANISM OF THE DAY.

BY

WILLIAM JOHNSON, Assoc. Inst. C.E.,
EDITOR OF "THE PRACTICAL MECHANIC'S JOURNAL."

SECOND EDITION,
CAREFULLY REVISED, WITH THE FRENCH MEASURES CONVERTED INTO ENGLISH.

"The PHILOSOPHER may very justly be delighted with the extent of his views, and the ARTIFICER with the readiness of his hands: but let the one remember, that without MECHANICAL performances, refined speculation is an empty dream; and the other, that without THEORETICAL reasoning, dexterity is little more than a brute instinct."—*Dr. Johnson.*

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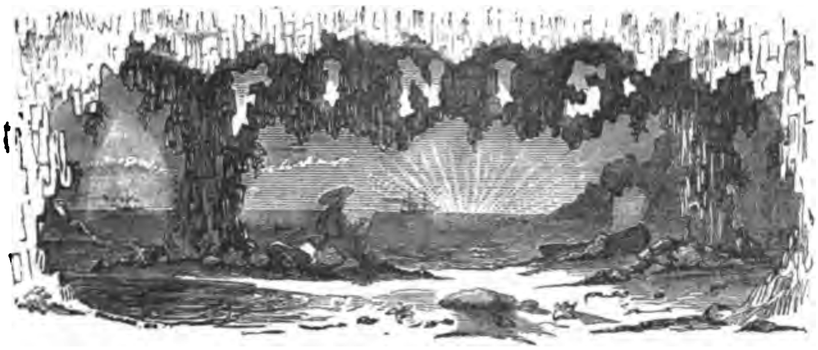


PREFACE TO THE FIRST EDITION.

INDUSTRIAL DESIGN is destined to become a universal language; for in our material age of rapid transition from abstract to applied Science—in the midst of our extraordinary tendency towards the perfection of the means of conversion, or manufacturing production—it must soon pass current in every land. It is, indeed, the medium between Thought and Execution; by it alone can the genius of Conception convey its meaning to the skill which executes—or suggestive ideas become living, practical realities. It is emphatically the exponent of the projected works of the Practical Engineer, the Manufacturer, and the Builder; and by its aid only, is the Inventor enabled to express his views before he attempts to realise them.

Boyle has remarked, in his early times, that the excellence of manufactures, and the facility of labour, would be much promoted, if the various expedients and contrivances which lie concealed in private hands, were, by reciprocal communications, made generally known; for there are few operations that are not performed by one or other with some peculiar advantages, which though singly of little importance, would, by conjunction and concurrence, open new inlets to knowledge, and give new powers to diligence; and Herschel, in our own days has told us that, next to the establishment of scientific institutions, nothing has exercised so powerful an influence on the progress of modern science, as the publication of scientific periodicals, in directing the course of general observation, and holding conspicuously forward models for emulative imitation. Yet, without the aid of Drawing, how can this desired reciprocity of information be attained; or how would our scientific literature fulfil its purpose, if denied the benefit of the graphic labours of the Draughtsman? Our verbal interchanges would, in truth, be vague and barren details, and our printed knowledge, misty and unconvincing.

Independently of its utility as a precise art, Drawing really interests the student, whilst it instructs him. It instils sound and accurate ideas into his mind, and developes his intellectual powers in compelling him to observe—as if the objects he delineates were really before his eyes. Besides, he always does that the best, which he best understands; and in this respect, the art of Drawing operates as a powerful stimulant to progress, in continually yielding new and varied results.

A chance sketch—a rude combination of carelessly considered pencillings—the jotted memoranda of a contemplative brain, prying into the corners of contrivance—often form the nucleus of a splendid invention. An idea thus preserved at the moment of its birth, may become of incalculable value, when rescued from the desultory train of fancy, and treated as the sober offspring of reason. In nice gradations, it receives the refining touches of leisure,—becoming, first, a finished sketch,—then a drawing by the practised hand—so that many minds may find easy access to it, for their joint counsellings to improvement—until it finally emerges from the workshop, as a practical triumph of mechanical invention—an illustrious example of a happy combination opportunely noticed. Yet many ingenious men are barely able even to start this train of production, purely from inability to delineate adequately their early conceptions, or furnish that transcript of their minds which might make their thoughts immortal. If the present Treatise succeeds only in mitigating this evil, it will not entirely fail in its object; for it will at least add a few steps to the ladder of Intelligence, and form a few more approaches to the goal of Perfection.

The study of Industrial Design is really as indispensably necessary as the ordinary rudiments of learning. It ought to form an essential feature in the education of young persons for whatever profession or employment they may intend to select, as the great business of their lives; for without a knowledge of Drawing, no scientific work, whether relating to Mechanics, Agriculture, or Manufactures, can be advantageously studied. This is now beginning to receive acknowledgment, and the routine of study in all educational establishments is being benefited by the introduction of the art.

The special object of the PRACTICAL DRAUGHTSMAN'S BOOK OF INDUSTRIAL DESIGN may almost be gathered from its title-page. It is intended to furnish gradually developed lessons in Geometrical Drawing, applied directly to the various branches of the Industrial Arts: comprehending Linear Design proper; the study of Projections; the Drawing of Toothed Wheels and Eccentrics; with Shading and Colouring; Oblique Projections; Parallel, Exact, and Isometrical Perspective; each division being accompanied by special applications to the extensive ranges of Mechanics, Architecture, Foundry-Works, Carpentry, Joinery, Metal Manufactures generally, Hydraulics, the construction of Steam Engines, and Mill-Work. In its compilation, the feeble attraction generally offered to students in elementary form has been carefully considered; and after every geometrical problem, a practical example of its application has been added, to facilitate its comprehension and increase its value.

The work is comprised within nine divisions, appropriated to the different branches of Industrial Design. The first which concerns Linear Drawing only, treats particularly of straight lines—of circles—and their application to the delineation of Mouldings, Ceilings, Floors, Balconies, Cuspids, Rosettes, and other forms, to accustom the student to the proper use of the Square, Angle, and Compasses. In addition to this, it affords examples of different methods of constructing plain curves, such as are of frequent occurrence in the arts, and in mechanical combinations—as the ellipse, the oval, the parabola, and the volute; and certain figures, accurately shaded, to represent reliefs, exemplifying cases where these curves are employed.

The second division illustrates the geometrical representation of objects, or the study of projections. This forms the basis of all descriptive geometry, practically considered. It shows that a single figure is insufficient for the determination of all the outlines and dimensions of a given subject; but that two projections, and one or more sections, are always necessary for the due interpretation of internal forms.

The third division points out the conventional colours and tints for the expression of the sectional details of objects, according to their nature; furnishing, at the same time, simple and easy examples, which may at once interest the pupil, and familiarise him with the use of the pencil.

In the fourth division are given drawings of various essentially valuable curves, as Helices, and different kinds of Spirals and Serpentines, with the intersection of surfaces and their development, and workshop applications to Pipes, Coppers, Boilers, and Cocks. This study is obviously of importance in many professions, and clearly so to Ironplate-workers, as Shipbuilders and Boiler-makers, Tinmen and Coppersmiths.

The fifth division is devoted to special classes of curves relating to the teeth of Spur Wheels, Screws and Racks, and the details of the construction of their patterns. The latter branch is of peculiar importance here, inasmuch as it has not been fully treated of in any existing work, whilst it is of the highest value to the pattern-maker, who ought to be acquainted with the most workmanlike plan of cutting his wood, and effecting the necessary junctions, as well as the general course to take in executing his pattern, for facilitating the moulding process.

The sixth division is, in effect, a continuation of the fifth. It comprises the theory and practice of drawing Bevel, Conical, or Angular Wheels, with details of the construction of the wood patterns, and notices of peculiar forms of some gearing, as well as the eccentrics employed in mechanical construction.

The seventh division comprises the studies of the shading and shadows of the principal solids—Prisms, Pyramids, Cylinders, and Spheres, together with their applications to mechanical and architectural details, as screws, spur and bevel wheels, coppers and furnaces, columns and entablatures. These studies naturally lead to that of colours—single, as those of China Ink or Sepia, or varied; also of graduated shapes produced by successive flat tints, according to one method, or by the softening manipulation of the brush, according to another.

The pupil may now undertake designs of greater complexity, leading him in the eighth division to various figures representing combined or general elevations, as well as sections and details of various complete machines, to which are added some geometrical drawings, explanatory of the action of the moving parts of machinery.

The ninth completes the study of Industrial Design, with oblique projections, and parallel, exact, and isometrical perspective. In the study of exact perspective, special applications of its rules are made to architecture and machinery by the aid of a perspective elevation of a corn mill supported on columns, and fitted up with all the necessary gearing.

A series of Plates, marked A, B, &c., are also interspersed throughout the work, as examples of finished drawings of machinery. The Letterpress relating to these Plates will form an appropriate Appendix to the Volume.

Such is the scope, and such are the objects, of the PRACTICAL DRAUGHTSMAN'S BOOK OF INDUSTRIAL DESIGN.

This is the course now submitted to the consideration of all who are in the slightest degree connected with the Constructive Arts. It aims at the dissemination of those fundamental teachings which are so essentially necessary at every stage in the application of the forces lent to us by Nature for the conversion of her materials. For "man can only act upon Nature, and appropriate her forces to his use, by comprehending her laws, and knowing those forces in relative value and measure." All art is the true application of knowledge to a practical end. We have outlived the times of random construction, and the mere heaping together of natural substances. We must now design carefully and delineate accurately before we proceed to execute—and the quick pencil of the ready draughtsman is a proud possession for our purpose. Let the youthful student think on this; and whether in the workshop of the Engineer, the studio of the Architect, or the factory of the Manufacturer, let him remember that, to spare the blighting of his fondest hopes, and the marring of his fairest prospects—to achieve, indeed, his higher aspirations, and verify his loftier thoughts, which point to eminence—he must give his days and nights to the study of

Industrial Design.

PREFACE TO THE SECOND EDITION.

THE First Edition of this Work having been exhausted, a Second is now offered to the Students of Mechanical and Architectural Drawing, which, it is hoped, will not be found less useful than it may be presumed the First has been. The whole has been very carefully revised, and various additions have been made throughout. French measures will no longer give trouble to the reader. The illustrated chapter on Drawing Instruments has been extended, so as to comprise all the additions which have been made to these important auxiliaries since the publication of the First Edition of this Work, as well as the improvements which have from time to time been introduced in the arrangement or form of existing instruments. We have deemed it the proper course to commence the Work with the descriptive matter relating to this part of our subject; so that the Student is, at the outset, put in possession of the necessary information connected with the requisites of the art. Many of the Instruments, we shall have occasion to describe, are not required by the young draughtsman, but the information connected therewith may prove highly useful. Our object is to anticipate the wants of the reader, by providing for his use all the information within our reach. To afford space for the extension of the introductory portion of our subject, it will be necessary to omit some of the Tabular Matter, which is to be found in every Work of Reference connected with Mechanics.

ABBREVIATIONS AND CONVENTIONAL SIGNS.

In order to simplify the language or expression of arithmetical and geometrical operations, the following conventional signs are used :—

The sign $+$ signifies *plus* or *more*, and is placed between two or more terms to indicate addition.

EXAMPLE : $4 + 3$, is 4 *plus* 3, that is, 4 added to 3, or 7.

The sign — signifies *minus* or *less*, and indicates subtraction.

Ex. : $4 - 3$, is 4 minus 3, that is, 3 taken from 4, or 1.

The sign $\times$ signifies *multiplied by*, and, placed between two terms, indicates multiplication.

Ex. : 5×3, is 5 multiplied by 3, or 15.

When quantities are expressed by letters, the sign may be suppressed. Thus we write indifferently :—

$$a \times b, \text{ or } ab.$$

The sign : or (as it is more commonly used) $\div$, signifies *divided by*, and, placed between two quantities, indicates division.

Ex. : $12 : 4$, or $12 \div 4$, or $\frac{12}{4}$, is 12 divided by 4.

The sign $=$ signifies *equals* or *equal to*, and is placed between two expressions to indicate their equality.

Ex. : $6 + 2 = 8$, meaning, that 6 plus 2 is equal to 8.

The union of these signs, : :: : indicates geometrical proportion.

Ex. : $2 : 3 :: 4 : 6$, meaning, that 2 is to 3 as 4 is to 6.

The sign $\sqrt{\ }$ indicates the extraction of a root ; as,

$$\sqrt{9} = 3, \text{ meaning, that the square root of 9 is equal to 3.}$$

The interposition of a numeral between the opening of this sign, $\sqrt{\ }$, indicates the degree of the root. Thus—

$$\sqrt[3]{27} = 3, \text{ expresses that the cube root of 27 is equal to 3.}$$

The signs $\angle$ and 7 indicate respectively, *smaller than* and *greater than.*

Ex. : $3 \angle 4$, $=$ 3 smaller than 4 ; and, reciprocally, 4 7 3, $=$ 4 greater than 3.

Fig. signifies figure ; and pl., plate.

EXPLANATION OF TECHNICAL TERMS IN GEOMETRY.

To *describe* a figure means to draw it on a plane surface, such as a sheet of paper.

To *produce* or *prolong* a straight line is to lengthen it in the same straight line.

A right-lined figure is said to be *circumscribed* about another, and the latter is said to be *inscribed* in the former when all the vertices of the inner figure are in the sides of the outer. In the case of circles circumscribing right-lined figures, the vertices of the latter are in the circumference ; and in the case of right-lined figures circumscribing circles, all the sides of the former are tangents to the latter.

To *bisect* signifies to divide into two equal parts ; to *trisect* to divide into three equal parts.

Two lines or figures are said to *coincide* when they agree in every respect.

The *point of intersection* of two lines is that where they cross each other.

A *given line* signifies a fixed or known line, the position or magnitude of which is given.

A *problem* is something required to be constructed or made ; a *theorem* is something required to be proved ; and both are *propositions*.

A *corollary* is something that follows as an additional conclusion from what has been constructed or proved.

An *axiom* is the statement of a truth admitted by all the world, and requiring no demonstration.

THE

PRACTICAL DRAUGHTSMAN'S

BOOK OF INDUSTRIAL DESIGN.

"We are not to forget the powerful effect which must be produced by the spread of elementary works and digests of what is actually known in each branch of science. * * * It is only by condensing, simplifying, and arranging in the most lucid possible manner the acquired knowledge of past generations, that those to come can be enabled to avail themselves to the full of the advanced point from which they will start."—*Sir John Herschel.*

CHAPTER I.

DRAWING INSTRUMENTS.

In the following pages we propose to teach the art of mechanical drawing, by going through such a complete course of instruction as will enable the student, with due attention and practice, to understand the subject thoroughly, and qualify him to produce drawings similar to our example plates. In carrying out our object we shall commence with the simplest rudiments of the art, and proceed onwards in a regular and carefully graduated course of lessons, until we arrive at the delineation of the finished subject. And, by giving, at an early stage of our labour, directions for producing a number of pleasing and useful combinations of geometrical figures, such as are delineated in the second of our illustrative plates, we hope to impart to the subject such an amount of interest as will greatly lighten the labour of the student, whilst he is overcoming the elementary difficulties.

Mechanical drawing is distinguished from "free hand drawing" by its lines and curves being drawn of uniform width and sharpness throughout, and with a degree of accuracy unattainable without the assistance of instruments. It will be therefore proper, before proceeding to the practical part of our work, to explain and illustrate the several instruments made use of by the draughtsman. Some of these instruments are indispensable to the beginner; the first requisite is:—

The Drawing Board: This, if of the ordinary kind, is made of well-seasoned deal of the best quality, and free from knots. The board is of a rectangular figure, the size being regulated according to the kind of paper to be used. The wood should be about ⅞ or 1 inch thick when planed up, the pieces of plank being carefully joined with tongue and groove joints; the edges of the board must also be accurately squared and at right angles to each other. A very useful size of board is 42 inches by 29; this size is adapted for double elephant paper, so as to leave a clear margin of one inch all round the edge.

Another size which is very well adapted for the beginner, and is more portable, is one of 24 by 18½ inches; this will take a sheet of demy or medium drawing paper, or a half-sheet of double elephant, either of which sizes answers well for generally useful drawings. The battens at the back of the drawing board should be from 1¼ to 1½ inches thick, and should have the outer edges bevelled off, for convenience in carrying the board. The screws which pass through the battens to connect them to the boards should not pass through simple holes, but have slots made for them in the battens, and these slots should be from about half an inch to ⅝ in length. When so fitted, the arrangement of the slots admits of the extension and shrinking of the board, without causing the joints to open, which always occurs when the battens are rigidly secured to the board. We have seen some very superior drawing boards made in narrow strips grooved and tongued together, the object being to prevent the casting or warping which sometimes occurs in the width of the plank forming the board.

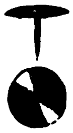
Some drawing boards are made with end pieces, with the grain at right angles to the length of the board, but these are objectionable, owing to the shrinking of these pieces, and so leaving unseemly openings between the contiguous edges, which are inconvenient when using thin drawing paper. To do away with the necessity of end pieces, and at the same time prevent the corners of the board from splitting off, it is well to have a small rectangular tenon of cross-grained wood let into a mortise made at each corner of the board. Without this precaution drawing boards are very liable to be split off at the corners in moving them about, or carrying them from place to place.

If lightness and portability are objects of importance to the student, he should select a French board. These drawing boards are made of thin but thoroughly seasoned wood, which does not warp, and they are very light and well adapted for carrying about.

In the drawing boards to which we have referred, the paper is strained by glueing its edges, when damped, down to the board, and this kind is by far the most generally used. But what is termed the "panel drawing board" is designed to obviate the necessity of using glue, or other adhesive material. These boards are generally made in mahogany, and they consist of an open frame to which is fitted accurately, a solid centre panel, and this panel is secured to the frame, when required, by moveable cross bars, or wedges. The disadvantages of these boards are, that the shrinking of the panel causes the paper to slacken at times, and so gives the draughtsman a good deal of trouble in his work. The paper in drying also frequently forms creases at the angles of the panel, the ridges of which are very inconvenient, if an attempt is made at drawing without re-straining the paper. Therefore, on the score of economy, as well as *bona fide* utility, we recommend the student to provide himself with an ordinary plain board of good quality.

When the drawing is to be only in outline, it is sufficient if the paper be fastened down at the corners with drawing pins. The English drawing pin is made with a circular brass head, the edges of which are bevelled off; the pin is about three-sixteenths of an inch in length, and is made of thick wire. The French drawing pin, which is greatly to be preferred, is shown in the accompanying engraving, fig. 1, in elevation and plan. The head of this drawing pin is made of very thin metal, the outer surface being slightly convex; the pin is of fine steel wire, and polished as highly as an ordinary sewing needle. These drawing pins hold the paper better than the ordinary English pin, the holes made in the paper are much smaller, and the T square glides over the heads of these pins without obstruction.

Fig. 1.



If the drawing is to be tinted, it is necessary to strain the paper on the board, and this, indeed, should be done in all cases where anything more than a mere rough drawing is to be made. The straining of the paper is best effected in the following manner: The sheet of paper is laid upon the drawing board, with the surface upon which the drawing is to be made, uppermost; about half an inch of the paper is turned in all round the margin, this inwardly turned part being pressed down upon the face of the paper with a folder. The paper is now laid with the face to the board, and the back is wetted all over by means of a sponge dipped into clean water, going over the whole surface excepting the inwardly turned margin. As soon as the paper has absorbed the water, so that it no longer shines upon the surface, the sheet is reversed and placed evenly upon the board. The folded edges are now raised from the face of the paper, and this part is gone over with a brush dipped in melted glue. Some have advised the use of a piece of solid glue dipped in hot water, so as to dissolve the surface portion of the glue, but this way is neither so expeditious or satisfactory, as the use of the glue pot and brush. As the sides of the sheet of paper are successively glued, they are rubbed down upon the board with a paper folder or other smooth surface. When the paper is dry it will be found to present a perfectly smooth and uninjured surface for the drawing to be made on. Animal and vegetable parchments are strained in like manner, only observing that in animal parchment there is a difference between the surfaces, the smooth side which is called the right side, being preferred by some for drawing on, whilst others consider that the back of the skin takes the ink and colour much better.

It is, however, with paper drawings that the young student will have to deal, and for his early efforts, cartridge paper, of good quality, will answer his purpose very well; at the same time it is much lower in price than drawing paper.

Drawing Paper is made in sheets of various sizes, to which the following distinctive names have been given; we have appended their respective dimensions in inches.

	Ins.		Ins.		Ins.		Ins.
Demy, ...	20	×	15½	Columbian, . . .	36	×	23½
Medium, ...	22¾	×	17½	Atlas, ...	34	×	26
Royal, ...	24	×	19¼	Double Elephant, .	40	×	27
Super Royal, . . .	27¼	×	19¼	Antiquarian, . . .	53	×	31
Imperial, ...	30	×	22	Emperor, ...	68	×	48
Elephant, ...	28	×	23				

These papers are made in two kinds, viz.:—rolled and unrolled; the former has a smooth glazed surface, produced by passing the paper between heated copper cylinders. This kind of paper is well adapted for drawings in outline, or for sectional work, but the smooth glossy surface does not do for tinting on. The unrolled paper has a fine granulated opaque surface, which is just the texture required for receiving colour and obtaining a pure and uniform tint. There are various makers of drawing paper, but the name which stands pre-eminent for excellence in this article, is that of Mr. Whatman, of Turkey Mills.

The sizes of paper which we have given are sufficiently large for all ordinary drawings, but for special work, such as railway plans, sections of canals, tunnels, sewers, &c., paper of much greater length than "Emperor" is required. For this purpose, machine-made paper is supplied to the dealers in continuous lengths or pieces, which may be purchased by the yard at a very moderate price.

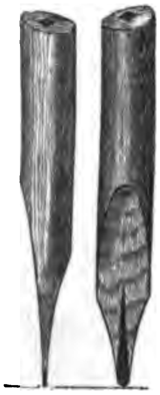
Before leaving this part of our subject we must say a word or two respecting tracing paper and tracing cloth. To be able to multiply quickly an original mechanical drawing, and to do this correctly, is at times of the utmost convenience to the draughtsman and engineer. These duplicates are obtained by means of tracing paper, or its substitute tracing cloth. Tracing paper is prepared by saturating large sheets or continuous lengths of tissue paper, with a preparation of Canada balsam, thinned with turpentine. This material renders the paper semi-transparent, so that when laid down upon a drawing, the lines may be readily traced over on the upper side of the superimposed paper, and in this manner a correct duplicate of the drawing is obtained. The tracing paper is kept steady by means of eight or more drawing pins put in round its margin. There is one very important objection to our English tracing paper, it gets opaque and very brittle by keeping. This is an evil which French draughtsmen do not experience, as the paper they use is very thin, pure in colour, and without any preparation on its surface. It is rendered sufficiently transparent for tracing purposes by rolling; the French makers indeed excel in the production of thin highly glazed papers. French tracing paper may be obtained from most of the leading artists' colourmen in this country.

Where a greater degree of durability is required, it is advisable to use tracing cloth. This material is made of fine muslin, rendered transparent and fit to draw on, by coating it with a preparation of wax. Tracing cloth is sold by the yard; its surface is admirably adapted for outline drawings, but it does not take colour well, as the water sinks through the prepared surface and destroys its brilliancy.

We now turn to the instruments required for setting out a drawing on the board, and as the lines must be put in first in pencil, we will show the best mode of preparing it for the draughtsman's use.

The pointing of a pencil may appear a very trivial matter, but the student may be reminded of Dr. Johnson's remark, "a thing that is worth doing at all, is worth doing well," and he will find that a due attention to small things, will wonderfully assist him in overcoming the difficulties of greater ones. For drawing lines with the assistance of the T, or set square, the point of the pencil should be cut flat and slightly rounded, in the manner shown in the subjoined illustration, fig, 2, which represents front and side views of the pointed end of the pencil. The wood at the sides of the point is cut away up to a moderate height, and these cuts are made at opposite angles of the enclosed rectangular slip of lead, so as to leave the wood extending down on each side of the lead towards the point. The front and back parts of the point are formed by cutting away the wood considerably above the sides, as shown in the figure, and in order to obtain the desired flat point, the wood is cut away freely, leaving a good portion of the lead denuded. This part of the lead is carefully cut to form a broad thin point, slightly rounded off at the sides, and quite sharp at the extremity as shown in the side view, fig. 2. A pencil pointed in this manner, is not only more convenient in use than the round point, but it wears longer, and is more easily kept in working order. And although a considerable portion of the lead is laid bare, there is less liability of fracture, on account of the lateral angles of the lead being well protected and supported by the cedar at the sides descending low down towards the point.

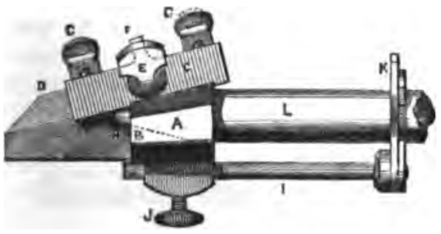
Fig. 2.



Various contrivances have from time to time been brought before the public for giving a finely rounded point to the pencil. Among those brought out some years ago, was Mr. Echstein's, which consists of two file cut metal surfaces, arranged at an angle of 45°, and forming a little trough, the metal being imbedded in a mahogany holder. The pencil is sharpened by rubbing it over either of the finely toothed metal surfaces, and finishing it in the apex of the angle. A homely substitute for this useful little instrument is fine glass paper, which produces a very good point, but speedily becomes filled with the particles of abraded lead.

A mechanical pencil sharpener of greater pretension has been introduced from France by Messrs. Marion, of Regent Street, London. Fig. 3, represents a side elevation of the instrument. A projection, A, is formed on the side of a piece of metal, sufficiently large to allow of a conical aperture, B, corresponding with the required cone of a pointed lead pencil. One side of this projection is slotted to receive the cutting edge of the small knife, C, which is attached to the inclined portion, D, of the metal block by the screw, E, passing through a slot in the knife. A short projection, F, is formed upon the knife for the convenience of adjustment, and when set, it is held in position by the two set-screws, G. A small handle is screwed into the block at H, from behind, for the convenience of holding the instrument when in use, and the end of this handle is hollowed to receive the small projection, F, on the knife, C, for the facility of holding it when detached for the purpose of sharpening the edge. An adjustable guide, I, is secured by the pinching screw, J, by one end, beneath the block, and is furnished with two arms, K, jointed on to the end of the rod of the guide, for embracing the pencil, L, during the cutting operation.

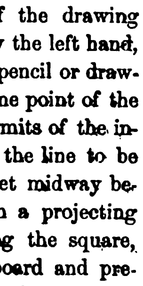
Fig 3.



In using this instrument, the pencil is simply passed between the two guide-arms, K, and its end is inserted in the conical hole, B. It is then turned round between the finger and thumb, and the knife-edge coming into contact with the end to be sharpened, quickly pares off the material. By this simple apparatus an excellent point is given to the pencil in a very short time.

Since that time we have bought for a single franc, in Paris, a still simpler form of cutter, consisting merely of a small brass holder with a conical hole for the pencil, and a little cutter screwed on outside. Now, however, the Americans supply us with something simpler and cheaper still, as shown in fig. 4. It is a cutter cast in a small conical holder of block tin. This sharpener is cast in a metal mould, the little knife being held in position by a small metal cam or retainer, whilst the block tin is poured round it. The whole is thus formed at one operation after the cutter is made: the holding ring at one end for the grasp of the fingers, the conical hole for the entry of the pencil, and the slot for the egress of the pencil shavings, being all produced in the mould. Our sketch shows this sharpener as in operation, with a pencil entered into it, and the shaving emerging from the slot. Such a sharpener may obviously be produced at an excessively cheap rate, and although when the cutter is blunted it must be thrown aside for a new one, as it cannot be sharpened, yet the very trifling expense of its primary production more than makes up for this defect.

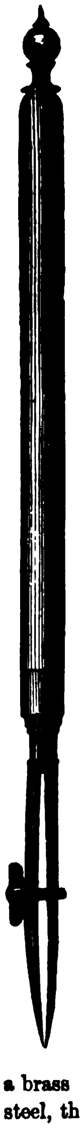
Fig. 4.



The T square is an instrument which is indispensable to the draughtsman. It is represented in its simplest form in fig. 6, plate 1. It consists of a head or stock, in which is fitted a long thin blade, E, set exactly at right angles to the head. The blade of the square is in this kind fitted flush with the upper face of the head, the edge of which is kept close up the edge of the drawing board or straight edge. The T square is operated by the left hand, and the upper edge of the blade against which the pencil or drawing pen works is bevelled off, so that only the extreme point of the pencil in drawing rests against the edge, which permits of the instrument being brought close up to the position of the line to be drawn. The English made T square has its blade set midway between the faces of the head, which is made with a projecting feather of about a quarter of an inch wide. In using the square, this feather rests upon the edge of the drawing board and prevents it from tilting or turning round in the hand. Another advantage is, that both edges of the blade are bevelled so that either may be used at pleasure. When working out at the end of the blade it is apt to deflect a little with the pressure against its edge. To avoid this, Messrs. Stanley and Robinson, of the Turnstile, London, introduced a T, in which the blade gradually increases in width from the outer end towards the bottom, so that at the junction with the head it is about four inches wide. This improvement leaves, of course, only one working edge like the French T square. In order to be enabled to set the blade of the square at any desired angle, the head is divided longitudinally, the two parts being connected by a set screw, which is adjusted by means of a milled head on the upper face of the head. The blade of the square is fixed to the upper half of the head, the lower portion being the moveable part. Upon loosening the screw slightly and turning the moveable part of the head about its axis, the blade may be set and fixed at any required angle. The addition of a graduated quadrant made of thin brass and let into the fixed part of the head so as to extend over the blade, enables the user to set the blade at once to the desired angle.

Set squares are constantly required for use in conjunction with the T square, for obtaining perpendicular lines, and for readily dividing circles into a given number of parts. These set squares are generally made out of thin pieces of pear tree or boxwood; they are made in various sizes, and the student will find it convenient to have three or four of different dimensions. Two sides of the set square invariably form a right angle, as shown in fig. 6, plate 1, whilst the third side is cut to an angle of 45°, 60°, or 70°, each kind being useful for special purposes.

Fig. 5.

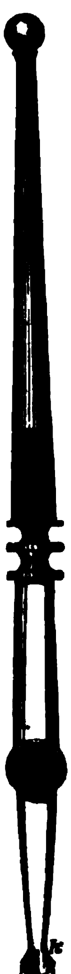


The ordinary English drawing pen is delineated in fig. 5; it consists of a pair of flat, tapered steel-blades, fixed to a handle of ivory or ebony. The ink is contained between the blades, and flows out from between the points, the thickness of line produced being dependent on the distance asunder of the points, which distance is regulated by a pinching-screw. In order to maintain a uniform thickness of line, care must be taken to clean the outsides of the points after each fresh supply of ink; this is best done by means of blotting paper or cloth, and the ink as it thickens between the points is cleared from time to time by passing the corner of a small piece of good writing paper between them.

It is often necessary to draw a number of lines, of different thicknesses, immediately succeeding each other. In this case, the inconvenience of repeatedly turning the adjusting-screw of the pen may be avoided by using a pen of the construction represented in figs. 6 and 7. This pen is the invention of M. Maubert, a French engineer, and it differs from the ordinary drawing-pen in the shape of the points, *g*, *h*, of which fig. 7 is an end view. These points are made broad and rounded, and are curved at the sides, so as to present convex surfaces towards each other; in other words, they touch each other at their centres, but gradually separate towards each side: and in using the pen, if a fine line is wanted, it is held vertically; if a thick line is needed, it is inclined more or less to either side, so as to bring the more separated portions of the acting edges in contact with the paper.

With the exception of the shape of the points, the pen represented in fig. 6 may be taken as an example of the best construction of a mathematical drawing-pen. The blades are formed of a single piece of well-tempered steel, and are fixed upon an ivory handle, by means of a brass socket. In some pens, the tips only of the blades are of steel, the remainder being of German silver, or of brass; and one blade is jointed at its root, so as to be capable of being opened out and cleaned, when necessary. A spring is fixed between the blades, so as to keep them open as far as the regulating screw will admit. Whilst, on the one hand, this facility in cleaning is an advantage, on the other, there is an accompanying liability of the joint getting loose, in which case the points can never be kept opposite to each other, and it is quite impossible to preserve uniformity and cleanness of lining with a pen in such a state.

Fig. 6.



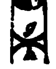


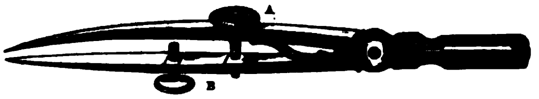
Fig. 7.

In making mechanical drawings in outline, and with shadow lines, two thicknesses of lining are necessary; and for this purpose, the secondary adjustment drawing-pen will be found very convenient. This modification of the common drawing-pen is the invention of Mr. G. P. Renshaw. It is represented in fig. 8. It has a regulating screw, A, like an ordinary drawing-pen; and by means of this screw the points are set for the thick or shadow line, whilst a secondary screw, B, is introduced, for regulating the thin or face line. The pen is made with a stronger spring than usual, and when a fine line is wanted, the points are pinched together by the grasp of the fingers, as closely as the screw, B, will permit; whilst a thick line is produced, by allowing the points to stand as far apart as the screw, A, is set for. We may here remark, that in using a drawing-pen of any construction, care must be taken not it press it against the straight edge, or ruler, as this will close the points; and if the pressure is not uniform, which is pretty certain to be the case, where any pressure at all is used, a line of irregular thickness will be the result.

In fig. 9 is represented a duplex pen, commonly known as "the road-pen" It is a very convenient instrument for drawing parallel lines, such as represent roads on maps. It consists of two pens, fixed upon one handle, their distance apart being regulated by a screw, with a central button, the portions entering the shanks of the pen having the thread in opposite directions, so as to open or shut the shanks according to the direction in which the button is turned. The two pens are of the usual construction, with regulating screws; and they may both be set for the same thickness of line or not, as convenient. Thus, one can be set for the face line, and the other for the shadow, or back-line, of a hedge, or other parallel-edged prominence. This instrument is mostly used in topographical drawing.

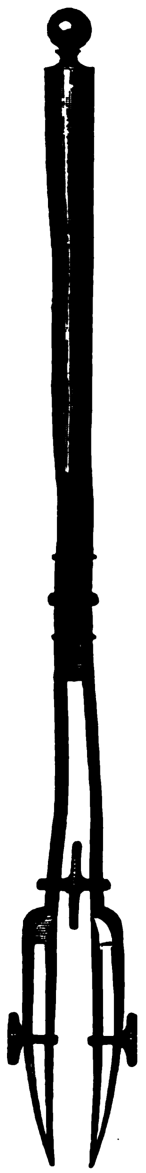
The pen for drawing dotted lines, resembles the ordinary lining-pen in shape, with the addition of a small wheel, like a spur-rowel, pivoted in the rounded points of the pen-blades. The ink is introduced between the blades, and the points of the rowel pass through it, and are intended to transfer to the paper beneath just sufficient ink to produce a line of dots. The action of the instrument is, however, very imperfect, and it is nearly discarded by modern draughtsmen, who prefer drawing dotted lines with the ordinary pen. Indeed, the time gained by using the rowel dotting-pen, when in perfect working order, is lost in bringing it to that condition. Besides, the pitch and depth of the dotted lining are liable to constant variation, according as the work in hand is on a large or small scale; and with the common pen, the operator can easily effect the necessary changes, whilst the dotting-wheel binds him down to one class of line.

Fig. 8.



We are, however, indebted to the ingenuity of a French draughtsman for a dotting-pen of very elegant action, and capable of producing a great variety of dotted lines. This instrument is represented in figs. 10 and 11, the former being a side elevation, and the latter a transverse vertical section. To an ebony or ivory handle, is attached a small frame, E, carrying two pulleys, or running wheels, F, G. The latter wheel is carried upon a steel stud, riveted to the plate, E, and is of considerable diameter, for the sake of steadiness. This stud, likewise, carries a disc, *b*, made to revolve with the wheel, G, by means of a pin, *a*, entering a socket in the latter. A nut is passed on to the screwed end of the stud, to retain the wheel and disc. The disc, *b*, is rather less in diameter than the wheel, G, and it is formed with indentations, to act as a rotatory cam. Immediately above the disc is the lever, H, which carries a pencil, or pen, of the ordinary description. The lever, H, vibrates on a screw-pin, securing it to the framing, E, and it is pressed down by a blade-spring, *d*. It is formed with a projection, *c*, which enters the indentations on the periphery of the disc, *b*; so that the rotation causes it, and with it the pen or pencil, to rise and fall. The action of the instrument is obvious. The back of the plate, E, is guided along the edge of a ruler, or square, the wheels being permitted to run on the paper; this motion turns the disc, *b*, and causes the pen to rise from or touch the paper at intervals, according to the character of the indentations on the disc. The pulley, F, is simply to preserve the level of the instrument, and is carried loose on a pin, screwed into the frame, E. A number of discs are provided with different patterns of indentations, so that either of them may be substituted for the disc, *b*, to correspond to the kind of dotted line the draughtsman desires to produce.

Fig. 9.



A draughtsman requires several kinds of compasses. The simplest are distinguished as dividers, and are used for transferring measurements from a drawing which is being copied, or from a scale; and also, as the name implies, for dividing lines and circles into equal parts. For this latter purpose, it is on the trial and error system that they are employed, if at all. Dividers consist of a pair of legs, pointed at one end, and jointed together at the other; the points, and a considerable portion of the leg, being of steel, whilst the shanks are of brass, German silver, or an amalgum known as electrum. German silver or electrum is to be preferred to brass, as the latter soils quickly, and is accompanied by an unpleasant odour. The joint of the dividers, and of all compasses, must be made free from all cross-play, or lateral looseness. The most ordinary kind are fastened simply by a common screw. We have, indeed, seen some very ordinary ones riveted together. The better kind have a steel pin passed through the leaves of the joint, upon which a flat brass or other metal nut is passed, at the further side. This nut has two small holes upon its face, for the introduction of the points of a turnscrew, to be met with in most sets of instruments. The joint-leaf of one leg of a pair of compasses is usually of steel, as this arrangement gives a smoother action than when both sides of the joint are of the same metal. The better kind are also made with two steel leaves on one side, which are introduced between three brass ones on the other; but some have only one leaf on one side, and two on the other. A perfect compasses-joint is a thing seldom met with, and draughtsmen are continually subject to annoyance, arising from the inequality of action of the joints of their compasses. Messrs. Bentley's "spiral spring compasses" aim at remedying this evil, by the adaptation of a small coiled spring to the joint, in such a manner as to equalise the pressure of the frictional surfaces throughout the entire movement. Our sketch, fig. 12, which represents a side view of the end of the centre joint of the compasses, explains the mode of application of the spring. The centre joint, A, which is sectioned to show the spring, has a recess bored out of one side of it, just large enough to receive the short coiled spring, B. When this centre joint is inserted between the two eyes forming the outer joints, the spring reacts from the bottom of its box against one of the eyes or cheeks of the outside joint, thus keeping up a regular smooth working pressure on the joint surface.

Fig. 10.

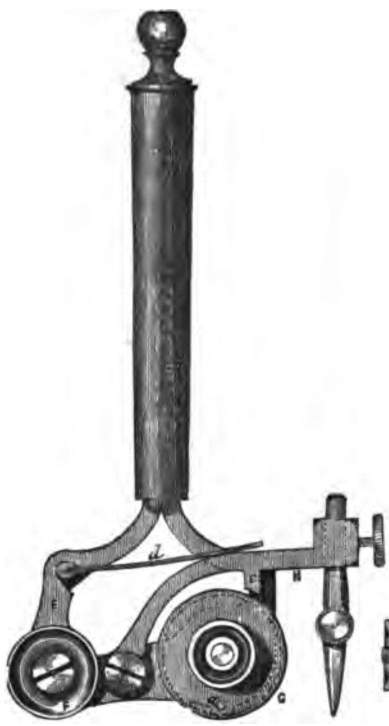


Fig 11.

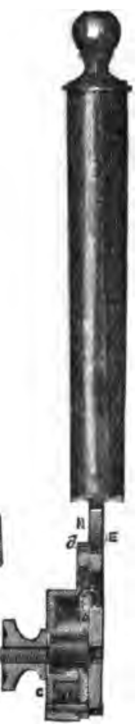
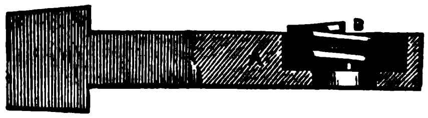


Fig. 12.



Externally, this little modification in no way affects the appearance of the instrument, as the spring, being entirely embedded in its recess, is not seen. At a mere trifle in the increase of the cost, an important objection is here remedied by very simple means.

Some dividers are made with one of the legs so fitted as to be capable of a slight adjustment independently of the main joint.

These are called "hair dividers," and are represented in fig. 13

Fig. 13.

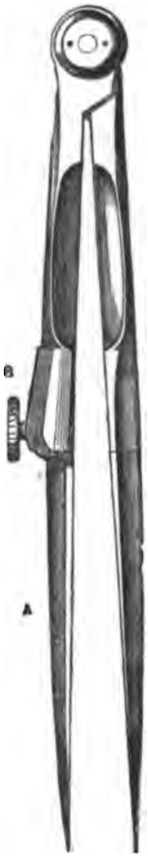


Fig. 14.

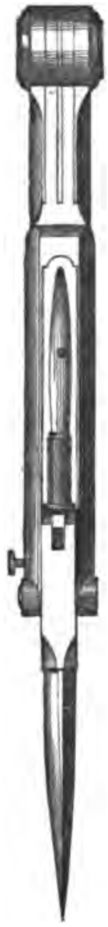
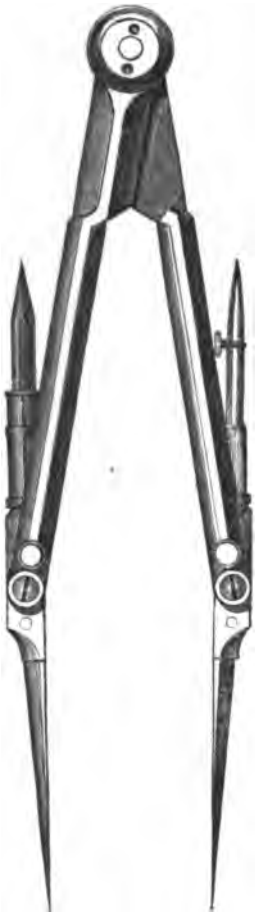


Fig. 15.



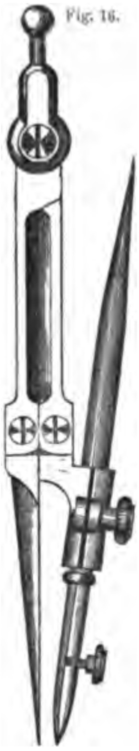


Fig. 16.

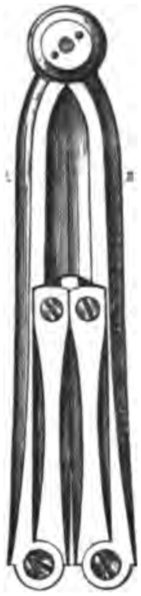
The leg, A, is not, like the other, soldered to the shank, but is formed with a long thin strip of metal, which lies in a groove on the inside of the shank, and is fixed to the latter by a screw, at its upper end, near the compasses-joint. This thin strip acts as a spring to bring the point, A, nearer to the point of the other leg. A screw, B, passed through the shank, adjusts the point, A, a slight distance in or outwards, thus affording a means of taking measurements more minutely accurate than with the mere direct action of the hand upon the main joint. Our fig. 13 may be taken as a representation of a very excellent style of dividers. The points should be strong, and not too finely tapered, and they should meet accurately when the instrument is closed.

All cases of instruments contain a large pair of compasses, in addition to the dividers, which is usually of similar construction, except that one of the legs is made to fit into a socket in the shank, and a pencil or pen may be substituted, as required. The pencil-holder and pen are both jointed, so that, in every case, they may be put in the best position for action. In the better kind, the fixed leg is also jointed; so that in describing circles of large diameter, the centre point may still be entered vertically into the paper. A lengthening bar is also provided, which can be fitted into the shank-socket, whilst the pen or pencil can be placed at the end of the bar, thus giving the compasses a greater range.

In figs. 14 and 15, we have represented a modification of the universal compasses. This tool has no separate pieces, but is so arranged that a pen, pencil, or point, may be brought into action as desired. The shanks are forked, and the leg pieces are jointed to their extremities. One of the leg pieces is formed with a steel point at one end, and a pen at the other; whilst the other leg has a steel point at one end, and a lead pencil at the other. The legs are jointed to the shanks by their longitudinal centres, and can be turned between the forks, so as to bring into action whichever end of the leg is required. A small pinching-screw is passed through one side of the shank, near its extremity, to fix the leg in position.

Fig. 16 is a representation of a somewhat smaller pair of compasses, in which the pen is formed in one piece with a steel point, which may be inverted, or a pencil-holder introduced instead. This is a very convenient instrument for small work.

Fig. 17.



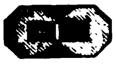


Fig. 20.

Fig. 18.

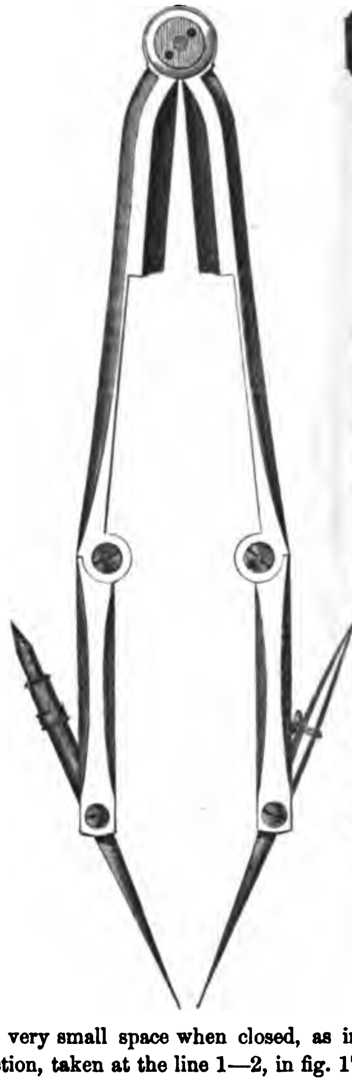
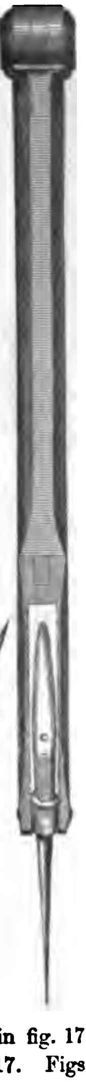


Fig. 19.



Another form of compass instrument known as "the Napier compasses," is represented in figs. 17, 18, 19, and 20. This is a "pocket" or "turn-in" compass, and takes up a very small space when closed, as in fig. 17. Fig. 20 is a cross section, taken at the line 1—2, in fig. 17. Figs.

18 and 19 are front and side views of the instrument when fully open. These compasses are, in some points, not unlike the pair just described. The shanks are each jointed, and their lower portions are forked to receive the swivelling points, which are jointed to them at their extremities. The swivelling points are formed respectively with a pen and pencil at their opposite extremities, and any one of these may be turned round into working position when required. A lateral screw for fixing the points, as in figs. 14 and 15, would be an improvement. Grooves are formed in the upper ends of the shanks, as shown in the section, fig. 20, and in these the points lie when the instrument is folded, as in fig. 17. When in this state the pen and pencil points lie within the forks of the lower joints. This tool is particularly convenient for those who require to carry drawing instruments with them from place to place.

We have next to describe the various forms of compasses of a smaller size, called "bow compasses." No mechanical draughtsman should be without these, for the larger sizes are far too heavy and cumbrous for the smaller and more delicate work which he is constantly called upon to execute. Fig. 21 is a front, and fig. 22 a side elevation of a very neat form of jointed bow compasses with a pen. The main joint is embraced by two eye plates, to which a small handle is attached. A portion of this handle is milled, to give control over the instrument and facility in turning it. It will also be observed that the centre point is a needle, held by a small screw, in a socket formed in the leg of the instrument. This arrangement affords a means of adjustment as to length, whilst the point can easily be replaced if accidentally broken. Compasses of the larger size are sometimes made with similar needles, but this refinement is considered by most draughtsmen as unnecessary, if the needle is not indeed inferior to the ordinary point, which, from its greater size and strength is less liable to injury. Both legs of the bow-pen compasses are jointed. Fig. 23 is a front elevation of an instrument precisely similar to the last, except that it has a pencil instead of a pen. These compasses require to be adjusted to any desired radius by the direct action of the hand. In work where great accuracy and minuteness is called for, it will often be found a very tedious matter to obtain a true adjustment in this manner, particularly if the joints of the instrument are not in perfect working condition. This difficulty is got over in what are termed "spring" or "screw" bow compasses. Fig. 24 is a front, and fig. 25 a side view of a pair of pen compasses of this class. The use of such instruments is confined to very small circles, of half or three quarters of an inch in radius at the most. In the example we have selected for illustration, the centre leg is of brass or German silver, and is in one piece with the milled handle. It is also provided with a needle point. The pen is made with a spring-tempered steel shank, K, which lies in a groove cut in the centre leg, or body, and which is fixed to the latter, at its top, by a screw. A small screw spindle, L, is passed through an opening in the pen shank, and is jointed to the centre leg, and a button, or nut, is passed on to the screw spindle outside the shank. This pen shank is so fixed as to have a tendency to stand out from the centre leg to the full extent of the instrument's range, and by turning the button of the screw, L, it may be forced in or allowed to open, so as to give the necessary adjustment. Fig. 26 is a side elevation of a pair of slightly modified spring-and-screw compasses; it is shown with a socket, carrying an engraver's *burin*. An ivory handle is fixed to the centre leg, or body of the instrument; and this arrangement is considered by some artists to give greater control over its action. The commoner kind of spring bow compasses consists of a single piece of steel forming the two legs, and having a small brass handle attached. The steel of the legs is so tempered as to give them a tendency to stand apart, and the radius distance is regulated by a screw in the same manner as in the instruments represented in figs. 24 and 25.

Fig. 21.

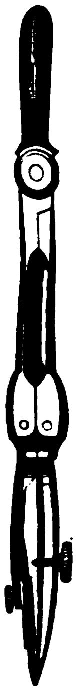


Fig. 22.

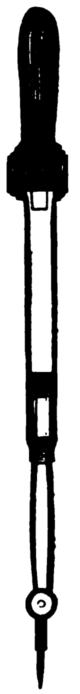


Fig. 23.

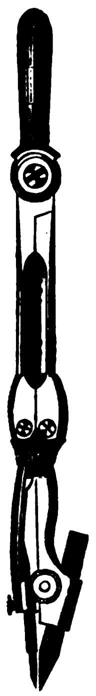


Fig. 24.

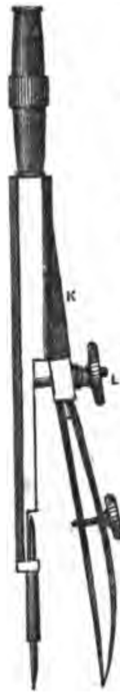


Fig. 25.

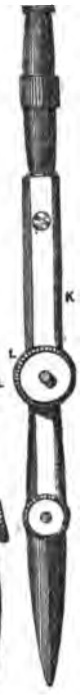
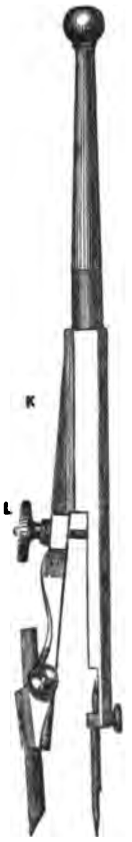


Fig. 26.



The draughtsman has frequently to delineate circles of a radius far exceeding the range of ordinary compasses, and for this purpose he must provide himself with "beam" compasses. A good form of this instrument is represented in side elevation, in fig. 27, and in transverse vertical section, in fig. 28. It consists of a wooden bar, or ruler, T, of considerable length, and of a T section, being formed of two strips united by a dovetail joint. This construction prevents warping or bending, and is necessary where a scale is cut on the bar, as any deviation from a straight line would render the measure inaccurate. The compasses are provided with a pen, or pencil leg, and a centre leg, these being fitted upon the bar with socket pieces, M, M¹. These socket pieces are fixed at any point along the bar, by pinching screws at the side; but to prevent the point of the screw from injuring the bar, a loose plate of metal is interposed next to the bar, as shown in the section, fig. 28. The socket, M, is in a solid piece with its pen, or pencil-holder; but the centre leg, N, is in a separate piece from its socket, M¹, and is capable of minute adjustment back or forward in the latter. The socket, M¹, has a cylindrical groove along its under side, in which slides the head of the leg, N. This head is formed with a horizontal screw passage, or nut, to receive the screw spindle, I, which is held by an eye at the end of the socket groove, and is actuated by means of a button on the outside. By turning the screw, I, in either direction, the centre leg may be adjusted with great accuracy at any part of the socket. In adjusting the instrument to any measurement, the pencil socket, M, is loosened and set pretty near the mark, and fixed; the exact length of radius is then obtained by adjusting the centre leg by means of the screw. A pair of beam compasses, or dividers, of somewhat different description, are represented in side elevation in fig. 29. In this instrument, which is entirely of metal, the centre leg is fixed to the bar, and the moveable leg is carried by a socket entirely embracing the bar, and sliding upon it. An additional socket is carried by the bar, and is connected to the first by a longitudinal screw spindle, whilst it may be fixed at any point on the bar by means of a pinching screw underneath. The bar is graduated, and in the larger socket an opening is made having a bevelled edge, upon which a vernier scale is cut, so that very minute measurements may be taken. In setting the instrument, the smaller socket is fixed at a convenient point on the bar, and then the larger socket, which carries the moveable point, pencil, or pen, is set back or forward, as may be necessary, by the longitudinal screw connecting it to the smaller socket.

Fig. 27.

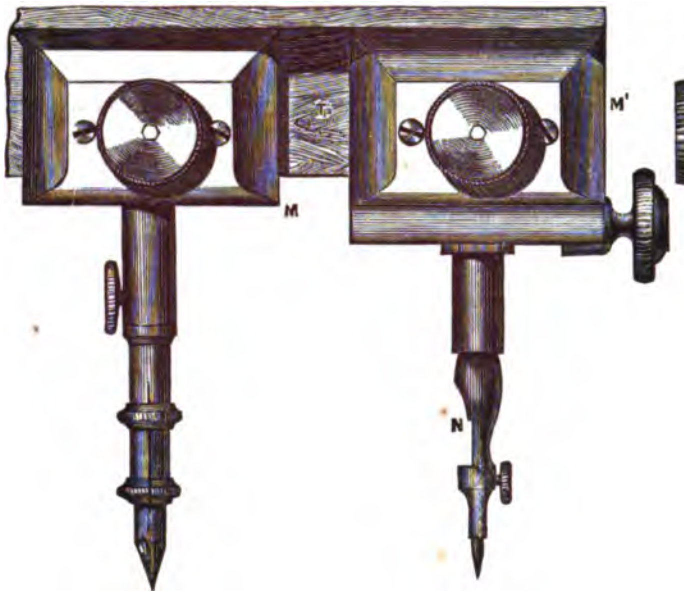


Fig. 28.

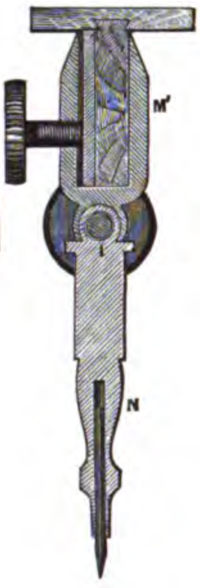
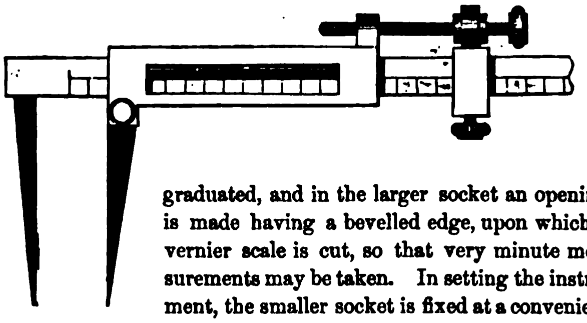
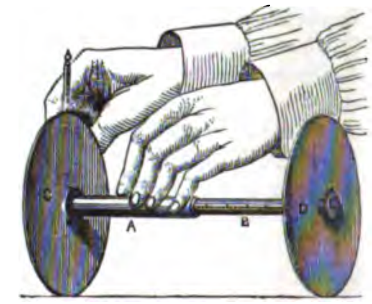


Fig. 29.



The accompanying sketch, fig. 30, represents a curve delineator, designed by P. B. Eassie, for tracing circles too large to be struck by trammels in the drawing office, as railway radii, curvilinear roofs, &c. A, is a tube, having fastened on its end a circular disc, C, 4 inches diameter; into this tube slides another, B, graduated for 3 inches in length to an inch scale.

Fig. 30.



On the outer end of this tube is placed a moveable disc, D, fastened by the set screw, F. Now, if the two discs were of equal size, the lines traced by the peripheries when rolling on their edges would be straight and parallel, but when one is less than the other, and consequently travelling over less space, the curves described by the larger disc will be the segment of a circle, the radius of which is found by taking the quotient derived from the division of the large disc by the reduction in size between the two, and multiplying it by their distances apart. Thus, when the disc at D is one-twelfth of an inch less than at B, their distance apart being 6 inches, we have

$$\frac{4}{1\cdot12\text{th}} = 48 \times 6 \text{ in.} = 288 \text{ in.} = 24 \text{ feet radius;}$$

and when the tube, B, is pushed in till the discs are 3 inches apart, we have

$$\frac{4}{1\ 12\text{th}} = 48 \times 3 \text{ in.} = 144 \text{ in.} = 12 \text{ feet radius,}$$

the scale marked on the tube, B, giving the intermediate radii. To make the instrument describe curves from 24 to 1 ft. 6 in. radius, 4 discs are required, fitting on the end of tube, B, of the following sizes:

$3\frac{11}{12}$ in. describes from 24 to 12 feet radius.
$3\frac{4}{6}$ " " " 12 " 6 " "
$3\frac{2}{3}$ " " " 6 " 3 " "
$3\frac{1}{3}$ " " " 3 " $1\frac{1}{2}$ " "

The curve is easily traced by holding either drawing pen or pencil to the disc, C, as shown in the figure.

Of this class of drawing instruments, there now remains to be described the proportional dividers. This instrument is represented, in front and side elevation, in figs. 31 and 32. Its use is to increase or reduce measurements to a scale different from that of the original drawing, of which a copy is being made; and a great deal of time may be saved by employing it, whilst there is much less risk of making mistakes with it, than when the draughtsman, in reducing a measurement, has first to take the distance, on the original drawing, in his common dividers, and by applying it to the scale of that drawing, ascertain how much it is arithmetically; and then to find the corresponding distance on the reduced scale, which he has again to take in his dividers, so as to apply it to the copy in hand. With the proportional dividers, on the other hand, the action of taking the measurement on the original drawing, at one end of the instrument, adjusts its other end to the measurement, as increased or reduced to the scale of the copy. The instrument consists of a couple of elongated slotted brass plates, connected by an adjustable point, J, and provided with steel points at both extremiities of each plate. It will be obvious that, if the joint, J, is adjusted exactly in the centre between the extreme points, on opening the dividers, the distances between the points, at each end, will be equal; but if the joint, J, is adjusted, as in the figure, so that the length of the legs on one side is only half that on the other, then the distance between the points at one end will be half that between the points at the other end. In the same manner, by shifting the joint, J, still nearer one side of the instrument, a still less distance will be measured by one end, as compared with the other; but the measurements will always bear the same proportion to each other as that which is, for the time being, between the portions of the instrument on each side of the joint, J. To enable the draughtsman to set the instrument to any desired proportion, the sides of the slot are graduated, and a projection on the joint has an index-line cut upon it, which is to be placed opposite the number corresponding to the desired proportion, in adjusting the instrument. When being adjusted the points of the instrument should actually coincide; and to secure this, a pin is fixed in one of the plates or legs, and a notch is cut in the other to receive it. The instrument is made to answer various purposes, in addition to that of reducing drawings. Thus, the joint may be so set, that when one end is applied to the radius of a circle, the other end will give the side of an inscribed polygon. One of the sides of the instrument is usually graduated for this purpose, and there are also divisions for finding the proportions between simlar plane figures, and for finding the proportions between cubes or spheres. The proportional dividers require to be very accurately made, and great care must be taken of them; for if the points become injured or bent, or if the joint gets loose, the instrument will be rendered useless.

Fig. 31.

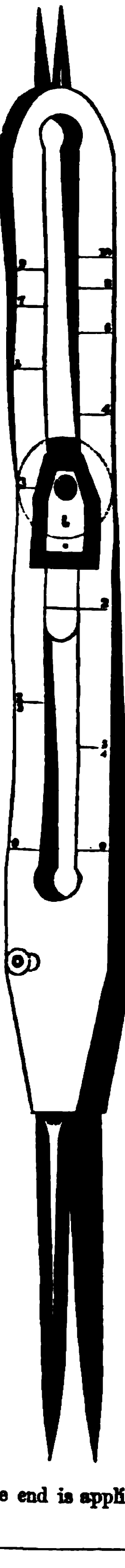
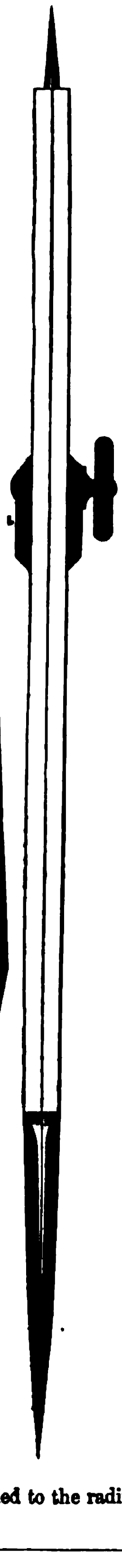


Fig. 32.



When the points of ordinary proportional compasses are broken or bent, the instrument is rendered perfectly useless, as its accuracy depends upon the length and position of the points. To remedy this evil, Mr. Joseph Roberts, of Hyde, near Manchester, has conceived the idea of rendering the points adjustable in such a manner that they may be accurately re-set by means of a gauge when broken or injured. The improved instrument is represented in edge view in fig. 33, whilst the gauge for the length is represented in fig. 34. The ends of the compass legs are made with small sockets, A, into which needles are fitted in angular positions, these needles being held in their sockets when adjusted, by small pinching screws. The gauge, fig. 34, is a straight piece of metal with its two ends bent at right angles, and with a mark at B, which corresponds with the index mark on the sliding piece, C, of the compasses, when this is set for giving an equal opening at both ends of the instrument. When the needle points are broken or inaccurately placed, the compasses must be applied to the gauge, the slider being first set to correspond with the mark, B, and the needles can then be lifted out or in, so as to bring their points to the proper positions coincident with the ends of the gauge. The inclination given to the needle sockets is such, that the points of each pair of needles only touch when the needles are accurately adjusted; and if the sockets are carefully made and their inclination accurately determined, and the needles perfectly straight, no gauge will be required, as all that will be necessary when the points are injured, will be to take new needles and to set them in the sockets, so that the individual points of each pair will just touch each other. The gauge, however, insures extreme accuracy.

Fig. 33.

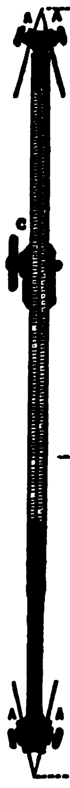


Fig. 34.

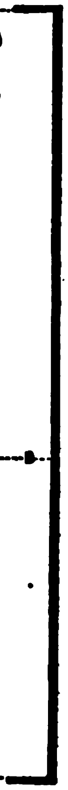
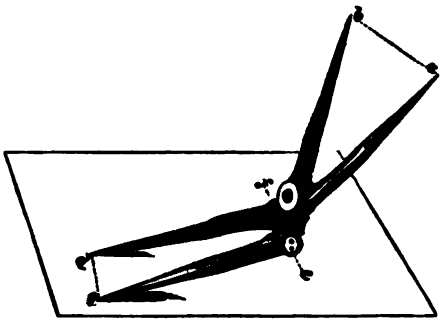
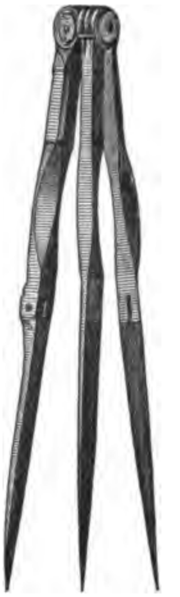


Fig. 35.



Another arrangement of the proportional compasses is shown in fig. 35. In this modification the adjustment of the width between the points is obtained by bending the compasses to a figure more or less approaching a right angle. The instrument consists of the legs, *a* and *b*, which are jointed like a pair of ordinary compasses, so that the legs move in a lateral direction. The legs, *a* and *b*, are connected at *e*, *f*, with the legs, *c* and *d*, the joints, *e* and *f*, being at right angles to the joint of the legs, *a* and *b*. With this arrangement of the parts, the points, *c* and *d*, are brought nearer together, as the legs, *a* and *b*, are bent upwards on the joints, *e* and *f*, and so caused to approach more or less the vertical position.

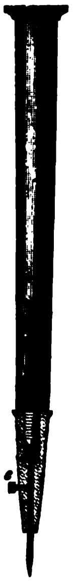
Fig. 36.



The triangular compasses which is represented in the subjoined illustration, fig. 36, is a very useful instrument in setting out and copying drawings. The figure is drawn one half the real size; it consists of an ordinary pair of compasses, to one face of the joint of which is jointed the third leg, which thus stands midway between the others when the legs are distended. With this arrangement any three points from a drawing may be transferred to the copy, and two of the points being again placed on two of the points on the drawing, a third point may be taken, and in like manner transferred to the copy that is being made. Numerous instances occur in which this instrument is of the greatest assistance to the draughtsman.

In copying drawings, where time is a matter of more consideration than the preservation of the original drawing, a great deal of labour may be saved by laying the original upon the paper for the copy and pricking the principal points through the former. A convenient instrument for this purpose is represented in partial section, in fig. 37; it consists of a needle point, held by a screw, *e*, in a brass socket, with an ivory handle. The socket is made to take off by unscrewing, and uncovers a small screwdriver, which will serve to turn the screw, *e*, or any of the smaller screws in the other instruments.

Fig. 37.

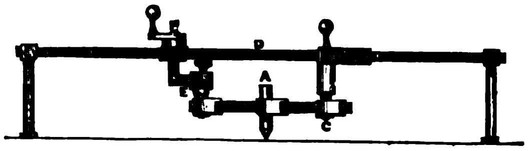


We shall, in its appropriate place in this volume, treat of the mode of drawing ovals, ellipses, parabolas, and volutes, but we shall here describe the several instruments by means of which these figures may be produced with ease and rapidity. Fig. 38 is an elevation, and fig. 39 a corresponding plan of an instrument for drawing the ove or egg-shaped figure.

The pencil, A, is placed in an adjustable holder, capable of sliding along the connecting-rod, B, one end of which is jointed at C, to a slider on the horizontal bar, D, set on two standards, which rest on the paper, whilst the opposite end is similarly jointed to the crank arm, E, revolving on the fixed centre, F, on the bar, D. By altering the length of the crank, and the position of the pencil on the connecting-rod, the shape and size of the ove may be varied as required.

Numerous instruments have been contrived for the regular oval or ellipse, one of the simplest of these is the "trammel." In figs. 40 and 41 we give an elevation and plan of a trammel of an improved form, in which the practical defects are much lessened, but the contrivances by which this approximate perfection is attained are of such a nature as to require more than ordinary excellence and accuracy of workmanship in the construction. The trammel consists of a metal bar, R, on which are fitted three sliding sockets, which can be adjusted at any points on the bar. Two of these sockets carry centre legs, P, O, and the third carries a pen, or pencil, S. In addition to these details, a guide-plate, Q, is required, having a couple of grooves cut in its upper face, at right angles to each other.

Fig. 38.



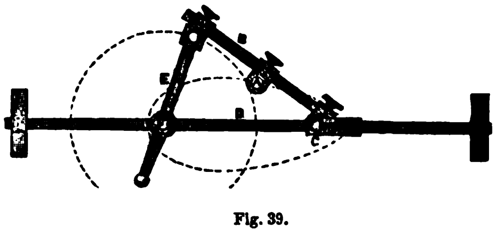
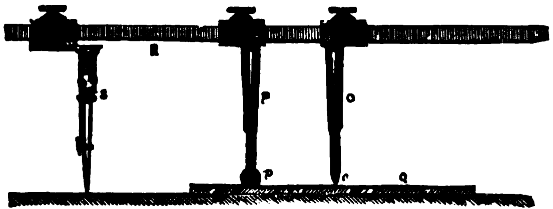


Fig. 39.

This guide-plate has two short pin points, on the under side, to prevent it from slipping on the paper upon which it is placed. In ordinary instruments the legs, O and P, terminate in simple points, which are respectively caused to traverse the grooves in the guide-plate, Q, in describing the ellipse. It is, however, found to be almost impossible to obtain a smooth action with this arrangement, as the pressure on the points, being oblique to their line of movement along the guide-grooves, the friction is apt to be irregular, and so cause a varying motion of the pen or pencil point, and produce an uneven outline. In the instrument represented in figs. 40 and 41 the parts which traverse the guide-grooves in the plate, Q, consist of small steel wheels, *o*, *p*, carried in the forked ends of steel spindles, which are entered loosely into the socket legs, O, P. Thus, whilst the wheels considerably alleviate the friction arising in traversing the grooves, they always maintain their position with regard to the grooves, whatever be the position of the bar, R, and pen, S. In adjusting the instrument, it is simply necessary

Fig. 40.



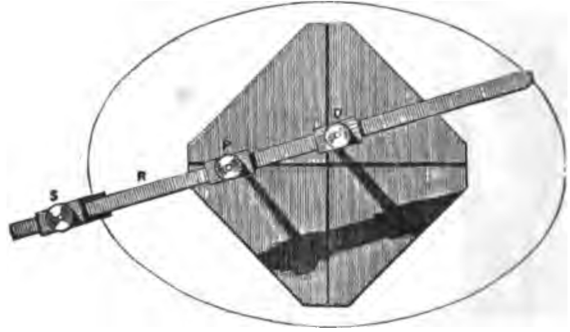
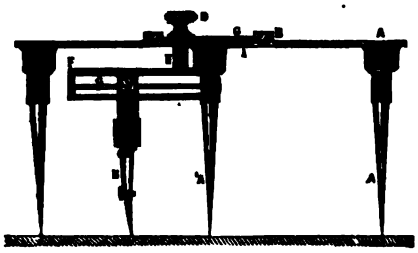


Fig. 41.

to set the pen and centre legs, so that the distance of the two latter from the former shall correspond respectively with the semi-transverse and semi-conjugate axes of the ellipse to be described. With the instrument represented in the engravings it will not be possible to describe any ellipse which does not lie wholly outside the guide-plate, Q; and where smaller ellipses are required, a smaller guide-plate must be used.

Fig. 42.



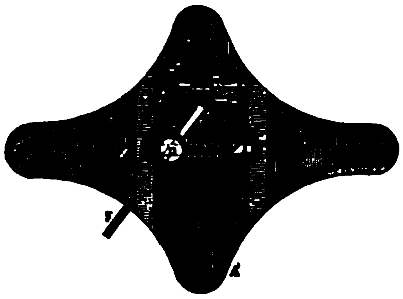


Fig. 43.

A superior instrument for drawing ellipses is that invented by Mr. Webb, and represented in elevation in fig. 42 and in plan in fig. 43. It consists of a lozenge-shaped table, A, of thin metal, supported upon four pointed legs, A^1. Two parallel guides, B, are fixed across the top of the table, A, and a disc, C, is fitted between them, in such a manner as to be just capable of turning and sliding between the guides, B. The disc has a slot at one side extending from the centre to the circumference, and in this is fixed, at any point, by a screw, D, a spindle, E, passing down through the table, A, below which it has fixed to it a slight frame, F, carrying a screw spindle, G. This screw serves to adjust the pen, or pencil, H, back or forward, on the frame, F. The spindle, E, works in a slot, I, in the table, A, which slot is at right angles to the disc guides, B. The instrument is caused to operate by turning the spindle, E, by means of the button, D, which action turns the carrier frame, F, and also the disc, B. It follows, that if the spindle, E, were fixed in the centre of the disc, B, the point of the pen would describe a circle. If, however, the spindle, E, is fixed eccentrically in the disc, the rotation of the latter, between its guides, B, will cause the spindle to traverse the slot, I, in the table, A, in such a manner that the point of the pen will describe a perfect ellipse. In adjusting the instrument, two lines are drawn on the paper at right angles to each other, and the points of the legs, A^1, are placed upon these lines, when the slot, I, will be immediately above one of them, which will answer for the transverse axis of the ellipse, whilst the other one will be immediately below a line midway between the two guides, B, and will serve for the conjugate axis. The disc, B, is then set with its slot at right angles to the slot, I, in the table, A, and the semi-conjugate axis being marked upon the paper beneath, the point of the pen, H, is adjusted to it by turning the screw, G. The disc, B, is then turned a quarter round, so that its slot may coincide with the slot, I, and the screw, D, being loosened, the spindle, E, is moved back or forward along the the disc slot, until the point of the pen, H, coincides with the end of the transverse axis of the ellipse. When so adjusted, the nut, D, is screwed down to fix the spindle, E, to the disc, B, and the ellipse may then be described. The legs, A^1, of the table are formed with telescopic socket joints, inside which very delicate springs are placed, of just sufficient strength to lift the pen's point off the paper, when not describing an ellipse, whilst a slight pressure of the hand, holding the table, A, during the drawing action, will overcome the resistance of the springs, and allow the pen, or pencil, to touch the paper. The legs, A, may be formed with a screw socket joint in addition, so that they may be accurately adjusted as to length, in order to preserve the table, A, perfectly level.

Fig. 44.

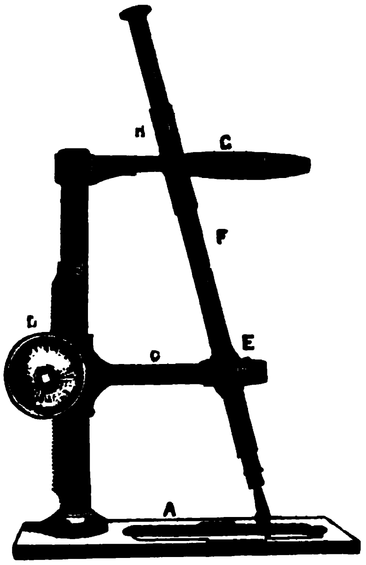


Fig. 44 represents an elevation of an ingeniously contrived instrument for drawing ellipses within a given range. It is shown as in working order, with a pen for drawing ellipses in ink. It consists of a rectangular base plate, A, having sharp countersunk points on its lower surface, to hold the instrument steady, and cut out to leave a sufficient area of the paper uncovered for the traverse of the pen. It is adjusted in position by four index lines, setting out the transverse and conjugate axes of the intended ellipse—these lines being cut on the inner edges of the base. Near one end of the latter, a vertical pillar, B, is screwed down, for the purpose of carrying the traversing slide-arm, C, adjustable at any height, by a milled head, D, the spindle of which carries a pinion in gear with a rack on the outside of the pillar. The outer end of the arm, C, terminates in a ring, with a universal joint, E, through which the pen or pencil holder, F, is passed. The pillar, B, also carries at its upper end a fixed arm, G, formed as an elliptical guide frame, being accurately cut out to an elliptical figure, as the nucleus of all the varieties of ellipse to be drawn. The centre of this ellipse is, of course, set directly over the centre of the universal joint, E, and the pen-holder is passed through the guide and through the joint, the flat-sided sliding-piece, H, being kept in contact with the guide, in traversing the pen over the paper. The pen thus turns upon its joint, E, as a centre, and is always held in its proper line of motion by the action of the slider, H. The distance between the guide ellipse and the universal joint determines the size of the ellipse, which, in the instrument here delineated, ranges from $2\frac{7}{8}$ inches by $1\frac{5}{8}$, to $\frac{7}{16}$ by $\frac{1}{4}$ inch.

A kindred arrangement of an elliptograph, the invention of Mr. P. B. Eassie, whose curve delineator we have described in a former page, is represented in perspective elevation in fig. 45. The instrument consists of a column, A, supported on two diverging feet, B, the points of which serve for setting the instrument by; and there is also a point at the foot of the column for the same purpose. On the column is fitted, by means of a socket, an arm, C, into the end of which is fitted a ball-and-socket joint for the pen or pencil-holder, D. A second arm, E, is fixed at the top of the column, A; and the outer end of this arm is forked, and supports a circular ring, F, which can be set at any inclination by means of a screw, G. The pen or pencil-holder, D, passes through the guide ring, F; and being kept in contact with it whilst being rotated, is made to describe an ellipse on the surface beneath, in consequence of the obliquity of the guide ring. An index may be jointed to the arm, E, so that the guide ring may thereby be set to any required angle, the index being turned out of the way after the ring is set. The arm, C, is made to shift up and down the column by means of a pinion working in a rack on the column, the pinion being turned by means of an external button. By shifting the arm, C, the size of the ellipse may be varied at pleasure; whilst the relative proportions of its axes depending on the inclination of the guide ring, F, may be varied by setting this ring accordingly.

Fig. 45.

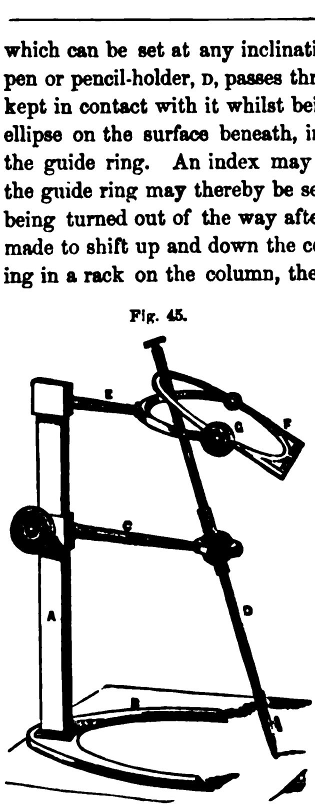
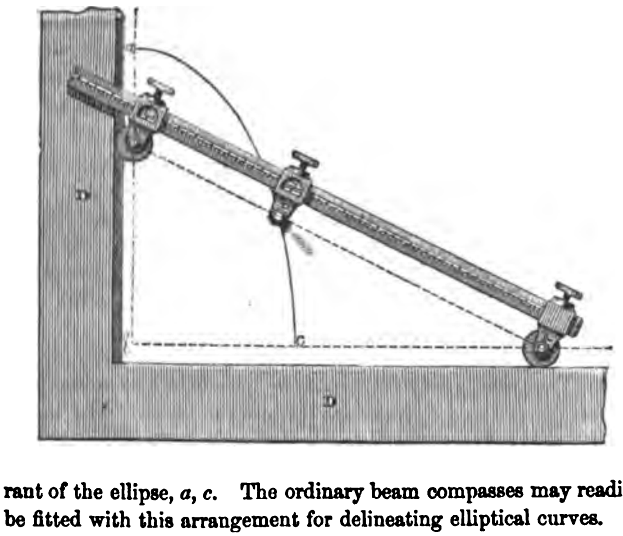


Fig. 46 is a plan of a simply arranged instrument for drawing ellipses. It consists of a divided ruler, having fitted thereto the adjustable sockets, A, B, and C, which are each fixed in the required position by means of a pinching screw. The sockets, A and C, are furnished with small rollers of say one inch in diameter; these rollers are flat at the bottom, and turn freely on their axes. The centres of these rollers are in a straight line with the point of the pen or pencil which is carried in the socket, B. The pen or pencil is adjusted so that the distance between A, B, is equal to half the major axis of the required ellipse; whilst the distance between B and C, is made equal to half the minor axis of the ellipse to be described. If, now, the rollers, A and C, are kept in contact with the edges of the square, D, and moved in the direction of the lines, *a*, *b*, *c*, the pen or pencil will describe the quadrant of the ellipse, *a*, *c*. The ordinary beam compasses may readily be fitted with this arrangement for delineating elliptical curves.

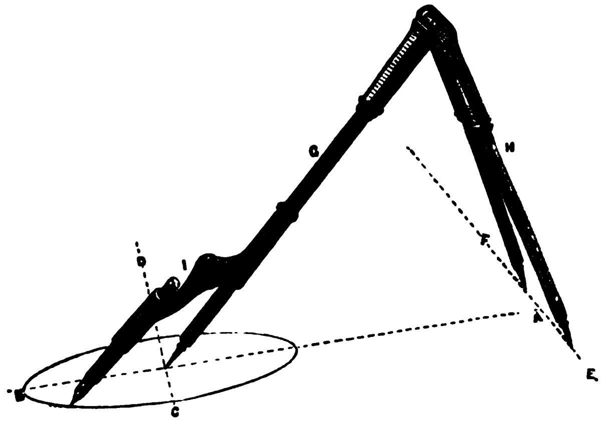
Fig. 46.



A convenient instrument for describing ellipses of small eccentricity, or which differ very little from circles, is the elliptical compasses, consisting of two round legs, jointed together like ordinary compasses, upon one of which a pencil-holder is fitted with a tubular socket, so as to move logitudinally and circularly upon the leg. The pencil-holder is jointed, so that the pencil point may be adjusted at any distance from the leg. In using the instrument, the point of the leg carrying the pencil-holder is placed in the centre of the ellipse, and the leg is inclined in the direction of the transverse axis, by stretching out the other leg along the prolongation of the axis. The legs are held steady by one hand, and the pencil is carried round with the other, being kept in contact with the paper, by causing it to move up and down the leg as well as to rotate. In order that the ellipse may be drawn accurately in any desired position, it is necessary that the leg carrying the pencil-holder should be held perpendicular to the conjugate axis. To secure this, one of the legs should have a point branching out on one, or on both sides of the point, which is placed on the transverse axis, and the additional point, or points, should be in such a position as, when touching the paper, to keep the legs of the compasses in the perpendicular position alluded to. It is for want of this important addition that elliptical compasses have hitherto been found to be fallacious, and, in fact, useless. The instrument is not applicable to very oblong ellipses, because the inclination of the pencil becomes too great for the production of a clean line.

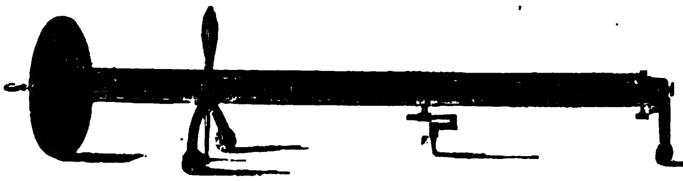
A useful modification of the elliptical compasses is shown in the elevation, fig. 47. It consists of a pair of triangular compasses, to one leg of which the pencil-holder is adapted. In using the instrument, a line, A, B, passing through the major axis of the ellipse to be described, is laid down; parallel to this, the line, C, D, is drawn to mark the centre of the ellipse. The line, A, B, is prolonged, and a second line, E, F, is drawn parallel to C, D. The point of the mid-leg of the compasses, is placed on the intersection of the lines, A, B, C, D, whilst the points of the legs, H, are placed equidistant from the intersection of the line, A, B, with the line, E, F. To the leg, G, is jointed the pencil-holder, I, and the movement of the marking-point round the leg, G, as well as up and down it, produces the required figure.

Fig. 47.



An instrument adapted for drawing ellipses, cylindrical curves, spirals, as well as reducing and enlarging drawings, is represented in fig. 48. This instrument is the invention of Dr. W. W. Wythes, of Philadelphia, and is named by him the cyclo-ellipto pentagraph. It consists of a beam of wood, A, about 14 inches in length, upon which is a brass frame, C, which slides along its whole length, and carries a point for the centre, supported by a couple of anti-friction rollers, which serve to steady the instrument. Another sliding frame, D, carries a pulley, F, upon the axis of which the sliding pencil arm, E, is attached. On one end of the beam is an axis upon which a disc of brass, B, with a milled edge, is made to revolve by the friction of the paper upon which the drawing is to be made. Upon the face of the disc is a bevelled wheel, which gears with another wheel, G, upon the beam, and gives motion to a pulley placed upon its shaft. An endless chain passes over the pulleys, G, H, and E, which is adjusted by means of the screw, at the end of the bar A, at the end of the beam. The leg, at the extremity of the bar, is only used when the instrument is arranged as a pentagraph.

Fig. 48.



It will be perceived that when the instrument is placed upon the drawing-board, the paper takes the place of a wheel, upon which the disc, B, rotates, and which may be of any radius, by sliding the centre frame nearer to, or farther from, the disc. Thus, the ratio of the velocity of the disc may be regulated by the graduation upon the beam—the first division upon the scale being equal to the radius of the disc—the second, twice the radius, etc. Now, if the centre be placed at the first division, and the beam be made to revolve, the pencil will make one revolution upon the axis, E, while the beam makes one upon the centre; if, at the second division, it will make two revolutions to one of the beam, and so on. The pencil may be made to move in the same direction as the disc, B, or otherwise, by placing the chain against the outer or inner edge of the pulley, E.

For the sake of clearer illustration, let us suppose the arm carrying the pencil to be the radius of a circle rolling upon the circumference of another circle which is fixed, and let us call the rolling circle the generating circle, the pencil point the generatrix, and the fixed circle the directrix. We will also call the distance betweeen the centre point and the disc, B, the radius of the fundamental circle. When the chain passes over the outer edge of the pulley, E, the generating circle is supposed to roll on the convex surface of the directrix, and when it presses against the inner edge of the pulley, it rolls on the concave side. When the generating circle has rolled once over, so that every point shall have been in contact with the directrix, the portion generated is called a branch. As every revolution of the disc, B, generates a branch of the curve, the number of branches will depend on the ratio between the fundamental circle and the disc, and is determined by the divisions of the graduation on the beam.

To construct the ellipse—the instrument is rectified by passing the chain over the inner edge of the pulley, E, and adjusting the centre to the second division of the scale. Then make the radius of the generating circle, *i.e.*, the pencil arm, equal to one-half the difference between the semi-axis of the ellipse required, and the instrument is adjusted.

To construct the epicycloid curves, &c.—Pass the chain over the outer edge of the pulley, E, and set the centre to the number on the scale corresponding with the number of branches required; then make the radius of the generating circle in the same ratio with the directrix as the number to which the centre is set, and the instrument will describe the proper epicycloid. If the chain is reversed upon the pulley, the curve generated will be the hypocycloid. When the pencil is without the proper circumference of the generating circle, the curve described will be the epitrocloid. These curves may be compounded and varied almost infinitely, and will supply a number of new and beautiful forms to the architect and designer.

If the circumference of the disc, B, be an aliquot part of that of the fundamental circle, there will be a finite number of branches—but if the ratio of the circumferences cannot be expressed in exact parts of one, the number of branches is infinite. These curves are produced by a continuons revolution of the beam, setting the centre at a distance from the disc that is not divisible by its radius. Among the various uses of the epicycloid curve, we may instance the construction of the teeth of spur-wheels, which is greatly facilitated by the instrument.

To draw the spiral, the pulley, E, is fastened by means of a small pin, and the frame, D, is unclamped; then as the beam revolves, the frame will be carried forward uniformly from the pole, the distance between the spires being regulated by the position of the centre as in the other curves.

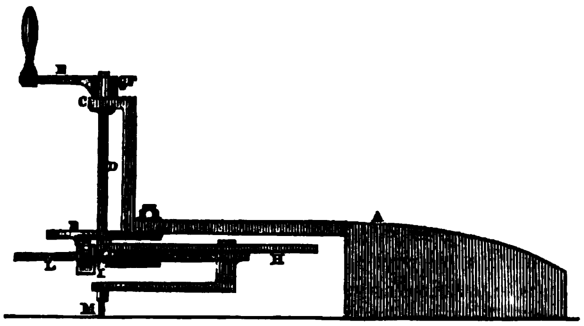
To use the instrument as a pentagraph, the handle is to be unscrewed, and the disc, B, removed. The end leg is to be attached to the other extremity of the beam, and the tracing point and pencil placed in the respective sockets on the under sides of the beam, A. The chain is then to be loosened, and the pulleys clamped by pins prepared for the purpose, in order to preserve the parallelism of the arms. The chain is then made tense—the pins removed, and then having adjusted the centre upon the scale, according to the size of the copy required, a right line is to be drawn upon the paper, and the tracing point, centre, and pencil, made to coincide with it, and the instrument is adjusted. As a pentagraph, the instrument has fewer centres of motion than those ordinarily made, and can be used upon a smaller drawing-board, having but one castor.

We shall in its proper place describe the arrangement and mode of using the ordinary pentagraph, and other modifications of the instrument, of which this is an example.

Closely allied with the instruments for producing elliptical figures, are those used for drawing volutes. The accompanying engraving, fig. 49, represents an elevation of one of the most practical of this class, it is the invention of Mr. T. E. Merritt, of Rochester.

This instrument consists of a heavy base, A, which is formed with an overhanging arm terminating in a vertical standard, C. To the outer part of the horizontal arm, of the base, A, is fixed a wheel, B, which is faced with India-rubber. Through this wheel, and the

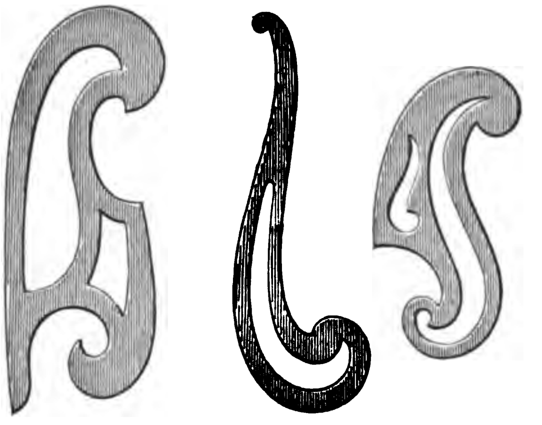
Fig. 49.



overhanging part of the standard, C, the spindle, D, passes, the upper part having fitted to it the handle, E, which is fixed on by the set screw, F. To the lower end of the spindle, D, is soldered a short tubular piece of brass, G, the backward part of which is prolonged, and forms a slotted guide, H. The tubular bar, G, is fitted with a tube, I, one end of which has a small jaw to receive the wheel, K. This wheel is set in the desired position by a set screw, and the centre of the wheel is tapped, so that it forms a running nut, in which the screw, L, works. The screw, L, is fast at one extremity to the guide block, which works to and fro in the slot of the part, H. The lower portion of this guide block extends forward, and carries at its front extremity the pencil, M. When the handle, E, is turned, the wheel, K, revolves by its friction against the disc, B, thereby causing the screw, L, to move outwards, and carry the pencil away from the centre toward the circumference of the volute to be drawn. The space between the spiral line thus produced depends on the adjustment of the wheel, K, the nearer it is to the centre of the disc, B, the narrower will be the space, and *vice versa*, whilst the size of the volute depends on the diameter of the disc, B.

In drawing sweeps and irregularly curved lines the draughtsman is greatly assisted, and much time is saved by the use of Curves. These are formed of thin pieces of pear tree, similar to the set squares; in fig. 49 is shown an example of three of the most useful forms. The figures of these curves are greatly varied, each being made to combine several curves, so as to enable the draughtsman, by using one part or the other, to produce the line he requires at once. The curve is laid down upon the drawing, and arranged so that its edge corresponds to the direction or curvature of the required line, which is put in by guiding the drawing pen along the edge. The young draughtsman will find the examples we have shown in fig. 49, as useful as any he can select, and all that he is likely to require. Curves are also made in sets which are specially designed for the use of naval architects and marine draughtsmen, to assist them in producing the curved lines required in drawing the hulls of ships. These curves may be readily obtained, if required, from the dealers in mathematical instruments.

Fig. 50.



Before quitting the subject of delineating curved lines by means of instruments, we may appropriately refer to one or two useful assistants to the draughtsman for measuring the lengths of curved or irregular lines upon plans or maps.

It is often necessary, particularly in topographical drawings, to make an estimate of the measurement of a series of curved undulating, or irregular lines, as the circumference of a field, or the outline of any other irregularly-shaped area; for which purpose an "Opisometer," or circumference measurer, is used. This instrument is represented in separate elevations, taken at right angles to each other, in figs. 51 and 52. It consists of a short screw spindle, fixed in a bracket frame, and attached to an ivory handle. Working upon the screw spindle, like a nut, is a small disc, with a tapered and finely-milled edge. This disc is made to roll along the line to be measured, and, in doing so, it necessarily traverses along the screw from end to end. When the end of the line is reached, the instrument is traversed along a graduated rectilinear scale in the reverse direction to that in which it was passed over the line to be measured. This movement brings back the disc to the end of the screw from which it started; and when this point is reached, the distance traversed on the scale is obviously equal to the length of the original line. Care mnst be taken to keep the handle of the instrument in a vertical position, and to assist in this one of the bracket arms is formed with a pin, projecting down almost to the level of the bottom of the disc. It is also necessary that the disc should roll with a uniform pressure. This instrument may be used for measuring curved surfaces as well as curved lines.

Fig. 51.

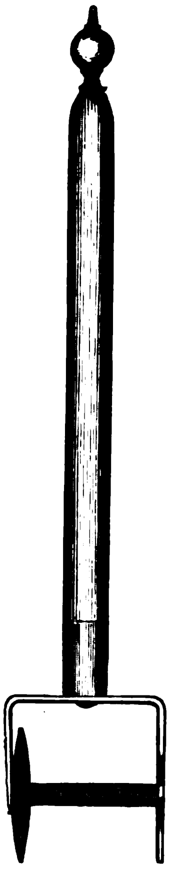


Fig. 52.

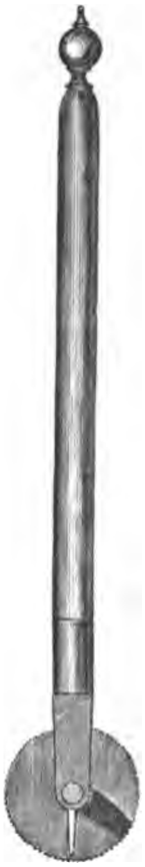
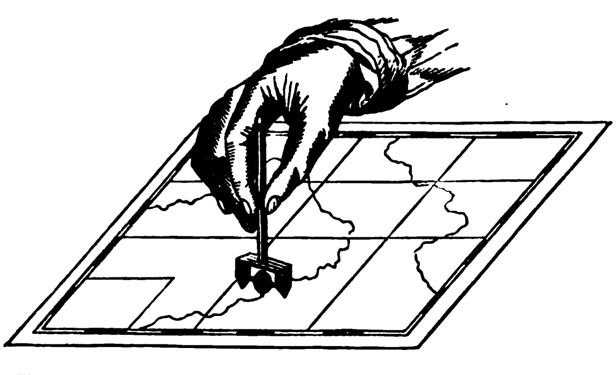


Fig. 53 shows the position of the hand and of the instrument when measuring lines from a map.

Fig. 53.

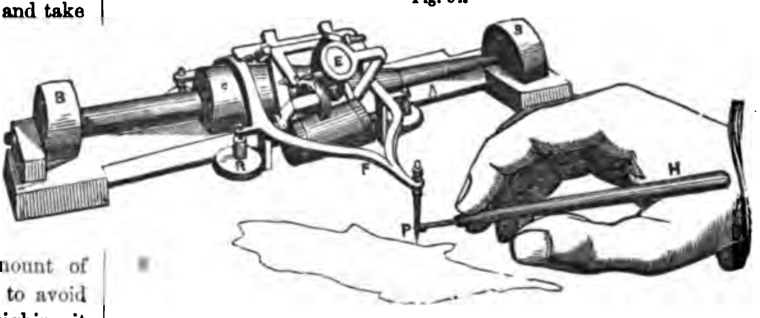


The accuracy of the result given by the opisometer is unaffected by the dimensions of the instrument itself, and depends entirely on the care with which it is used. The chief point is to see that the handle of the instrument is perpendicular to the surface at the beginning and end of each step of the measurement.

Several very ingenious instruments have been invented for a purpose similar to that for which the opisometer is used, but presenting more serious difficulties. The usual method of discovering the area of a figure drawn on a plan, is to divide it in a number of triangles or trapeziums—to measure the base and altitude of each, and take the sum of their products. By a careful process of this kind, the area may be discovered with great accuracy; but as it is necessary to revise the calculations several times, both for the purpose of obviating faults in the arithmetical part of the work, and in order, by taking the average of a few independent measurements, to increase the probable accuracy of the result, this method of calculation, especially when the figure is irregular, entails a considerable amount of labour of an irksome kind. Attempts have been made to avoid this by cutting the figure from the sheet of paper, and weighing it in a delicate balance against weights consisting of parts of the same paper, of determinate sizes; but this method—at first sight simple and practical—is rendered of little use by the impossibility of obtaining paper of uniform thickness throughout the sheet, the variation of thickness—and hence of weight—being greater than the amount of error that could be allowed in the results.

Several "planimeters," or instruments for mechanically measuring the area of plane surfaces, were exhibited in the Great Exhibition of 1851, and are noticed in Mr. Glaisher's admirable report on Class X., "Philosophical instruments, and processes depending upon their use." All, or nearly all of these, aimed at the solution of the problem by integrating the differential expression of a curve, traced on a plane surface, being conceived on the old, and now almost forgotten, view of the differential calculus, which regarded the differential of a magnitude as a measure of the velocity of its increase at any instant. Suppose a straight line to be carried, with a uniform motion, along the base line or abscissa, of any curvilinear area, remaining always parallel to itself, and perpendicular to the base line, and that, during this motion, a moveable point in the line, so carried, is always kept on the circumference or boundary line of the area. Then it is clear, that the velocity of increase of the area will be proportional to, and therefore measured by, the length of the ordinate, or portion of the moveable line included between the base line and the describing point. Again, a disc or wheel can be supposed to revolve with an angular velocity always proportionate to the same ordinate; in which case the total angle of revolution described by it will increase by similar increments with the curvilinear area, and will, consequently, always be proportionate to, and a measure of, that area. The area may therefore be read off, upon its circumference, by any method which keeps account of the number of revolutions made by this wheel, which may be called the integrating wheel, disc, or roller. If the circumferences of two circles be connected by teeth, or by simple contact, so as to work together, their angular velocities will be inversely as their radii; so that, if the radius of one of them be constant, the angular velocity of that one will be directly as the radius of the other. Thus, any mechanical arrangement securing the condition that a roller, or disc, shall be carried round on its centre, by contact with a uniformly revolving circle of a radius always equal to the length of the variable ordinate, will at once be a solution of the problem. The condition alluded to may be obtained by employing a couple of discs at right angles to each other, or by using a cone and a disc, with their axes parallel to each other. The former construction is adopted in one or two instruments, invented by continental mathematicians, the latter by Mr. Sang, of Kirkcaldy.

Fig. 54.



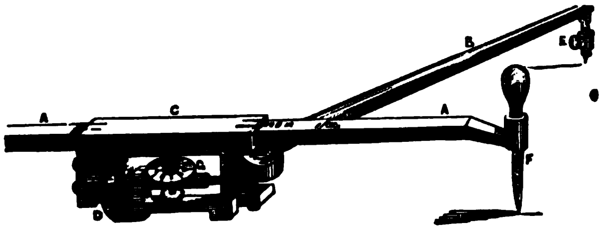
Mr. Sang's instrument, which is represented in perspective in fig. 54, indicates the area of any figure, however irregular, on merely carrying the point of a tracer round its boundary; and, besides the advantage of not injuring the drawing, it possesses that of speed and accuracy. A frame, A, carries an axle, which has on it two rollers, B, of equal size, and a cone, C. It is heavy, so that it maintains its parallelism on being pushed along the paper. The sides of the frame are parallel to the edge of the cone, and are fitted to receive the circumference of four friction rollers, R, which move along A, and carry a light frame, F, terminating on the tracing point, P, to which the handle, H, is attached by a universal joint. The frame, F, also carries a wheel, I, which, by means of a weight, is pressed on the surface of the cone, and receives motion from it as the tracer is carried along the paper. The index-wheel, I, only touches the cone by a narrow edge, the rest of its circumference being of smaller diameter, and containing a silver ring divided into 200 parts, which are again sub-divided by a vernier into 2,000 parts. The value of each of these divisions is the $\frac{1}{100}$th part of a square inch; so that one turn of the wheel represents 20 inches. Another index-wheel, T, moved by I, is divided into five parts, each of which represents 20 inches, so that a complete revolution of T values 100 inches. The eye-glass, E, assists in reading the divisions and vernier.

It is apparent, from the construction of this instrument, that if the tracer be moved forward, it will cause the index to revolve, not simply in proportion to that motion, but in proportion to the motion of the tracer, multiplied by the distance of the edge of the index-wheel, from the apex of the cone; and that the revolving motion of the index will be positive or negative, according as the tracer is carried backwards or forwards. Hence, if the tracer be carried completely round the outline of any figure—on arriving at the end of its journey, the index-wheel will show the algebraic sum of the breadth of the figure at every point, multiplied by the increment of the distance of the points from the apex of the cone; that is to say, the area of the figure.

This instrument possesses great simplicity of construction. Both factors of the continuous multiplication are directly transmitted from the motion of the tracing point in the simplest manner. The influence of the elasticity of the parts of the machine on the accuracy of its indications, may be discovered by moving the tracer a second time over the boundary of the figure, after having turned the whole instrument round 180°. The effects of the imperfections in the mechanism will now have changed signs, and one of the results will probably be found to be a little too large, and the other a little too small. The average between the two is the exact area of the figure, and is more to be depended on than the results of measurements made by scale and calculation in the usual way. A careful operator, in using the planimeter, will always take the average of two tracings in this manner; but when he experiences the rapidity with which this may be done, he will find the trouble as nothing in comparison with the harassing labour of calculating by scale and multiplication.

The accompanying figure (fig. 55) is an elevation of Amsler's planimeter. This singularly beautiful instrument is the invention of Professor Amsler, of Schaffhausen, by means of which the area of any portion of a map, or plan, drawn to scale, is readily and accurately measured, however irregular the boundaries may be.

Fig. 55.

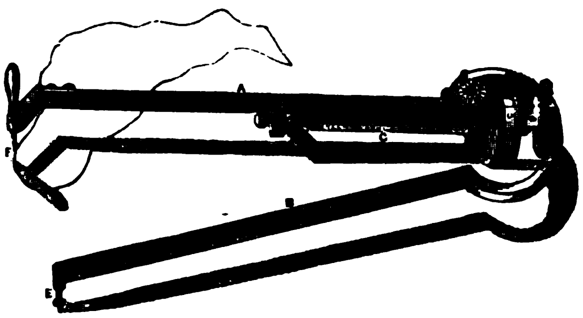


This planimeter, when ready for use as in the annexed figure, rests upon three points, D, E, F; these are respectively, 1st, a point of the circumference of the divided wheel, D; 2ndly, a point of the tracer F, at the end of the arm, A; 3rdly, a point, E, at the end of the other arm, B, which is kept fixed during the time of operation. To calculate contents, or areas, in square inches, set the slide, A, to 10 in., as shown, which means that the result multiplied by 10, gives the content, or area, in square inches. To obtain this result, place the point, E, at a convenient distance from the figure to be measured, so that the tracer, F, may traverse the entire periphery of the figure. But if the figure is too large to allow this, it can be subdivided by drawing straight lines through it, and the contents, or areas, of the several parts computed separately, and added together. Then place the point of the tracer on any convenient starting point in the periphery. When the instrument is thus adjusted, read off the division on the horizontal disc, G; also that on the perpendicular wheel and vernier, H. Suppose that the horizontal disc gives 3, and the vertical wheel gives 905, namely 90 on the wheel and 5 on the vernier; this reading must be put down thus, 3·905. Then carry the tracing point round the figure in the direction of the hands of a watch; and when the whole circuit has been made, observe the readings again. Suppose them 5·763; then subtract the former reading from the latter; the result will be 1·858; multiply this by 10, and you will get the content of the figure in square inches, namely 18·58 square inches. Notice must be taken whether the disc, G, has made an entire circuit; if so, 10 must be added, for every revolution, to the whole number. If the disc, in the above case, had gone once round, the second reading would have been 15·763. If twice round, 25·763; and so on. If the result is required in acres, multiply the above result by the number of acres in the square inch, according to the scale used in drawing the plan.

The other divisions on the arm, A, may be easily applied to any scale, by finding the result of one square acre or chain, and using that number as the coefficient to give the content of the surface required.

Fig. 56 is a modification of Prof. Amsler's planimeter, showing the form in which the instrument was first exhibited at the Society of Arts, London. Its action is similar to that we have just described.

Fig. 56.



CHAPTER II.

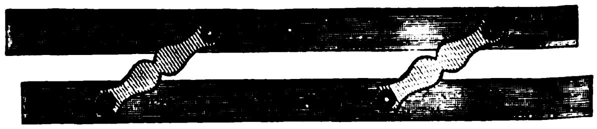
DRAWING INSTRUMENTS CONTINUED.

Accompanying every case of drawing instruments are one or more scales, made either of ivory or boxwood. Scales are indispensable for measuring straight lines, and laying down distances, the several vertical divisions on the scale corresponding to the foot, yard, or chain. On the ordinary simply divided scale there are usually six scales, produced by dividing the surface of the scale into six equal divisions extending longitudinally. Each of these divisions is divided by vertical lines into equal parts, the upper scale being the smaller, and increasing in proportion to the lowest of the series. The first division to the left hand of the scale is divided into ten equal parts; and if thirty of these parts are equal to one inch, it is called a scale of thirty. The six scales are generally divided into scales of 30, 35, 40, 45, 50, and 60.

The mode of constructing plain and diagonal scales, and their practical applications in drawing, will be described in a subsequent chapter, when referring to figs. 13, 14, and 15, on plate 1.

The parallel-ruler, which is shown in fig. 57, consists of two flat rulers, similar and equal in all respects, which are so connected together by means of two equal link pieces of brass, working loosely round fixed pins in the rulers, that, when the rulers are separated, both their outer and inner edges are parallel to each other.

Fig. 57.

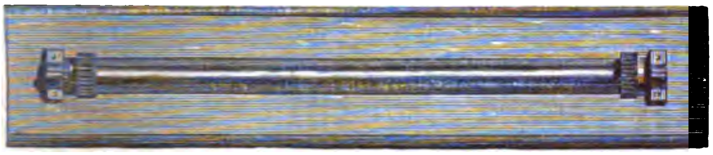


It is requisite, not only that one link should be equal to the other, but also that the distance between the pins in one ruler be equal to the distance between the pins in the other. In which case the lines joining the points always form a parallelogram; and as these points are equidistant from both the outer and inner edges in each ruler, those edges will always be parallel.

Hence it is plain, this instrument may be used for drawing any number of parallel straight lines, or for drawing one or more straight lines parallel to a straight line already drawn.

Fig 58 represents another form of parallel ruler, which consists of a rectangular piece of ebony, or ivory, on the upper side of which is fitted a longitudinal bar, carrying near its extremities two rollers, the peripheries of which project through the under side of the ruler, and rest upon the paper.

Fig. 58.



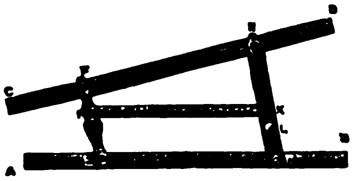
This arrangement enables the user to move the ruler over the paper with a smooth rolling motion so that parallel lines may be drawn in with great rapidity and accuracy.

In certain cases, in perspective drawings for instance, a great number of lines converging to a point not within the drawing is required, and it would be troublesome to repeat the construction over and over again. Consequently an instrument has been invented, something after the fashion of a parallel-ruler, to save all this trouble to the draughtsman. It is called the centrolinead.

The centrolinead, fig. 59, is a ruler by which we are enabled to draw through any given point a straight line, which, if produced, will pass through the point of intersection of any two or more given straight lines, when the latter point is not determined.

It consists of two flat-rulers, A B and C D, connected together by two bars, E F, G H, moveable round joints at E F G and H. E F is of a fixed length, which never varies in the same instrument, as in the parallel-ruler; but G H is made up of two parts, one sliding on the other by means of a groove, so that G H admits of different lengths, and is adjusted to some particular length by a screw at L, which clamps the two parts of G H together, when required. G, is a fixed point, but the point at H is carried in a groove, and slides on the ruler, C D. There is also another ruler, I K, fixed as in the figure, with joints at I and K, so that E G K I is always a parallelogram, as shown by the dotted lines. The centres of motion at E and G are equidistant from both edges of the ruler, A B, and the points where E G produced meets the ends of the ruler are marked on the ruler. But the centres at F and H are so made, that they are in a line with the edge of the ruler, C D.

Fig. 59.



In using the instrument, A B is first made to coincide with one of the two given converging lines (not the edge of A B but the two marks on the ends of A B, mentioned before). The screw, L, is slackened, and holding A B tight, C D is brought to coincide with the second given line. Then the screw is clamped tight; and while A B is still held in its original position, the edge of C D can be made to pass through any proposed point within certain limits, and all lines drawn along it will converge to the same point as the two given lines.

For measuring angles, the instruments represented in figs. 4 and 5, on plate 1, are used. They are designated Protractors. The ordinary form of the instrument is shown in fig. 4. It consists of a semicircle of brass, the central part being cut out. The limb, or semicircular band, is divided into 180 equal parts, all of which, if produced, pass through the centre of the circle at B.

In fig. 5, the protractor forms a complete circle, which is divided into 360 degrees, or equal divisions. The practical application of these instruments will be adverted to in their proper place.

Semicircular protractors, of from two to five inches in diameter, may be obtained in a thin transparent composition of gelatine. These protractors are of French manufacture, and are exceedingly portable, and moderate in price.

The protractor is frequently made in the form of a flat ruler, as shown in fig. 60. A, B, C, D, is a thin ivory ruler of a rectangular figure. To construct this scale, A B is bisected in O; with centre O, and radius O A or O B, describe a semicircle. Divide this semicircle into eighteen equal parts by lines through O, and mark the points of intersection of these lines with the edge of the ruler. This being done, it is obvious that the semicircle may be erased, leaving the ruler divided, as shown in the figure, so as to admit of its being used for drawing straight lines.

These protractors have frequently the diagonal scale shown in fig. 14, plate 1, engraved on the inner portion of the ruler; and some are made with a longitudinal roller, similar to fig. 60, so that the instrument then forms a convenient ruler for drawing a series of lines parallel to each other.

Fig. 60.

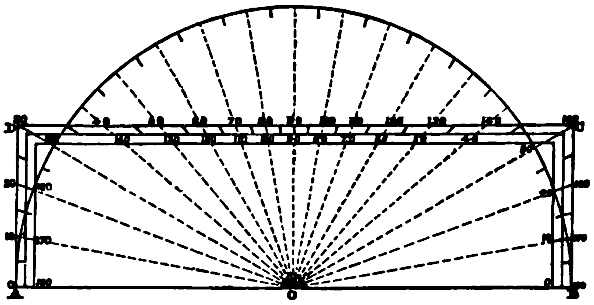
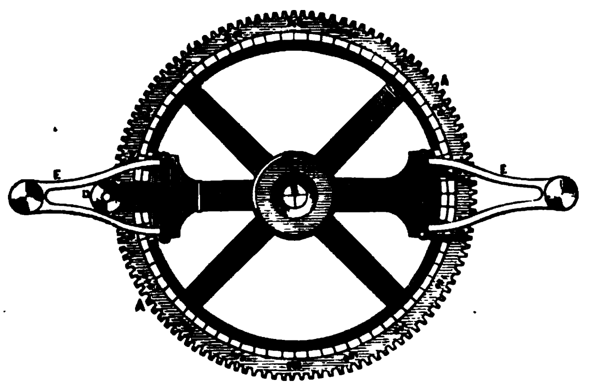


Fig. 61 represents the most approved arrangement of the circular protractor, as made by our leading instrument makers.

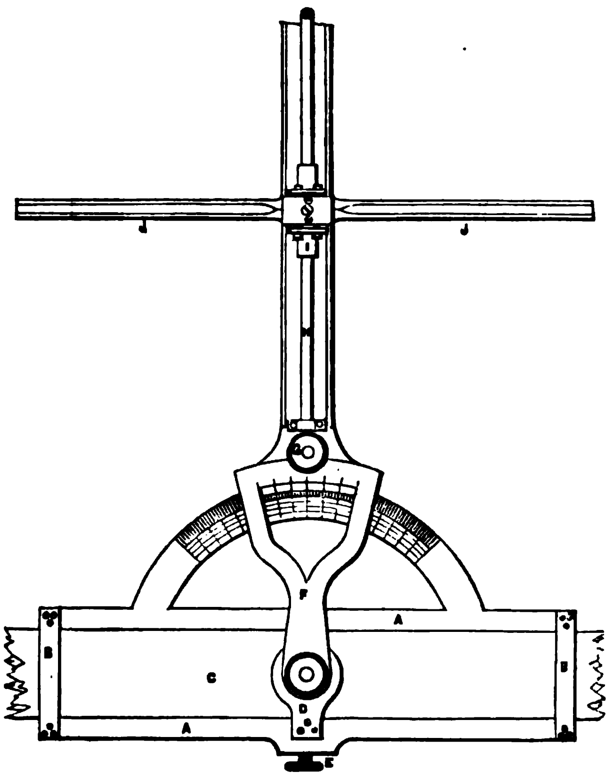
In this form of protractor, the circle, A, is connected with its centre by four radial bars. The centre is left open and is surrounded by the concentric ring or collar, B, which carries the two diverging arms, C. To the extremity of one of these arms is fitted a pinion, D, which gears with the circular rack extending round the circumference of the instrument. The extremity of the other arm, C, is fitted with a vernier which divides the primary divisions on the protractor to single minutes, and by estimation to 30 seconds. Upon each arm, C, is jointed an open branch, E, which carries at its extremity a fine steel pricker, the point of which is kept above the surface of the paper by a small spring placed under it; the spring gives way when the end is pressed down upon the paper and allows the point to enter. The centre of the instrument is shown by the intersection of two lines drawn at right angles to each other on a piece of plate glass fitted in the ring, B, which enables the user to place the instrument so that its centre coincides with any point on the plan. In using the instrument the vernier should first be set to zero, or the division marked 360 on the divided limb, and then placed on the paper so that the two steel points are on the given line, from whence the other angular lines are to be drawn. This done, press the protractor gently down, which will fix it in position, by means of the points on the under side. It is now ready to lay off the given angle or any number of angles required, which is done by turning the pinion, D, till the opposite vernier reads the required angle. Then press down the ends of the branches, E, which will cause the points to make punctures in the paper at opposites sides of the circle. These points being afterwards connected, the line will pass through the given angular point, if the instrument was correctly adjusted at the outset. In this way at one setting of the instrument a great number of angles or a complete circular protractor may be laid off from the same point.

Fig. 61.



Mr. Simpson produced an elegant duplex straight-edge protractor, which is represented in plan in fig. 62. A graduated quadrant, forming an index for angles, is made in one piece with a parallel bar, A, which is connected at each extremity by means of transverse bars, B, with a similar bar, A, the space between the two being of such a width as to admit the straight-edge, C, at any part of which it can be fixed by the pinching screw, E. A central transverse piece, D, which is screwed to the two bars, A, carries a pivot, or joint stud, for the eye of a radial straight-edge, F, having its edges bevelled off for ruling close to the paper. A screw, G, and clamping plate are provided to fix the radial straight bar, F, at any angle. Immediately above the centre of the radius bar, F, and fixed to it at both ends, is a smooth rod, H, upon which slides the socket, I. This socket carries the two straight-edges, J, lying in one straight line, and united together by a bridge-piece embracing the socket, I, and adjusted upon it by means of screws, so as to be capable of being kept at all times accurately at right angles with the radius bar, F. At a short distance from the centre of motion of the radius bar, F, is a segmental opening, graduated to 30°, on each side of the centre, or zero point, forming two verniers, for instruments differently divided, and for the minute subdivision of the graduations of the quadrant on the piece, A. By this apparatus, any angles may be measured and laid off by the quadrant, and transferred to any point on the paper without the aid of any additional instrument, as the whole instrument may be moved to any desired point on the straight edge, C, without any shift of position with reference to the meridian line or starting point. With additional scales on the right angle straight-edges, J, the instrument may be employed, as an offset scale, for plotting surveys and laying down sections.

Fig. 62.



Mr. J. M. Balfour, C.E., recently communicated to the Royal Scottish Society of Arts a paper on an improved form of protractor, of which the following is an abstract:—

All who are in the habit of protracting surveys of considerable extent, especially if they be marine surveys, are aware that it is often necessary, or at least very desirable, to prick off one or more protracting circles on the paper on which the work is to be laid down. This has been hitherto done either by means of a circular protractor, or by dividing a circle on the paper by means of chords, somewhat after the method of graduation invented by the celebrated instrument-maker, Bird. These methods are, however, somewhat tedious, especially when the circle has to be minutely divided; and as they are, besides, both more or less liable to inaccuracy, I have been led to adopt the method of graduation which was invented by the late Mr. Edward Troughton, and applied by him to the graduation of his circular-dividing engine and to other important instruments.

The instrument consists simply of a small wheel, A, fig. 63, of thin steel, cut into 90 sharp-edged teeth, though any other number would answer nearly as well; after the teeth are cut, the steel is filed away from one side, so as to reduce the whole to a series of sharp points, every tenth tooth being made with a double point, to facilitate counting. The wheel is then hardened, and attached by screws to a brass disc, B, to prevent the teeth penetrating the paper too far, and the whole is mounted in a frame, C, so as to fit upon a beam compass. The wheel thus mounted is simply Mr. Troughton's dividing roller, adapted for working on paper, and for marking the divisions at the same time; and all that is further necessary before using it, is to find experimentally the proper radii for which the distance between the points will represent degrees, half degrees, fifteen or ten minutes, or any other quantities which may be desired, and to mark the radii on the beam in a permanent manner. Of course the influence of temperature will here come into play, but if the roller be set to the radii found by trial, and the instrument run round 180° or 360° on a piece of waste paper, any little error arising from a change of temperature may easily be corrected by the slow motion screw of the beam. I am aware that, as pointed out by Mr. Troughton, there will be small errors in intermediate quantities, though the wheel turns up 360° accurately, but these errors I have found quite inappreciable in practice, and they are, I believe, when the wheel is well divided, much smaller than the errors resulting from graduating circles in the common way. It is also worthy of notice that these are absolute errors, which will not increase with the increase of the radius employed, so that the divisions may be rendered more accurate when required by doubling the radius of the beam, and using only every second mark. In pricking off the divisions it is of importance that the beam be always held in one division, as, if it be more inclined at one time than another, the length of the radius is altered, and I have in consequence found it convenient to have a centre, D, fig. 64, made in such a manner as always to retain the centre point, E, of the beam in an upright position. The centre, D, has fine points on its under surface, to prevent it from slipping, and it is divided into four equal parts on the chamfered edge, to facilitate its being laid down correctly over any required point.

Fig. 63.

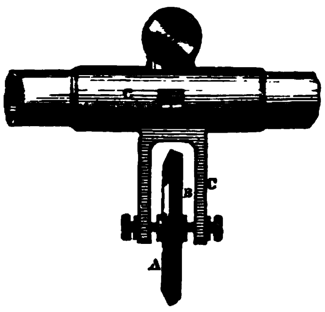
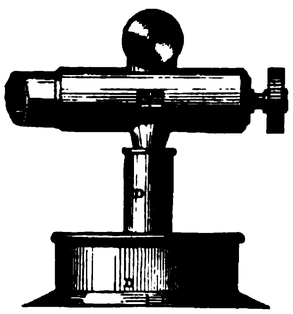


Fig. 64.



This instrument was principally designed to facilitate the protraction of the positions of such soundings as have been fixed by sextant angles, by the method of intersecting circles, and I have found it to answer exceedingly well for the arcs of circles which require to be divided for this purpose. The saving of time thus occasioned is very considerable.

Fig. 65 represents one side of a sector. This is a highly useful instrument for measuring angles, dividing lines equally and unequally, as well as solving questions in proportion. The angular functions, viz., chords, sines, tangents, &c., may be set off or measured to any radius whatever; plans and drawings may be reduced or enlarged in any required proportion; and, in short, every operation in geometrical drawing may be performed by the aid of this instrument and the compasses only.

Sectors are made of different sizes, and their length is usually denominated from that of the legs when shut together. Thus, a sector of six inches, such as is supplied in the common pocket cases of instruments, forms a rule of twelve inches, when opened; and this circumstance is taken advantage of, by filling up the spaces not occupied by the sectoral lines with such lines as it is most important to lay down upon a greater length than the six-inch plain scale will admit. Among these the most usual are the lines of logarithmic numbers, sines, and tangents; a scale of 12 inches, in which each inch is divided into ten equal parts; and a foot divided into ten equal primary divisions, each of which is subdivided into ten equal parts, so that the whole is divided into 100 equal parts. The last-mentioned is called the decimal scale, and is placed on the edge of the instrument.

The *sectoral lines* proceed in pairs from the centre, one line of each pair on either leg, and are, upon one face of the instrument, a pair of scales of equal parts, called the *line of lines*, and marked L; a pair of lines of chords, marked C; a pair of lines of secants, marked S; a pair of lines of polygons, marked POL. Upon the other face, the sectoral lines are—a pair of lines of sines marked S; a pair of lines of tangents up to 45°, marked T; and a second line of tangents to a lesser radius, extending from 45° to 75°.

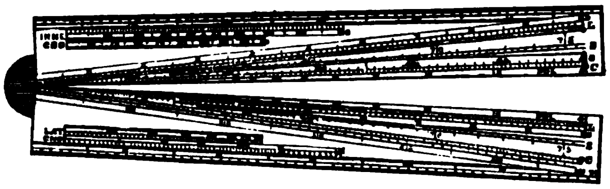
Each pair of sectoral lines, except the line of polygons, should be so adjusted as to make equal angles at the centre, so that the distances from the centre to the corresponding divisions of any pair of lines, and the transverse distance between these divisions, may always form similar triangles. On many instruments, however, the pairs of lines of secants, and of tangents from 45° to 75°, make angles at the centre equal to one another, but unequal to the angle made by all the other pairs of lines.

The solution of questions on the sector is said to be *simple*, when the work is begun and ended upon the same pair of lines; *compound*, when the operation is begun upon one pair of lines and finished upon another.

In a compound solution the two pairs of lines used must make equal angles at the centre, and, consequently, in the exceptional case mentioned above, the lines of secants and of tangents above 45° cannot be used in connection with the other sectoral lines.

When a measure is taken on any of the sectoral lines beginning at the centre, it is called a *lateral* distance; but, when a measure is taken from any point on one line to its corresponding point on the line of the same denomination on the other leg, it is called a *transverse* or *parallel* distance.

Fig. 65.



The divisions of each sectoral line are contained within three parallel lines, the innermost being the line on which the points of the compasses are to be placed, because this is the only line of the three which goes to the centre, and is therefore the sectoral line.

THE USES OF THE LINE OF LINES.—*To find a Fourth Proportional to three given Lines.*—Set off from the centre a lateral distance equal to the first term, and open the sector till the transverse distance at the division thus found, expressing the first term, is equal to the second term; again, extend to a point whose lateral distance from the centre is equal to the third term, and the transverse distance at this point will be the fourth term required.

If the legs of the sector will not open far enough to make the lateral distance of the second term a transverse distance at the division expressing the first term, take any aliquot part of the second term, which can conveniently be made such transverse distance, and the transverse distance at the third term will be the same aliquot part of the fourth proportional required.

A third proportional to two given lines is found by taking a third line equal to the second, and finding the fourth proportional to the three lines.

Example.—To find a fourth proportional to the numbers 2, 5, and 10. Open the sector till the lateral distance of the second term 5 becomes the transverse distance 2, the first term; then the transverse distance at 10 will extend, as a lateral distance, from the centre to 25, the fourth proportional required.

To bisect a given Straight Line.—Take the extent of the line in the compasses, and open the sector till this extent is a transverse between 10 and 10 on the line of lines: then the transverse distance from 5 to 5, on the same pair of sectoral lines, gives the half of the line, and this extent set off from either end will bisect it.

To open the Sector so that the Line of Lines may answer for any required Scale of equal Parts.—Take one inch in the compasses, and open the sector, till this extent becomes a transverse distance at the division indicating the number of parts in an inch of the required scale; or, if there be not an integral number of parts in one inch, it will be better to take such a number of inches as will contain an integral number of parts, and make the extent of this number of inches, if it be not too great, a transverse distance at the division indicating the number of parts of the required scale in this extent.

Example.—To adjust the Sector as a Scale of One Inch to Four Chains.—Make one inch the transverse distance of 4 and 4; then the transverse distances of the other corresponding divisons and subdivisions will represent the number of chains and links indicated by these divisions: thus, the transverse distance from 3 to 3 will represent three chains; the transverse distance at 4·7, or the seventh principal subdivision after the primary division marked 4, will represent 4 chains 70 links, and so on.

To construct a Scale of Feet and Inches in such a manner that an extent of Three Inches shall represent Twenty Inches.—1. Make three inches a transverse distance between 10 and 10, and the transverse distance of 8 and 8 will represent 16 inches. 2. Set off this extent from A to B, divide it by continual bisection into 16 equal parts, and place permanent strokes to mark the first 12 of these divisions, which will represent inches. 3. Place the figure 1 at the twelfth stroke, and set off again the extent of the whole 12 parts, from 1 to 2, 2 to 3, etc., to represent the feet.

As an Example of the Use of the Line of Lines in reducing Lines, let it be required to reduce a drawing in the Proportion of 5 *to* 8.—Take in the compasses the distance between two points of the drawing, and make it a transverse distance at 8 and 8; then the transverse distance of 5 and 5 will be the distance between the two corresponding points of the copy. 2. These two points having been laid down, make the distance between one of them and a third point a transverse distance at 8, and with the transverse distance at 5 describe, from that point as centre, a small arc. 3. Repeat the operation with the other point, and the intersection of the two small arcs will give the required position of the third point in the copy. In the same manner all the other points of the reduced copy may be set off, each one from two points previously laid down.

The Line of Chords.—The double scales of chords upon the sector are more generally useful than the single line of chords described on the plain scale; for, on the sector, the radius with which the arc is to be described may be of any length between the transverse distance of 60 and 60 when the legs are close, and that of the transverse of 60 and 60 when the legs are opened as far as the instrument will admit off: but, with the chords on the plain scale, the arc described must be always of the same radius.

The Line of Polygons.—The line of polygons is chiefly useful for the ready division of the circumference of a circle into any number of equal parts from 4 to 12; that is, as a ready means to inscribe regular polygons of any given number of sides, from 4 to 12, within a given circle. To do which, set off the radius of the given circle (which is always equal to the side of an inscribed hexagon) as the transverse distance of 6 and 6, upon the line of polygons. Then the transverse distance of 4 and 4 will be the side of a square; the transverse distance between 5 and 5, the side of a pentagon; between 7 and 7, the side of a heptagon; between 8 and 8, the side of an octagon; between 9 and 9, the side of a nonagon, &c., all of which is too plain to require an example.

If it be required to form a polygon, upon a given right line set off the extent of the given line, as a transverse distance between the points upon the line of polygons, answering to the number of sides of which the polygon is to consist; as for a pentagon between 5 and 5; or for an octagon between 8 and 8; then the transverse distance between 6 and 6 will be the radius of a circle whose circumference would be divided by the given line into the number of sides required.

The line of polygons may likewise be used in describing, upon a given line, an isosceles triangle, whose angles at the base are each double that at the vertex. For, taking the given line between the compasses, open the sector till that extent becomes the transverse distance of 10 and 10, then the transverse distance of 6 and 6 will be the length of each of the two equal sides of the isosceles triangle.

All regular polygons, whose number of sides will exactly divide 360 (the number of degrees into which all circles are supposed to be divided) without a remainder, may likewise be set off upon the circumference of a circle by the line of chords. Thus, take the radius of the circle between the compasses, and open the sector till that extent becomes the transverse distance between 60 and 60 upon the line of chords; then, having divided 360 by the required number of sides, the transverse distance between the numbers of the quotient will be the side of the polygon required. Thus for an octagon, take the distance between 45 and 45; and for a polygon of 36 sides, take the distance between 10 and 10, etc.

Lines of Sines, Tangents, and Secants.—*Given the radius of a circle (suppose equal to two inches) required the Sine and Tangent of* 28° 30′ *to that radius.*—Open the sector, so that the transverse distance of 90 and 90 on the sines, or of 45 and 45 on the tangents, may be equal to the given radius, viz., two inches; then will the transverse distance of 28° 30′, taken from the sines, be the length of that sine to the given radius, or, if taken from the tangents, will be the length of that tangent to the given radius.

But if the Secant of 28° 30′ *is required,* make the given radius of two inches a transverse distance of 0 and 0 at the beginning of the line of secants, and then take the transverse distance of the degrees wanted, viz., 28° 30′.

A Tangent greater than 45° (suppose 60°) is thus found:—Make the given radius, suppose two inches, a transverse distance to 45 and 45, at the beginning of the line of upper tangents, and then the required degrees (60) may be taken from the scale.

The tangent, to a given radius, of any number of degrees greater than 45° can also be taken from the line of lower tangents, if the radius can be made a transverse distance to the complement of those degrees on this line.

Example.—To find the tangent of 78° to a radius of two inches. Make two inches a transverse distance at 12 on the lower tangents, then the transverse distance of 45 will be the tangent of 78°.

In like manner the secant of any number of degrees may be taken from the sines, if the radius of the circle can be made a transverse distance to the complement of those degrees upon this sine. Thus making two inches a transverse distance to the sine of 12°, the transverse distance of 90 and 90 will be the secant of 78°.

To find, by means of the lower tangents and sines, the degrees answering to a given line, greater than the radius which expresses the length of a tangent or secant to a given radius. For a tangent, make the given line a transverse distance at 45 on the lower tangents; then take the extent of the given radius, and apply it to the lower tangents; and the complement of the degrees at which it becomes a transverse distance will be the number of degrees required. For a secant make the given line a transverse distance at 90 on the sines; then the extent of the radius will be a transverse distance at the complement of the number of degrees required.

Given the Length of the Sine, Tangent, or Secant of any Degrees, to find the Length of the Radius to that Sine, Tangent, or Secant- Make the given length a transverse distance to its given degrees on its respective scale. Then,

If a sine	the transverse distance of	90 and 90 on the sines	will be the radius sought.
If a tangent under 45°		45 and 45 on the tangents	
If a tangent above 45°		45 and 45 on the upper tangents	
If a secant		0 and 0 on the secants	

To find the Length of a versed Sine, to a given Number of Degrees, and a given Radius.—1. Make the transverse distance of 90 and 90 on the sines equal to the given radius. 2. Take the transverse distance of the sine of the complement of the given number of degrees. 3. If the given number of degrees be less than 90, subtract the distance just taken, viz., the sine of the complement from the radius, and the remainder will be the versed sine; but if the given number of degrees are more than 90, add the complement of the sine to the radius, and the sum will be the versed sines.

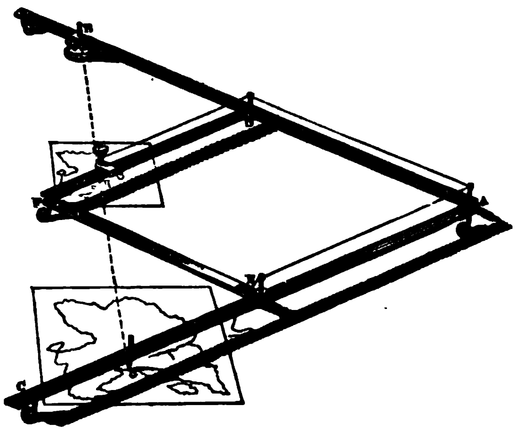
To open the legs of a Sector, so that the corresponding double Scales of Lines, Chords, Sines, and Tangents may make each a right Angle.—On the line of lines make the lateral distance 10, a transverse distance between 8 on one leg, and 6 on the other leg.

On the line of sines make the lateral distance 90, a transverse distance from 45 to 45; or from 40 to 50; or from 30 to 60; or from the sine of any degree to their complement.

On the line of sines make the lateral distance of 45 a transverse distance between 30 and 30.

The Pentagraph is an exceedingly useful instrument for reducing and enlarging drawings. By means of it, drawings may be copied on an enlarged or a reduced scale, by the mere action of carrying a tracing point over the lines of the original drawing. The motion of the tracing point is communicated to the delineating pencil by the angular movements of a series of levers oscillating upon a fixed centre. If one arm of a lever is twice as long as the other, the arc described by its extremity will be twice as great, whatever be the extent of the movement. In this case, however, the radii of the arcs would be uniform, and an arrangement is required, providing for the increase or decrease in length of the two radii, but in such a manner that they shall continually be in the same proportion to each other. This is effected by making the main lever with joints midway between the extremities and the centre of motion. It is, necessary, however, that the outer joints of the lever should be maintained constantly parallel to each other, and it is likewise desirable that the instrument should be capable of adjustment for different proportions. Fig. 66 represents a perspective view of the instrument, as prepared for reducing a plan or drawing to one-half the original size. It consists of four bars of metal, A B, A C, D F, and E F; the two longer bars are jointed at A, round which they move as a centre. The shorter bars are jointed in like manner at F, and to the bars A and B, at D and E, so as to form, in whatever direction they are moved, an accurate parallelogram. The instrument is supported on small castors to admit of its moving easily in any desired direction. One of the longer arms, A B, and one of the shorter, D F, are graduated and divided into aliquot parts, each having a sliding index, which can be fixed at either of the divisions by means of a clamping screw. The sliding indices have each a tube adapted to slide on a pin rising from a heavy circular weight, which forms the fulcrum, or to receive a sliding holder fitted with a pencil or pen, or a tracing point, as may be required.

Fig. 66.



When the instrument is properly set, the tracing point, G, the pencil, H, and the fulcrum, I, are in a straight line. To reduce a drawing to half size, the sliding indices are fixed at the divisions on the graduated bars marked $\frac{1}{2}$; the tracing point is then fixed in the socket at G, the original drawing being placed beneath the point. The pencil is arranged in the holder on the bar, D F, and the paper to receive the copy placed below it, the fulcrum being fixed to the clamp on the bar, A B.

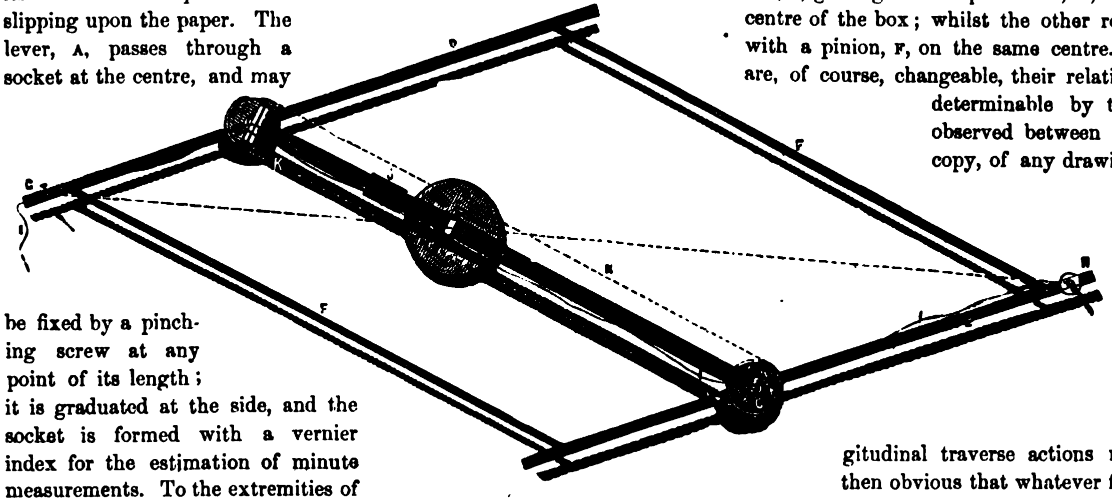
In this position of the parts the tracing point, G, is passed steadily and accurately over each line of the original drawing. By this movement of the tracer, the pencil is caused to mark a true copy of the drawing, but reduced to one-half the size of the original. The fine silk cord which is carried from the pencil round the instrument to the tracer, enables the draughtsman to raise the pencil off the paper beneath, when it is requisite to move the tracer from one part of the drawing to another and avoid making false lines.

To enlarge a drawing, the tracer is placed on the arm, D F, and the pencil at G, on the arm, A C. If the drawing is required the same size as the original, the fulcrum is placed at the central station, and the tracing point and pencil on the longer bars, A B, and A C, of the instrument.

To cause the pencil to press more or less heavily on the paper, the holder is fitted with a small cup on the upper part to contain shot, by which means the pressure of the pencil point is adjusted.

Fig. 67 is a Pentagraph of an approved design and construction. The main lever, A, turns upon a centre carried by the weight, B, which has fine points upon its under side to prevent its slipping upon the paper. The lever, A, passes through a socket at the centre, and may be fixed by a pinching screw at any point of its length; it is graduated at the side, and the socket is formed with a vernier index for the estimation of minute measurements. To the extremities of the lever, A, are jointed the discs, C, which are formed with sockets on their under sides to receive the bars, D E. These bars are graduated in a similar manner to the main lever, and vernier indices are formed in the plates, C, to correspond. The parallelism of these bars is maintained by the rods, F; the tracing point is at G, and the delineating pencil at H, or *vice versa*, as the case may be. A string, I, is passed from the tracing point through guide eyes at the joints, C, to a small bell-crank lever at H, by means of which the pencil is raised when it is not wished to mark; this being effected by drawing the string, I, at the tracing point. As represented in the engraving, the instrument is adjusted to copy a drawing upon an enlarged scale. To obtain the correct action of the instrument it is necessary that the tracer, G, pencil, H, and centre of motion, B, be in a straight line. The proportion of reduction or enlargement being determined on, the main lever, A, is so adjusted in its socket, B, that the portions of the lever on each side of the centre may have this proportion to each other; this being indicated by the graduated scale on the side of the lever. The bars, D E, must then be correspondingly adjusted in their carrying sockets, the distance between the tracer, or pencil, and joint being always equal to the distance between the joint and turning centre on the respective side of the main lever, A. The instrument, when adjusted, is balanced on its centre by means of the sliding weight, J, upon the lever, A. In some pentagraphs the rods, F, are dispensed with, and the parallelism of the bars, D E, maintained by a belt, K, indicated in dotted lines, passed round the peripheries of the discs, C. grooved for the purpose. This belt is usually of thin flat steel wire, similar to that used for watch springs—a belt of ordinary material causing inaccuracies, owing to its elasticity. The arrangement in which the rods are used is, however, superior, as the belt is apt to slip, or, if it is too tight, it occasions an injurious strain on the joints.

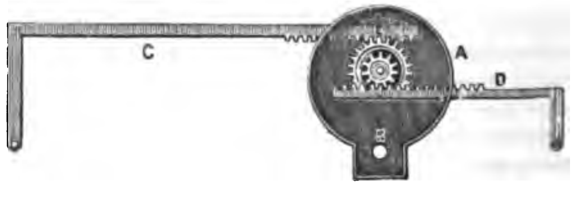
Fig. 67.



Another form of Pentagraph has been suggested by Mr. R. Forster, jun., of Dublin, which seems susceptible of being rendered a very efficient instrument. It is delineated in fig. 68. The small and shallow circular box, A, contains the actuating mechanism, and is arranged to turn at pleasure upon the fixed centre stud, B; and from each side of the box, a rod, C D, projects, the points of the rods being brought into a horizontal line with the stud centre, B. The box is in horizontal section, to exhibit the internal gearing. The end of each rod has rack-teeth upon it, the teeth on the rod, C, gearing with a spur-wheel, E, fast on a stud in the centre of the box; whilst the other rod, D, similarly gears with a pinion, F, on the same centre. These two wheels are, of course, changeable, their relative radii being always determinable by the proportion to be observed between the original and the copy, of any drawing to be reduced or enlarged by the instrument. The same relation is also to be kept up between the lengths of the two rods, in order that both the angular and longitudinal traverse actions may coincide. It is then obvious that whatever figure is traced out by the point on one rod, will be delineated by the pencil on the other, in the proportion determined by the wheels and the leverage of the rods. The rods may be either worked on opposite sides, or both on the same side.

Another mode of reducing or enlarging drawings is by means of squares ruled upon the original drawing to an equal size, and to the proportional size upon the sheet of paper on which the copy is to be made. Thus, if the original drawing is divided by faintly ruled lines into squares, the sides of which are two inches in length, and it is required to produce a copy one half the size. The sheet of paper on which the copy is to be made, is divided by faint lines, the sides of which are one inch in length. Commencing then at the upper part of the drawing, the student notices the position of the outline of the drawing in one of the squares, and he proceeds to lay down in the corresponding square of his copy a similar line. The position of the line in his copy he may either form with the assistance of the proportional compasses, or verify the accuracy of his eye and hand by their aid. This being done, he proceeds to the next square and copies the portion of the outline contained therein.

Fig. 68.



Having gone over the whole of the outline, the details are then filled in like manner. If all this has been carefully done, the copy will be a *fac simile* of the original, but contained in a space equal to one fourth of its area.

Although the practical application of surveying instruments forms no part of our course of instruction, yet as a knowledge of the construction of the more important surveying instruments is at all times useful to the young draughtsman, we propose to close this portion of our subject with a few illustrations of this kind.

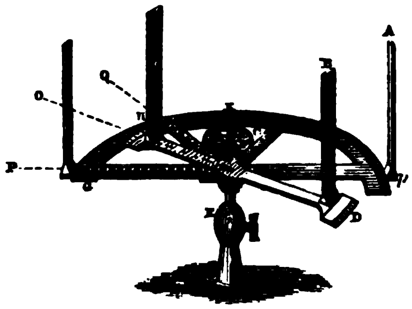
The Land Chain is used for measuring distances. It consists of 100 links, each being, with its connecting ring, 7·92 inches in length. Thus the hundred links measure 66 feet, or four poles. Every tenth link has attached to it a brass mark so as to readily count off the number of links.

The Level is used for ascertaining the differences of level or vertical differences between distant places or stations. There are several arrangements of levels which are named from some peculiarity, or after the designers. Thus, there is the Y level, Gravatt's level, Elliot's Dumpy level, Troughton's level, the Water level, and others. The Spirit levels consist of a telescope with the level either suspended thereto or arranged upon the upper part of its tube. The instrument is furnished with a compass which is generally arranged below the telescope, but in Troughton's level is placed above it, and the whole is carried upon a tripod stand.

The Prismatic Compass is a very portable and convenient instrument for observing horizontal angles. It consists of a brass box, containing the compass. To the edge of the box is fitted the sight vane, which is a small open frame, having a fine silk thread stretched vertically in the opening. Opposite to the sight vane is arranged a moveable prism, which is attached to a metal plate having a small slit in it. Through this aperture the observer looks when using the instrument. Upon looking through the sight aperture the prism is so adjusted that the divisions upon the compass card are distinctly seen; the instrument is held so that the object to be observed is seen through the sight aperture, and is brought in exact coincidence with the thread of the sight vane. The compass card is brought to rest by touching a spring at the side of the instrument. The division on the card, which coincides with the prolongation of thread, gives the angle, which the object makes with the magnetic meridian.

The Graphometer is an instrument consisting of a semicircle, or, which is much better, a complete circle of brass, the rim of which is divided into 180°, in the one case, and 360° in the other, with two diametral bands, one fixed, and the other moveable about the centre. The advantages of the complete circle over the semicircle are well known to practical observers. And where great accuracy of measurement is of importance, the semicircular instrument ought never to be used. At right angles to the extremities of each of these diametral bands is placed a pinnule, or sight. This consists of a thin, oblong, flat piece of brass, as represented at A and B, about 12 inches high, and having a slit pierced lengthways, and in the middle. One half of this slit is very narrow; the other is much broader, and is bisected lengthways by a wire, which, if continued, would also bisect the narrow part of the slit. At one end of each diametral band the narrow slit is uppermost; at the other it is reversed. Each end of the moveable diametral band is furnished with a vernier for reading off angles. The whole is attached centrally to a pivot, which works by a universal joint in an upright pillar resting upon a tripod. A compass and a spirit-level are attached, so that the diametral bands of the instrument can be placed in any required position, with respect to the points of the compass, and the plane of the circle be made horizontal.

Fig. 69.



The graphometer is used for taking angles, most commonly, either in an horizontal, or vertical plane; but it may be turned in any direction, so as to bring it into the plane of any two or more objects whose bearings are required.

The observer looks through the narrow slit, and therefore has the larger opening of the opposite sight in the direction of the object observed. He finds the object through that latter opening, and brings the wire which bisects it into the same plane with the narrow slit close to the eye, so that the plane passing through the wires bisects the object.

Let E be the place of observation; O, P, and Q, objects in an horizontal plane, whose relative positions the observer wishes to ascertain.

Place the circle in the plane passing through O, P, and Q, and let the fixed diametral band be directed to P, so that to the observer at *p* the vertical plane, passing through the wires of A and its opposite sight, may bisect P. Direct the moveable diametral band, D H, so that the vertical plane through B, and its opposite sight may bisect O. Then the number of degrees and the arc, G H, will measure the angular distance of O from P. The moveable diametral band is next directed to Q, and in like manner is obtained the angle, P E Q; subtracting the previously obtained angle, O E P, we obtain the difference, O E Q.

An improvement on the ordinary surveying instrument has been recently introduced into this country by Mr. J. H. Johnson. It is the invention of Mr. E. A. Crandall, of New York.

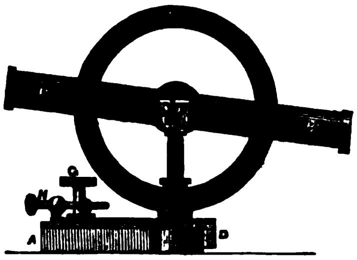
This instrument consists of two telescopes, sight tubes, or other sighting devices, placed at a certain fixed distance apart, on a suitable table or base, one of the telescopes or sighting devices being stationary relatively to the table or base, the axis lying at right angles to a line joining the two telescopes, and the other being moveable on a fixed pivot, so as to be capable of being brought to bear upon the same point or object. It also contains certain accurately arranged levers, which successively come into action as the telescopes approach very nearly to parallel positions, and multiply the motion so as to render very slight changes of position in the moveable telescope very clearly readable on the several scales attached.

The operation of the instrument is based upon the well-known principle in trigonometry, that when the length of the base of a right angled triangle is given, the adjacent angle formed by the hypothenuse serves to determine the length of the perpendicular.

Fig. 70 of the engravings is a side view of the instrument. Fig. 71 is a plan of the same, with a portion of the top plate of the hollow box, serving as the base, broken away to show the interior. Figs. 72 and 73 are diagrams to illustrate the operation of the instrument.

A, is a rectangular box, forming the horizontal table or base of the instrument. B, is a telescope or sighting tube, placed on a stand, B^1, at one corner of the box, A, and arranged to be capable of receiving motion on a centre in a vertical plane on trunnions, *b*, fitted to bearings in the stand, B^1. C, is a second moveable telescope or tube, carried by a stand similar to B^1, but free to turn horizontally on a pivot, *c*, in the base. The centre of this pivot, *c*, forms the termination of the right line, *a*, which is the base line on which all the measurements are founded, and which must be exactly at right angles to the axis of the telescope, B.

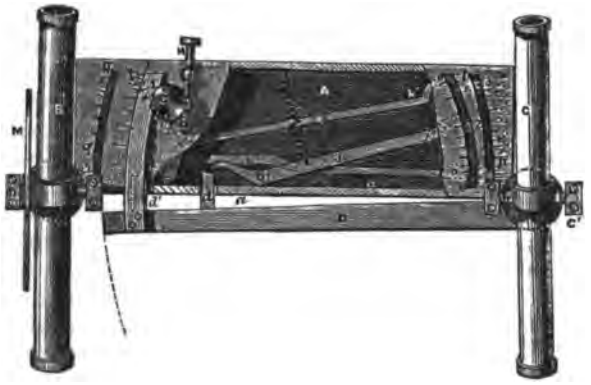
Fig. 70.



The telescope, C, is also arranged to turn vertically on trunnions, *b*. D, is a horizontal index bar, carrying an index, 1, and attached to the stand, C^1, of the telescope, C, in such a manner that when the telescopes are parallel, the said index bar will be parallel to the side, *a*, of the base, A. The index, 1, works on a graduated scale of distances, $e\,e^1$, on the face of the top plate of the base of the instrument. This index is furnished with a screw clamp, G, by which it is attached at pleasure to a screw, H, so that by turning the screw, H, the telescope, C, will be partially revolved in a horizontal plane, and adjusted with accuracy in any desired position. The base of the instrument, which can be attached to a tripod similar to other surveying instruments, is furnished with a horizontal limb, a compass, and spirit level.

Let it be supposed that the distance from B to T, diagram fig. 72, is to be measured. A staff is placed at the point, T, unless some object already stationary at that point be used as a sight point. The instrument is placed with the axis of the stand, B^1 (figs. 70 and 71), on the point, B (fig. 72), and with the tube, B, sighted to the point, T. The line, *a*, of the instrument then forms a base to which the line, B, T, is perpendicular. The tube, C, is then turned on its pivot, *c*, at the end of the line, *a*, until it also sights the pivot, T; and the length of the base, *a*, being constant, the angle that is formed at C serves to determine the length of the perpendicular, B, T.

Fig. 71.



The index, D, moves with the telescope, C, so that the scale, $e\,e^1$, being graduated in distances corresponding to the varying angles at C, the true distance from B to T is indicated by the index on the scale, $e\,e^1$. The spaces between the graduations on the scale, $e\,e^1$, diminish according to a certain law at one extremity of the scale.

In measuring long distances, C, stands nearly parallel with B, and as they approach a state of parallelism the prolonged axis of C, or the line, O T, will take a long range along the line, B T, without producing any very perceptible movement in the index. This renders it impossible that the scale, $e\,e^1$, the width of graduations of which diminish progressively from the commencement, can either be graduated or read up to a termination at the end, e^1, with so great a degree of exactness as is desirable for precise measurement. It is with a view to remedy this deficiency that the inventor has added to the instrument multiplying levers, and additional scales. The multiplying levers, of which three, J, K, L, are shown, but, if desired, there may be any other number, are all arranged to work within the box or hollow base, A, so as to be out of the way of the surveyor, but are furnished respectively with indices, 2, 3, and 4, working through slots in the top plate of the base over their respective graduated scales, $o\,o^1$, $p\,p^1$, $q\,q^1$, on the upper surface of the plate. The lever, J, which is the first of the series, is a lever of the first order, working on a fulcrum, *j*, and has a long and a short arm; the index, 2, being attached to the long arm j^2, whilst the short arm, j^1, is fitted at its extremity with a roller or shoe; or it is made of a pointed or rounded form, and is so placed, relatively to the index bar, D, that when the index, 1, has moved far enough up the scale, $e\,e^1$, to afford accurate indications, and arm, *d*, on the bar, D, comes in contact with the end of the short arm of the lever, J, and by a continuous movement of the bar, D, towards the line, *a*, the lever, J, is put in motion, and the index, 2, is caused to move over the scale, $o\,o^1$.

Fig. 72.

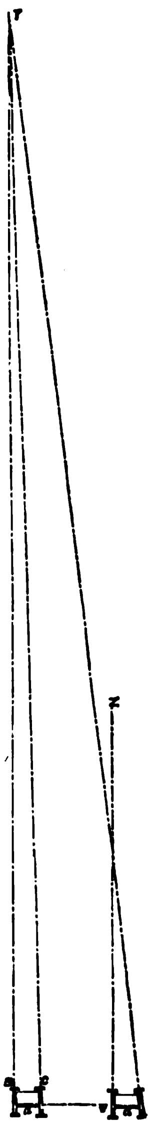
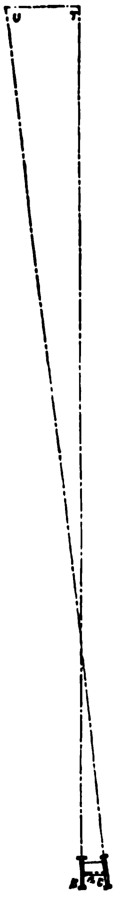


Fig. 73.



The scale, $o\,o^1$, is graduated to suit the movement of the index, 2, the spaces between each graduation bearing the same proportion to the corresponding graduations on the index, $e\,e^1$, as the rate of motion of the index, 2, bears to that of the index, 1. The lever, K, carrying the index, 3, working over the scale, $p\,p^1$, is placed on a fulcrum, *k*, and is similar to the lever, J, except that its long arm, k^2, carrying the index is so proportioned to the short arm, k^1, that any motion of the index bar, D, causes a greater movement of its index than that of the index, 2. It is operated upon by a second arm, d^1, on the index bar, D, but the parts are so proportioned that it is not touched till the index, 2, approaches near to the end, o^1, of its scale, where the divisions become very minute.

The lever, L, carrying the index, 4, may be similar to the lever, K, except that there must be a still greater difference in the lengths of its long and short arms, represented in the engravings. The effect in either case is to multiply the slight motion of the index bar in some great and unchanging ratio. As represented, it works on a fulcrum, *l*, over the scale, $q\ q^1$, near the opposite end of the base, A, and derives its motion from the lever, K, at a point near the index, 3. When the index, 3, arrives near the end, p^1, of its scale, the lever, K, comes in contact with the arm, l^1, of the lever, L, and gives an increased motion to the index, 4.

The levers, J, K, L, have severally applied to them springs, V V, to throw their indices back to the commencement of their respective scales, when the bar, D, is moved away from the edge of the base, A.

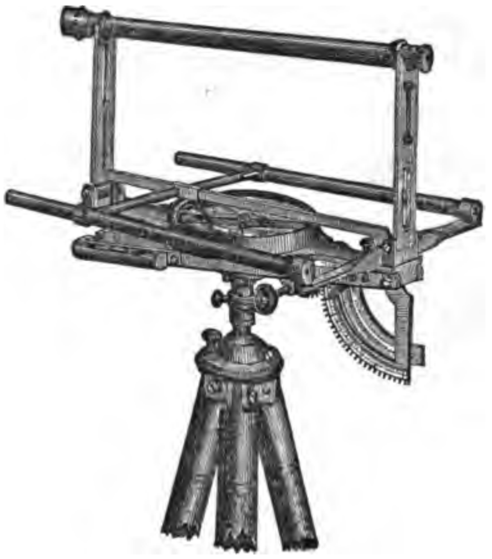
By an arrangement of multiplying levers and scales such as described, a sufficient motion of the index may be obtained to show the movement of the telescope, C, to any desired point in the line of sight of the telescope, B. The three levers, proportioned as shown in the figures, multiply the scale one hundred times. To multiply the motion to the same extent, with the indices all in operation the whole time, would compel the quickest indices to make more than a complete revolution, and would thus confound the graduations, or else would require so much room as to be almost useless in practice; but by arranging the indices to come successively into operation, each as the graduation of the scale of its predecessor becomes too small to be easily distinguishable, the multiplication is not effected till it becomes requisite, and the separate indices are caused to require only a short range.

In measuring long distances it may be advisable, in some instances, practically to increase the length of the base, *a*, by making an offset of any convenient length. In operating in this manner, the telescope, B, is first sighted at the object, then the whole instrument is transferred bodily to a new position at the extremity of the offset, and the telescope, B, is placed parallel to its first position, or in the line, W Z (fig. 72.) The telescope, C, being next brought to bear on the object, T, the distance indicated by the indices must be multiplied by the amount of the offset in feet, plus one; the product is the true distance, B T. A similar result may be obtained at one observation, by employing a horizontal rod in the hands of the staff-holder, as represented in fig. 73, where B, *a*, C, represents the actual width of the instrument, B T the distance to be measured, and U T the rod referred to. A staff being fixed on each end of this rod, and the rod being of a determinate length, nine feet, for example, and fixed by any well-known means in a position at right angles to the line, B T, the telescope, B, is sighted at T, and the telescope, C, sighted at U. Under these circumstances the distance indicated on the scales being multiplied by the length of the rod, U T, in feet, plus one, the true distance desired is obtained.

The distances indicated by this instrument are measured in a horizontal plane—that is, the instrument indicates the absolute distances to points on its own level, directly above or below the object, T. To the telescope, B, is attached a circle, M, described from the trunnions, *b b*, and graduated on a part of its circumference in degrees and minutes, and a stationary index or vernier, N, is provided, as represented, so that when both the angular altitude of an object, and its horizontal distance from the point of observation, may be thus indicated, the difference in level in feet may be readily ascertained by the aid of proper tables, which are intended to accompany each instrument.

A highly useful instrument, calculated to save much labour in underground surveying, has been introduced by Mr. Rennie, mining engineer. Fig. 74 is a perspective elevation of the instrument.

Fig. 74.



It is attached by means of a ball and socket joint to a tripod stand of the ordinary kind; it consists of a fixed or main frame, to one side of which is fitted a spirit tube, that serves to show when the instrument is level. Above the main frame is fitted a moveable frame, which is hinged to the lower frame at the back part of the instrument. To the moveable frame are attached the vertical limbs which carry the upper tube and the graduated central bar to which the lower tubes are connected. These tubes are hinged to the moveable frame, and are provided with eye-glasses and cross hair sights. The tubes are connected together by a cross bar, which slides to and fro on the graduated central bar. This cross bar is made somewhat shorter than the distance between the tubes. By this means, when the connecting bar is traversed upon the central bar towards the back of the instrument, the front ends of the tubes are caused to converge, so that they may be simultaneously brought to bear upon a distant object. In using this instrument, it is first brought to a level, which is shown by the spirit tube at the side of the main frame. The distance of the object may then be ascertained, by bringing the lower tubes to bear upon it, when the distance will be shown on the graduated central bar by the position of the slide of the connecting bar. During the taking of an observation, the main frame remains level, but the moveable frame may be elevated until the object is brought into view. The angle of elevation is shown by an index or pointer attached to the main frame, and which shows upon the quadrant attached to the moveable frame the number of degrees the tubes are elevated. By means of this arrangement, much labour may be saved in underground surveying.

Fig. 75 represents a perspective elevation of "a portable or field transit instrument, for finding the time on shore, and laying down meridian lines in extensive surveys." This instrument is the invention of Mr. Alexander Gerard, of Gordon's Hospital, Aberdeen.

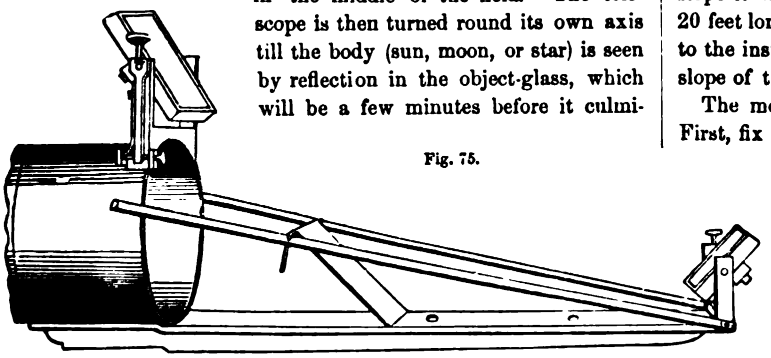


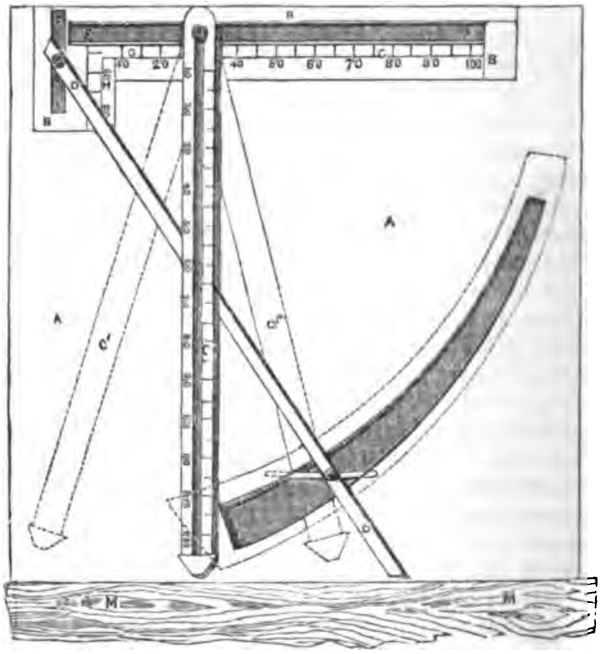
Fig. 75.

It consists mainly of two mirrors, arranged as in the sextant, but requires no graduated arc. The mirrors are so fixed as to measure an angle of nearly 90°—say 89° or 91°. Two stations are selected—the one about 100 yards nearly east or west of the other, and on the same level. A mark resembling a target, with equidistant concentric circles, is set up at one of the stations, and the instrument taken to the other, and so placed as to have the mark in the middle of the field. The telescope is then turned round its own axis till the body (sun, moon, or star) is seen by reflection in the object-glass, which will be a few minutes before it culminates. It will appear to pass the different circles on the mark in succession, and the times are to be accurately noted. The observer then takes the instrument to the second station, and an assistant sets up the mark at the first—a set of observations is then taken as before. The mean of all the times will obviously be the time of the body's transit for the middle point of the line joining the two stations, which, if the stations be correctly chosen and levelled, will be the time of the meridian passage. But as this is not at first to be expected, the stations may be shifted, or their deviation from the true position found as in the common transit instrument.

The instrument exhibited has the advantage over the common transit instrument in point of steadiness, derived from the principle of double reflection, which makes the quadrant and sextant so useful. It might be used with advantage for rating chronometers at ports where there is no observatory. For this purpose, two stands should be erected in some open space not far from the harbour, either of stone, or casks filled with sand, fitted with V's for the instrument and mark. As the time could thus be found to a greater degree of accuracy than with the sextant and artificial horizon—on account of the greater magnifying power of the telescope that might be employed—a rate might be obtained in a shorter interval.

The kremnometer is an instrument invented by Mr. A. S. Wilson, C.E., of Aberdeen; its name is compounded of two Greek words signifying slope-measurer. It is applied to the setting out of the widths of cuttings and embankments of railways and other similar works. The stock or frame, A, of the instrument, consists of a piece of mahogany or other hard wood, 9 inches square and 1¼ inch thick. B, is a thin plate of German silver, sunk flush with the polished surface of the frame. C, is a plummet of the same metal ⅝ of an inch broad and $\frac{1}{40}$ of an inch in thickness, having a projecting piece of metal at the end to insure its hanging vertically. There is a slit in the centre of the plummet from end to end, and it is graduated into feet along its whole length, (the engraving shows only every fifth foot) 16 feet to an inch is a convenient scale, and sufficiently large for practice. D, is the slope bar of the instrument, ¼ of an inch wide and $\frac{1}{50}$ of an inch thick. E, is a groove cut in the frame, in which the pivot block, L—on which the plummet is suspended—moves up and down. F, is also a groove at right angles to the other, in which the centre of the slope-bar moves. G and H, are scales of the same graduation as that on the plummet. The scale, G, must be perfectly parallel to the sole plate of the instrument; and the scale, H, exactly at right angles to the same. K, is a segmental slot cut in the frame with a shoulder on each side, which, with the cross-bar shown in the engraving, serves by a clamp-screw to keep the slope-bar at any slope to which it is set. M, is a straight edge of wood, from 10 to 20 feet long, 6 inches broad, and ½ an inch thick. It is not attached to the instrument, but serves for placing it in proper relation to the slope of the ground.

Fig. 76.



The mode of applying the instrument may be briefly described. First, fix the slope-bar at the required slope of the work to be set out; this is done by having a mark at the bottom of the instrument, perpendicularly under any point, say 30 on the scale, G. Lay the plummet over this line; then place the slope-bar at the half-width of the formation level of the cutting or embankment, shown on the scale, H. This half-width, added to the ratio of the slope to the base, will show the half-width of a horizontal cutting on the plummet scale at any depth of cutting to which the plummet is set. If, as in the engraving, the half-width of the formation level be 15 feet and the slope 1½ to 1, then a cutting of 30 feet deep of a horizontal cross-section gives a half-width of 60 feet. The scale, H, which represents the formation, is counted from a point the same distance from the edge of the groove, E, as the centre point on which the plummet swings. When the slope bar is adjusted it is fixed by the clamp-screws behind the slots, K and F. Unscrew now the clamp which holds the sliding block, L, and place the plummet on the scale, G, at the depth of the cutting or embankment where the slopes are to be set out, and there screw it tight; then place the straight edge, M, along the slope of the ground and set the instrument upon it. According to the amount of slope the plummet will hang, for example, at C C¹, and the reading on the scale where it cuts the slope-bar is the width for one side measured—not horizontally but along the straight edge or slope of the ground. Now turn the instrument half-round, and the plummet will hang as at C², and the reading on the plummet scale where it cuts the slope-bar, as seen through the slot in the plummet, will be the width for the other side. Proceed to the next point; and again setting the plummet to the depth shown in the section, lay out the widths as before.

Another instrument for effecting the same object has been designed by Mr. James Dillon, C.E., who has named it the Pleonometer. Fig. 77 represents a 12-inch pleonometer, drawn to a scale of 3 inches to a foot. The arms, A and B, turn on a centre at C; the arms, D, E, and C¹, slide along a groove in the arm, A. The divisions on the instrument are all equal, representing so many feet, the arm, D E, may be set to any inclination, as it moves on a centre at D¹.

Fig. 77.

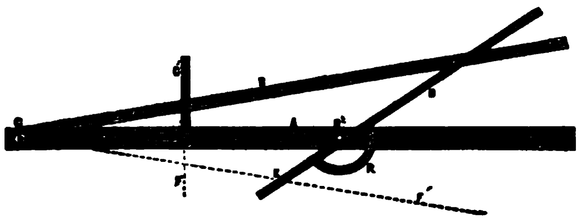
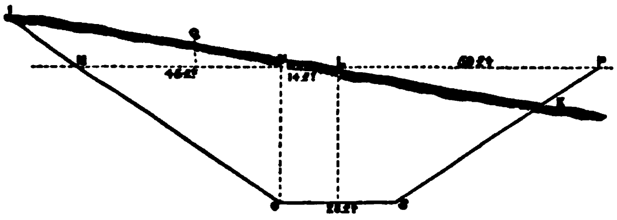


Fig. 78 represents a cross section of a railway cutting through the side of a hill. I K, is the natural surface and L the centre line of railway. The depth of the cutting is 30 feet, and the width at the bottom 28 feet, the sides having a batter of 1½ base to 1 perpendicular; that is, M N is 1½ times the depth of N O. Therefore, assuming the ground to be horizontal, as represented by the dotted line, M N P, then the half-width of the cutting at the point, L, will equal half the base (14 feet), added to the batter, M N, (45 feet) equal 59 feet:—so the horizontal distance, M L, or L P, equals 59 feet. Now at the point, Q, 30 feet from centre, L, the ground rises 5 feet ascertained with the level, the arm, C¹, of the pleonometer is moved 30 feet from the centre, C; the arm, B, is raised until its lower edge cuts 5 feet on the arm, C¹; then the angle, D C B, equals I L M, in fig. 78. The arm, D E, is the next to be moved until the centre, D¹, is 59 feet from the centre, C; then D¹ C equals M L, fig. 78. According to the scale, D E, the batter of 1½ to 1, which is marked on the semicircle at E, being the same angle of slope as I O. It will now be evident that the number of feet to be read off on the scale, B, at the point of intersection, D, expresses the distance, I L, fig. 78, because the triangle, D C B, is similar to the triangle, I L M.

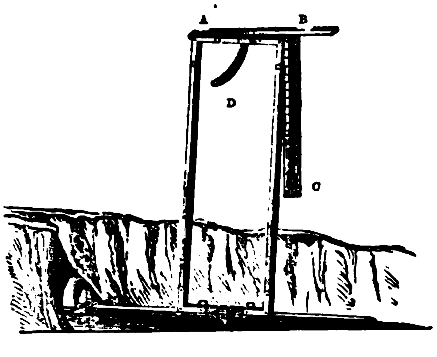
Fig. 78.



In the same manner, the distance, L K, fig. 78, is found by reversing the arms, C¹ and B, as shown by the dotted lines, F and F¹; and the number of feet read off at the point of intersection, E, expresses the distance, L K, because the triangle, C B E, is similar to L P K, fig. 78. Ranging rods are placed in the points, I and K thus found, and the same operation repeated wherever the depth of the cutting or slope of the cross section varies; a line is then lock-spitted in the direction of the points thus found, from which the slopes, I O and K S, are cut downwards;—thus giving the proper breadth at bottom.

The Inclinometer is a simple contrivance, originally devised by W. Gillespie, Esq., of Torbanehill, Linlithgowshire, for the purpose of facilitating the formation of a drain to carry off the water from the foundations of his house. The circumstances of the case demanded especial exactness and uniformity of slope, and the quantity of water to be removed was very considerable; for, on going down 2½ feet, it was found that the house was actually standing on a hydrostatic bed. This accumulation of moisture was to be discharged by a drain, sunk direct 5½ feet at the very door-step. Commencing at such a depth, it was, of course, essential to guide the slope with accuracy, so as to preserve the outfall at the other extremity; and it was evident than any misdirection might endanger the house by causing the unpleasant result of backwater. During the progress of the work, Mr. Gillespie, being dissatisfied with its appearance, conceived the idea of the apparatus which is here shown, and the working model of impromptu construction at once set the matter right. The instrument, which is now in established use, is nothing more than a parallelogram of timber and a plummet in combination. Fig. 79 shows it as pointing out the slope of a line of drain pipes. From the nature of the parallelogram, it is obvious that the top must be parallel with the base; and to show the deviation of the upper of these coinciding slopes from the level, the instrument is provided with the means of determining what the true level is. It has a duplicate top, A B, hinged to the angle, A. The other extremity of this duplicate top being a little protracted, is formed into the well-known T square by insertion through a slit (in which a slight range is given to accommodate the working of the implement) of a depending limb, C, at right angles to A B. C, is graduated downwards for several inches in sixteenths of an inch. The face of the depending limb is likewise grooved for the reception of a plummet or pendulum of wire playing upon its graduated front. A quadrant, D, moved by turning a ratchet-pin, is employed to elevate or depress the duplicate top spar, A B, until the plummet rests from its oscillations, in exact accordance with a vertical line drawn down the face of the T square. This shows the top spar, A B, to have been adjusted to the proper level. On the other side of the implements, behind the ratchet-pin, will be found an inverted pinch or pressing screw, by turning which backwards, the implement is set, and the square top fixed on the horizontal or true level of the parallelogram. The limb, A, being now upon the level, whilst the limb, B, still con-

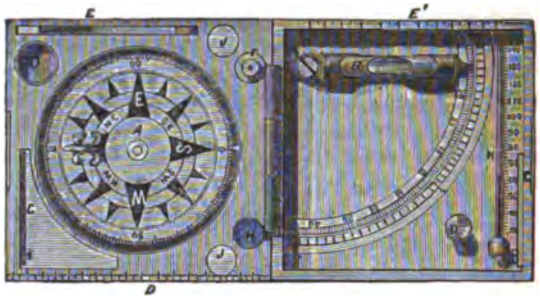
Fig. 79.



tinues to indicate the slope, the difference intervening betwixt the level and the slope is necessarily denoted on the graduated scale, which, being fixed upon the inner edge of the plummet style, measures the exact rate of slope to which the instrument is applied. A sight telescope, for extending the range of the level, may be fitted on the upper limb, A B. By means of it, the outfall or depth of slope can be determined throughout any distance within the scope of vision, and the heights of objects may be measured where their distances can be ascertained. An extra base piece is provided for protracting the slope, and giving the rate of it with greater certainty of precision.

A very ingenious instrument, the contrivance of the late Mr. Herbert Mackworth, and made for the inventor by Mr. J. D. King, of Bristol, is represented in the accompanying figures. The instrument is named the Metra, and within its very ilmited space it offers to the engineer and traveller, the means of taking most of the measurements it is desirable to record. Without sacrificing the strength necessary for constant use, or interfering with the accuracy of each portion, this instrument contains, in a brass case three inches square, and litttle more than one inch thick, eleven different instruments. The traveller in distant lands can ascertain by its means the temperature, the force of the wind, the latitude, the position of the rocks, or survey and map his route. The geologist can determine and draw the direction and amount of dip of the rocks, the angles of cleavage and crystallisation, the temperature of springs, or examine by a plate of tourmaline, the bottoms of pools or shallow depths along coast lines, otherwise invisible to the eye. The miner, for whom chiefly this little instrument is intended, can, moreover, survey and level the roof or floor of his workings, and requires only a pencil to map them upon paper. It contains also a magnifying lens, and a measure of the wires in a Davy lamp. The miner can ascertain the temperature of the air under ground, and discover whether the ventilation is deficient, or if the foul air from abandoned workings leaks into the life-supporting current. By the anemometer, a thin plate of mica suspended so as to be frictionless, he may measure the quantity of air brought in, and determine whether it is proportioned to the wants of the mine. Beneath the transparent mica, a table of constants exhibits the weight of gases, liquids, and solids, besides some thirty measures and formulæ for steam, boilers, engines, ropes, and air. To exhibit the ingenuity displayed in this pocket instrument, it is sufficient to mention that it contains a Clinometer, Goniometer, Level, double Compass, plotting Scales of various kinds, and an Anemometer. The engravings, Figs. 80 and 81, represent the plan and side view of the metra when open and ready for use. A, is the double compass, and B, the level. The arc is graduated in degrees and in inches fall per yard. C, the sights, D, the scales, E, the goniometer arm, and E¹ the goniometer scale; F, the plummet, G, the lens, with a telescopic slide to measure the wire gauze; H, the tourmaline, J, the pivots on which the instrument stands; K, are the two joints of the brass leg, by which the horizontality of the instrument can be obtained; L, is a flat chisel point for entering joints of rock or masonry. This end unscrews, exposing a wood screw, M, by which the leg can be secured to a tree or a stand. N, is the thermometer, O, the screw which secures the top and bottom of the instrument together. Beneath the bottom, P, are placed the anemometer and the table of constants. The adjustments of the metra possess several novelties, with the view of shortening the operator's work. On this head we may allude to the plan of laying down the surveys on paper, without the aid of anything but a pencil, by first adjusting the north and south line of the plan by the compass, and fastening the paper down by weights. The compass then serves as a protractor, and by the engraved scales the distances are measured off. This method saves calculation and ruling parallel lines, and obviates some instrumental errors. Any dial or theodolite can be fixed to a scale, and used in a similar way as a protractor.

Fig. 80.



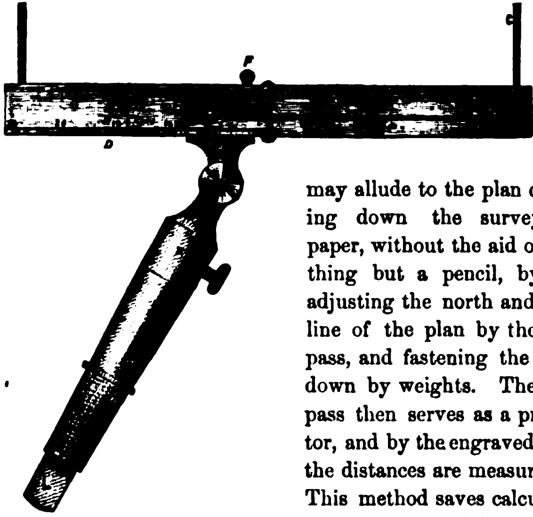


Fig. 81.

With this instrument we shall conclude this portion of our subject. The student who desires further information relative to surveying instruments, will find it appended to several of the treatises which are exclusively devoted to that branch of mathematical science.

We have a word to say to the student who purposes following us through our course of instruction, and that is let him make himself acquainted with the use of the Slide Rule. In the examples we shall place before him, as well as in others which will occur to him in his studies, this instrument will be of the utmost service in facilitating and verifying the results of his calculations; and where arithmetical operations are required at the moment the rule will be found invaluable in reducing the labour of the mind and pen. It may be used with advantage in working out the elementary and higher rules of arithmetic; mensuration, superficial and solid; land measuring, levelling, hydrostatics, velocity of falling bodies, weights of metal, pumping engines, mill work, mechanical powers, calculating brickwork, gauging, and many other useful purposes, which are detailed in the small explanatory work which is usually with the instrument.

INSTRUMENTS FOR THE CONSTRUCTION

Fig. 1.

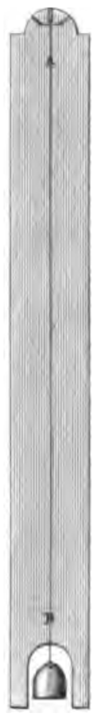


Fig. 2.

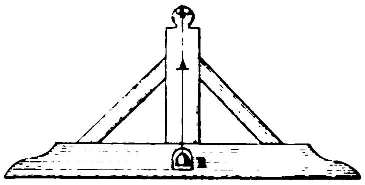


Fig. 3.

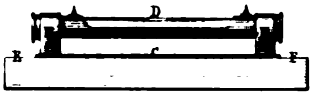


Fig. 4.

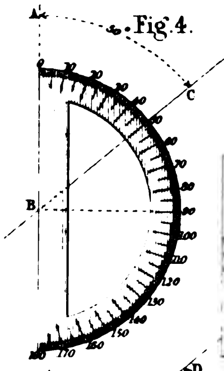


Fig. 5.

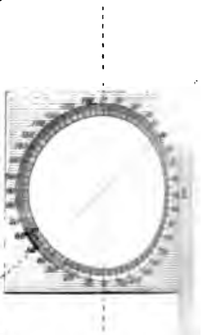


Fig. 7.

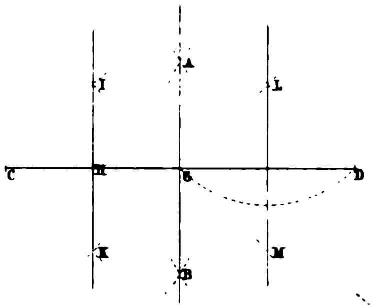


Fig. 8.

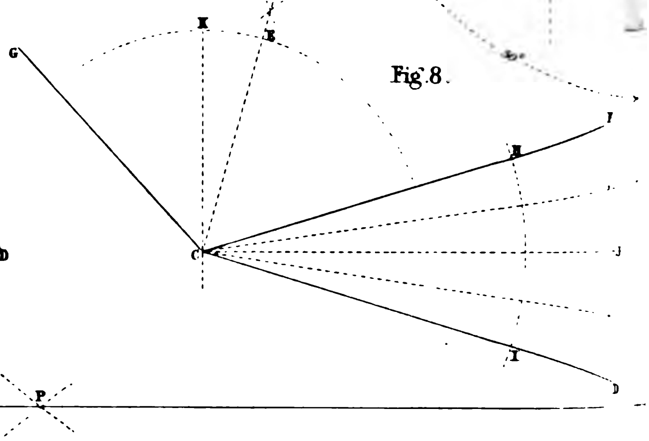


Fig. 11.

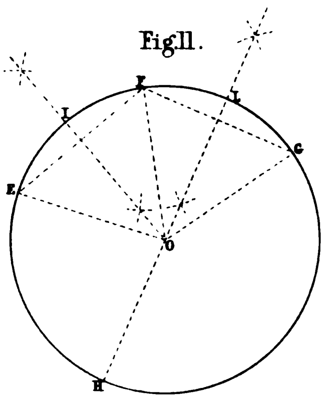


Fig. 12.

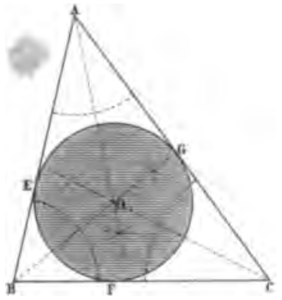


Fig. 1

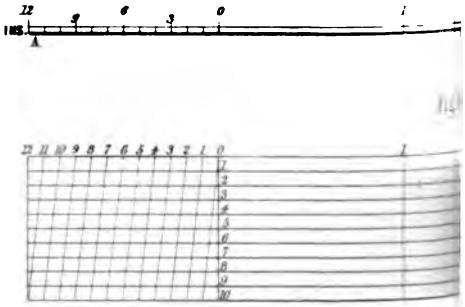


Fig. 18.

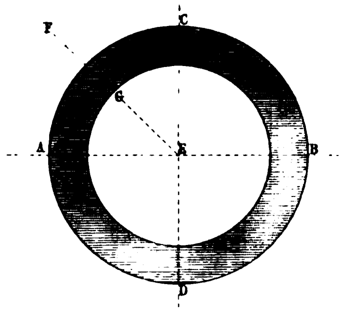


Fig. 19.

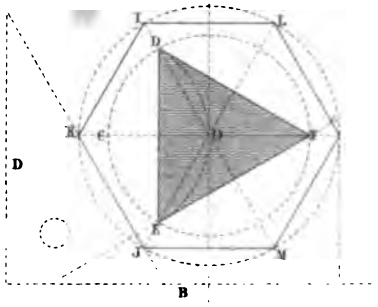


Fig. 20.

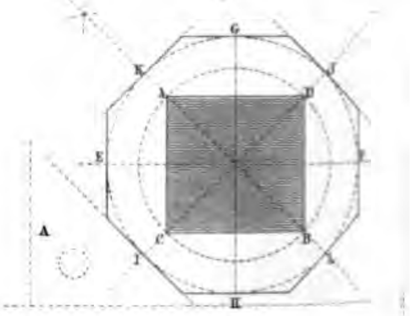


PLATE .1.

F SURFACES & LINEAL FIGURES.

Fig. 6.

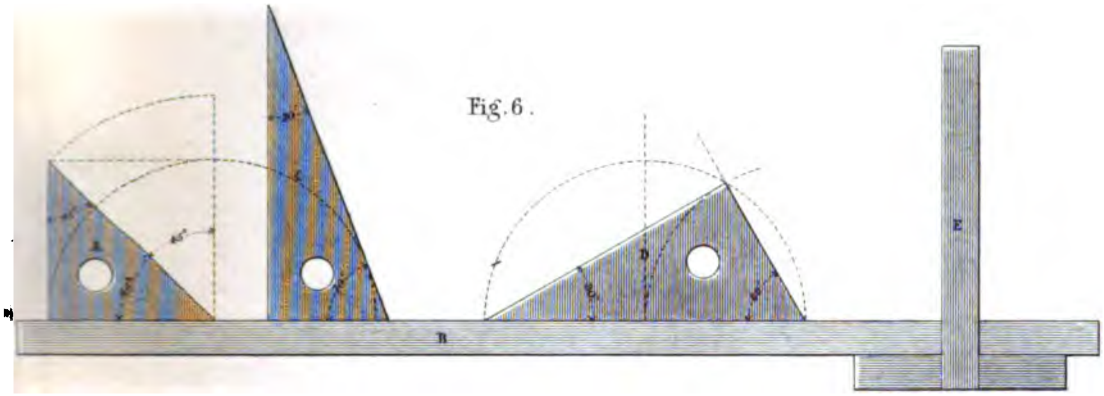


Fig. 15.

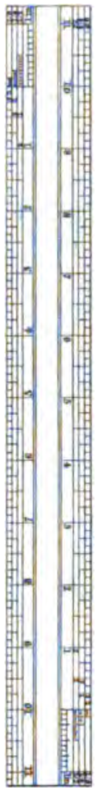


Fig. 9.

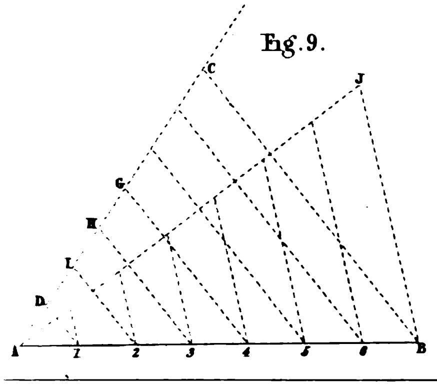
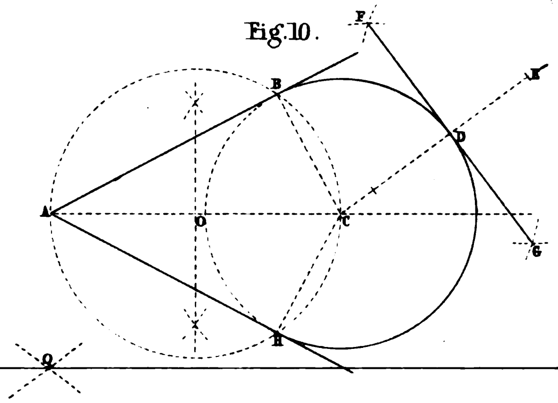


Fig. 10.



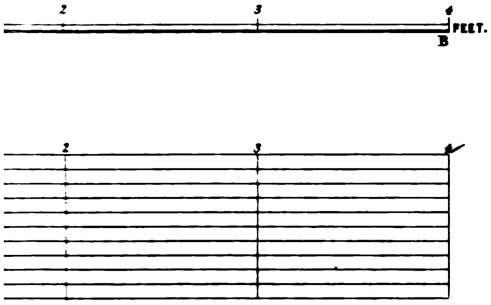


Fig. 16.

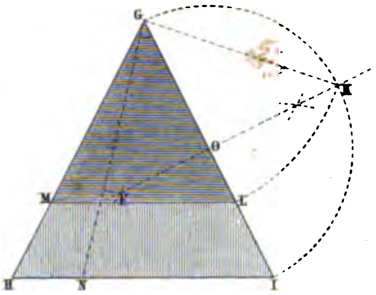


Fig. 17.

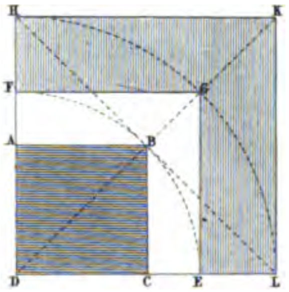


Fig. 21.

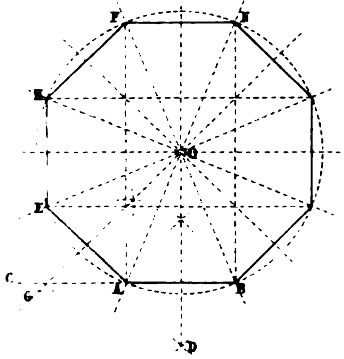


Fig. 22.

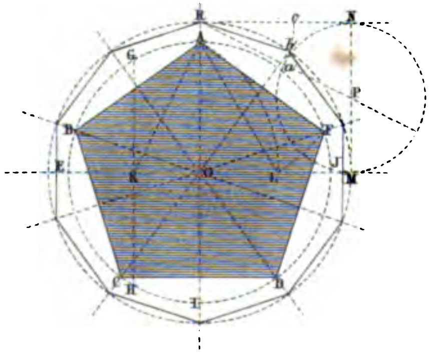
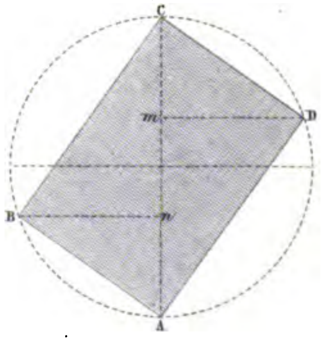


Fig. 23.



Mackay & Kirkwood Glasgow

CHAPTER III.

LINEAR DRAWING.

Drawing is the art of representing the forms of objects upon a flat surface, such as a sheet of paper.

The principles and methods concerned in that division of the art which is termed *linear drawing*, and which is the foundation of all drawing, whether industrial or artistic, are, for the most part, derived from elementary geometry.[*] This branch of drawing has for its object the accurate delineation of surfaces and the construction of figures, by the appropriate combination of lines; and, with a view to render the study easier, and at the same time more attractive and intelligible to the student, the present work has been arranged to treat successively of definitions, principles, and of problems or practical applications.

DEFINITIONS.

LINES AND SURFACES.

PLATE 1.

In Geometry, *space* is described in the terms of its three dimensions—length, breadth or thickness, and height or depth.

The combination of two of these dimensions represents *surface* and one dimension takes the form of a *line*.

Lines.—There are several kinds of lines used in drawing—*straight* or *right lines*, *curved lines*, and *irregular* or *broken lines.*

Right lines are *vertical*, *horizontal*, or *inclined.* Curved lines are *circular*, *elliptic*, *parabolic*, *&c.*

Surfaces.—Surfaces, which must always be bounded by lines, are *plane*, *concave*, or *convex.* A surface is plane when a straight-edge is in contact with it at every point, in whatever position it is applied to it. If the surface is hollow so that the straight-edge only touches it at each extremity, it is concave; and if it swells out so that the straight-edge only touches it in one point, it is convex.

Vertical lines.—By a vertical line is meant a line parallel to the direction assumed by a flexible thread suspended at one extremity, and having a weight attached to the other; such is the line A B, represented in Fig. 1. This line is always a straight line, and straight lines are the shortest that can be drawn between their extreme points. A vertical line is perpendicular to all lines drawn in the plane of, or parallel to, the horizon at any given place.

Plumb-line.—The instrument represented at Fig. 1 is called a plumb-line. It consists of a piece of wood, having parallel sides, and a line drawn up its centre parallel with its edges; to the one end of this piece of wood is attached a cord corresponding with the said line, the other end being allowed to hang freely with a weight or plumb-bob attached to it. This instrument, or the surface to which its edge is applied, is said to be plumb when the whole length of the line coincides with the line drawn on the piece of wood. This instrument is much employed in building and the erection of machinery, as a guide to the construction of vertical lines and surfaces.

Horizontal line.—When a liquid is at rest in an open vessel, its upper surface forms a horizontal plane, and all lines drawn upon such a plane, or any plane parallel thereto, are called horizontal lines. A line is horizontal when it is at right angles to a vertical one at any given place.

Levels.—The simplest form of this instrument is that represented at Fig. 2. It is constructed in accordance with the principle of the last definition of a horizontal line, and consists simply of a plumb-line, A, as before described, attached to a piece of wood, B, having its lower edge at right angles to it, so that when the plumb-line, A, is vertical, the lower edge of B is horizontal, or the surface on which it rests is level. This instrument is much used by masons, carpenters, joiners, and other artificers.

Liquid or fluid levels are constructed on the principles involved in the first definition of a horizontal line. One description of fluid level consists of two upright glass tubes, connected by a pipe communicating with the bottom of each. When the instrument is partly filled with water, the water will stand at the same height in both tubes, and thereby indicate the true level. Another form, and one more generally used, denominated a spirit level from spirit being usually employed, consists of a glass tube, Fig. 3, enclosed in a metal case, A, attached by two supports, B, to a plate, C. The tube is almost filled with a liquid, and the bubble of air, D, which remains, is always exactly in the centre of the tube when any surface, E F, on which the instrument is placed, is perfectly horizontal.

Perpendiculars.—If the vertical line, A B, Fig. 7, be placed on the horizontal line, C D, the two lines will be perpendicular to, and form right angles with, each other. If now we suppose these lines to be turned round on the point of intersection as a centre, always preserving the same relative position, they will in every position be perpendicular to, and at right angles with, each other. Thus the line, I O, Fig. 11, is at right angles to the line, E F, although neither of them is horizontal or vertical.

Broken lines.—It is usual to call a line a "broken line," which consist of a series of right lines lying in different directions—such as the lines B, A, E, H, F, N, Fig. 21.

The Circle and its Circumference.—The continuous line, E F G H, Fig. 11, drawn on a plane surface with one of the points of a pair of compasses, whilst the other point remains fixed, is called the circumference, and the space enclosed thereby is a circle. It is evident that every point of the circumference is equally distant from the fixed *centre*, O. Circles are concentric when they have a common centre, Fig. 18.

Radius.—The extent of opening of the compasses, or the distance between the two points, O, F, Fig. 11, is called a radius. All lines, as O E, O F, O G, drawn from the centre to the circumference are equal.

Diameter.—Any right line, L H, passing through the centre, O, and limited each way by the circumference, is a *diameter.* The diameter is therefore double the length of the radius. A circle is divided by any diameter into two semi-circles.

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[*] For those to whom geometry is an entirely new study no better commencing book can be recommended than Mr. Grant's "Elements of Practical Geometry, for schools and workshops." It is eminently plain and sound in its teaching. Another excellent elementary work, but more formally scientific, is Mr. Lund's "Elements of Geometry and Mensuration, with easy exercises, designed for schools and adult classes."

Arc.—Any part of the circumference, E I F, or F L G, is called an *arc.*

Chords.—Right lines, E F, F G, connecting the extremities of arcs, are *chords;* such lines extended beyond the circumference become *secants.*

Tangent.—A right line, A B, Fig. 10, which touches the circumference in a single point, however much it may be prolonged, is a *tangent.* Tangents are always at right angles to the radius which meets them at the point of contact, B.

Sector.—Any portion of a circle, as B O H C, Fig. 10, comprised within two radii and the arc which connects their outer extremities, is called a *sector.*

Segment.—A segment is any portion, as E F I, Fig. 11, of a circle, comprised within an arc and the chord which subtends it.

Right, continuous, and broken lines, are drawn by the aid of the square and angle; circular lines are delineated with compasses.

Angles.—It has been already stated that, when right lines are perpendicular to each other, right angles are formed by their intersections. When such lines meet or cross each other without being perpendicular, they form *acute* or *obtuse* angles. An acute angle is one which is less than a right angle, as F C D, Fig. 8; and an *obtuse* angle is greater than a right angle, as G C D. By *angle* is generally understood the extent of opening of two intersecting lines, the point of intersection being called the *apex,* and the lines, *legs.* An angle is *rectilinear* when formed by two right lines, *mixtilinear* when formed by a right and a curved line, and *curvilinear* when formed by two curved lines.

Measurement of Angles.—It is customary to divide the circumference of a circle into 360 equal parts called degrees. Now, if, with the apex of an angle as the centre, we describe an arc extending from leg to leg, the size of the angle may be expressed by the number of degrees contained in such arc, and this is the usual way of measuring angles and arcs. Instruments called protractors, represented in Figs. 4 and 5, have been constructed, whereby the number of degrees contained in any angle may be readily ascertained. The first, Fig. 4, which is to be found in almost every set of mathematical instruments, being that most in use, consists of a semicircle divided into 180 parts. In making use of this instrument, its centre, B, must be placed on the apex of the angle in such a manner that its diameter coincides with one side, A B, of the angle, A B C, when the measure of the angle will be indicated by the division intersected by the other side, A C, of the angle. Thus the angle, A B C, is one of 50 degrees—abbreviated, or usually marked thus, 50°; and this will always be the measure of this particular angle or arc, whatever be the length of radius of the arc, or legs of the angle, since the measuring arc must always be the same fraction of the entire circumference. The degree is divided into 60 minutes, and the minute (or 1') into 60 seconds (or 60"). Thus 47° 15' 30", means an angle of forty-seven degrees, fifteen minutes, and thirty seconds.

The other protractor, Fig. 5, of modern invention, possesses the advantage of not requiring access to the apex of the angle. It consists of a complete circle, each half being divided on the inner side into 180 degrees, but externally the instrument is square. It is placed against a rule, R, made to coincide with one side, C E, of the angle, E C D, the other side, D C, crosses two opposite divisions on the circle indicating the number of degrees contained in the angle. It will be seen that the angle, D C E, is one of 50°.

Amongst the numerous uses of the protractor are those of drawing any required angle, dividing an angle into any number of equal parts, and cutting off any number of degrees from an angle or arc. Several most useful protractors have already been described in the second chapter on instruments.

Oblique lines.— Right lines, which do not form right angles with those they intersect, are said to be oblique, or inclined to each other. The right lines, G C and F C, Fig. 8, are oblique, considered with reference to the vertical line, K C, or the horizontal line, C J.

Parallel lines.—Two right angles are said to be parallel with each other when they have the property of never approaching to or receding from each other, however much they may be prolonged; thus, the lines, I K, A B, and L M, Fig. 7, are parallel. A right line, drawn across a series of parallel lines, will strike every one at the same angle; thus, a line which is perpendicular to the first of a series of parallel lines, will, if prolonged, be perpendicular to each one of the rest.

Triangles.—The space enclosed by three intersecting lines, forming a trilateral figure, is called a triangle; when the three sides, as D E, E F, and F D, Fig. 19, are equal, the triangle is *equilateral;* if two sides only, as G H, and G I, Fig. 16, are equal, it is *isosceles;* and it is *scalene,* or irregular, when the three sides are unequal, as in Fig. 12. The triangle is called *rectangular* when any two of its sides, as D L and L K, Fig. 17, form a right angle; and in this case the side, as D K, opposite to, or subtending the right angle, is called the *hypothenuse.* An instrument constantly used in drawing is the set-square, more commonly called an *angle;* it is in the shape of a rectangular triangle, and is constructed of various proportions. Three of the most common are shown at Fig. 6. A, is one having two angles of 45°, represented as having one of the sides containing the right angle laid against a straight edge, B; C, is one containing angles of 70°, and 20°; whilst D contains angles of 60°, and 30°. From these numbers, it will be seen that these angles, with the aid of the straight-edge, B, or T square, E, divide the circle into a great variety of equal parts, and, consequently, are of great use to the draughtsman in the construction of regular polygons and other figures. The angle of 45° marked A, with the aid of the T square, divides the circle into four or eight parts; and the angle of 60° and 30°, marked D, divides the circle into six or twelve parts, and, with the aid of the 45°, will divide it into twenty-four parts; while the one marked C will divide the circle into eighteen parts.

Square.—A square is a four sided or *quadrilateral* figure, the sides of which, as A B, B C, C D, and D A, Fig. 17, are equal, and each pair of adjacent sides perpendicular to one another, the angles consequently being equal, and all right angles.

A *rectangle* is a quadrilateral, having two sides equal, as A B and F N, Fig. 21, and perpendicular to two other equal and parallel sides, as A F and B N.

A *parallelogram* is a quadrilateral figure, as A B C D, Fig. 23, of which the opposite sides and angles are equal; and a *lozenge* is a quadrilateral figure having all the sides equal, but only the opposite angles equal, as in the shaded portions, Fig. 17, Plate 2.

A *trapezium* is a quadrilateral figure, of which only two sides, as H I and M L, Fig. 16, are parallel.

Polygon.—A space enclosed by any number of right lines more than four, lying at any angle to each other, and forming a *multilateral* figure, is called a *polygon;* and its outline, the *perimeter.* Polygons are *regular* when all their sides and angles are equal, otherwise they are *irregular.* All regular polygons are capable of being inscribed in a circle, so that the apex of every angle shall touch the circumference. Hence the great facility with which they may be accurately delineated.

OBSERVATIONS.

We propose now to proceed to the solution of those elementary problems with which, from their frequent occurrence in practice, it is important that the student should be well acquainted. The paper being well stretched on the board, as before described, and perfectly dry and clean, the drawing is then first made in faint pencil lines, and afterwards put in with ink, by means of the drawing pen, and the other instruments already described. To distinguish those lines which may be termed working or pencil lines, as being but guides to the formation of the actual outlines of the drawing, we have, in the plates, represented the former by dotted lines, and the latter by full continuous lines.

PROBLEMS.

1. *To erect a perpendicular on the centre of a given right line, as* C D, Fig. 7.—From the extreme points, C D, as centres, and with a radius greater than half the line, describe the arcs which cross each other in A and B, on either side of the line to be divided. A line, A B, joining these points, will be a perpendicular bisecting the line, C D, in G. Proceeding in the same manner with each half of the line, C G and G D, we obtain the perpendiculars, I K and L M, dividing the line into four equal parts, and we can thus divide any given right line into 2, 4, 8, 16, &c., equal parts. This problem is of constant application in drawing. For instance, in order to obtain the principal lines, 1, 2, and 3, 4, which divide the sheet of paper into four equal parts; with the points, R, S, T, U, taken as near the edge of the paper as possible, as centres, we describe the arcs which intersect each other in P and Q; and with these last as centres, describe also the arcs which cut each other in V and W. The right lines, 1, 2, and 3, 4, drawn through the points, P, Q, and V, W, respectively, are perpendicular to each other, and serve as guides in drawing on different parts of the paper, and are merely pencilled in to be afterwards effaced.

2. *To erect a perpendicular on any given point, as* H, *in the line* C D, Fig. 7.—Mark off on the line, on each side of the point, two equal distances, as C H and H G, and with the centres C and G describe the arcs crossing at I or K, and the line drawn through the intersecting points, and through the point, H, will be the line required.

3. *To let fall a perpendicular from a point, as* L, *apart from the right line,* C D.—With the point L, as a centre, describe an arc which cuts the line C D, in G and D, and with these points as centres, describe two other arcs cutting each other in M, and the right line joining the points, L and M, will be the perpendicular required. In practice, such perpendiculars are generally drawn by means of an angle and a *square*, or T square, E, Fig. 6.

4. *To draw parallels to any given right lines, as* 1, 2, *and* 3, 4, *at a given perpendicular distance from such lines.*—To insure accuracy in our drawings, it is usual to begin by constructing a marginal rectangle, such as A B, C D, on the paper that is being drawn upon, which is thus done:—From the points 1 and 2, as centres, describe arcs at A, B, C, and D, with the given distance as a radius; and applying the rule tangentially to the two first, drawing the line A B, and then in the same manner tangent to the two latter, draw the line C D; these lines, A B and C D, will be parallel to one of the given right lines, viz., 1, 2, and at the given perpendicular distance from it. The lines, A C, and B D, may be drawn parallel to, and at the given distance apart from, the line 3, 4, in a similar manner. In general, however, such parallels are more quickly drawn by means of the T square, the cross piece or stock being slid along the edge of the board, and the longer arm or blade being placed on the paper. Short parallel lines may be drawn with the angle and rule, or with any of the angles, A, C, or D, set on the edge of the square.

5. *To divide a given right line, as* A B, Fig. 9, *into several equal parts.*—We have already shown how a line may be divided into 2 or 4 equal parts. We shall now give a simple method for dividing a line into *any* number of equal parts. From the point, A, draw the line, A C, making any convenient angle with A B; mark off with the compasses, on A C, as many equal distances as it is wished to divide the line A B into; in the present instance, seven. Join C B, and from the several points marked off on A C, draw parallels to C B, using the rule and angle for this purpose. The line A B will be divided into seven equal parts by the intersections of the parallel lines just drawn. Any line making any angle with A B, as A J, may be employed instead of A C, with exactly the same results. This is a very useful problem, especially applicable to the formation of scales for the reduction of drawings.

6. A *scale* is either a straight line, divided and subdivided into feet, inches, and parts of inches, for the purpose of directly measuring actual distances; or it is a straight line, divided and subdivided into parts representing feet, inches, and parts of inches, but greater or smaller in a constant proportion.

7. *To construct a plain scale, as shown at* Fig. 13.—Suppose we wish to draw an object one-twelfth of the size of the original, we begin by drawing the straight line, A B, and from a point, as O, or zero, we set off as many divisions to the right as we choose; each equal to one-twelfth of a foot, or one inch in length, as 1, 2, 3, 4, which will represent one, two, three, or four feet from the point, O; then set off one division to the left equal to one of the divisions to the right, and subdivide this one into twelve; then each of these divisions, to the left of O, will represent inches. Each of these again may be subdivided into eight or ten equal parts, which will give the eighths or tenths of inches. In representing on paper any of the dimensions of a real object, we first ascertain the actual size required with the foot rule in the usual way, and then take from the scale the length corresponding thereto. For example, suppose we want to draw a line to represent 3 feet 9 inches to an inch scale, which is the one shown and just described at Fig. 13, we take, with the compasses, three of the divisions to the right and nine to the left of the point, O, or zero, at one span. This will be the size required to make the line. Scales of this class are made on strips of card board with the divisions near the edge, so that they can be laid to the line required, and the lengths marked off with the pencil.

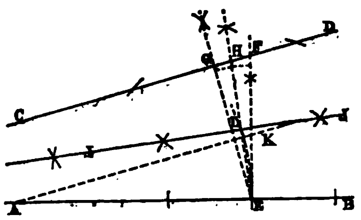
The diagonal scale.—This scale, which is shown at Fig. 14, is very simple, and most useful where great precision and many small measurements are required, as it gives the third measurements, or the decimal parts of the inch, with greater ease and accuracy than the simple scale, just described at Fig. 13. To construct the diagonal scale, we draw eleven long parallel lines at equal distances apart; then, suppose it is an inch scale that is required, we proceed as before, and set off on the top line from the point, *o*, as many inches, 1, 2, 3, and 4, to the right, as required, to represent feet, and one inch to the left, which is again subdivided into twelve equal parts, marked 1, 2, 3, to 12, to represent inches; through the points, *o*, 1, 2, 3, and 4, to the right, and the point, 12, to the extreme left, draw perpendiculars through the eleven parallel lines, and at the intersections of the perpendicular lines, from the point, *o*, with these lines mark them off in succession, respectively, 1, 2, 3, 4, 5, and so on to 10, which is the intersection on the last line of the whole; then, on this line, and to the left of the point, 10, mark off one space equal to one of the small divisions to the left of *o*, on the top line, join the point, *o*, with this division on the lower line by a right line, and parallel to it draw lines through the several points, 1, 2, on to the 11th point, to the left of *o*, which will meet and coincide with the extreme point to the left of the lowest line. The intersection of these oblique lines with the long parallel lines, marked 1 to 10, will give the tenths of an inch, thus: suppose we wish to take a measure of three feet, nine inches, and eight-tenths of an inch, we set the point of one leg of the compasses on the point of intersection formed by the third vertical line to the right of *o*, for the feet, with the horizontal line marked 8, for the tenths, then open the other leg till the point reaches the intersection of the oblique line, marked 9, for the inches to the left of *o*, with the same horizontal line, and this will be the measurement required. When there is no tenths of inches required, the measurement is taken on the top line, as in the simple scale. This scale is frequently used as a decimal scale entirely, to measure to the hundredth, or thousandth part of the integral part, or measure.

Fig. 15 represents a useful scale, drawn to one-third of the real size. It is formed of a thin strip of ivory or boxwood, about 1¼ inch broad, and 12 inches long, thinned towards the edges, so as to allow the required distances to be directly measured off upon the paper from the edge of the scale. There are four scales on each side towards each edge on four parallel lines, the one set being inch, half-inch, one-fourth, and one-eighth of an inch, each being one-half of the other, so that every second division is squared back to the line immediately behind it, and this forms a scale double the size of the one immediately in front of it; thus giving four scales as simply and easily worked from as one, since the integral parts of the smaller scales form the fractional parts of the larger ones. On the opposite side and edge, corresponding to that of the former, there is another set of four scales, viz., one-and-a-half inch, three-fourths, three-eighths, and three-sixteenths of an inch, each; these two sets are again subdivided into twelfths, in the usual way, to represent inches. On the other two sides, and corresponding edges respectively, there are two other sets of scales, similar to the others, except that they are divided into tenths of feet, instead of twelfths, as the former.

8. *To divide a given angle, as* F C D, Fig. 8, *into two equal angles.*—With the apex, C, as a centre, describe an arc, H I, and with the two points of intersection, H I, as centres, describe arcs cutting each other in J; join J C, and the right line, J C, will divide the angle, F C D, into two equal angles, H C J and J C I. These may be subdivided in the same manner, as shown by dotted lines in the figure. An angle may also be divided by means of either of the protractors, Figs. 4, 5.

To bisect an angle formed by two lines, the apex of which is inaccessible. Let A B and C D, Fig. 8A, be the two lines forming an acute angle with each other, so that the apex is inaccessible or cut away from any point, E, in the line, A B; erect the perpendicular, E F, and from the same point without the line, C D, let fall a perpendicular, E G; bisect the angle, F E G, formed by these two perpendiculars, the apex, E, of which is accessible, by the first part of this problem, by the line, E H; then bisect E H by the perpendicular, I J, produced both ways, and it will bisect the given angle formed by the two lines, A B, and C D. That is, the angle formed by B A, I J, is equal to that formed by D C, I J, each half of the given angle formed by B A, C D. Otherwise, through any point, A, in A B, draw the line, A K, parallel to C D, then, with A as a centre, and with A E as a radius, describe the arc, E L, cutting A K in the point, L; join E L, and produce it till it cuts the other given line, C D, in H, then, as before, bisect H E by the perpendicular, I J, and it bisects the given angle.

Fig. 8A.



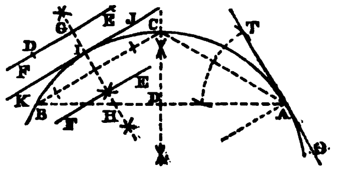
If a given angle is required to be divided into an odd number of equal parts, the only practicable method is to describe an arc from the apex, as a centre, and to divide the part lying between the legs, or its chord, with the compasses, into the number of parts required.

9. *To draw a tangent to a given circle,* O B D H, Fig. 10.—If it is required to draw the tangent through a given point, as D, in the circumference of the circle, a radius, C D, must be drawn meeting the point, and be produced beyond it, say to E. Then, by the method already given, draw a line, F G, perpendicular to C E, cutting it in D, and it will be the tangent required. If, however, it is required to draw the tangent through a given point, as A, outside the circle, a straight line must be drawn, joining the point, A, and the centre, C, of the circle. After bisecting this line in the point, O, with this point as a centre, describe a circle passing through A and C, and cutting the given circle in B and H; right lines joining A B and A H will both be tangents to the given circle, and the radii, C B, and C H will be perpendiculars to A B and A H respectively.

Observe that a right angle is always formed by a rectilineal tangent, and the radius drawn from the point of contact. Hence, if there be two or more tangents to the same circle, straight lines forming right angles with them will meet at the centre of the circle.

If it is required to draw a tangent to a given point in the arc of a circle, the centre of which is inaccessible, we proceed thus: draw a chord from the given point, A, Fig. 10A, to any other point, B, in the arc, and bisect the chord, A B, by the perpendicular, D C. Draw the chord, C A, and through the point, A, draw the line, T A O, making the angle, C A T, equal to the angle, C A D; T A O, will be the tangent required. To draw a tangent to a given circle or arc, parallel to a given line either within or without the given circle, whether the centre is accessible or not, let A C B, Fig. 10A, be the given arc, and E F, the given line, within or without the arc, A C B; draw parallel to the line, E F, problem 4, any chord, C B, cutting the arc, A C B, in C and B, bisect C B by the perpendicular, G H; and through the point of intersection, I, of the perpendicular, G H, with the arc, A C B, draw the line, I J, perpendicular to G H, or parallel to B C, and produce it in the direction of K, then J K is the tangent required, touching the arc, A C B, and parallel to the given line, E F, within or without the circle.

Fig. 10A.



10. *To find the centre of a given circle, or that with which a given arc, as* E F G, Fig. 11, *is drawn.*—Take any three points, E F G, in the arc, and draw the lines, E F, F G. From the centre of each chord draw the perpendicular lines, I O and L O; O, the point of intersection of these two lines, is the required centre.

11. *To describe a circle through any three points not in a right line.*—Since only one circle can pass through the same three points, and since any circle may be described when the centre is found and a point in the circumference given, this problem is solved in exactly the same manner as the preceding.

12. *To inscribe a circle in a given triangle, as* A B C, Fig. 12.—A circle is said to be *inscribed* in a figure, when all the sides of the latter are tangents to it. Bisect any two of the angles by right lines, as A O, B O, or C O; and from the point of intersection, O, let fall perpendiculars to the sides, as O E, O F, and O G. These perpendiculars will be equal, and radii of the required circle, O being the centre.

13. *To divide a triangle, as* G H I, Fig. 16, *into two equal parts.*—If the parts are not required to be similar, bisect one side, as G I, in the point, O, with which, as a centre, describe the semicircle, G K I, of which G I, is the diameter. This semicircle will be cut in the point, K, by the perpendicular, K O; mark off on G I, a distance, G L, equal to G K, and draw the line, L M, parallel to H I. The triangle, G L M, and the trapezium, H I L M, will be equal to each other, and each equal to half the triangle, G H I. If the given triangle were G N I, it would also be divided into two equal parts by the line, L M.

14. *To draw a square double the size of a given square,* A B C D, Fig. 17.—After producing from different corners any two sides which are at right angles to each other, as D A and D C, to H and L, with the centre, D, and radius, D B, describe the *quadrant* or quarter of a circle, F B E, and through the points of intersection, F and E, with the lines, D A and D L; draw parallels to D L and D A respectively, or tangents to the quadrant, F B E; the square, F G E D, will be double the area of the given square, A B C D; and in the same manner a square, H K L D, may be drawn double the area of the square, F G E D. It is evident that the diagonal of one square is equal to one side of a square twice the size.

15. *To describe a circle half the size of a given circle, as* A C B D, Fig. 18.—Draw two diameters, A B and C D, at right angles to each other; join an extremity of each by the chord, A C. Bisect this chord by the perpendicular, E F. The radius of the required circle will be equal to E G. It follows that the annular space shaded in the figure is equal to the smaller circle within it.

16. *To inscribe in given circles, as in* Fig. 19, *an equilateral triangle and a regular hexagon.*—First, as to the triangle. C D F E, being the given circle, draw any diameter, C F, and with C as a centre, describe the arc, D O E, its radius being equal to that of the given circle; join D E, E F, and F D, and D E F will be the triangle required. As to the hexagon. The side of a regular hexagon is equal to the radius of the circumscribing circle, and, therefore, in order to inscribe it in a circle, such as K L M, all that is necessary is to mark off on the circumference the length of the radius, and, joining the points of intersection, as K I L H M J, the resulting figure will be the hexagon required. To inscribe figures of 12 or 24 sides, it is merely necessary to divide or subdivide the arcs subtended by the sides obtained as above, and to join the points of intersection. It is frequently necessary to draw very minute hexagons, such as screw-nuts and bolt-heads. This is done more quickly by means of the angle, D, Fig. 6, of 60° (60° = one-sixth of the circumferenc which is 360°), the angle being placed against a rule, B, or the T square, E, Fig. 6, in different positions, as indicated in Fig. 19.

17. *To inscribe a square in a given circle, as* A C B D, Fig. 20.—Draw two diameters, as A B, C D, perpendicular to one another, and join the points of intersection with the circle, and A C B D will be the square required.

18. *To describe a regular octagon about a circle having a given radius, as* O E, Fig. 20.—Having, as in the last case, drawn the two diameters, E F, G H, draw other two, I J, K L, bisecting the angles formed by the former; through the eight points of intersection with the circle, draw the tangents, E, K, G, J, F, L, H, and I, these tangents will cut each other and form the regular octagon required. This figure may also be drawn by means of the square, and angle of 45°, A, Figs. 6 and 20.

19. *To construct a regular octagon of which one side is given, as* A B, Fig. 21.—Draw the perpendicular, O D, bisecting A B; draw A F parallel to O D, produce A B to C, and bisect the angle, C A F, by the line, E A, making E A equal to A B. Draw the line, O G, perpendicular to, and bisecting E A; O G will cut the line, O D, in O, which will be the centre of the circle circumscribing the required octagon. This circle being drawn, the octagon may at once be inscribed by simply marking off arcs, as E H, H F, &c., equal to A B, and joining the points, E, H, F, &c. By dividing and subdividing the arcs thus obtained we can draw regular figures of 16 or 32 sides. The octagon is a figure of frequent application, as for drawing bosses, bearing brasses, &c.

20. *To construct a regular pentagon in a given circle, as* A B C D F, *also a decagon in a given circle, as* E R M, Fig. 22.—The pentagon is thus obtained; draw the diameters, A I, E J, perpendicular to each other; bisecting O E in K, with K as a centre, and K A, as radius, describe the arc, A L; the chord, A L, will be equal to a side of the pentagon, which may accordingly be drawn by making the chords which form its sides, as A F, F D, D C, C B, and B A, equal to A L. By bisecting these arcs, the sides of a decagon may be at once obtained. A decagon may also be constructed thus:—Draw two radii perpendicular to each other, as O M and O R; next the tangents N R and N M. On the latter, describe a circle having N M for its diameter; join R, and the centre, P, of this circle, the line, R P, cutting this circle in *a*; R *a*, is the length of a side of the decagon, and applying it to the given circle, as R *b*, &c., the required figure will be obtained.

Note that the distance, R *a*, or R *c*, is a *mean proportional* between an entire radius, as R N, or R O, and the difference, *c* N, between it and the radius. A *mean proportional* between two lines is one having such relation to them that the square of which it is the one side, is equal to the rectangle, of which the other two are the dimensions. Expressed arithmetically, it is the square root of the product of two sums: *e.g.*—4, *is the mean proportional, or geometrical mean, between* 2 and 8, since $\sqrt{2 \times 8} = 4$; or as $2:4::4:8$.

21. *To construct a rectangle of which the sides shall be mean proportionals between a given line, as* A C, Fig. 23, *and one a third or two-thirds of it.*—A C, the given line, will be the diagonal of the required rectangle; with it as a diameter describe the circle, A B C D. Divide A C into three equal parts in the points, *m*, *n*, and from these points draw the perpendiculars, *m* D, and *n* B; the lines which join the points of intersection of these lines with the circle, as A B, A D, C B, C D, will form the required rectangle, the side of which, C D, is a mean proportional between C *m* and C A, or—

$$\mathrm{C}m : \mathrm{CD} :: \mathrm{CD} : \mathrm{CA};$$

that is to say, the square of which C D is a side, is equal to a rectangle of which C A is the length, and C *m* the height, because

$$\mathrm{CD} \times \mathrm{CD} = \mathrm{C}m \times \mathrm{CA}$$ [*]

In like manner, A D is a mean proportional between C A and *m* A. This problem often occurs in practice, in measuring timber. Thus the rectangle inscribed in the circle, Fig. 23, which may be considered as representing the section of a tree, is the form of the beam of the greatest strength which can be obtained from that tree.

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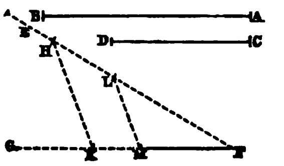
[*] See the notes and rules given at the end of this chapter.

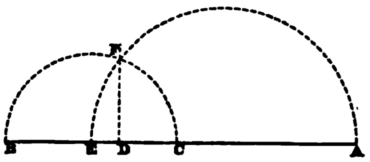
22. *To find a third proportional to two given lines.*—A third proportional is thus found: A B, and C D, being the given lines, Fig. 24, draw any angle, as E F G, and from the point, F, cut off from F E, a part, F H, equal to A B; and, again, from F E and F G cut off F L and F K, equal to C D. Join H K, and through the point, L, draw L M parallel to H K. F M is the third proportional required; that is, the ratio of A B to C D, is the same as that of C D to F M. Thus, in the instance of two lines, 8 inches and 2 inches in length, the third proportional will be a line $\frac{1}{2}$ an inch in length, for as 8 : 2 : : 2 : $\frac{1}{2}$. The third proportional in the other direction is easily found to be 32; for as 2 : 8 : 8 : 32.

Fig. 24.



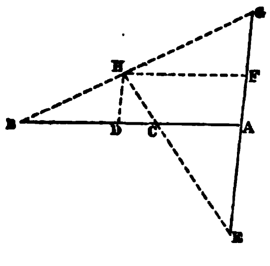
23. *To find the arithmetical and harmonical means between two given lines.*—Two other *means* are employed in geometry, viz., the *arithmetical mean*, and the *harmonical mean.* The former is half the sum of two lines, or, using an arithmetical expression, of two sums; thus, 5 is the arithmetical mean between 2 and 8, since $\frac{2+8}{2} = 5$. The harmonical mean between two lines is a third line, the difference between which and the greater of the two given lines bears the same ratio to the difference between it and the less, as the greater given line bears to the less. To find the harmonical mean between the two lines, A B, A C, Fig. 25, describe on B C, the difference between the two given lines, the semicircle, B F C; E being its centre; also on A E, as a diameter, describe another semicircle, cutting the first in F, and draw D F perpendicular to A B. Now, A D will be the harmonical mean between A B and A C, for A B : A C : : B D : C D.

Fig. 25.



24. *To divide a given line,* A B, Fig. 26, *in harmonical proportion, and in a given ratio; that is to find two points,* C, D, *in a given line,* A B, *so that* A B : A C : : B D : C D : : *as the one term : the other of the given ratio.*—This mode of dividing lines is of frequent application in linear perspective.

Fig. 26.



Through either end of A B draw a line, E G, at pleasure, nearly at right angles thereto, and make A E, A G, each equal to the larger term, and A F equal to the smaller term of the given ratio, and join B G. Through F, draw a line parallel to A B, cutting B G in H, then a line joining E H, will cut A B in C one point; and another line through H, parallel to E G, will cut A B in D, the other point of harmonical division of A B.

APPLICATIONS.

DESIGNS FOR INLAID PAVEMENTS, CEILINGS, AND BALCONIES.

PLATE 2.

The problems just considered are capable of a great variety of applications, and in Plate 2 will be found a collection of some of those more frequently met with in mechanical and architectural constructions and erections. In order, however, that the student may perfectly understand the different operations, we would recommend him to draw the various designs to a much larger scale than that we have adopted, and to which we are necessarily limited by space. The figures are arranged in double rows, the upper row, or odd numbers, represent the complete designs, while the under row, or even numbers, show the method of forming the outlines of the several patterns, which are shown in sharp lines, and to a much larger scale than those of the upper row. The working or pencil lines required in drawing the patterns are shown in dotted lines throughout.

25. *To draw a pavement consisting of equal squares,* Figs. 1 *and* 2.—Taking the length, *a b,* equal to half the diagonal of the required squares, mark it off a number of times on a horizontal line, as from A to B, B to C, &c. Through A, draw I H perpendicular to A C, and draw parallels to it, as D E, G F, &c., through the several points of division. On the perpendicular, I H, mark off a number of distances equal to A B, and draw parallels to A B, through the points of division, as H G, I F, &c. A series of small squares will thus be formed, and the larger ones are obtained simply by drawing the diagonals to these, as shown.

26. *To draw a pavement composed of squares and interlaced rectangles,* Figs. 3 *and* 4.—Let the side, as *c d,* of the square be given, and describe the circle, L M Q B, the radius of which is equal to half the given side. With the same centre, O, describe also the larger circle, K N P I, the radius of which is equal to half the side of the square, plus the breadth of the rectangle, *a b.* Draw the diameters, A C, E D, perpendicular to each other; draw tangents through the points, A, D, C, E, forming the square, J H F G; draw the diagonals J F, G H, cutting the two circles in the points, I, B, K, L, M, N, P, Q, through which draw parallels to the diagonals. It will be perceived that the lines, A E, E C, C D, and D A, are exactly in the centre of the rectangles, and consequently serve to verify their correctness. The operation just described is repeated, as far as it is wished to extend the pattern or design, many of the lines being obtained by simply prolonging those already drawn. In inking this in, the student must be very careful not to cross the lines. This design, though analogous to the first, is somewhat different in appearance, and is applicable to the construction of trellis-work, and other devices.

27. *To draw a Grecian border or frieze,* Figs. 5 *and* 6.—On two straight lines, as A B, A C, perpendicular to each other, mark off, as often as necessary, a distance, A 1, &c., representing the width, *e f,* Fig. 5, of the ribbon forming the pattern. Through all the points of division, as 1, 2, 3, &c., draw parallels to A B, A C—thus forming a series of small squares, guided by which the pattern may be at once inked in; equal distances being maintained between the sets of lines, as shown shaded in Figs. 5 and 6. This ornament is frequently met with in architecture, being used for ceilings and cornices; also, in cabinet work, the construction of borders, wood and iron gratings, &c.

28. *To draw a pavement composed of squares and regular octagons,* Figs. 7 *and* 8.—With a radius, E O, equal to half the width,

DESIGNS FOR INLAID PAVEM

Fig. 1.

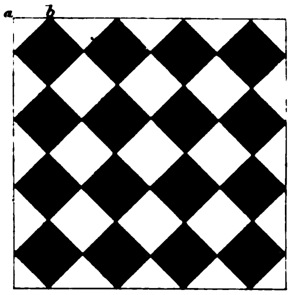


Fig. 3.

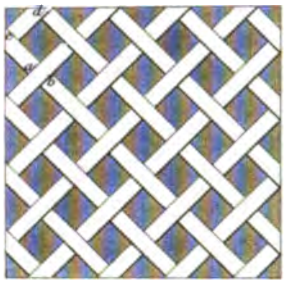


Fig. 5.

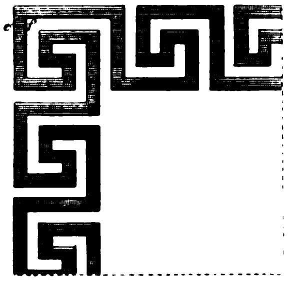


Fig. 2.

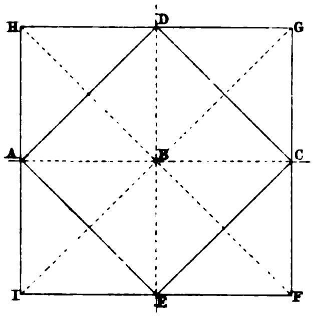


Fig. 4.

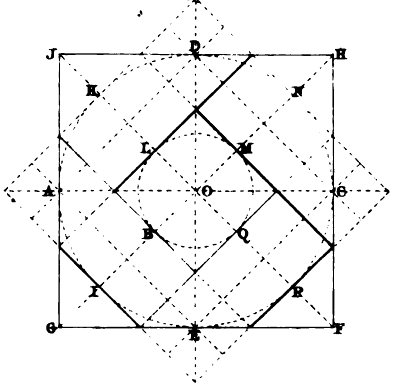


Fig. 6.

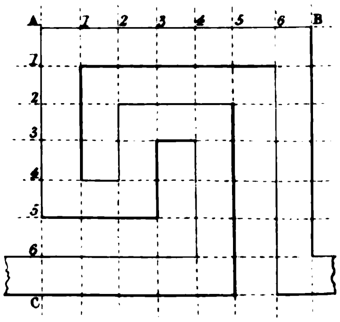
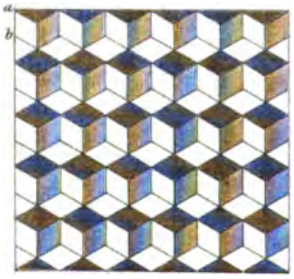


Fig. 13.



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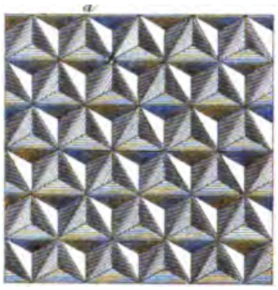


Fig. 17.

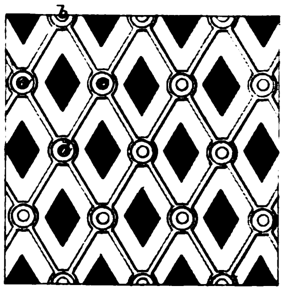
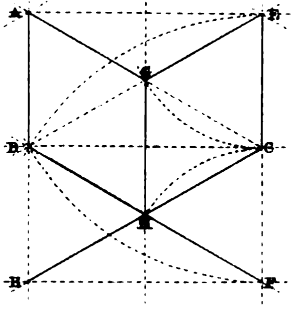


Fig. 14.



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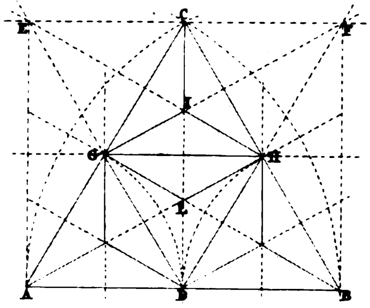
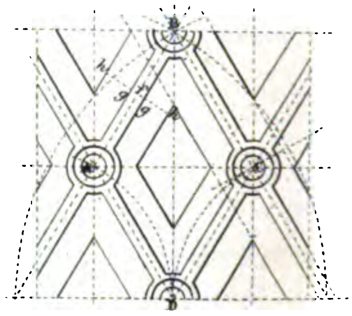


Fig. 18.



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NTS, CEILINGS & BALCONIES.

Fig. 7.

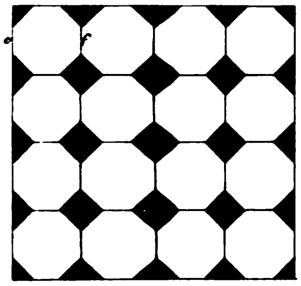


Fig. 9.

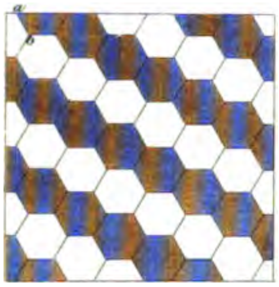
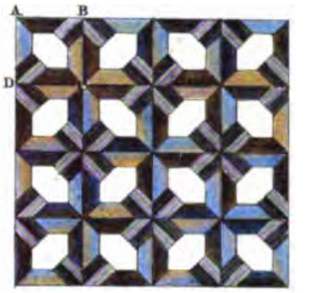


Fig. 11.



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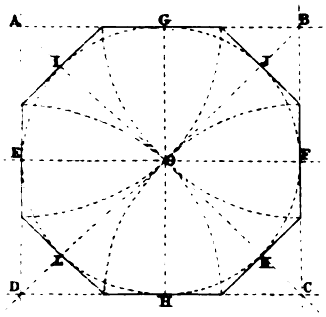


Fig. 10.

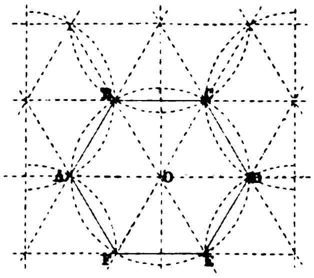


Fig. 12.

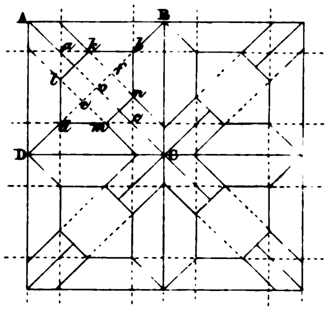


Fig. 19.

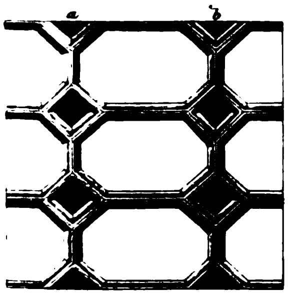


Fig. 21.

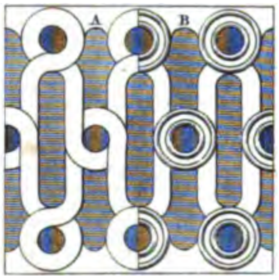


Fig. 23.

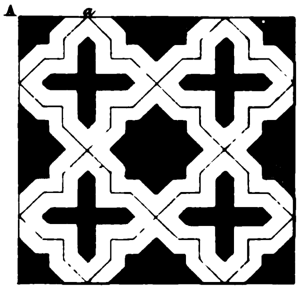


Fig. 20.

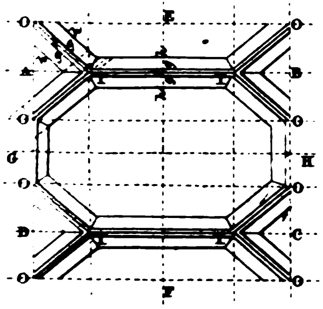


Fig. 22.

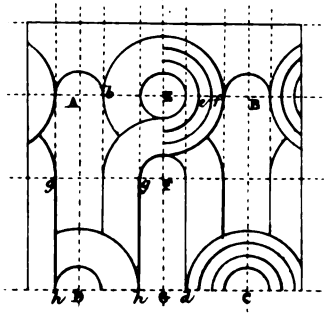
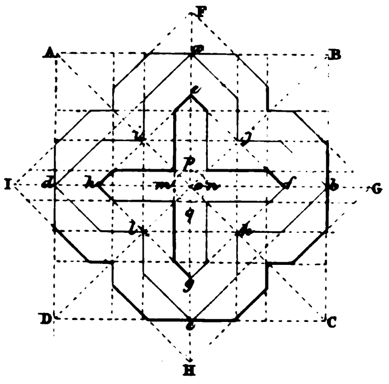


Fig. 24.



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E F, of the octagon, describe a circle, E G F H, and draw the octagon circumscribing it (Problem 18, Fig. 20, Plate 1), the square, A B C D, being first obtained, and its diagonals, A C, B D, drawn cutting the circle in the points, I, J, K, L, tangents being then drawn through these points. The octagon may also be formed by marking off from each corner of the square, A, B, C, D, a distance equal to A O, or half its diagonal, and thereby will be obtained the points of junction of the sides of the octagon. The pattern is extended simply by repeating the above operation, the squares being formed by the sides of four contiguous octagons, which are inclined at an angle of 45° to the horizontal lines. This pattern is generally produced in black and white marble, or in stones of different colours, whereby the effect is distinctly brought out.

29. *To draw a pavement composed of regular hexagons*, Figs. 9 *and* 10.—With a radius, A O, equal to a side, *a b*, of the hexagon required, describe a circle, in which inscribe the regular hexagon, A B C D E F, Problem 16, Fig. 19, Plate 1. The remaining hexagons will readily be obtained by producing, in different directions, the sides and diagonals of this one. In Fig. 9, the hexagons are plain and shaded alternately, to show their arrangement; but in practice they are generally all of one colour.

30. *To draw a pavement composed of trapeziums, combined in squares*, Figs. 11 *and* 12.—Draw the square, A B C D; also its diagonals, A C, B D; construct the smaller square, *a b c d*, concentric with the first. On the diagonal, B D, mark the equal distances, *o e*, *o f*, and through *e* and *f* draw lines parallel to the diagonal, A C; join the points of intersection of these with the smaller squares by the lines, *k l*, *m n*, which will give all the lines required to form the pattern, requiring merely to be produced and repeated to the desired extent. Very beautiful combinations may thus be formed in different kinds of wood for furniture and panels.

31. *To draw a panel design composed of lozenges*, Figs. 13 *and* 14.—On a straight line, A B, mark off the lengths, A D, and D B, each equal to a side of the lozenge, as *a b*, Fig. 13; and on A B, construct the equilateral triangle, A B C; draw the line, C D, perpendicular to A B; and draw A E and B F parallel to D C, and E F parallel to A B. Construct the equilateral triangle, E D F, cutting the triangle, A B C, in G and H. and join G H. In this manner are obtained the lozenges, A G H D and E G H C, and by continuing the lines and drawing parallels at regular distances apart, the remainder of the pattern will be readily constructed.

32. *To draw a panel pattern composed of isosceles triangles*, Figs. 15 *and* 16.—If we take the construction as shown in Figs. 13 and 14, and draw the longer diagonal of each lozenge, we shall obtain two of the patterns shown at Fig. 15. Let us, however, suppose that the base, *a b*, of the triangle is given, instead of the side of the lozenge:—Mark off this length twice on the line, A B, and construct the equilateral triangle, A B C, just as in the preceding case; also the second similar triangle, D E F, thus obtaining the points G and H. Join A H, G B, E H, and G F, &c., and each point of intersection, as I, L, &c., will be the apex of three of the isosceles triangles. The pattern, Fig. 15, is produced by giving these triangles various tints.

The patterns we have so far given are a few of the common arrangements of various regular polygons. An endless variety of patterns may be produced by combining these different figures, and these are of great use in many arts, particularly for cabinet inlaid mosaic work, as well as for pavements and other ornamental constructions.

33. *To draw an open-work casting, consisting of lozenges and rosettes*, Figs. 17 *and* 18.—The lozenge, *a b c d*, or A B C D, being given, the points, A, B, C, and D, are each the centre of a rosette; draw and indefinitely produce the diagonals, A C, and B D, which must always be perpendicular to each other. Through the points, A, B, C, D, draw parallels to these diagonals, also an indefinite number of such parallels at equal distances apart. The intersections of these lines will be the centres of rosettes and lozenges alternately, and the former may accordingly be drawn, consisting merely of circles with given radii. The centres of the rosettes are joined by straight lines, and to right and left of these, at the given distances, *f g*, *f h*, parallels to them are drawn, thereby producing the concentric lozenges completing the pattern.

34. *To draw a pattern for a ceiling, composed of small squares or lozenges, and irregular but symmetrical octagons*, Figs. 19 *and* 20. – The rectangle, A B C D, being given, its corners forming the centres of four of the small lozenges, draw the lines, E F, G H, dividing the rectangle into four equal parts; next mark off the semi-diagonals of the lozenges, as A I, A O, and join I and O. The centre lines of the pattern being thus obtained, the half-breadths, *f g*, *f h*, are marked on each side of these, and the appropriate parallels to them drawn. In extending the pattern by repetition, the points corresponding to I and O will be readily obtained by drawing a series of parallel lines, as I I and O O. By varying the proportions between the different dimensions of each, a number of patterns may be produced of very different appearance, although formed of these simple elements.

35. *To draw a stone balustrade of an open-work pattern, composed of circular and straight ribbons interlaced*, Figs. 21 *and* 22.—Construct the rectangle, A B C D, its corners being the centres of some of the required circles, which may accordingly be drawn, with given radii, as A *b*, C *d*; after bisecting A B in E, and drawing the vertical E G, make E F equal to E A, and with F as a centre, draw the circle having the radius, F *g*, equal to A *b*, drawing also the equal circles at C, B, E, &c. Draw verticals, such as *g h*, tangents to each of the circles, which will complete the lines required for the part of the pattern, Fig. 21, to the left. The rosettes to the right are formed by concentric circles of given radii, as E *e*, E *f*. The duplex pattern, shown at Fig. 21, may be supposed to represent the pattern on the opposite sides of a stone balustrade. Where straight lines are run into parts of circles, the student must be careful to make them join well, as the neatness of the drawing depends greatly on this. It is better to ink in the circles first, as it is practically easier to draw a straight line up to a circle than to draw a circle to suit a straight line.

36. *To draw a pattern for an embossed plate or casting, composed of regular figures combined in squares*, Figs. 23 *and* 24.—Two squares being given, as A B C D and F G H I, concentric, but with the diagonals of one parallel to the sides of the other, draw first the square, *a b c d*, and next the inner and concentric one, *e f g h*. The sides of the latter being cut by the diagonals, A C and B D, in the points, *i*, *j*, *k*, *l*, through these draw parallels to the sides of the square, A B C D, and finally, with the centre, O, describe a small circle, the diameter of which is equal to the width of the indented crosses, the sides of these being drawn tangent to this circle. Thus are obtained all the lines necessary to delineate this pattern; the relievo and intaglio portions are contrasted by the latter being shaded.

In the foregoing problems, we have shown a few of the many varieties of patterns producible by the combination of simple regular figures, lines, and circles. There is no limit to the multiplication of such designs; the processes of construction, however, being similar to those just treated of, the student will be able to produce them with facility.

SWEEPS, SECTIONS, AND MOULDINGS.

PLATE 3.

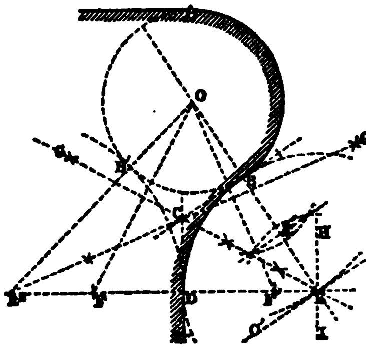
37. *To draw in a square a series of arcs, relieved by semicircular mouldings*, Figs. 1 *and* 2.—Let A B be a side of the square; draw the diagonals cutting each other in the point, C, through which draw parallels, D E, C F, to the sides; with the corners of the square as centres, and with a given radius, A G, describe the four quadrants, and with the points, D, F, E, describe the small semicircles of the given radius, D *a*, which must be less than the distance, D *b*. This completes the figure, the symmetry of which may be verified by drawing circles of the radii, C G, C H, which should touch, the former the larger quadrants, and the latter the smaller semicircles. If, instead of the smaller semicircles, larger ones had been drawn with the radius, D *b*, the outline would have formed a perfect *sweep*, being free from angles. This figure is often met with in machinery, and may represent the section of a beam, connecting-rod, or frame standard.

38. *To draw an arc tangent to two straight lines, making any angle with each other.*—First, let the radius, *a b*, Fig. 3, be given; then with A, the point of intersection or apex of the angle, formed by the two lines, A B, A C, as a centre, and a radius equal to *a b*, describe arcs, B and C, cutting the given lines, and continued for some distances on each side of them. Tangent to these arcs, A and C, draw (Problem 4 or 9, Fig. 10A) the lines, B O and C O, parallel to A C and A B, respectively, and cutting each other in O, which will be the centre of the required arc; in the case of the angle, B A C, being an obtuse angle, these lines, B O, C O, will touch the arcs, B and C, in the inside of the given lines or angle, but when it is an acute angle, they will touch the arcs on the outside of the given lines and angle. Draw perpendiculars from the point, O, to the straight lines, A B, A C, meeting them in D and E; O D or O E, will each be radii of the required arc, and D and E will be the points of contact with the given lines. Secondly, if a point of contact be given, as B, Fig. 5, the lines being A B, A C, making any angle with each other, bisect the angle, B A C, by the straight line, A D; draw B O perpendicular to A B, from the point B, and the point, O, of its intersection with A D, will be the centre of the required arc. If, as in Figs. 3 and 5, we draw arcs, G H, of radii, somewhat less than O B, we shall form figures, which stand out from, instead of being tangents to, the given straight lines. This problem, as well as the following, are necessary for drawing such a figure as that shown at Fig. 4, representing the section of a cast-iron beam or frame.

39. *To draw a circle tangent to three given straight lines, which make any angles with each other*, Fig. 6.—Let B A, A C, and C D, be the given lines. Bisect the angle formed by the first of these two lines, viz., B A C, by the straight line, A E, and also the angle, D C A, contained in the two latter, formed by the line, C F. A E and C F will cut each other in the point O, which is at an equal distance from each side, and is consequently the centre of the required circle, which may be drawn with a radius, equal to a line from the point, O, perpendicular to any of the sides.

40. *To draw the section of a stair rail, as represented in* Fig. 7.—This gives rise to the problems considered in Figs. 8 and 9. Let it be required to draw an arc tangent to a given arc, as A B, and to the given straight line, C D, Fig. 9. Let the point of contact D, in the straight line, C D, be given; secondly, let the point of contact, B, in the arc, A B, be given; and, thirdly, let the radius, B E, of the required arc be given:—In the first case, then, through D, the point of contact given in the line, C D, draw E F perpendicular to C D; make F D equal to O B, the radius of the given arc, to the right or left of C D, as the case may be, and join O F, through the centre of which draw the perpendicular, G E, and the point, E, of its intersection with E F, will be the centre of the required arc, and E D the radius. Further, join O E, and the point of intersection, B, with the arc, A B, will be the point of junction of the two arcs. In the second case, join the centre, O, of the given arc, Fig. 9A, and the point of contact, D, in its circumference, producing it in the direction of E, through the point of contact, B, draw the tangent, B C, and if it is not parallel to the given line, D C, it must form an angle with it, and cut it in a point, C; bisect this angle, B C D (Problem 8, Figs. 8 or 8A), and it will cut the line, O B, produced in a point, E, which will, in this case, be the centre of the arc or circle, required, and E B, or a perpendicular, E D, from E, to the line, C D, will be its radius. But should B C, be parallel to the given line, then O B will be at right angles to both the given line and B C, and, consequently, will cut the given line when produced in the point of contact, as E, in the line, C^1 D^1, and the centre of the required circle will be found by simply bisecting the line, B E. In the third case, let the radius, B E^1 or B E, of the required arc be given; then the given lines, C^1 D^1 or C D, must not be further from the circumference of the given arc, A B, in a perpendicular line, than twice the given radius, but either exactly that or less. Let B E^1, be exactly the half of the distance, B E, of the perpendicular from the point or centre, O, to the given line, C^1 D^1, cut by the circumference of the given arc, in B, then, as in the latter case, the centre of the required circle will be found in the centre of the line, B E, by bisecting it. Let the given radius, B E, of the required arc, be greater than half the perpendicular distance between the circumference of the given arc, and the given line, C D. Then draw the line, H I (Problem 4), parallel to C D, and at a distance from it equal to the radius, B E, of the required arc, and from the centre, O, of the given arc, A B, with a radius equal to that of the required arc, plus or minus, as the case may be, of that of the given arc, viz., B E, plus O B, or minus O B, when it is the concave side, as at A, that it is required to draw the arc tangent to, describe an arc cutting the parallel line, H I, in the point, E, which will be the centre of the required arc, and E B or E D, its radius. It will be seen in these cases, a circle being given instead of an arc, and the several lines produced on both sides of it, that two such arcs as that required, and two similar ones, can be drawn, viz., one on each side of the given line or circle, as the one D B^1, subtends that of D B, and G^1 E^2, bisects the angle, G E^2 F^1, as G E did that of O E F, on the opposite side of the given line, C D. The two similar ones can be drawn in the same way at the side, A, of the given circle, the line, D C, being produced beyond it. When the point, B, is given, the four arcs will be on the same side of the given line D C, which, being produced in the direction of A, will have two of the arcs tangent to it, and to the inner or concave side,

Fig. 9A.



SWEEPS, SECTION

Fig. 1.

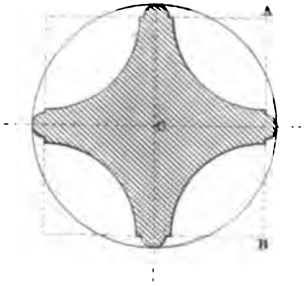


Fig. 2.

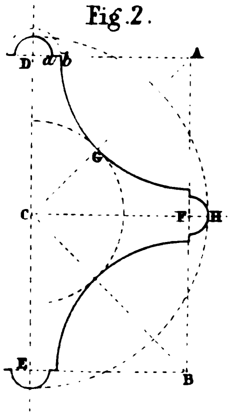


Fig. 3.

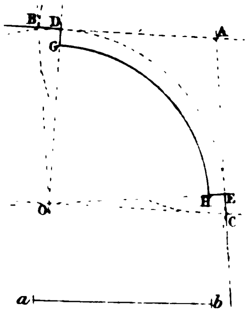


Fig. 4.

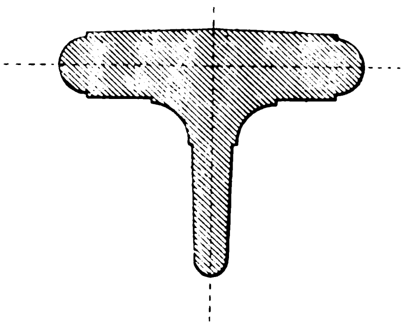


Fig. 5.

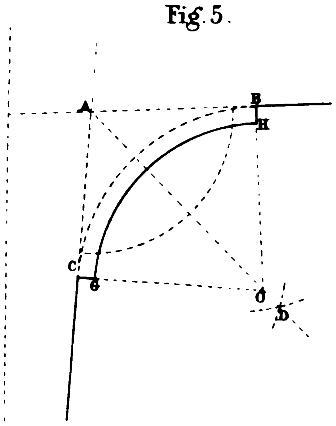


Fig. 6.

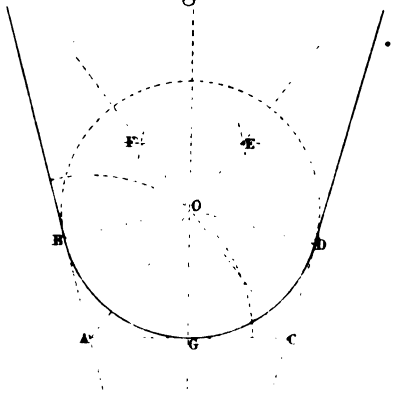
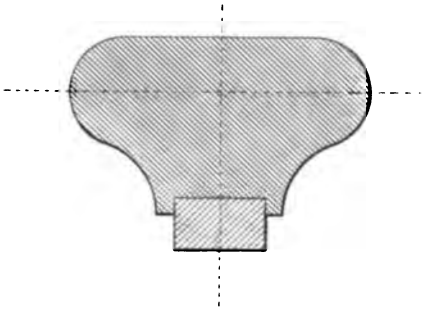


Fig. 7.



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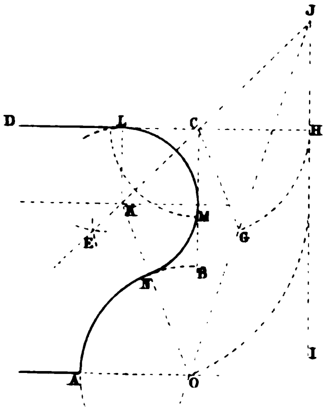

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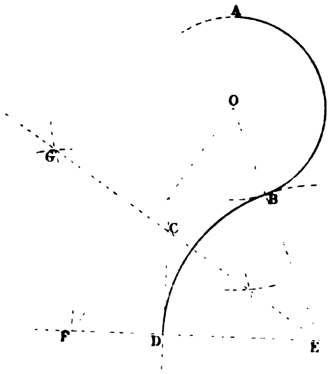

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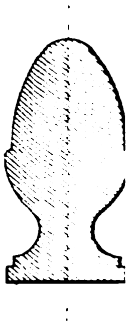

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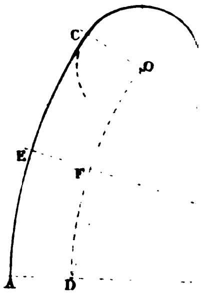

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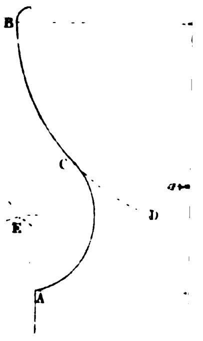

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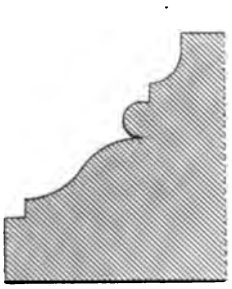
Fig. 13.



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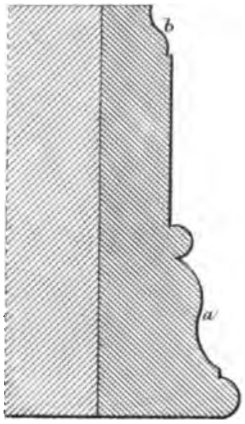


Fig. 17.

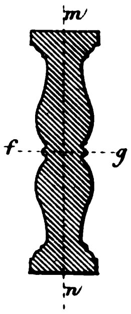
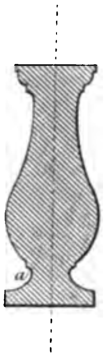


Fig. 19.



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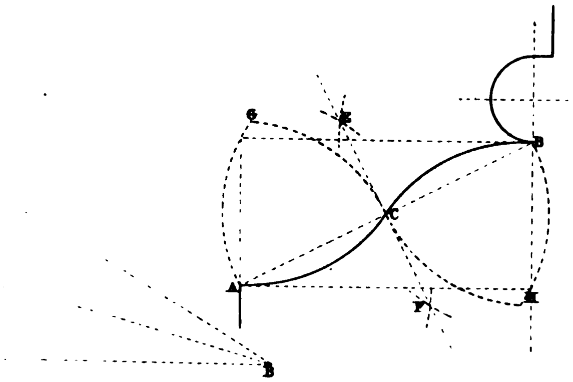
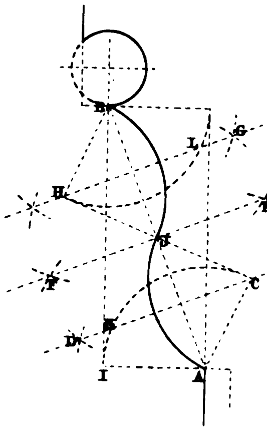


Fig. 16.



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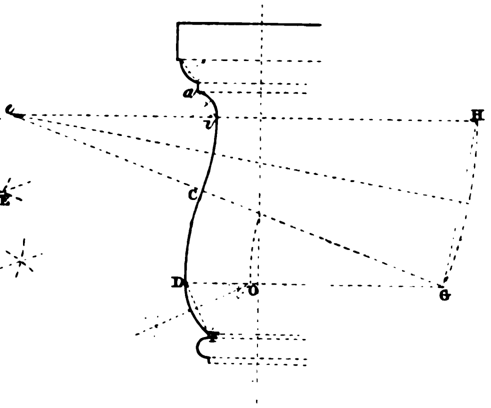

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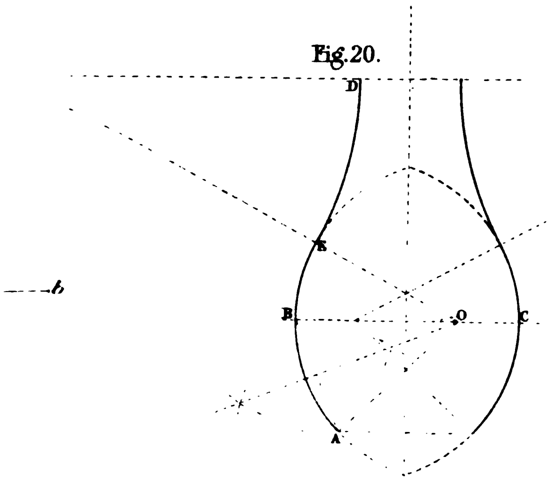


Fig. 21.

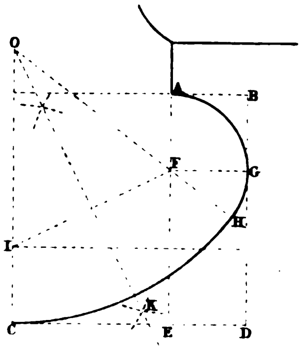
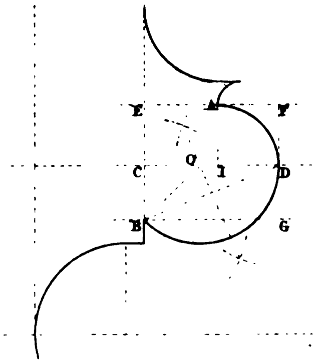


Fig. 22.



subtending two on the outer or convex side. Secondly, let it be required to draw an arc tangential to a given arc, as A B, and to two straight lines, making any angle with each other, Fig. 8; some part of such arc must fall within the angle formed by these two lines. Bisect the angle, B C D, by the straight line, C E; with the centre, C, and the radius, C H, equal to that of the given arc, O A, describe the arc, H G: tangent to H G, and parallel to B C draw I H J, cutting E C produced in J. Join O J, the line, O J, cutting the arc, H G, in G; join C G, and draw O K, parallel to G C; the point, K, of its intersection with E J, will be the centre of the required arc, and a line, K L, or K M, perpendicular to either of the given straight lines will be the radius. An arc drawn from the centre, K, with K O, as radius, will join the line, I J, tangentially, as well as the arc, G H.

41. *To draw the outline of an acorn as shown in section at* Fig. 10.—This figure calls for the solution of the two Problems considered in Figs. 11 and 12. First, it is required to draw an arc, passing through a given point, A, Fig. 11, in a line, A B, in which also is to be the centre of the arc, this arc at the same time being a tangent to the given arc, C. This is another form of the first case of Problem 40, Fig. 9, in which the portion equal to the radius of the given arc is cut off to the right hand instead of the left. Make A D equal to O C, the radius of the given arc; join O D, and draw the perpendicular, F B, bisecting it. B, the point of intersection of the latter line, with A B, is the centre of the required arc, A E C, A B being the radius. Secondly, it is required to draw an arc passing through a given point, A, Fig. 12, tangential to a given arc, B C D, and having a radius equal to *a b*. With the centre, O, of the given arc, and with a radius, O E, equal to O C, plus the given radius, *a b*, Problem 40, Fig. 9A, radii being given, draw the arc, E; and with the given point, A, as a centre, and with a radius equal to *a b*, describe an arc cutting the former in E, then E will be the centre of the required arc, and its point of contact with the given arc will be in C, on the line, O E. It will be seen that in Fig. 10, these Problems arise in drawing either side of the object. The two sides are precisely the same, but reversed, and the outline of each is equidistant from the centre line, which should always be pencilled in when drawing similar figures, it being difficult to make them symmetrical without such a guide. This is an ornament frequently met with in machinery, and in articles of various materials and uses.

42. *To draw a wave curve, formed by arcs, equal and tangent to each other, and passing through given points,* A, B, *their radius being equal to half the distance,* A B, Figs. 13 *and* 14.—Join A B, and draw the perpendicular, E F, bisecting it in C. With the centres, A and C, and radius, A C, describe arcs cutting each other in G, and with the centres, B and C, other two cutting each other in H; G and H will be the centres of the required arcs, forming the curve or sweep, A C B. This curve is very common in architecture, and is styled the *cyma recta.*

43. *To draw a similar curve to the preceding, but formed by arcs of a given radius, as* A I, Figs. 15 *and* 16.—Divide the straight line into four equal parts by the perpendiculars, E F, G H, and C D; then, with the centre, A, and given radius, A I, which must always be greater than the quarter of A B, describe the arc cutting C D in C; also with the centre, B, a similar arc cutting G H in H; C and H will be the centres of the arcs required; join C H, and it will cut the line, A B, or the two arcs, A J, and B J, in their point of contact, J, where they join each other tangentially and form the curve, A J B, required. Whatever be the given radius, provided it is not too small, the centres of the arcs will always be in the lines, C D and G H. It will be seen that the arcs, C I and H L, cut the straight lines, C D and G H, in two points respectively. If we take the second points, K, L, as centres, we shall form a similar curve to the last, but with the concavity and convexity transposed, and called by architects the *cyma reversa.* The two will be found in Fig. 15, the first at *a*, and the second at *b*. This figure may represent the section of a door, or window frame, and is one well known to carpenters and masons.

44. *To draw a baluster of a duplex contour as represented at* Figs. 17 *and* 18.—It is here necessary to draw an arc tangent to, or sweeping into two known arcs, *a i* and C D, and having its centre in a given horizontal, *e i*. Extend *e i* to H, making *i* H equal to G D, the radius of the arc, C D. Join G H, bisecting G H by a perpendicular; this will cut *e* H in the point, *e*, which is the centre of the required arc, C *i*—*e i*, being its radius. A line joining *e* G cuts C D in C, the point of contact of the two arcs. The arc, D F, which is required to be a tangent to C D, and to pass through the point, F, is drawn with the centre, O, obtained by bisecting the chord, D F, by a perpendicular which cuts the radius of the arcs, C D. This curve has, in Fig. 17, to be repeated both on each side of the vertical line, *m n*, and of the horizontal line, *f g*.

45. *To draw the section of a baluster of simple outline, as represented at* Fig. 19.—We have here to draw an arc passing through two points, A, B, Fig. 20, its centre being in a straight line, B C; this arc, moreover, requiring to join at D, and form a sweep with another, D E, having its centre in a line, F D, parallel to B C. Joining B A, a perpendicular bisecting B A, will cut B C in O, which will be the centre of the first arc, and that of the second may now be obtained, as in Problem 41, Fig. 9.

46. The base of the baluster, Fig. 19, is in the form of a curve, termed a *scotia.* It may be drawn by various methods. The following are two of the simplest. According to the first, the curve may be formed by arcs sweeping into each other, and tangents at A and C to two given parallels, A B, C D, Fig. 21. Through A and C draw the perpendiculars, C O and A E, and divide the latter into three equal parts. With one division, F A, as a radius, describe the first arc, A G H; make C I equal to F A, join I F, and bisect I F by the perpendicular, O K, which cuts C O in O. O, will be the centre of the other arc required. The line, O H, passing through the centres, O and F, will cut the arcs in the point of junction, H. It is in this manner that the curve at *a*, in Fig. 19, is obtained. The second method is to form the curve by two arcs sweeping into each other and passing through the given points, A B, Fig. 22, their centres, however, being in the same horizontal line, C D, parallel to two straight lines, E F and B C, passing through the given points. Through A, draw the perpendicular, A L I, its point of intersection with C D, is the centre of one arc, A D. Next draw the chord, B D, the perpendicular bisecting, which will cut C D, in O, the centre of the other arc, the radius being O D, or O B. This curve is more particularly met with in the construction of bases of the Ionic, Corinthian, and Composite orders of architecture.

With a view to accustom the student to proportion his designs to the rules adopted in practice in the more obvious applications we have drawn the small sections in the top row of this plate in proportion, but to a small scale, and the corresponding outlines of the various parts, to much larger scales. It must, however, be understood, that the various problems given are equally capable of solution with other data; and, indeed, the number of applications of which the forms considered are susceptible, may be said to be innumerable.

ELEMENTARY GOTHIC FORMS, OGIVES, AND ROSETTES.

PLATE 4.

47. Having solved the foregoing problems, the student may now attempt the delineation of more complex objects. He need not, however, as yet, anticipate much difficulty, merely giving his chief attention to the accurate determination of the principal lines, which serve as guides to the minor details of the drawing.

It is in Gothic architecture that we meet with the most numerous applications of outlines formed by smoothly joined circles and straight lines, and we give a few examples of this order in Plate 4. . Fig. 1 represents the upper portion of a window, composed of a series of arcs, combined so as to form what are denominated *cuspid* arches. The width or span, A B, being given, and the apex, C; joining A C, C B, draw the bisecting perpendiculars, cutting A B in D and E. These latter are the centres of the concentric arcs, which, severally cutting each other on the vertical, C F, form the arch of the window. The small interior cuspids are drawn in the same manner, as indicated in the figure; the horizontal, G H, being given, also the span and apices. These interior arches are sometimes surmounted by the ornament, M, consisting simply of concentric circles.

48. Fig. 2 represents a quadrant of a Gothic rosette, distinguished as *radiating*. It is formed by a series of cuspid arches and radiating *mullions*. In the figure are indicated the centre lines of the several arches and mullions, and in the detail, Fig. 3, the capital, connecting the mullion to the arch, is represented drawn to double the scale. With the given radii, A B, A C, A D, A E, describe the different quadrants, and divide each into eight equal parts, thus obtaining the centres for the *trefoil* and *quatrefoil* ornaments in and between the different arches. These ornaments are drawn to a larger scale, in Figs. 4, 5, and 6, in which are indicated the several operations required.

49. Fig. 7 also represents a rosette, composed of cuspid arches, and trefoil and quatrefoil ornaments, but disposed in a different manner. The operations are so similar to those just considered, that it is unnecessary to enter into details.

50. Fig. 8 represents a cast-iron grating, ornamented with Gothic devices. Fig. 9 is a portion of the details on a larger scale, from which it will be seen that the entire pattern is made up of arcs, straight lines, and sweeps formed of these two, the problems merely comprehending the division of lines and angles, and the obtaining of the various centres.

51. Fig. 10 represents a rosette, formed by concentric circles, the outer ring containing a series of smaller circles, forming an interlaced fillet or ribbon. The radius, A O, of the circle, containing the centres of all the small circles, is supposed to be given. Divide it into a given number of equal parts. With the points of division, 1, 2, 3, &c., as centres, describe the circles tangential to each other, forming the fillet, making the radii of the alternate ones in any proportion to each other. Then, with the centre, O, describe concentric circles, tangential to the larger of the fillet circles of the radius, A *b*. The central ornament is formed by arcs of circles, tangential to the radii, drawn to the centres of the fillet circles, their convexities being towards the centre, O; and the arcs, joining the extremities of the radii, are drawn with the actual centres of the fillet circles.

52. Figs. 11 and 12 are sections of pendants or ornaments, such as are suspended from the centres of Gothic ceilings. In order to draw them, it is only necessary to determine the radii and centres of the various arcs composing them.

Several of the figures in Plate 4 are partially shaded, to indicate the degree of relief of the various portions. In this plate are presented a few minor difficulties, our object being to familiarize the student to the use of his instruments, especially the compasses. These exercises will, at the same time, qualify him for the representation of a vast number of forms met with in architecture and machinery.

OVALS, ELLIPSES, PARABOLAS, VOLUTES, &c.

PLATE 5.

53. The *ova* is an ornament of the shape of an egg, and is formed of arcs of circles. It is frequently employed in architecture, and is thus drawn:—The axes, A B and C D, Fig. 5, being given, perpendicular to each other, with the point of intersection, O, as a centre; first describe the circle, C A D E, half of which forms the upper portion of the ova. Joining B C, make C F equal to B E, the difference between the radii, O C, O B. Bisect F B by the perpendicular, G H, cutting C D in H. H will be the centre, and H C the radius of the arc, C J; and I, the point of intersection of H C with A B will be the centre, and I B the radius of the smaller arc, I B K, which, together with the arc, H K, described with the centre, L, and radius, L D, equal to H C, form the lower portion of the required figure; the lines, G H, L K, which pass through the respective centres, also cut the arcs in the points of junction, J and K. This ova will be found in the fragment of a cornice, at *a*, Fig. 1.

54. The *oval* differs from the ova in having the upper portion symmetrical with the lower; and to draw it, it is only necessary to repeat the operations gone through in obtaining the curve, L B H, Fig. 5.

55. The *ellipse* is a figure which possesses the following property: —The sum of the distances from any point, A, Fig. 6, in the circumference, to two constant points, B, C, in the longer axis, is always equal to that axis, D E. The two points, B, C, are termed *foci*. The curve forming the ellipse is symmetrical with reference both to the horizontal line or axis, D E, and to the vertical line, F G, bisecting the former in O, the centre of the ellipse. Lines, as B A, C A, B F, C F, &c., joining any point in the circumference with the foci, B and C, are called *vectors*, and any pair proceeding from one point are together equal to the longer axis, D E, which is called the *transverse*, F G being the *conjugate* axis. There are different methods of drawing this curve, which we will proceed to indicate.

56. *First Method.*—This is based on the definition given above, and requires that the two axes be given, as D E and F G, Fig. 6. The foci, B and C, are first obtained by describing an arc, with the extremity, G or F, of the conjugate axis as a centre, and with a radius, F C, equal to half the transverse axis; the arc will cut the latter in the points, B and C, the foci. If now we divide D E unequally in H, and with the radii, D H, E H, and the foci as centres, we describe arcs severally cutting each other in I, J, K, A; these four points will lie in the circumference. If, further, we again unequally divide D E, say in L, we can similarly obtain four other points in the circumference, and we can, in like manner, obtain any number of points, when the ellipse may be traced through them by hand. The large ellipses which are sometimes required in constructions are generally drawn with a *trammel* instead of compasses, the trammel being a rigid rule with adjustable points.—The *gardener's ellipse:* To obtain this, place a rod in each of the foci of the required ellipse; round these place an endless cord, which, when

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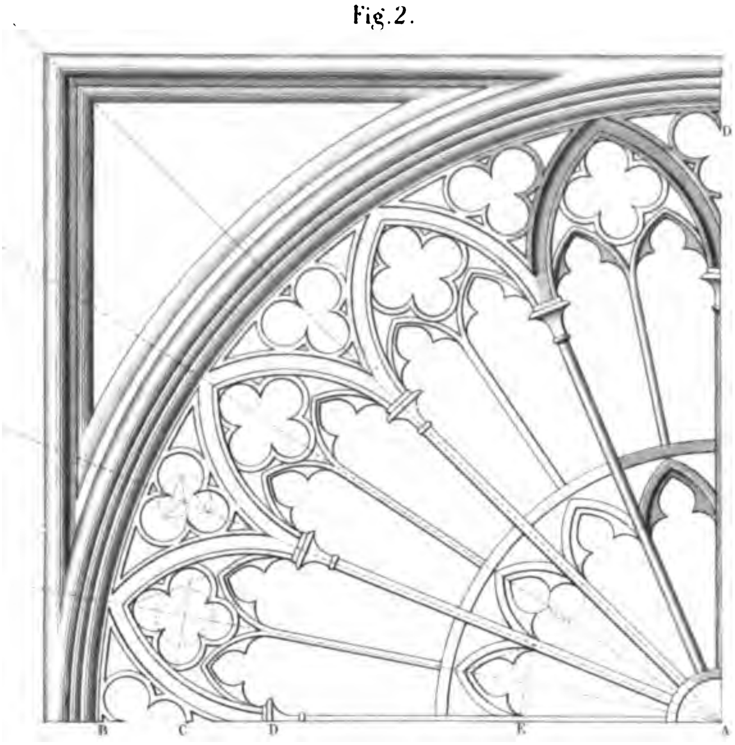
Fig. 2.



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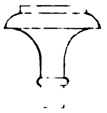

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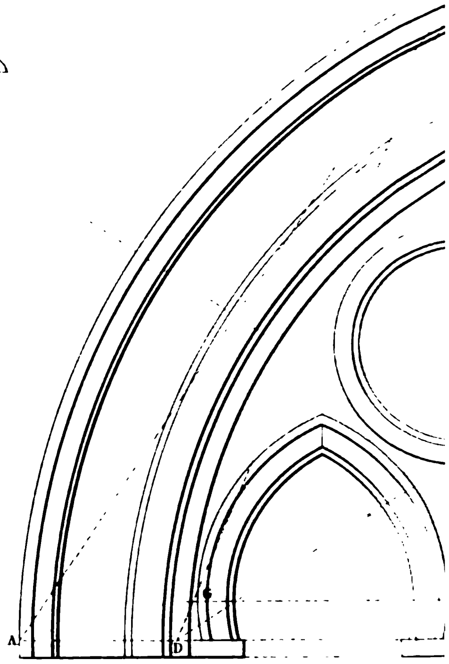


Fig. 7.

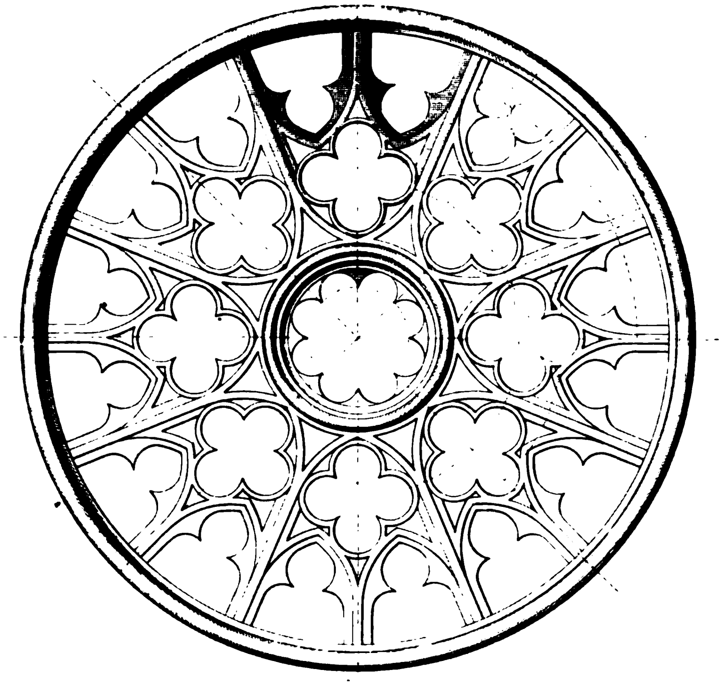
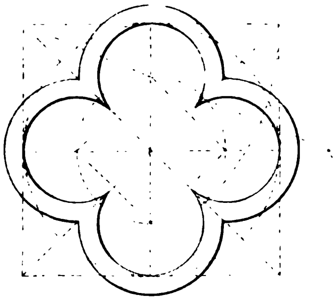


Fig. 4.



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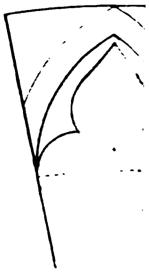

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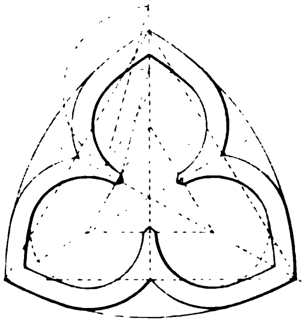
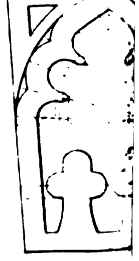


Fig. 9



Scale for Fig.s 1. 2 7 8 & 10

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FORMS & ROSETTES.

I.

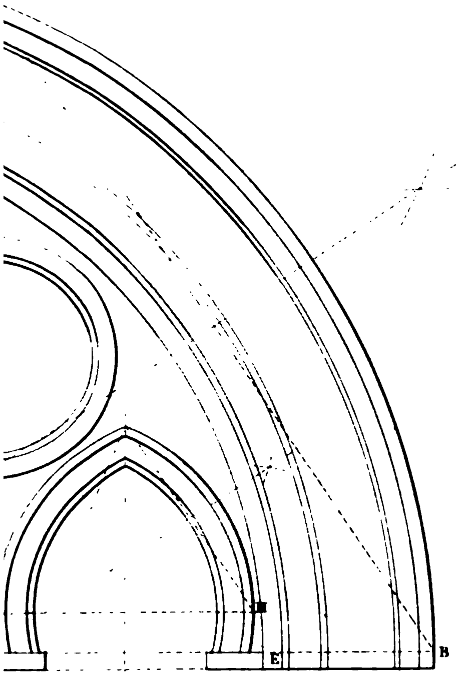
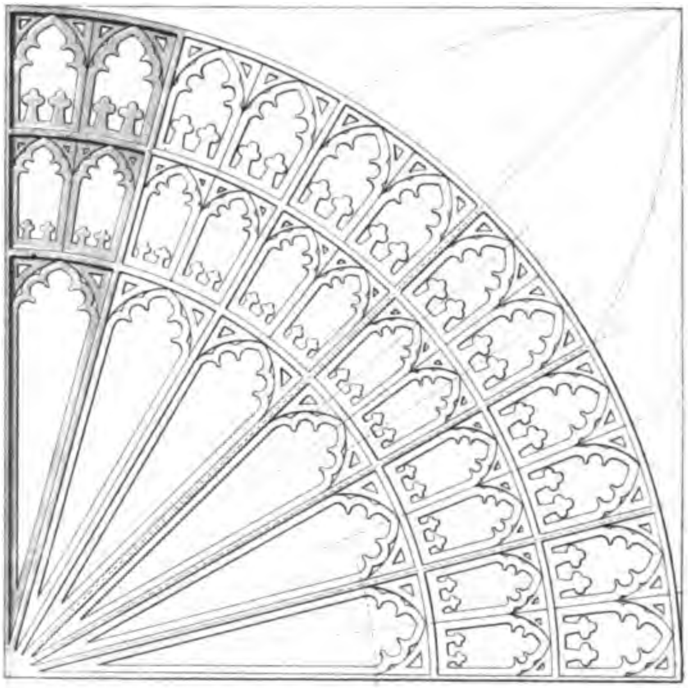


Fig.8.



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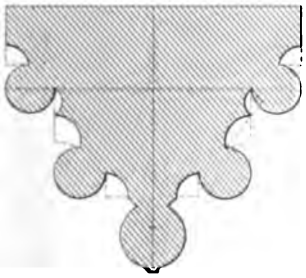

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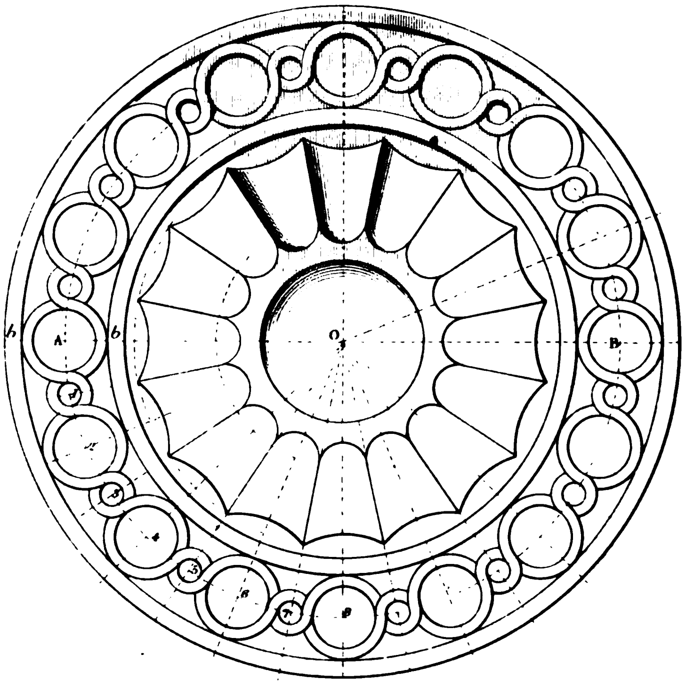

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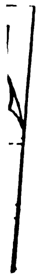

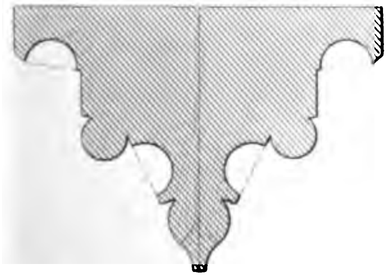

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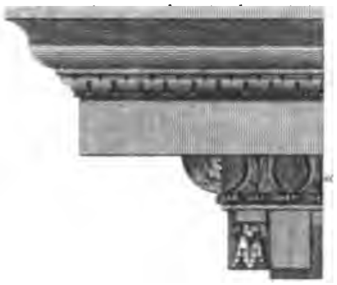
OVALS, ELLIPSES, PA

Fig. 1.



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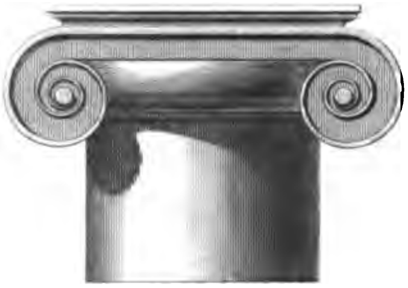

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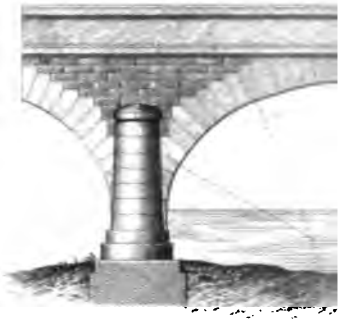

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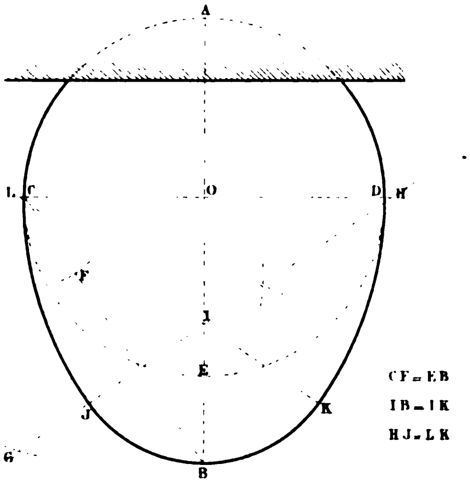

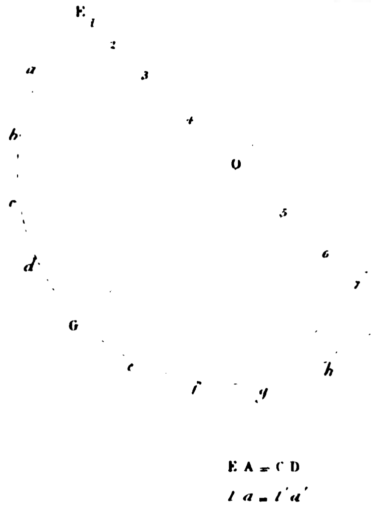

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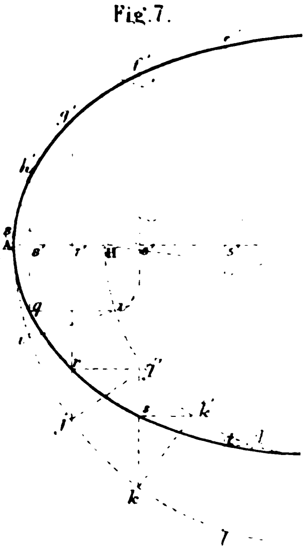


Fig. 6.

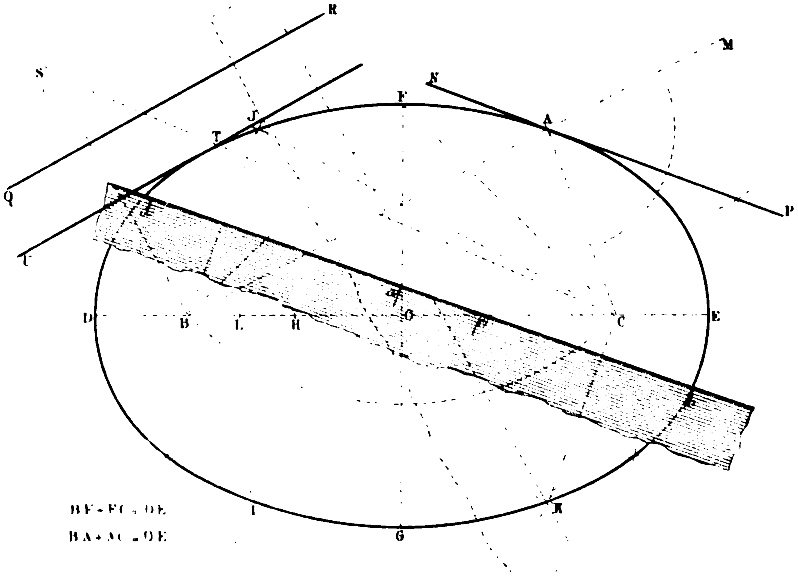
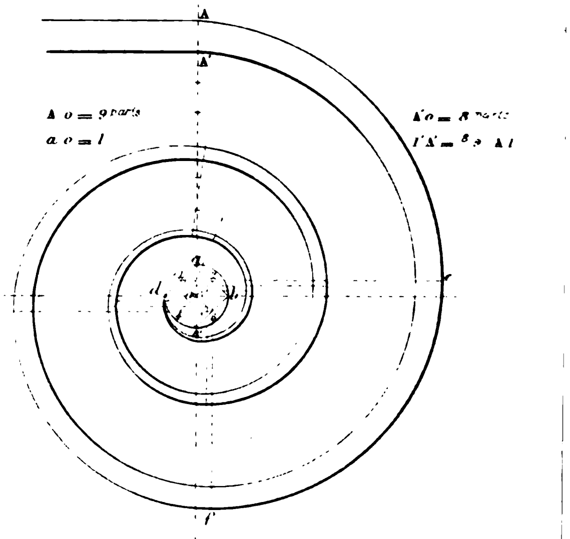


Fig. 10.



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ABOLAS & VOLUTES.

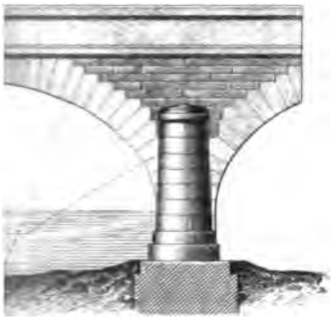
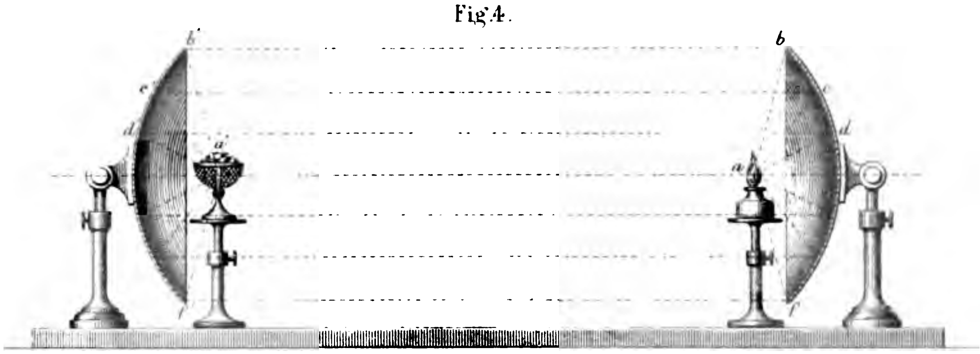


Fig.4.



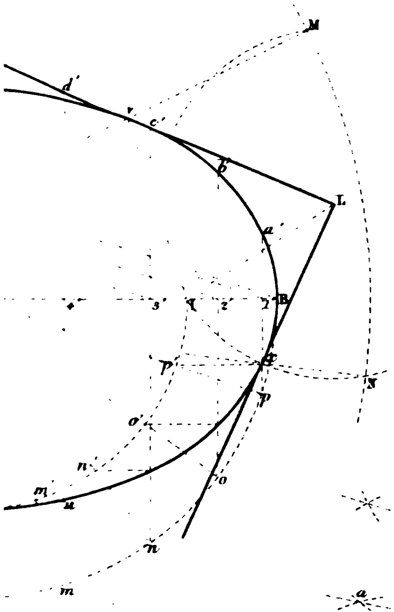


Fig.8.

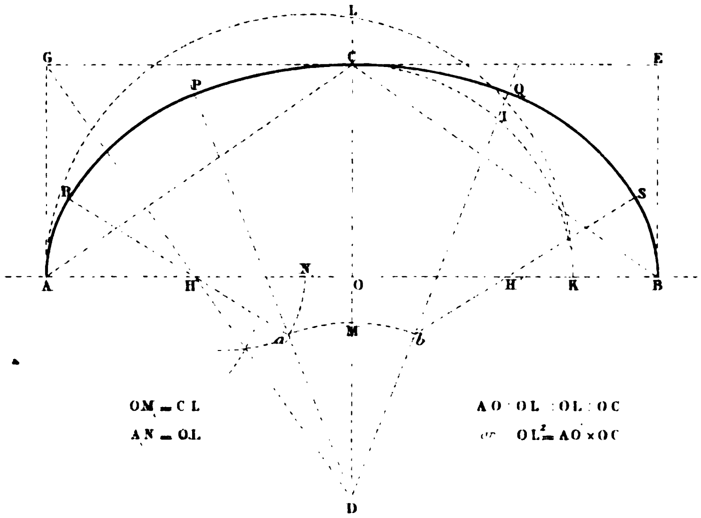


Fig.11.

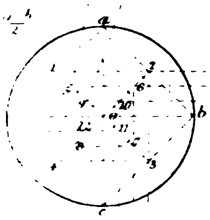


Fig.13.

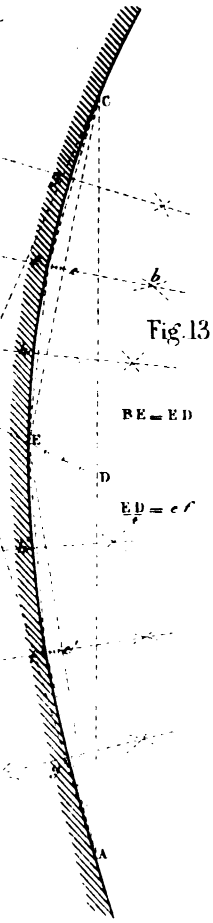


Fig.9.

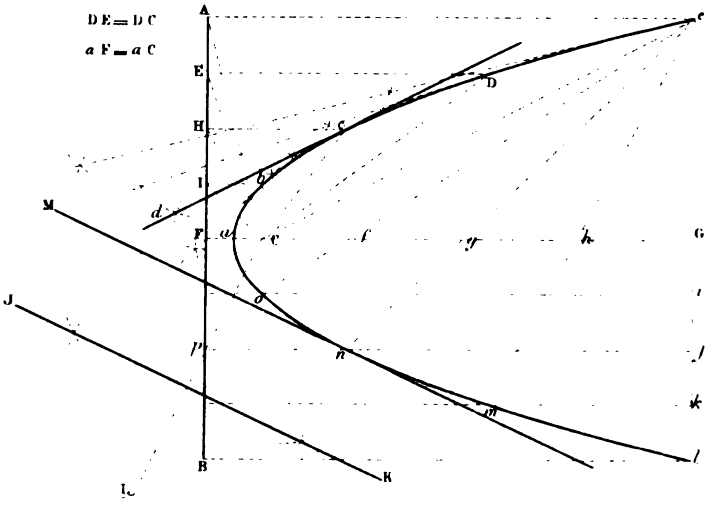
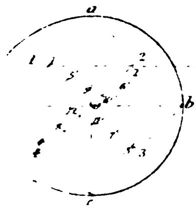


Fig.12.



stretched by a tracer, will form the vectors; and the ellipse will be drawn by carrying the tracer round, keeping the cord always stretched.

57. *Second Method.*—Take a strip of paper, having one edge straight, as *d b*, and on this edge mark a length, *a b*, equal to half the transverse axis, and another length, *b c*, equal to half the conjugate axis. Place the strip of paper so that the point, *a*, of the longer measurement lies on the conjugate, F G, and the other point, *c*, on the transverse axis, D E. If the strip be now caused to rotate, always keeping the two points, *a* and *c*, on the respective axes, the other point, *b*, will, in every position, indicate a point in the circumference which may be marked with a pencil, the ellipse being afterwards traced through the points thus obtained.

58. *Third Method*, Fig. 7.—It is demonstrated, in that branch of geometry which treats of solids, as will be seen farther on, that if a cone, or cylinder, be cut by a plane inclined to its axis, the resulting section will be an ellipse. It is on this property that the present method is based. The transverse and conjugate axes being given, as A B and C D, cutting each other in the centre, O, draw any line, A E, equal in length to the conjugate axis, C D, and on A E, as a diameter, describe the semicircle, E G A. Join E B, and through any number of points, taken at random, on E A, as 1, 2, 3, &c., draw parallels to E B. Then, at each point of division, on E A, erect perpendiculars, 1 *a*, 2 *b*, 3 *c*, &c., cutting the semicircle; and, at the corresponding divisions obtained on A B, erect perpendiculars, as $1^1\,a^1$, $2^1\,b^1$, $3^1\,c^1$, &c., and make them equal to the corresponding perpendiculars on E A. A line traced through the various points thus obtained—that is, the extremities, a^1, b^1, c^1, &c., of the lines—will form the required ellipse.

59. *Fourth Method.*—On the transverse axis, A B, and with the centre O, describe the semicircle, A F B, the axis forming its diameter; and with the diameter, H I, equal to the conjugate axis, describe the smaller semicircle, H D I. Draw radii, cutting the two semicircles, the larger in the points, *i*, *j*, *k*, *l*, &c., and the smaller in the points, i^1, j^1, k^1, l^1, &c. It is not necessary that the radii should be at equiangular distances apart, though they are drawn so in the plate for regularity's sake. Through the latter points draw parallels to the transverse axis, A B, and through the former, parallels to the conjugate axis, C D, the points of intersection of these lines, as *q*, *r*, *s*, *t*, &c., will be so many points in the required ellipse, which may, accordingly, be traced through them. It follows from this, that, in order to draw an ellipse, it is sufficient to know either of the axes, and a point in the circumference. Let the axis, A B, be given, and a point, *r*, in the circumference, which must always lie within perpendiculars passing through the extremities of the given axis. Through *r* draw a line, $r\,j^1$, parallel to A B, and a line, *r j*, perpendicular to it; with the centre, O, and radius, O A, equal half the given axis, describe the arc, cutting *r j* in *j*; join *j* O, and the line *j* O will cut $r\,j^1$ in j^1: O j^1 will be equal to half the conjugate axis, C D. If the conjugate axis, C D, be given, proceed as before; the arc, however, in this case, having the smaller radius, O D, and cutting $r\,j^1$ in j^1; then join O j^1, producing the line till it cuts *r j*, which will be in *j*, and O *j* will equal half the transverse axis, A B. It has already been shown how to describe an ellipse, when the two axes are given.

We may here give a method invented a short time back by Mr. Crane, of Birmingham, for constructing an ellipse with the compasses. This method applies to all proportions, and produces as near an approximation to a true ellipse as it is possible to obtain by means of four arcs of circles.

By applying the compasses to any true ellipse, it will be seen that certain parts of the curve approach very near to arcs of circles, and that these parts are about the vertices of its two axes; and by the nature of an ellipse, the curve on each side of either axis is equal and similar; consequently, if arcs of circles be drawn through all the vertices, meeting one another in four points, the opposite arcs being equal and similar, the resulting figure will be indefinitely near an ellipse. Four circles, described from four different points, but with only two different radii, are then required. These four points may be all within the figure; the centres of the two greater circles may either be within or without, but the centres of the two circles at the extremities of the major axis must always be within, and, consequently, the whole four points can never be without the figure. Again, the proportions of the major and minor axes may vary infinitely, but they can never be equal; therefore, any rule for describing ellipses must suit all possible proportions, or it does not possess the necessary requirements. Moreover, if any rule apply to one certain proportion and not to another, it is evident that the more the proportions differ from that one, whether greater or smaller, the greater will be the difference of the result from the true one. From this it follows, that if a rule applies not to all, it can only apply to one proportion; and, also, that if it apply to a certain proportion and not to another, it can only be correct in that one case.

Let A B be any major axis, and C D any minor axis, in the accompanying Figs. 14 and 15; produce them both in either direction, say towards F and H, and make A F equal to C G; then join C A, and through F draw F H parallel to C A. Set off B I, A J, and C K, equal to H C; join J K, and bisect it in R, and at R erect a perpendicular, cutting C D, or C D produced, at M; then make G E equal to G M; J, E, I, M, will be the centres of the four circles required. Through the points, J and I, draw M N, M O, E P, and E Q, each equal to M C; then M N and E P will be the radii of the greater circles, and J N, I O, of the less: the points of contact will therefore be at N, O, P, Q, and the figure drawn through A, N, C, O, B, Q, D, P, will be the required ellipse.

Fig. 14.

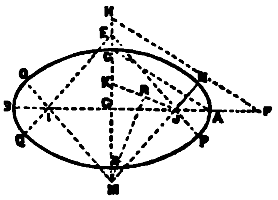
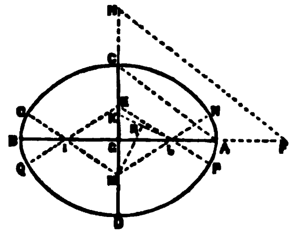


Fig. 15.



60. *Tangents to ellipses.*—It is frequently necessary to determine the position and inclination of a straight line which shall be a tangent to an elliptic curve. Three cases of this nature occur: when a point in the ellipse is given; when the given point is outside the ellipse; and when a straight line is given, to which it is necessary that the tangent should be parallel.

First, then, let the point, A, in the ellipse, Fig. 6, be given; draw the two vectors, C A, B A, and produce the latter to M; bisect the angle, M A C, by the straight line, N P; this line, N P, will be the tangent required; that is, it will touch the curve in the point, A, and in that point alone.

Secondly, let the given point, L, be outside the ellipse, Fig. 7. Join L with I, the nearest focus to it, and with L as a centre, and a radius equal to L I, describe an arc, M I N. Next, with the more distant focus, H, as a centre, and with a radius equal to the transverse axis, A B, describe a second arc, cutting the first in M and N. Join M H and N H, and the ellipse will be cut in the points *v* and *x*; a straight line drawn through either of these points from the given point, L, will be a tangent to the ellipse.

61. Thirdly, let the straight line, Q R, Fig. 6, be given, parallel to which it is required to draw a tangent to the ellipse. From the nearest focus, B, let fall on Q R the perpendicular, S B; then with the further focus, C, as a centre, and with a radius equal to the transverse axis, D E, describe an arc cutting B S in S; join C S, and the straight line, C S, will cut the ellipse in the point, T, of contact of the required tangent. All that is then necessary is, to draw through that point a line parallel to the given line, Q R, the accuracy of which may be verified by observing whether it bisects the line, S B, which it ought to do.

62. *The oval of five centres*, Fig. 8.—As in previous cases, the transverse and conjugate axes are given, and we commence by obtaining a mean proportional between their halves; for this purpose, with the centre, O, and the semi-conjugate axis, O C, as radius, we describe the arc, C I K, and then the semi-circle, A L K, of which A K is the diameter, and further prolong O C to L, O L being the mean proportional required. Next construct the parallelogram, A G C O, the semi-axes constituting its dimensions; joining C A, let fall from the point, G, on the diagonal, C A, the perpendicular, G H D—which, being prolonged, cuts the conjugate axis or its continuation in D. Having made C M equal to the mean proportional, O L, with the centre, D, and radius, D M, describe an arc, *a* M *b*; and having also made A N equal to the mean proportional, O L, with the centre, H, and radius, H N, describe the arc, N *a*, cutting the former in *a*. The points, H, *a*, on one side, and H^1, *b*, obtained in a similar manner on the other, together with the point, D, will be the five centres of the oval; and straight lines, R H *a*, S H^1 *b*, and P *a* D, Q *b* D, passing through the respective centres, will meet the curve in the points of junction of the various component arcs, as at R, P, Q, S.

This beautiful curve is frequently adopted in the construction of arches, bridges, and vaults; an example of its use is given in Fig. 3.

63. The *parabola*, Fig. 9, is an open curve, that is, one which does not return to any assumed starting point, to however great a length it may be extended; and which, consequently, can never enclose a space. It is so constituted, that any point in it, D, is at an equal distance from a constant point, C, termed the *focus*, and in a perpendicular direction, from a straight line, A B, called the *directrix*. The straight line, F G, perpendicular to the directrix, A B, and passing through the focus, C, is the *axis* of the curve, which it divides into two symmetrical portions. The point, *a*, midway between F and C, is the apex of the curve. There are several methods of drawing this curve.

64. First method:—This is based on the definition just given, and requires that the focus and directrix be known, as C, and A B. Take any points on the directrix, A B, as A, E, H, I, and through them draw parallels to the axis, F G, as also the straight lines, A C, E C, H C, I C, joining them with the focus. Draw perpendiculars bisecting these latter lines, and produce them until they cut the corresponding parallels, and the points of intersection, *b*, *c*, D, *e*, will be in the required curve, which may be traced through them.

65. The straight lines which were just drawn, cutting the parallels in different points of the curve, are tangents to the curve at the several points. If, then, it is required to draw a tangent through a given point, *c*, it is obtained simply by joining *c* C, making H *c* equal to *c* C, and bisecting the angle, H *c* C, by the straight line, *c d*, which will be the required tangent. If the point given be apart from the curve, the procedure will be the same, but the line coresponding to H *c* will not be parallel to the axis.

66. Second method:—We have here given the axis, *a* G, the apex, *a*, and any point, *l*, in the curve. From the point, *l*, let fall on the axis the perpendicular, *l* G, and prolong this to *e*, making G *e* equal to *l* G. Divide *l* G into any number of equal parts, as in the points, *i*, *j*, *k*, through which draw parallels to the axis; divide also the axis, *a* G, into the same number of equal parts, as in the points, *f*, *g*, *h*; through these draw lines radiating from the point, *e*, and they will intersect the parallels in the points, *m*, *n*, *o*, which are so many points in the curve.

67. If it is required to draw a line tangent to a given parabola, and parallel to a given line, J K, we let fall a perpendicular, C L, on this last; this perpendicular will cut the directrix in *p*, and *p n* drawn parallel to the axis will cut the curve in the point of contact, *n*.

We find frequent applications of this curve in constructions and machinery, on account of the peculiar properties it possesses, which the student will find discussed as he proceeds.

68. The objects represented in Fig. 4 are an example of the application of this curve. They are called *Parabolic Mirrors*, and are employed in philosophical researches. The angles of incidence of the vectors, *a b*, *a c*, *a d*, are equal to the angles of reflection of the parallels, *b* b^1, *c* c^1, *d* d^1. It follows from this property, that if in the focus, *a*, of one mirror, *b f*, the flame of a lamp, or some incandescent body be placed, and in the focus, a^1, of the opposite mirror, $b^1 f^1$, a piece of charcoal or tinder, the latter will be ignited, though the two foci may be at a considerable distance apart; for all the rays of caloric falling on the mirror, *b f*, are reflected from it in parallel lines, and are again collected by the other mirror, *b* f^1 and concentrated at its focus, a^1.

69. *To draw an Ionic volute*, Fig. 10.—The vertical, A *o*, being given, and this being the length from the summit to the centre of the volute, divide it into nine equal parts, and with the centre, *o*, and a radius equal to one of these parts, describe the circle, *a b c d*, which forms what is termed the *eye* of the volute. In this circle, represented on a larger scale in Fig. 11, inscribe a square, its diagonals being vertical and horizontal; through the centre, *o*, draw the lines 1—3, and 4—2, parallel to the sides, and divide the half of each into three equal parts. With the point, 1, as a centre, and the radius, 1 A, Fig. 10, draw the arc, A *e*, extending to the horizontal line, 1 *e*, which passes through the point, 2. With this latter point as a centre, and a radius equal to 2 *e*, draw the next arc, extending to the vertical line, 2 *f*, which passes through the point, 3, the next centre. The points, 4, 5, 6, &c., form the subsequent centres; the arcs in all cases joining each other on a line passing through their respective centres. The internal curve is drawn in the same way; the points, 1^1, 2^1, 3^1, &c., Fig. 12, being the centres of the component arcs. The first arc is drawn with a radius, 1^1 A^1, a ninth less than 1 A, and the others are consequently proportionately reduced, as manifest in Fig. 10. An example of the application of the volute will be found in the capital of an Ionic column at Fig. 2. An instrument for drawing volutes was illustrated and described at Fig. 49, page 19.

70. *To draw a curve tangentially joining two straight lines*, A B *and* B C, Fig. 13, *the points*, A *and* C *being the points of junction.*—Join A C, and bisecting A C in D, join D with B, the point of inter-

section of the lines A B, B C. Bisect B D in E, which will be a point in the curve. Join E C, E A, and bisect the lines, E C, E A, by the perpendiculars, $a\ b\ c\ d$; make $e\,f$, and $e^1\,f^1$, equal to a fourth part of E D; f and f^1 will be other two points in the curve. Proceed in the same way to obtain the points, $g\ h$, and $g^1\ h^1$, or more if desirable, and then trace the curve through these several points. This method is generally adopted by engineers and constructors, and examples of the curve will be met with in railways, bridges, and embankments, and wherever it is necessary to connect two straight lines by as regular and perfect a curve as possible. It is also particularly applicable where the scale is large.

RULES AND PRACTICAL DATA.

LINES AND SURFACES.

71. The standard British unit of length in ascertaining the dimensions of bodies is the yard; but in calculations as to motions and forces the unit of length commonly used is the foot, the third part of a yard; and in expressing the dimensions of machinery the unit of length usually employed is the inch—the twelfth part of a foot. The fractions of an inch are stated either in eighths of an inch or in decimal parts. Calculations would be much facilitated if all our measures were constructed on the decimal principle.

A square yard contains nine square feet and 1296 square inches. A square foot contains 144 square inches.

72. ***Measurement of surfaces.***—The surface or area of a square, as well as of all rectangles and parallelograms, is expressed by the product of the base or length, and height or breadth measured perpendicularly from the base. Thus the area of a rectangle, the base of which measures 12 feet and the height 4 feet, is equal to

$$12 \times 4 = 48 \text{ square feet.}$$

The area of a rectangle being known, and one of its dimensions, the other dimension may be obtained by dividing the area by the given dimension.

Example.—The area of a rectangle being 48 square feet, and the base 12 feet, the height is

$$\frac{48}{12} = 4 \text{ feet.}$$

This operation is constantly needed in actual construction; as, for instance, when it is necessary to make a rectangular aperture of a certain area, one of the dimensions being predetermined.

The area of a *trapezium* is equal to the product of half the sum of the parallel sides into the perpendicular breadth.

Example.—The parallel sides of a trapezium being respectively 5 feet and 7 feet, and the breadth 8 feet, the area will be

$$\frac{5+7}{2} \times 8 = 48 \text{ feet.}$$

The area of a *triangle* is obtained by multiplying the base by half the perpendicular height.

Example.—The base of a triangle being 12 inches, and the perpendicular height 10 inches, the area will be

$$12 \times \frac{10}{2} = 60 \text{ sq. inches.}$$

The area of a triangle being known, and one of the dimensions given—that is, the base or the perpendicular height—the other dimension can be ascertained by dividing double the area by the given dimension. Thus, in the above example, the division of 60 sq. inches × 2, by the height 10 inches, gives for quotient the base 12 inches, and its division by the base 12 inches gives the height 10 inches.

73. It is demonstrated in geometry that the square of the hypothenuse, or longest side of a right-angled triangle, is equal to the sum of the squares of the two sides forming the right angle. It follows from this property, that if any two of the sides of a right-angled triangle be given, the third may be at once ascertained.

First, if the sides forming the right angle be given, the hypothenuse is determined by adding together their squares, and extracting the square root.

Example.—The side, A B, of the triangle, A B C, Fig. 16, Plate 1, being 3 feet, the side, B C, 4 feet, the hypothenuse, A C, will be

$$\text{A C} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ feet.}$$

Secondly, If the hypothenuse, as A C, be known, and one of the other sides, as A B, the third side, B C, will be equal to the square root of the difference between the squares of A C and A B.

Thus, assuming the above measures—

$$\text{B C} = \sqrt{25 - 9} = \sqrt{16} = 4 \text{ feet.}$$

The diagonal of a square is always equal to one of the sides multiplied by $\sqrt{2}$; therefore, as $\sqrt{2} = 1{\cdot}414$ nearly, the diagonal is obtained by multiplying a side by 1·414.

Example.—The side of a square being 6 feet, its diagonal

$$= 6 \times 1{\cdot}414 = 8{\cdot}484 \text{ feet.}$$

The sum of the squares of the four sides of a parallelogram are equal to the sum of the squares of its diagonals.

74. ***Regular polygons.***—The area of a regular polygon is obtained by multiplying its perimeter by half the *apothem* or perpendicular, let fall from the centre to one of the sides.

TABLE OF ANGLES AND MULTIPLIERS FOR REGULAR POLYGONS OF FROM 3 TO 12 SIDES.

NAMES.	No. of Sides.	A Length of Radius, Apothem = 1.	B Length of Side, Radius = 1.	C Length of Radius, Side = 1.	D Multiplier for Areas, Side = 1.	E Angle of Centre	F Internal Angle of Adjacent Sides.	G Length of Apothem, Radius = 1.
Triangle, Trigon,..................................	3	2·000	1·730	·579	·433	120° 0′	60° 0′	·5000
Square, Tetragon,..................................	4	1·414	1·412	·705	1·000	90° 0′	90° 0′	·7071
Pentagon,..................................	5	1·238	1·174	·852	1·720	72° 0′	108° 0′	·8090
Hexagon,..................................	6	1 156	radius.	side.	2·598	60° 0′	120° 0′	·8660
Heptagon,..................................	7	1·111	·867	1 160	3·634	51° 25′ $\frac{5}{7}$	128° 34′ $\frac{2}{7}$	·9010
Octagon,..................................	8	1·080	·765	1·307	4·828	45° 0′	135° 0′	·9239
Enneagon,..................................	9	1·062	·681	1·470	6·182	40° 0′	140° 0′	·9397
Decagon,..................................	10	1·050	·616	1·625	7·694	36° 0′	144° 0′	·9511
Hendecagon,..................................	11	1·040	·561	1 777	9·365	32° 43′ $\frac{7}{11}$	147° 16′ $\frac{4}{11}$	·9595
Dodecagon,..................................	12	1 037	·516	1·940	11·196	30° 0′	150° 0′	·9659

A regular polygon of 5 sides, one of which is 9 inches, and the perpendicular distance from the centre to one of the sides 6 inches, will have for area—

$$9 \times 5 \times \frac{6}{2} = 135 \text{ inches.}$$

The area of an irregular polygon may be obtained by dividing it into triangles, rectangles, or trapeziums, and then adding together the areas of the various component figures.

By means of this table, we can easily solve many interesting problems connected with regular polygons, from the triangle up to the dodecagon. Such are the following:—

First, *The width of a polygon being given, to find the radius of the circumscribing circle.*—When the number of sides is even, the width is understood as the perpendicular distance between two opposite and parallel sides; when the number is uneven, it is twice the perpendicular distance from the centre to one side.

Rule.—Multiply half the width of the polygon by the factor in column A, corresponding to the number of sides, and the product will be the required radius.

Example.—Let 18·5 feet be the width of an octagon; then,

$$\frac{18{\cdot}5}{2} \times 1{\cdot}08 = 9{\cdot}99 \text{ feet.}$$

or say 10 feet, the radius of the circumscribing circle.

Second, *The radius of a circle being given, to find the length of the side of an inscribed polygon.*

Rule.—Multiply the radius by the factor in column B, corresponding to the number of sides of the required polygon.

Example.—The radius being 10 feet, the side of an inscribed octagon will be—

$$10 \times {\cdot}765 = 7{\cdot}65 \text{ feet.}$$

Third, *The side of a polygon being given, to find the radius of the circumscribing circle.*

Rule.—Multiply the side by the factor in column C, corresponding to the number of sides.

Example.—Let 7 feet be the side of an octagon; then,

$$7 \times 1{\cdot}307 = 8{\cdot}149 \text{ feet.}$$

Fourth, *The side of a polygon being given, to find the area.*

Rule.—Multiply the given side by the factor in column D, corresponding to the number of sides.

Example.—The side of an octagon being 7 feet, the area will be—

$$7 \times 4{\cdot}828 = 33{\cdot}796 \text{ sq. feet.}$$

Fifth, *Required to describe a pentagon in a given circle.*—Take 1·174 in column B from the scale, the unit of which is equal to the radius, and step round the circumference five times. The angular points of the pentagon will thus be found.

Sixth, *Required to describe a heptagon on a given line.*—The given line being the unit of a scale, take off from it the distance 1·160 in column C, and construct on the line an isosceles triangle, having that distance for the sides. The vertex will be the centre of the circle circumscribing the required heptagon.

Seventh, *Required to ascertain the perimeter of a decagon.*—The radius of the circle being unity, multiply the number ·616 in column B by 10, and the product will be the perimeter.

THE CIRCUMFERENCE AND AREA OF A CIRCLE.

75. If the circumference of any circle be divided by its diameter, the quotient will be a number which is called *the ratio of the circumference to the diameter.* This ratio cannot be expressed in whole numbers, but it is found to be, approximately—

$$3{\cdot}1416, \text{ or } 22 : 7;$$

that is, the circumference equals 3·1416 times the length of the diameter. This number is expressed, in algebraic formulas, by the Greek letter π (*pi*). Thus, if C represents the circumference of a circle, and D its diameter, the formula,

$$C = \pi D, \text{ or } C = 3{\cdot}1416 \times D,$$

expresses the development of the circumference. For example, if the diameter of a circle, or D, = 24 feet, or the radius, R = 12 feet, the circumference will be equal to—

$$3{\cdot}1416 \times 24, \text{ or } 3{\cdot}1416 \times 12 \times 2 = 75{\cdot}3984 \text{ feet.}$$

The circumference of a circle being known, its diameter, or radius is found by dividing this circumference by 3·1416 for the former, or 6·2832 for the latter. Thus, the diameter, D, of a circle, the circumference of which is 75·3984, is—

$$\frac{75{\cdot}3984}{3{\cdot}1416} = 24 \text{ feet.}$$

and the radius, R, is—

$$\frac{75{\cdot}3984}{6{\cdot}2832} = 12 \text{ feet.}$$

The area of a circle is found by multiplying the circumference by half the radius.—This rule is expressed in the following formula:—

$$\text{The area of a circle} = 2\,\pi\,R \times \frac{R}{2} = \pi\,R^2.$$

This term, πR^2, is merely the simplification of the formula. The number 2, being both multiplier and divisor, may be cancelled, and the product of R into R is expressed by R^2, or the square of the radius. It follows, then, that the area of a circle is equal to the square of the radius multiplied by the circumference, or 3·1416.

Example.—The radius of a circle being 12 feet, the area will be—

$$3{\cdot}1416 \times 12 \times 12 = 452{\cdot}3904 \text{ sq. feet.}$$

The area of a circle being known, the radius is determined by dividing the area by 3·1416, and extracting the square root of the quotient.

Example.—The area of a circle being 452·3904 sq. feet, the radius is—

$$\sqrt{\frac{452{\cdot}3904}{3{\cdot}1416}} = 12 \text{ feet.}$$

The area of a circle may be derived from the diameter; thus—

$$\text{Area} = \frac{\pi\,D \times D}{4}, \text{ or } \frac{\pi\,D^2}{4};$$

$$\text{then, since } \frac{\pi}{4} \text{ or } \frac{3{\cdot}1416}{4} = {\cdot}7854,$$

the formula resolves itself into

$$\text{area} = {\cdot}7854 \times D^2.$$

That is to say, if we multiply the fraction, ·7854, by the square of the diameter, the product will be the area.

Example.—The area of a circle, the diameter of which measures 2 feet, is—

$${\cdot}7854 \times 2 \times 2 = 3{\cdot}1316 \text{ sq. feet.}$$

It follows from this, that if the area of a square is known, that of an inscribed circle is obtainable by multiplying by ·7854; that is, the area of a square is, to the area of the inscribed circle, as—

$$4 : 3{\cdot}1416, \text{ or } 1 : {\cdot}7854.$$

TABLE OF FACTORS RELATING TO THE CIRCLE WHEN DIAMETER = 1.

Circumference (π) = 3·1416.	Unity ÷ circumf. = 0·3183.
Circumference × 2 = 6·2832.	2 ÷ circumf. = 0·6366.
Circumference ÷ 2 = 1·5708.	360 ÷ circumf. = 114·5915.
Circumference ÷ 12 = 0·2618.	Area = 0·7854.
Circumference ÷ 360 = 0·0087.	

TABLE OF APPROXIMATE RATIOS BETWEEN CIRCLES AND SQUARES.

1.	The diameter of a circle,	×	·8862	=	the side of a square of nearly equal area.
2.	The circumference of a circle,	×	·2821		
3.	The diameter,	×	·7071	=	the side of the inscribed square.
4.	The circumference,	×	·2251		
5.	The area of a circle,	×	·6366	=	the area of the inscribed square.
6.	The side of an inscribed square,	×	1·4142	=	the diameter of the circumscribing circle.
7.	" " "	×	4·4430	=	the circumference of the circumscribing circle.
8.	The side of a square,	×	1·1280	=	the diameter of an equal circle.
9.	" "	×	3·5450	=	the circumference of an equal circle.

The above table affords a ready solution of the following amongst other problems:—

First, the diameter of a circle being 125 feet, the side of a square of equal area is

$$125 \times {\cdot}8862 = 110{\cdot}775 \text{ feet.}$$

Second, The circumference of a circle being 860 feet, the side of the inscribed square is

$$860 \times {\cdot}2251 = 193{\cdot}586 \text{ feet.}$$

Third, The side of a square being 215·86 feet, the diameter of the circumscribing circle is

$$215{\cdot}86 \times 1{\cdot}4142 = 305{\cdot}27 \text{ feet.}$$

The radii and diameters of circles are to each other as the circumferences, and *vice versâ*. The areas, therefore, of circles are to each other as the squares of their respective radii or diameters.

It follows, that if the radius or diameter be doubled, the circumference will only be doubled, but the area will be quadrupled; thus, a drawing reduced to one-half the length, and half the breadth, only occupies a quarter of the area of that from which it is reduced.

76. *Sectors—Segments.*—In order to obtain the area of a sector or segment, it is necessary to know the length of the arc subtending it. This is found by multiplying the whole circumference by the number of degrees contained in the arc, and dividing by 360°.

Example.—The circumference of a circle being 12 feet, an arc of 45° will be

$$\frac{12 \times 45}{360} = 1{\cdot}5 \text{ feet.}$$

The length of an arc may be obtained approximately when the chord is known, and the chord of half the arc, by subtracting the chord of the whole arc from eight times the chord of the semi-arc, and taking the third of the remainder.

Example.—The chord of an arc being 344 inches, and that of half the arc 198 inches, the length of the arc is

$$\frac{198 \times 8 - 344}{3} = 413{\cdot}33 \text{ inches.}$$

The area of a sector is equal to the length of the arc multiplied into half the radius.

Example.—The radius being 10 inches, and the arc 4·8;

$$\frac{10 \times 4{\cdot}8}{2} = 24 \text{ inches, the area of the sector.}$$

The area of a segment is obtained by multiplying the width; that is, the perpendicular between the centre of the chord, and the centre of the arc, by ·626, then adding to the square of the product the square of half the chord, and multiplying twice the square root of the sum by two-thirds of the width.

Example.—Let 48 inches be the length of the chord of the arc, and 18 inches the width of the arc, then we have

$18 \times {\cdot}626 = 11{\cdot}268$, and $(11{\cdot}268)^2 = 126{\cdot}9678$; whilst $\left(\frac{48}{2}\right)^2 = 576$; therefore, $2 \times \sqrt{126{\cdot}9678 + 576} \times \frac{2 \times 18}{3} = 636{\cdot}24$ sq. inches, the area of the segment.

The area of a segment may also be obtained very nearly by dividing the cube of the width by twice the length of the chord, and adding to the quotient the product of the width into two thirds of the chord. Thus with the foregoing data, we have

$$\frac{18^3}{48 \times 2} = 60{\cdot}75$$

$$\text{and, } \frac{48 \times 2 \times 18}{3} = \underline{636{\cdot}75}$$

Total, 636·75 sq. inches.

A still simpler method, is to obtain the area of the sector of which the segment is a part, and then subtract the area of the triangle constituting the difference between the sector and segment.

To find the area of an annular space contained between two concentric circles, multiply the sum of the diameters by their difference, and by the fraction ·7854.

Example.—Let 100 inches and 60 inches be the respective diameters; then, $(100 + 60) \times (100 - 60) \times {\cdot}7854 = 5026{\cdot}56$ sq. inches, the area of the angular space.

The area of a fragment of such annular space will be found by multiplying its radial breadth by half the sum of the arcs, or, more correctly, by the arc which is a mean proportional to them.

CIRCUMFERENCE AND AREA OF AN ELLIPSE.

77. The circumference of an ellipse is equal to that of a circle, of which the diameter is a mean proportional between the two axes; therefore it will be obtained by multiplying such mean proportional by 3·1416, the ratio between the diameter and circumference of a circle.

Example.—Let 10 inches and 6·4 inches be the lengths of the respective axes; then,

$$\sqrt{10 \times 6{\cdot}4} \times 3{\cdot}1416 = 25{\cdot}1328 \text{ sq. inches.}$$

The area of the ellipse is obtained by multiplying the product of the two axes by ·7854, the ratio between the diameter and the area of the circle.

Example.—$10 \times 6{\cdot}4 \times {\cdot}7854 = -\,50{\cdot}2656$ sq. inches.

These rules meet with numerous applications in the industrial arts, and particularly in mechanics, as will be seen further on. The examples given will enable the student to understand the various operations, as well as to solve other analogous problems.

CHAPTER IV.

THE STUDY OF PROJECTIONS.

78. To indicate all the dimensions of a solid object by pictorial delineation, it is necessary to represent it under several different aspects. These various views are comprehended under the general denomination of *Projections*, and usually consist of *elevations*, *plans*, and *sections*. The object, then, of the study of projections, or *descriptive geometry*, is the reproduction on paper of the appearances presented by any body of several dimensions when viewed from different positions.

It is customary to determine the projections of a body on two principal planes, one of which is distinguished as the *horizontal plane* or *plan*, and the other as the *vertical plane* or *elevation*. These two views are also called *geometric* projections or plans. They are annexed to each other, the horizontal plan being the lower; the line intersecting them is called the *base line*, and is always parallel to one of the sides of the drawing.

It is of great importance to have a knowledge of the elementary principles of descriptive geometry, in order to be able to represent, in precise and determinate forms, the contours of objects; and we shall now enter upon such explanatory details as are necessary, commencing with the projections of a point and of a line.

ELEMENTARY PRINCIPLES.

THE PROJECTIONS OF A POINT.

PLATE 6.

79. The young student ought to lay off the marginal line, A B C D, and also the centre lines, 1 2, and 3 4, as described in reference to Plate 1, on a large sheet of paper, and project the figures given in the first and third rows of this Plate, to a scale of twice or three times that shown in the second or fourth rows. Although this might at first seem too simple a task; yet, still great good will result from the simple practice of projecting these figures, as it will familiarise the mind with the principles, and also with the transfer of the several measurements to their proper plane of projection.

Let A B C D, Figs. 1 and 2, be a horizontal plane—representing, for example, the board on which the drawing is being made, or the surface of a piece of pavement. Also, let A B E F, be a vertical plane, such as a wall at one side of the piece of pavement; the straight line, which is the intersection of these two planes, is the *base line*. These two planes, namely, the horizontal and vertical, are at right angles to each other, and are represented by the same letters in all the figures of this small plate. Finally, let G, be any point in space, the representation of which it is desired to draw. If, from this point, G, we suppose a perpendicular, G g, to be let fall on the horizontal plane, the point of contact, g, or the foot of this perpendicular, will be what is understood as the horizontal projection of the given point. Similarly, if from the point, G, we suppose a perpendicular, G g^1, to be let fall on the vertical plane, A B E F, the point of contact, g^1, or foot of this perpendicular, will be the vertical projection of the same point. These perpendiculars are reproduced in the vertical and horizontal planes, by drawing lines, g^1 a, and a g, respectively parallel and equal to G g, and G g^1.

80. It follows from this construction, that, when the two projections of any point are given, the position in space of the point itself is determinable, it being necessarily the point of intersection of perpendiculars erected on the respective projections of the given point.

In drawing, only one surface is employed, namely, a sheet of paper. We are consequently limited to one and the same plane, and it is customary to make the vertical plane, A B E F, Fig. 1, form a continuation of the horizontal plane, A B C D, being turned on the base line, A B, as on a hinge, so as to coincide with it—just as a book, half open, can be fully opened and laid flat on a table. We thus obtain the figure, D C E F, Fig. 2, representing on the paper the two planes of projection, separated by the base line, A B, and the points, g, g^1, Fig 2, represent the horizontal and vertical projections of the given point.

It will be remarked, that these points lie in one line, perpendicular to the base line, A B; this is because, in the turning down of the previously vertical plane, the line, a g^1, becomes a prolongation of the line, a g. It is necessary to observe, that the line, a g^1, measures the distance of the point from the horizontal plane, whilst a g, measures its distance from the vertical plane. In other words, if on g, we erect a perpendicular to the horizontal plane, and measure the distance, a g^1, on this perpendicular we shall obtain the exact position of the point in space. It is thus obvious that the position of a point in space is determinable by means of two projections, these being in planes at right angles to each other.

THE PROJECTIONS OF A STRAIGHT LINE.

81. As a general rule, if, from several points in the given line, perpendiculars be let fall on to each of the planes of projection, and their points of contact with these planes be joined, the resulting lines will be the respective projections of the given line.

When the line is straight, it will be sufficient to find the projections of its extreme points, and then join these respectively by straight lines.

82. Let G H, Fig. 3, represent a given straight line in space, which we shall suppose to be, in this instance, perpendicular to the horizontal, and, consequently, parallel to the vertical plane of projection. To obtain its projection on the latter, perpendiculars, G g^1, H h^1, must be let fall from its extremities, G, H; the straight line, g^1 h^1, joining the extremities of these perpendiculars, will be the required projection in the vertical plane, and in the present case it will be equal to the given line.

The horizontal projection of the given line, G H, is a mere point, g, because the line lies wholly in a perpendicular, G g, to the plane, and it is the point of contact of this line which constitutes the projection. In drawing, when the two planes are converted into one, as indicated in Fig. 4, the horizontal and vertical projections of the given right line, G H, are respectively the point, g, and the right line, g^1 h^1.

83. If we suppose that the given straight line, G H, is horizontal, and at the same time perpendicular to the vertical plane, as in Figs. 5 and 6, the projections will be similar to the last, but transposed; that is, the point, h^1, will be the vertical, whilst the straight line, g h, will be the horizontal projection.

In both the preceding cases, the projections lie in the same perpendicular line, g^1 g, Fig. 3, and h^1 h, Fig. 6.

DIAGRAMS ON

Fig. 1.

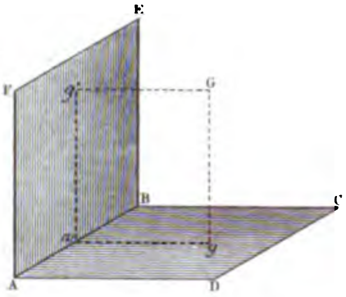


Fig. 3.

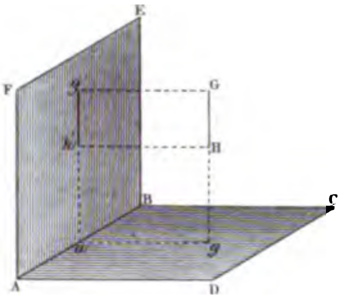


Fig. 5.

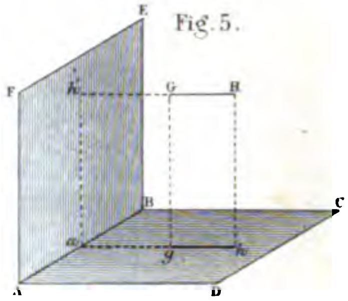
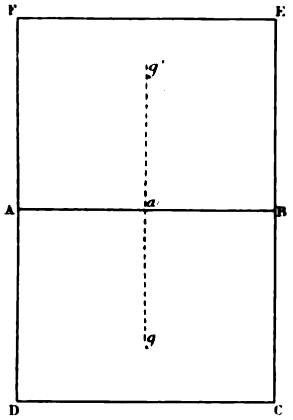


Fig. 2.



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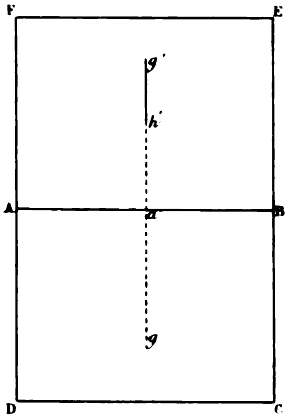


Fig. 6.

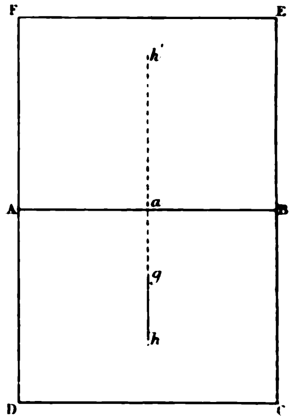


Fig. 13.

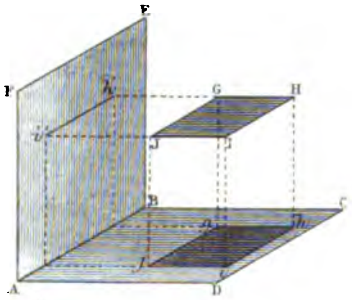


Fig. 15.

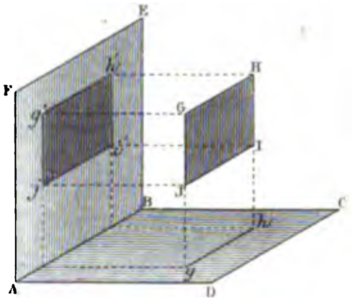


Fig. 17.

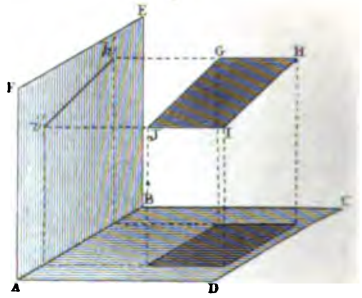


Fig. 14.

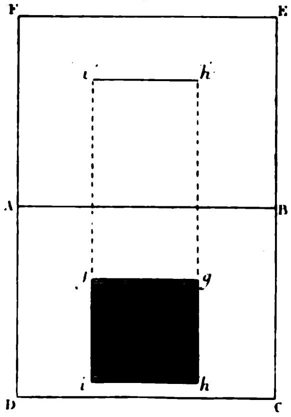
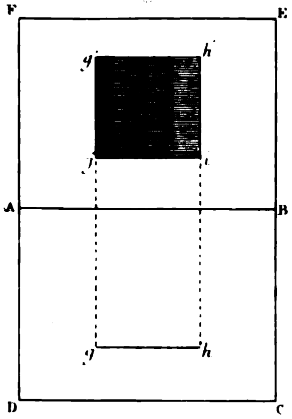


Fig. 16.



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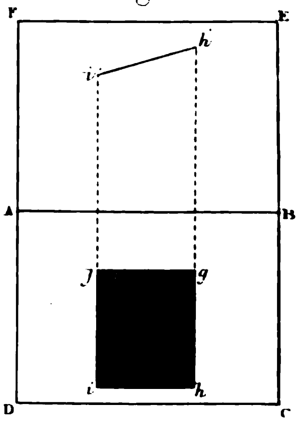

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ROJECTIONS,

Fig.7.

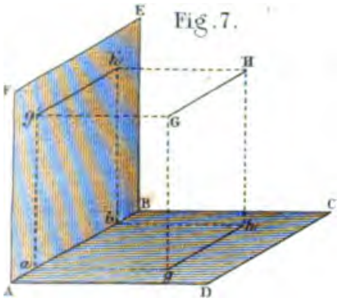


Fig.9.

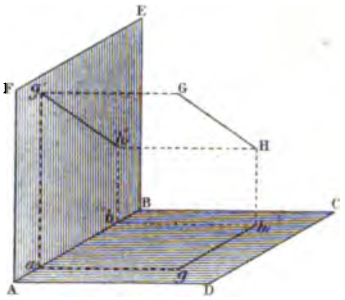


Fig.11.

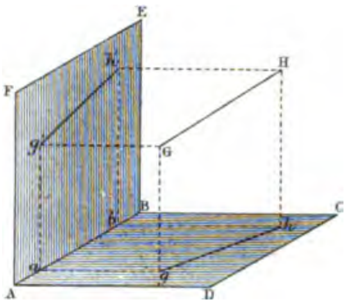


Fig.8.

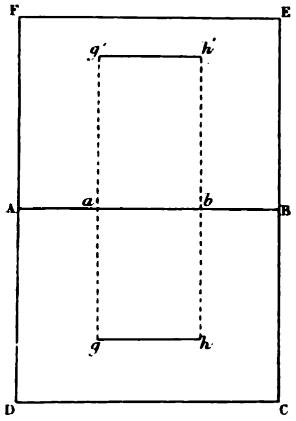


Fig.10.

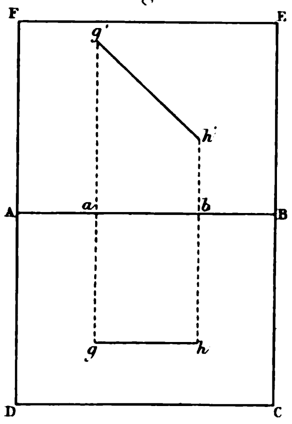


Fig.12.

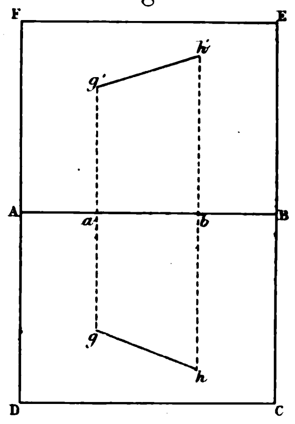


Fig.19.

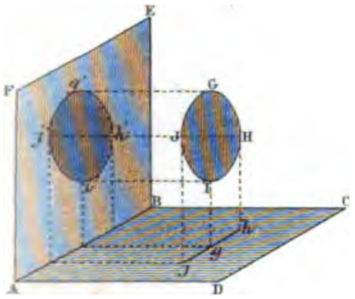
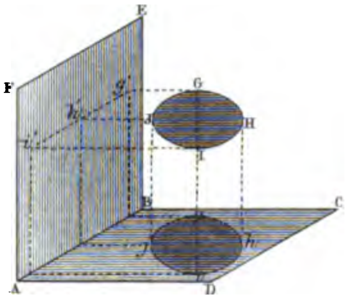


Fig.21.



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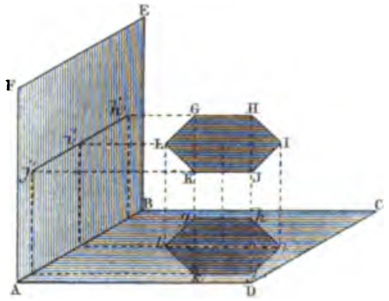


Fig.20.

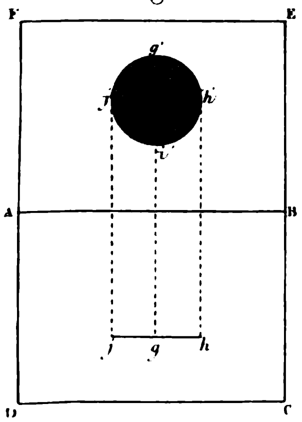
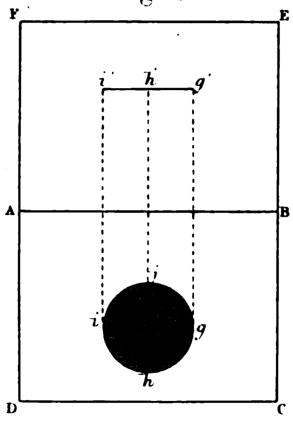


Fig.22.



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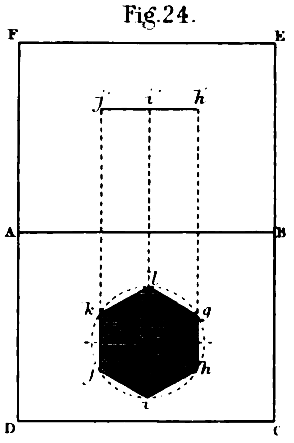

PRISMS & C

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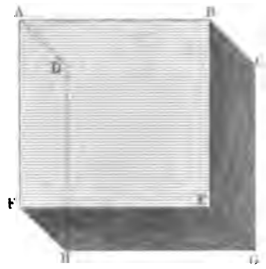
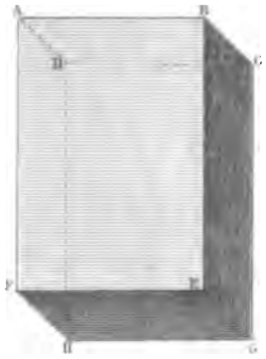


Fig. 4.



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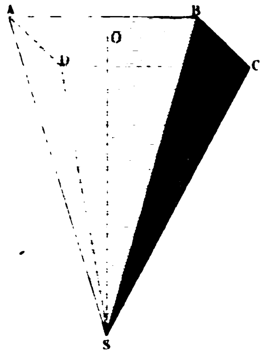

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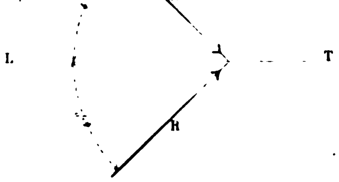

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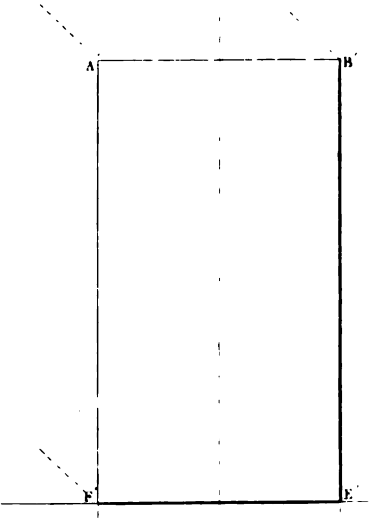

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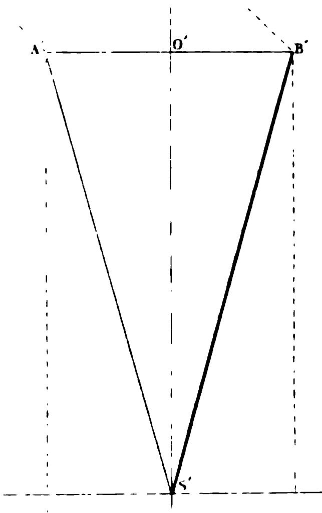


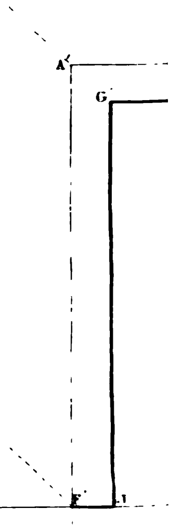
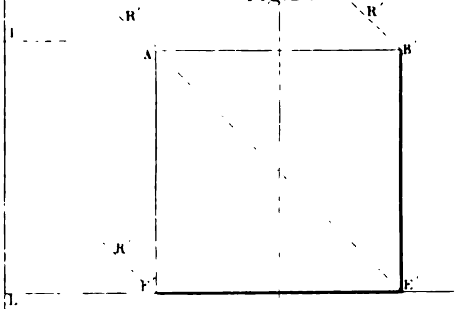


Fig. 2.



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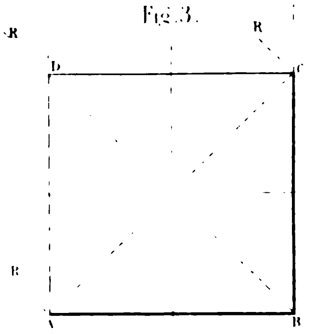

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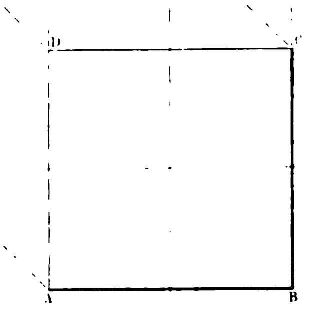
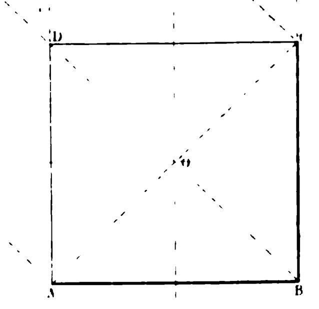
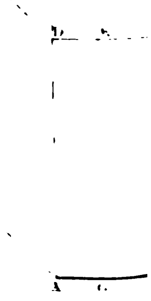


Fig. 9.





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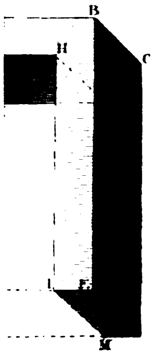


Fig.13.

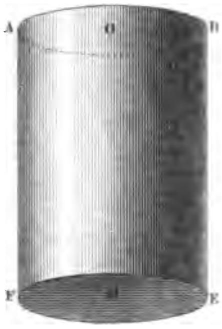


Fig.16.

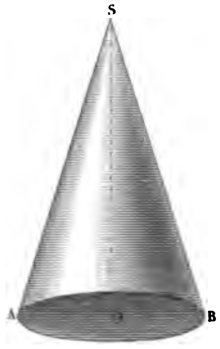


Fig.19.

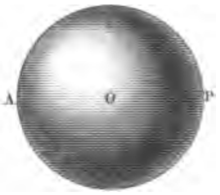
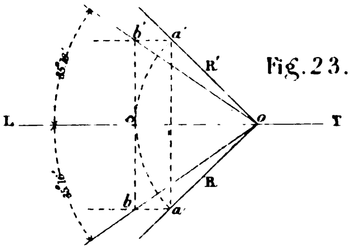


Fig.23.



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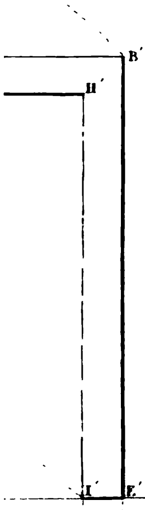


Fig.14.

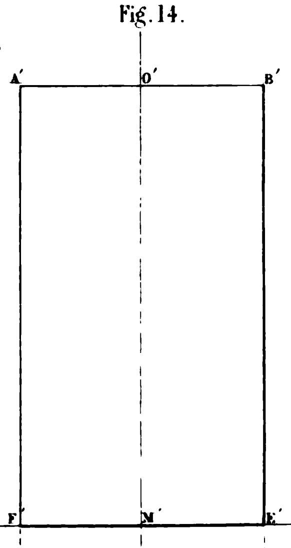


Fig.17.

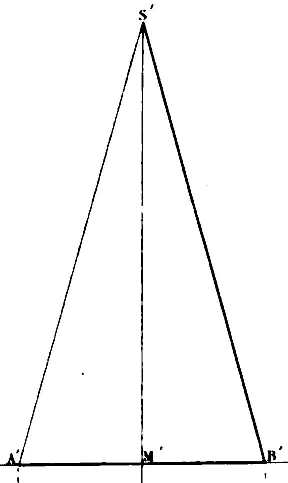
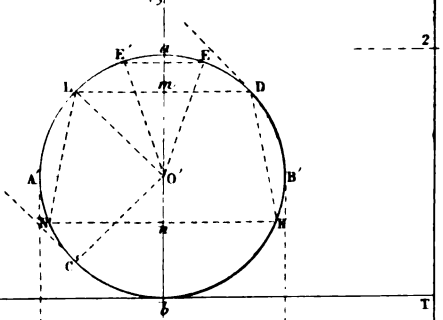


Fig.20.



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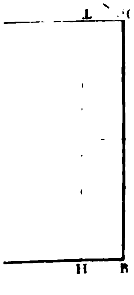


Fig.15.

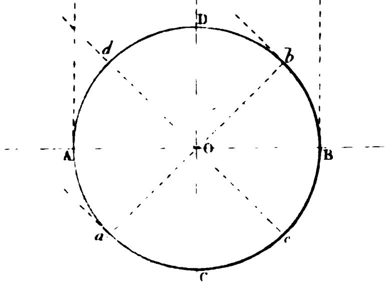


Fig.18.

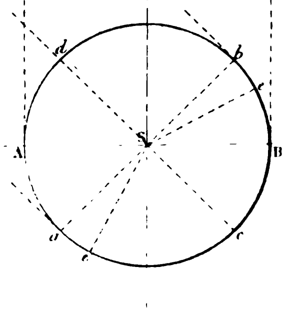
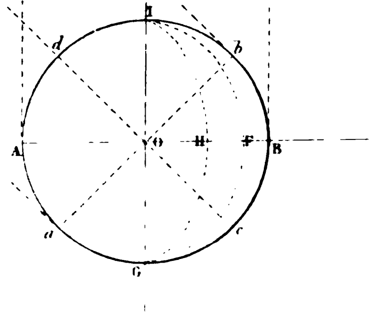


Fig.21.



84. When the given straight line, G H, is parallel to both the horizontal and the vertical plane, as in Figs. 7 and 8, its two projections, *g h* and $g^1\ h^1$, will be parallel to the base line, and they will each be equal to the given line.

85. When the given straight line, G H, Figs. 9 and 10, is parallel to the vertical plane, A B E F, only, the vertical projection, $g^1\ h^1$, will be parallel to the given line, whilst the horizontal projection, *g h*, will be parallel to the base line. Inversely, if the given straight line be parallel to the horizontal plane, its horizontal projection will be parallel to it, whilst its vertical projection will be parallel to the base line.

86. Finally, if the given straight line, G H, Figs. 11 and 12, is inclined to both planes, the projections of it, *g h*, $g^1\ h^1$, will both be inclined to the base line, A B. These projections are in all cases obtained by letting fall, from each extremity of the line, perpendiculars to each plane.

The projections of a straight line being given, its position in space is determined by erecting perpendiculars to the horizontal plane, from the extremities, *g h*, of the projected line, and making them equal to the verticals, $a\ g^1$ and $b\ h^1$. The same result follows, if from the points, $g^1\ h^1$, in the vertical plane, we erect perpendiculars, respectively equal to the horizontal distances, *g a* and *h b*. The free extremities of these perpendiculars meet each other at the respective extremities of the line in space.

THE PROJECTIONS OF A PLANE SURFACE.

87. Since all plane surfaces are bounded by straight lines, as soon as the student has learned how to obtain the projections of these, he will be able to represent any plane surface in the two planes of projection. It is, in fact, merely necessary to let fall perpendiculars to each of the planes, from the extremities of the various lines bounding the surface to be represented; in other words, from each of the angles or points of junction of these lines, by which means the corresponding points will be obtained in the planes of projection, which, being joined, will complete the representations. It is by such means that are obtained the projections of the square, G H I J, represented in different positions in Figs. 13, 14; 15, 16; and 17, 18. It will be remarked, that, in the two first instances, the projection is in one or other of the planes an exact counterpart of the given square, because it is parallel to one or other of the planes.

88. Thus, in Fig. 13, we have supposed the given surface to be parallel to the horizontal plane; consequently, its projection in that plane will be a figure, *g h i j*, equal and parallel to itself, whilst the vertical projection will be a straight line, $h^1\ i^1$, parallel to the base line, A B.

89. Similarly, in Fig. 15, the object being supposed to be parallel to the vertical plane, its projection in that plane will be the equal and parallel figure, $g^1\ h^1\ i^1\ j^1$, whilst that in the horizontal plane will be the straight line, *g h*. When the two planes of projection are converted into one, the respective projections will assume the forms and positions represented in Figs. 14 and 16.

90. If the given surface is not parallel to either plane, but yet perpendicular to one or the other, its projection in the plane to which it is perpendicular will still be a straight line, as $h^1\ i^1$, Figs. 17 and 18, whilst its projection in the other plane will assume the form, *g h i j*, being a representation of the object somewhat foreshortened in the direction of the inclination.

The cases just treated of have been those of rectangular surfaces, but the same principles are equally applicable to any polygonal figures, as may be seen in Figs. 23 and 24, which will be easily understood, the same letters in various characters indicating corresponding points and perpendiculars. Nor does the obtainment of the projections of surfaces bounded by curved lines, as circles, require the consideration of other principles, as we shall proceed to show, in reference to Figs. 19 and 20.

91. In the first of these, Fig. 19, the circular disc, G H I J, is supposed to be parallel to the vertical plane, A B E F, and its projection on that plane will be a circle, $g^1\ h^1\ i^1\ j^1$, equal and parallel to itself, whilst its projection on the horizontal plane, A B C D, will be a straight line, *j g h*, equal to its diameter. If, on the other hand, the disc is parallel to the horizontal plane, as in Figs. 21 and 22, its vertical projection will be the straight line, $i^1\ h^1\ g^1$, whilst its horizontal projection will be the circle, *g h i j*.

If the given circular disc be inclined to either plane, its projection in that plane will be an ellipse; and if it is inclined to both planes, both projections will be ellipses. This will be made evident by obtaining the projections of various points in the circumference.

92. When constructing the projections of regular figures, it facilitates the process considerably if projections of the centres and centre lines be first found, as in Figs. 19, 21, and 23.

In general, the projection of all plane surfaces may be found, when it is known how to obtain the projections of points and lines. And, moreover, since solids are but objects bounded by surfaces and lines, the construction of their projections follow the same rules.

PRISMS AND OTHER SOLIDS.

PLATE 7.

93. Before entering upon the principles involved in the representation of solids, the student should make himself acquainted with the descriptive denominations adopted in science and art, with reference to such objects; and we here subjoin such as will be necessary.

DEFINITIONS.—A *solid* is an object having three dimensions; that is, its extent comprises *length*, *width*, and *height*. A solid also possesses magnitude, volume, or capacity.

There are several forms of solids. The *polyhedron* is a solid, bounded by plane surfaces; the *cone*, the *cylinder*, and the *sphere*, are bounded by curved surfaces; and those are termed *solids of revolution*, which may be defined as generated by the revolution of a plane about a fixed straight line, termed the *axis*. Thus, a ring, or annular *torus*, is a solid, generated by the revolution of a circle about a straight line, lying in the plane of the circle, and at right angles to the plane of revolution. A *prism* is a polyhedron, the lateral faces of which are parallelograms, and the ends equal and parallel polygons. A prism is termed *right*, when the lateral faces, or *facets*, are perpendicular to the ends; and it is *regular*, when the ends are regular polygons. A prism is also called a *parallelopiped*, when the ends are rectangles, or parallelograms; and when it is formed of six equal and square facets, it is termed a *cube*, or regular *hexahedron*. This solid is represented in Fig. 1. Other regular polyhedra, besides the cube, are distinguished by appropriate names; as, the *tetrahedron*, the *octahedron*, and the *icosahedron*, which are bounded externally, respectively, by four, eight, and twenty equilateral triangles; and the *dodecahedron*, which is terminated by twelve regular pentagons. A pyramid is a polyhedron, of which all the lateral facets are triangles, uniting in one point, the *apex*, and having, as bases, the sides of a polygon, which is the base of the pyramid, as Fig. 7. The prism and pyramid are triangular, quadrangular, pentagonal, hexagonal, &c., according as the polygons forming the bases are triangles, squares, pentagons, hexagons, &c.

By the *height* of a pyramid is meant the length of a perpendicular let fall from the apex on the base; the pyramid is a *right* pyramid when this perpendicular meets the centre of the base.

A *truncated* pyramid, or the *frustum* of a pyramid, is a solid which may be described as a pyramid having the apex cut off by a plane parallel, or inclined to the base.

The term *cylinder* is sometimes applied to the surface generated by a straight line, revolving about, and at a given distance from, a rectilinear axis to which it is parallel; and sometimes to the solid, generated by a rectangle revolving about one of its sides as an axis. It is called a right cylinder when the axis is perpendicular to the base, as that shown at Fig. 13.

A *cone*, Fig. 16, is a solid generated by an angle revolving about one of the limbs containing that angle as an axis. A *truncated* cone is one which is terminated short of the apex by a plane parallel, or inclined to the base. This solid is also called the *frustum* (*i.e.*, the fragment) of a cone. A cone is said to be right when its base is a circle, and when the axis is at right angles to, and falls in the centre of, the base.

A *sphere* is a solid generated by the revolution of a semicircle about its diameter as an axis, as Fig. 19.

A *spheric sector* is a solid generated by the revolution of a plane sector, as O^1 L E^1 Fig. 20, about an axis, *a b*, which is any diameter of the sphere of which the sector forms a part. When the axis of revolution is exterior to the generating sector, the spheric sector obtained will be annular or zonic. The *zone* described by the arc, L E^1, is the base of the spheric sector. The zone becomes a *spheric arc* when the axis of revolution is one of the radii forming the sector.

A *spheric wedge*, or *ungula*, is any portion, as I H G F, Fig. 21, comprised between two semicircular planes inclined to each other and meeting in a diameter, as I G, of the sphere. That portion of the surface of the sphere which forms the base of the ungula, is termed a *gore*.

A *spheric segment* is any part of a sphere cut off by a plane, and may be considered as a solid of revolution generated by the revolution of a plane segment about its centre line. The plane surface is the base of the segment. When the dividing plane passes through the centre of the sphere, two equal segments are obtained, termed *hemispheres*.

A *segmental annulus* is a solid generated by the revolution of a plane segment, D B^1 K, Fig. 20, about any diameter, *a b*, of the sphere, apart from the segment. D K, is the chord, and *m n*, its projection on the axis, is the height of the segmental annulus.

A *zonic segment* of a sphere is the part, L N K D, of a sphere comprised between two parallel planes.

A *spheric pyramid*, or *pyramidal sector*, is a pyramid of which the base is part of the surface of a sphere, of which the apex is the centre; the base may be termed a *spheric polygon*.

THE PROJECTIONS OF A CUBE, FIG. 1.

94. A cube, of which two sides are respectively parallel to the planes of projection, is represented in these planes by equal squares, A B C D, and A^1 B^1 E^1 F^1, Figs. 1, 2, and 3.

This is indeed but a combination of some of the simple cases already given. We have seen that when a side, such as A B E F, Fig. 1, is parallel to the vertical plane, its projection on the horizontal plane is reduced to a straight line, A B, Fig. 3, its projection on the vertical plane being a figure, A^1 B^1 E^1 F^1, Fig. 2, equal to itself.

Similarly, the side, A B C D, which is parallel to the horizontal plane, is projected on the vertical plane in the line, A^1 B^1, Fig. 2, and by the figure, A B C D, Fig. 3, in the horizontal plane. The sides, A D H F and B C G E, Fig. 1, which are perpendicular to both the horizontal and the vertical plane, are represented in both by straight lines, as A D and B C, Fig. 3, and A^1 F^1 and B^1 E^1, Fig. 2, being respectively in the same straight lines perpendicular to the base line, L T, which should be drawn parallel to the two marginal lines, A B, C D, as before described, or with the square. It will also be perceived, that the base, F E G H, Fig. 1, cannot be represented in the horizontal projection, nor the side, D C G H, in the vertical, since they are respectively immediately behind and hidden by the sides, A D C D and A B E F, represented in the projections by the squares, A B C D, Fig. 3, and A^1 B^1 E^1 F^1, Fig. 2. They are, however, indicated in the planes to which they are perpendicular, by the straight lines, F^1 E^1 and D C.

95. It will be evident from these remarks, that in order to design a cube so that a model may be constructed, it is sufficient to know one of the sides, for all the sides are equal to each other.

When the plans are intended to be used in the actual construction of machinery or buildings, the objects should be represented in the projections as having their principal sides parallel or perpendicular to the horizontal and vertical plane respectively, in order to avoid the *foreshortening* occasioned by an oblique or inclined position of the object with reference to these planes, because the actual measurements of the different parts cannot be readily ascertained where there is such foreshortening.

To obtain, then, the projections of the cube, Fig. 1, a square must be constructed, as A B C D, Fig. 3, having its sides equal to the given side or edge, the sides A B and D C being disposed parallel to the base line; next, the square must be reproduced as at A^1 B^1 E^1 F^1, Fig. 2, on the prolongations of the sides, A D and B C, which are perpendicular to the base line.

THE PROJECTIONS OF A RIGHT SQUARE-BASED PRISM, OR RECTANGULAR PARALLELOPIPED, FIG. 4.

96. The representation of this solid is obtained in precisely the same manner as that of the cube, the sides being supposed to be parallel or perpendicular to the respective planes of projection. The base of the prism being square, its horizontal projection is necessarily also a square, A B C D, Fig. 6; but its vertical projection will be the rectangle, A^1 B^1 E^1 F^1, Fig. 5, equal to one of the sides of the prism. For the construction of these projections, the same datum as in the preceding case is required; namely, a side of the base, and, in addition, the height of the parallelopiped, or prism.

THE PROJECTIONS OF A QUADRANGULAR PYRAMID, FIG. 7.

97. This pyramid is supposed to be inverted, and having its base, A B C D, parallel to the horizontal plane: it follows upon this assumption, that its horizontal projection is represented by the square, A B C D, Fig. 9. The axis, or centre line, O S, which is supposed to be vertical, and consequently passes through the centre of the base, is projected on the horizontal plane as a point, O, Fig. 9, and on the vertical plane as a line, O^1 S^1; drawing through the point, O^1, of this line, the horizontal line, A^1 B^1, equal to a side of the base, which is supposed to be parallel to the vertical plane, we

shall obtain the vertical projection of the base; and joining A^1 S^1, B^1 S^1, that of the whole pyramid, the points A^1 and B^1 may be found by prolonging the parallels, A D, B C, Fig. 9. This may be conveniently done with the T, or set square, and the operation is usually termed *squaring* over a measurement—that is, from one projection to another. The lateral facets, S B C and S A D, are represented in the vertical projection by the straight lines, A^1 S^1, B^1 S^1, Fig. 8, since they are perpendicular to the vertical plane; and the projection of the facet, D S C, is identical with A^1 S^1 B^1, that of the front facet, A S B, immediately behind which it is. Since each of the inclined converging facets are hidden by the base, they cannot be drawn in sharp lines in the horizontal projection; we have, however, indicated their positions in dotted lines in Figs. 7 and 9. Were these lines full, in the latter figure, it would be the horizontal projection of a pyramid with the apex uppermost, or of a hollow, baseless pyramid, in the same position as Fig. 7.

THE PROJECTIONS OF A RIGHT PRISM, PARTIALLY HOLLOWED, AS FIG. 10.

98. The vertical and horizontal projections of the exterior of this solid, are precisely the same as those of Fig. 4; they are represented respectively by the square, A B C D, Fig. 12, and the rectangle, A^1 B^1 E^1 F^1, Fig. 11. It will be perceived, that the internal surfaces of this figure are such as may be supposed to form some of the sides of a smaller prism; the sides, G H I J and K L M N, are parallel to the vertical plane, and G K N J and H I M L, perpendicular to it, and it follows that the projections of this lesser figure will assume the forms, G^1 H^1 I^1 J^1, Fig. 11, and G H L K, Fig. 12.

99. The lines, K G, L H, are faint dotted lines, instead of being sharp and full, as being hid by the base, A B C D, of the external prism. It is usual to indicate those parts of an object which are hidden behind more prominent portions, by regular dotted lines. This distinction has been adhered to throughout the entire series of plates.

100. On examining the examples just treated of, it will be observed, from the horizontal projections, that the contour, or outline, is in every case square, whilst, from the vertical projections, it will be seen that each object is different. This demonstrates that one projection is not sufficient for the determination of all the dimensions of a solid object; and that, even in the simplest cases, two different projections are absolutely necessary. It will, moreover, be seen, as we advance, that in many cases, three, and at times more, projections are required, as well as sections through two or more planes.

THE PROJECTIONS OF A RIGHT CYLINDER, FIG. 13.

101. The axis, O M, of this cylinder is supposed to be vertical, and its bases, A B, E F, will consequently be horizontal. Its projections in Figs. 14 and 15, are represented by the rectangle, A^1 B^1 E^1 F^1, on the one hand, and the circle, A C B D, on the other. It is evident, that to draw these figures, it is quite sufficient to know the radius, O A, of the base, and the height, O M; with the given radius, we describe the circle, A C B D, which is the horizontal projection of the whole cylinder; then making the vertical, O^1 M^1, equal to the given height, and "squaring over" by means of the parallels, A A^1, B B^1, the diameter of the circle, we draw, through O^1 and M, the horizontals, A^1 B^1, E^1, F^1, completing the parallelogram, A^1 B^1 E^1 F^1, which is the vertical projection of the cylinder.

THE PROJECTIONS OF A RIGHT CONE, FIG. 16.

102. The projections of a right cone differ from those of the cylinder solely as far as regards the vertical plane. Thus it will be seen, in Figs. 17 and 18, that the horizontal projection of the cone, S A B, is exactly the same as that of a cylinder having an equal base; but the vertical projection, S^1 A^1 B^1, in place of being a rectangle, is an isosceles triangle, of which the base is equal to the diameter of the circle, forming the horizontal projection, whilst the height is that of the cone. Similarly to the preceding case, in order to construct these projections, it is sufficient to know the radius of the circular base and the height.

THE PROJECTIONS OF A SPHERE, FIG. 19.

103. A sphere, in whatever position it may be with reference to the planes of projection, is invariably represented in each by a circle, the diameter of which is equal to its own; consequently, if the two projections, O^1 and O, Figs. 20 and 21, of the centre be given, we have merely to describe circles with these centres, with a radius equal to that, O A, of the given sphere.

It would seem from this, that one projection should be sufficient to indicate that the object represented is a sphere; but, on referring to Figs. 15, 18 and 21, it will be seen that a circle is one projection of three very different solids—namely, the cylinder, the cone, and the sphere. This is a further illustration of the inadequacy of one projection to give a faithful representation of any solid form. It is true, that by shading the single projection we approach nearer to the desired representation; but still, such shaded projection would equally represent that of a cylinder with a hemispherical termination. The same remark applies to the shaded projections of cylinders and cones, and, indeed, to all solid bodies.

OF SHADOW-LINES.

104. To distinguish and relieve those parts of a drawing which are intended to indicate the more prominent portions of the object represented, it is customary to employ fine sharp lines for that part of the outline on which the light strikes in full, and strong heavy lines for the parts which are at the same time in relief and in the shade; the latter description of lines are called *shadow-lines.*

For the maintenance of uniformity, it is obviously expedient to suppose the light to strike any object in some constant and particular direction. The assumed direction should be oblique, in order that some of the visible parts of the object may be thrown into shade, whilst others are more strongly illuminated. Hitherto no one system has been universally adopted with regard to the assumed direction of the rays of light. The system most generally adopted in Great Britain, however, is, that the direction of the rays should be parallel to the diagonal, A G, of the cube, Fig. 1, in the vertical plane of projection, of which the projections are A^1 E^1, in Fig. 2, and D B, in Fig. 3, respectively, and this is the system we have adopted throughout the work, that is to say, the light is supposed to come in the same direction in both planes of projection, the direction of which is shown by the arrow, R^1, Figs. 22 and 23, and also by the short dotted lines, R, (for ray of light,) in most of the figures. Some draughtsmen however, cause the ray of light to take the direction A G, Fig. 1, both in the vertical and horizontal planes of projection, thus having the light indicated by the arrow, R^1, Fig. 22, in the vertical, and R, in the horizontal planes of projection respectively.

The lines, which we take as representing the direction of the light, are the diagonals of the cube, which extend from the corner, A, of the front facet of the cube, Fig. 1, to the extreme opposite corner, G, of the posterior facet in the vertical plane, and from the upper corner, D, to the lower corner, E, in the horizontal plane of projection. The projections of these straight lines, viz., A G and D E, in the representations of the cube, Figs. 2 and 3, are respectively the lines A E^1 and D B, lying each at an angle of 45° with the base line. Thus, in our drawings the objects are supposed to receive the light in the direction of the arrow, R^1, in Fig. 22, both in the vertical and horizontal plane of projection.

105. We must observe, however, that the actual inclination of these straight lines thus adopted, is not that of 45°; the angle of inclination is in fact somewhat less, and may be determined by means of the diagram, Fig. 23. For this purpose, it is necessary to suppose the perpendicular plane in which the ray or line lies, as turned or folded down upon the vertical or horizontal plane, the turning axis being perpendicular to the base line. Let us, in the first place, suppose the projection, R^1, of the ray, to meet the base line, L T, in the point, *o*, taking any point in this ray, as projected in the vertical plane at a^1, and with the point, *o*, as a centre, and radius, $a^1\ o$, describe the arc, $a^1\ c\ a$, cutting the base line in the point, *c;* through this point draw the perpendicular, $b^1\ b$, limited each way by the lines, $a^1\ b^1$, *a b*, drawn parallel to the base line through the points, $a^1\ a$. Join $o^1\ b$, and the line thus obtained indicates the position and inclination of the ray, when folded down, as it were, on either plane of projection; and on applying a protractor, it will be found that the actual angle of inclination is one of 35° 16′ nearly. Having, then, fixed upon the direction of the rays of light, which are, of course, supposed to be parallel amongst themselves, it will be easy to determine which part of an object is illuminated, and which is in the shade. It will be perceived, for example, in Figs. 2 and 3, that the illuminated portions are those represented by the lines, $A^1\ B^1$, and $A^1\ F^1$ on the one hand, and D C and D A, on the other; and that those in the shade are represented respectively by the lines, $E^1\ B^1$, $E^1\ F^1$, and B A, B C. The lines, R, *a o*, *a*, *b*, and *b o*, Fig. 23, in the horizontal plane, all correspond on the French system to the similar lines, R^1, $a^1\ o$, $a^1\ b^1$, and $b^1\ o$, respectively, in the vertical plane, as before referred to at Fig. 22. What has just been said with reference to the cube, is equally applicable to all prisms or solids bounded by sharp definite outlines, care being taken to employ heavy shadow-lines only on the outlines of those parts which are both prominent and in the shade—such shadow-lines separating the facets which are illuminated, from those which are not.

106. With regard to round bodies, the projections of the lateral portions being bounded by lines which do not indicate prominent and sharply defined edges, so full a shadow-line should not be employed as that forming the outline of a plane and prominent surface. Thus, in Figs. 14, 17, and 20, the lines, $B^1\ E^1$, $S^1\ B^1$, and C^1 B, D, are not so strong as the corresponding lines, $B^1\ E^1$, in Figs. 2 and 5. Nevertheless, these lines should not be as fine as those on the illuminated side of the object, but of a medium strength or thickness, to indicate the portion of the object which is in the shade. In other words, a sharp fine line indicates the fully illuminated outline, a fuller line the portion in shade, and a shadow-line still stronger that portion which is both in the shade, and has a prominent sharply defined edge. The straight lines, $F^1\ E^1$ and $A^1\ B^1$, Figs. 14 and 17, will necessarily be full shadow-lines, as representing the edges of planes entirely in the shade.

In the horizontal projection of the cylinder, Fig. 15, the illuminated portion corresponds to the semi-circle, *a d b*, whilst that in the shade is the other semi-circle, *a c b;* the points, *a*, *b*, of separation of the two halves, are obtained by drawing through the centre, O, a diameter, *a b*, perpendicular to the ray of light, *d* O, or by drawing a couple of tangents to the circle parallel to this ray. The straight line, *a b*, is inclined to the base line at an angle of 45°. Great care is necessary in producing the circular shadow-line, *a c b*, and the nibs of the drawing-pen should be gradually brought closer as the extremities, *a* and *b*, of the shadow-line are approached, so that it may gradually die away into the thickness of the illuminated line. By inclining the drawing-pen, or by pressing it sideways against the paper, the desired effect may be produced; the exact method, however, being obtained rather by practice than by following any particular instructions. A very good effect may also, in some instances, be produced, by first drawing the entire circle with the fine line, and then retracing the part to be shadow-lined with a centre slightly to one side of the first centre, and repeating this until the desired strength of the shadow-line is obtained.

107. In the plan of a cone, Fig. 18, the part in the shade is always less than the part illuminated; but it requires an especial construction, which will be found in the chapter treating of Shadows, for the determination of the lines of separation, S *e*; and it is seldom that such extreme nicety is observed in outline drawings, the shadow-line of the plan of the cone being generally made the same as that of a cylinder, or perhaps a little less, according to the judgment of the artist. Yet, if the height of the cone be less than the radius of the base, the whole conical surface will be illuminated, and consequently its outline should have no shadow-line.

108. In explanation of the reasons which have guided us in the adoption of the diagonal of a cube, as projected in the line, R^1, Figs. 22 and 23, as the direction of the rays of light, in preference to the other systems proposed, we shall proceed to point out some of the inconveniences attending the latter.

In reality there does not seem much to choose between the English and the French systems in point of merit, as they agree in the more essential parts as regards the light falling on the object in the direction of the diagonal of the cube, in both the elevation and plan, that is in the direction of the projected line, R^1, Figs. 22 and 23; and the French system only differs from the English in supposing the light to come from below, or in the direction of R, in the same figures, instead of from above in the plan or horizontal view. Thus, having the light coming in two different directions, and throwing cross shadows on the same sheet of drawings, while the system we adopt, has the light, and consequently, the shadows all thrown in the same direction.

If, however, on the other hand, as proposed by some draughtsmen, the rays of light are supposed to be perpendicular to either plane, such confusion will result as to render it impossible to ascertain, by any reference to the shadow-lines, what is, or what is not, illuminated, and thus the object of employing shadow-lines would be lost sight of. Let us suppose, for example, that the light is perpendicular to the vertical plane, whence it follows that the whole of the anterior facet, Figs. 2, 5, 8, and 11, is fully illuminated; but, at the same time, all the facets perpendicular to the vertical plane are equally in the shade, and it would consequently be necessary to use shadow-lines all round, or else not at all; and whichever plan was adopted, would be quite unintelligible. Besides this, it is unnatural to suppose that the spectator should place himself between the light and the object.

GROOVED CYLINDERS

Fig. 1.

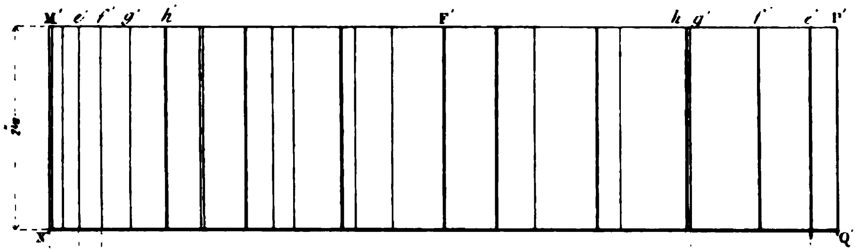


Fig. 2.

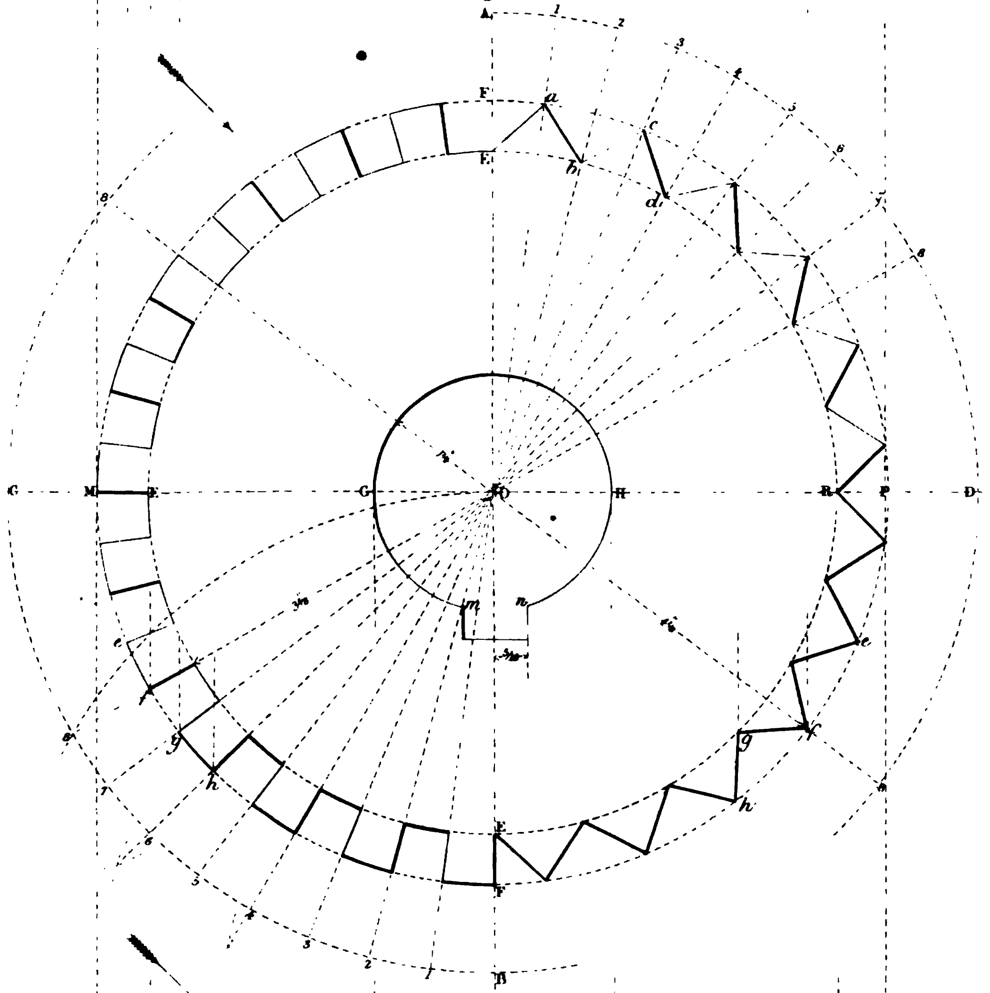
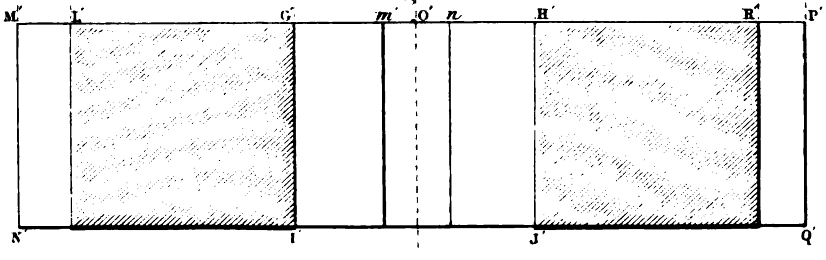


Fig. 3.



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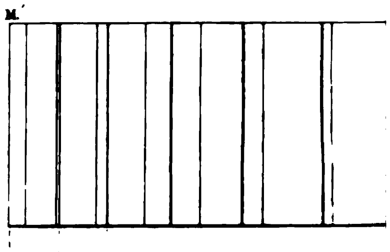

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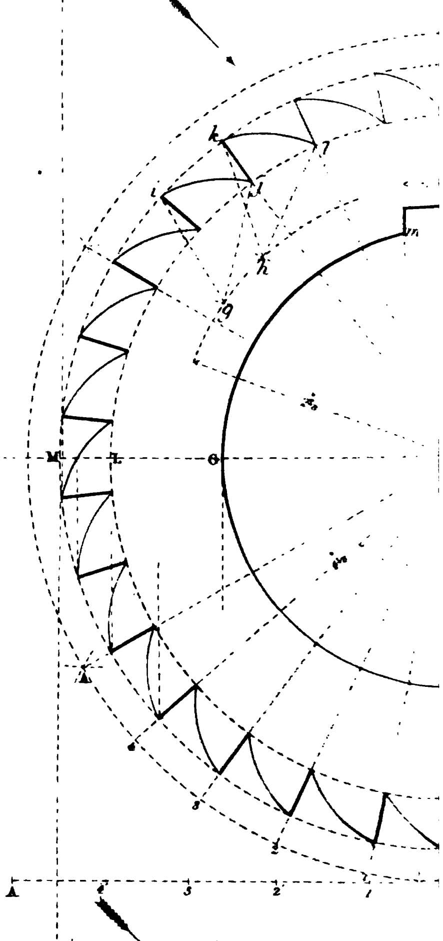

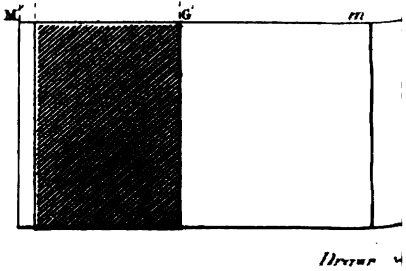

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RATCHET WHEELS.

l.

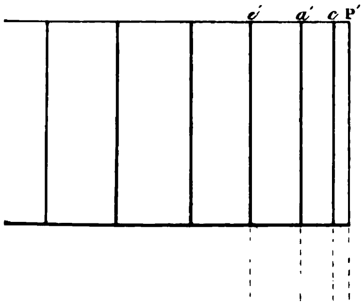
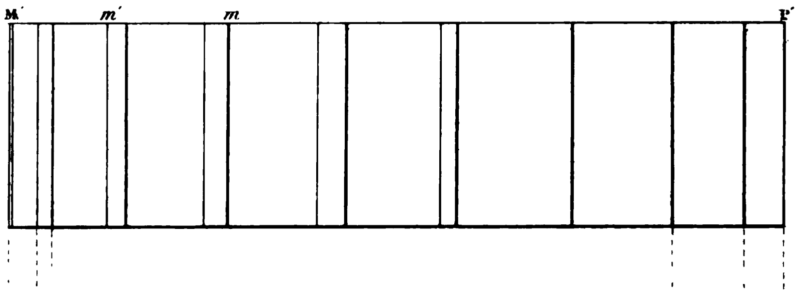


Fig. 7.



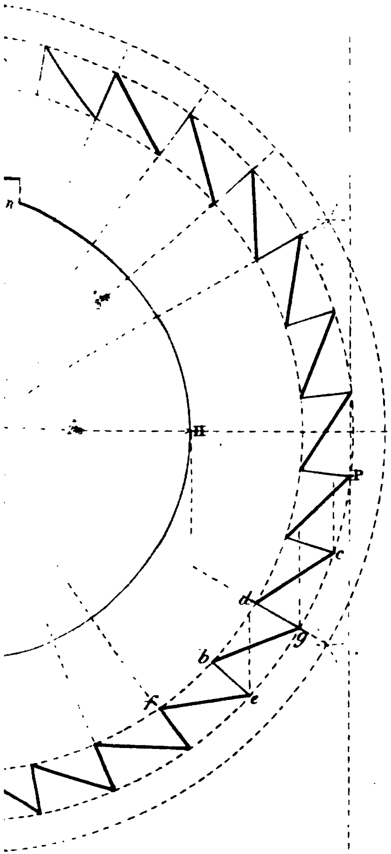
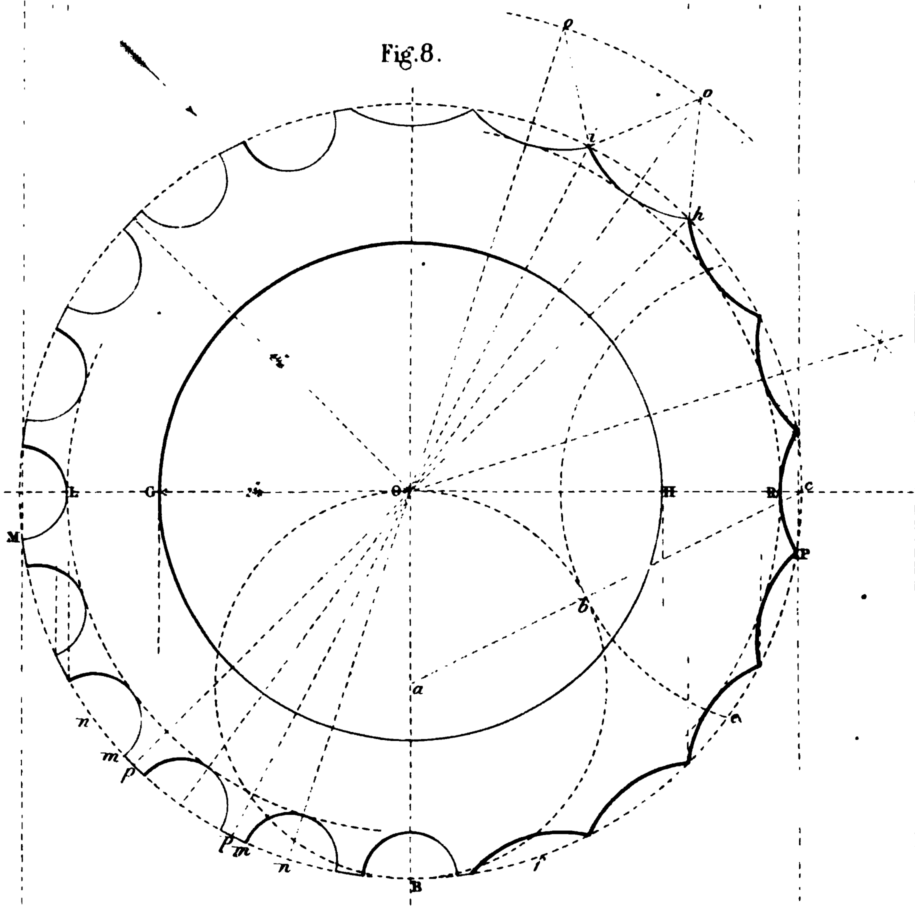


Fig. 8.



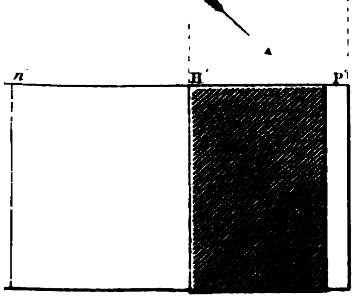
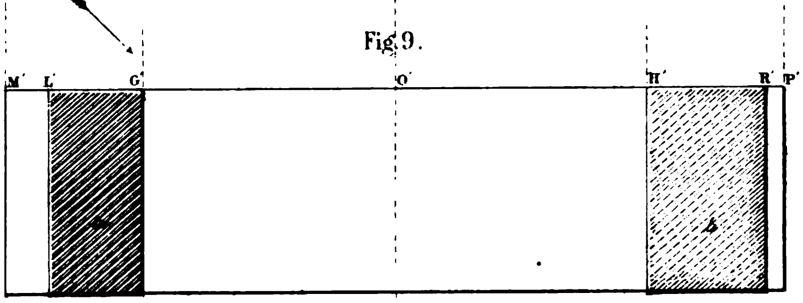


Fig. 9.



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PROJECTIONS OF GROOVED OR FLUTED CYLINDERS AND RATCHET WHEELS.

PLATE 8.

109. The various diagrams in this plate are designed principally with the view of making the student practically conversant with the construction of the projections of objects; and, besides teaching him how to delineate their external contours, to enable him to represent them in *section*, that their internal structure may also be recorded on the drawing.

Figs. 1 and 2 are, respectively, the elevation and plan of a right cylinder, which is grooved on its entire external surface. The grooves on one-half of the circumference are supposed to be pointed, being formed by isosceles triangles of regular dimensions, and may represent the rollers used in flax machinery, in apparatus for preparing food for animals, and in many other machines. The other half of the circumference is formed into square or rectangular grooves, the lateral faces of which are either portions of radii of the circle, or are parallel to radii from the centre of the circle passing through the middle of each rectangle.

110. To construct the horizontal projection of this cylinder, that is, as seen from above, we must first ascertain how many grooves are contained in the whole circumference; then drawing a circle with a radius, A O, which should be greater than that of the given cylinder, divide it into twice as many equal parts as there are grooves. If the student will refer back to the section treating of linear drawing, illustrated in Plate 1, he will find simple modes of dividing circles into 2, 3, 4, 6, 8, and 12 equal parts, and, further, of subdividing these. Thus, as the cylinder, Fig. 2, contains 24 grooves, its circumference must be divided into 48 equal parts. To obtain these, begin by drawing two diameters, A B, C D, perpendicular to each other; then, from each extremity, mark off the length of the radius, A O, thus obtaining the four points numbered 8 on one side, and the points numbered 4 on the other—making, with the points of intersection of the two diameters with the circumference, in all, 12 points. There remains simply to bisect each space, as A—4, B—4, or 4—8, &c., as well as the lesser spaces thus found; this will give the 48 divisions required. Through the points of division draw a series of radii, which will divide the inner circle described, with the radius, O F, into the same number of equal parts. The depth of the grooves is limited by the circle described with the radius, O E, whilst the outside of the intervening ridges is defined by the circle of the radius, O F. All the operations which we have so far explained, are called for in the construction of both the triangular and rectangular grooves. In proceeding, we must, for the angular grooves, join the points of intersection, *a*, *b*, *c*, *d*, which are in each circumference alternately; whilst in the case of rectangular grooves we require no fresh lines, but have simply to ink in alternate portions of the two circles, as well as the radial lines joining these.

111. To draw the vertical projection, Fig. 1; it is necessary that the depth should be given, say $M^1 N^1 = 2\frac{1}{2}$ inches. First set out the two horizontals, $M^1 P^1$, $N^1 Q^1$, limiting the depth of the figure; then, to obtain the projection of the grooves and ridges, square over each of the points, *e*, *f*, *g*, *h*, &c., and draw parallels through the points thus found in Fig. 1, as e^1, f^1, g^1, h^1. Then, to complete the elevation, it is only necessary to square over the outer extremities, M and P, of Fig. 2; it will be seen that $F^1 M^1$, thus squared over, is equal to the radius, O M, of the wheel; but on the other side it is a groove and not a tooth that is in a line with the centre line of Fig. 2, parallel to the vertical plane of projection. The line, $F^1 P^1$, is somewhat less than a radius, $P^1 Q^1$, being, in fact, the apex of the tooth immediately in front of the centre line squared over. This completes the elevation, Fig. 1, which represents the front half, M F P, of the cylinder below the horizontal line, $M^1 P^1$.

112. It has already been observed, that two projections are not always sufficient to form a complete representation of an object; thus it will be evident, from a consideration of Figs. 1 and 2, that a third view is necessary to explain the interior of the cylinder. The radius, $O G = 1\frac{1}{2}$ inches, of the central circular opening, is not apparent in Fig. 1, it is only to be found in the plan; whereas we have already seen, that, to determine its exact position, it should be represented in two projections. From Figs. 1 and 2 it is impossible to see if the opening exists throughout the depth of the cylinder, or if its radius be the same down to the bottom; and the same remark applies to the key-way. In consequence of this, it is expedient to draw the object in section—for example, through the centre line, M P—by a plane parallel to the projection. Such a section is represented in Fig. 3; and from it, it is at once manifest that the central opening, as well as the key-way, extend equally throughout the depth of the cylinder. The outline of these parts is formed by the verticals drawn through the points, G^1, m^1, n^1, H^1, obtained by squaring over the corresponding points, G, *m*, *n*, H, in the plan, Fig. 2. This view also shows that the external grooves are equal throughout their depth, as indicated by the verticals drawn through M^1, L^1, E^1, P^1. When the outlines of the interior of an object are few and simple, they may be indicated in an elevation, such as Fig. 1, by dotted lines. But if the outlines are numerous or complex, too great a confusion would result from this method; and it is far better, in such case, to give a sectional view.

That portion of the solid mass of the cylinder, through which the sectional plane passes, is indicated in Fig. 3, by a flat-tinted shading, so as to distinguish it from the parts which the plane does not meet: this is the plan generally adopted to show the parts in section; the strength of the shade, or *sectioning*, is varied according to the nature of the material. Thus, for cast-iron, a darker shade is used, whilst a lighter one indicates wood or stone; and as an example of this distinctive use of various degrees of shade, we may point out that the sectioning in Fig. 3, indicates the object to be made of copper, malleable iron, and other light metals, whilst that in Fig. 6 corresponds to cast-iron, and at *a* and *b*, in Fig. 9, it represents, respectively, wood and masonry.

113. It must be observed, that *section lines*, of whatever description they may be, are always inclined at an angle of 45° with the base line; this is to distinguish the sectioning from flat tints frequently employed in elevations, to show that one surface is less prominent than another: this latter flat-tinting is generally produced by perpendicular or horizontal lines. The line, $I^1 J^1$, which indicates the base of the internal cylinder, G *m* H, should not be a shadow-line equal in strength to the bases of the sectional parts, for the latter are more prominent. This point is seldom attended to as it should be; greater beauty and effect, however, would result if it were. This remark applies equally to all projections of objects, of which one portion is more prominent than another. Thus, in Figs. 1, 4, and 7, the vertical lines passing through F^1 are considerably thicker than those passing through $P^1 Q^1$, and lying in a posterior plane. These distinctions should always be observed in the representations of complex objects, since by their means a comprehension of the drawing is much facilitated. After an attentive study of Figs. 1, 2, and 3, on this plate, the remaining Figs. 4, 5,

6, 7, 8, and 9, which represent ratchet wheels and also fluted cylinders, such as the columns of the Doric and the other higher orders of architecture, will be quite intelligible to the student; such operations as are additional, being rendered obvious in the views themselves.

THE ELEMENTS OF ARCHITECTURE.

THE FIVE ORDERS OF ARCHITECTURE.

PLATE 9.

114. In plate 9 are shown the five ancient orders of architecture. [*]

1. The Tuscan.
2. The Doric.
3. The Ionic.
4. The Corinthian.
5. The Composite.

Fig. 1 is an elevation of the Tuscan order, complete with pedestal, drawn to a scale of three-eighths of an inch to the foot, and supposing the radius of the column at *a b*, (which is called a module or unit of measurement in all the orders) to be equal to one foot. In this order, the module is subdivided into twelve parts, so that in this instance, the scales of feet and inches for the several figures of this order will respectively represent modules and parts of modules in the several figures to which they belong. Down the side of this figure, as well as that of the other orders, are two rows of figures, enclosed within parallel lines, and crossed by lines squared over from the several members of the orders. These rows are marked, H and P, at the top, respectively, for height and projection; the figures in the first, represent the height of that part of the order squared over by the cross lines, within which they are enclosed, as 12', represents the length of the shaft, E, from the astragal to the base, which is twelve modules. While the figures in the second row, represent the projections of the part or member, within the parallel cross lines enclosing them, from the centre line marked at the top and bottom, *c d*, to the outermost point of projection, as 1', 8½', represents *d* A, the projection of the lower plinth; that is one module, eight and a half parts.

Fig. 2 is an elevation of the ancient Grecian Doric, drawn to the same scale as the Tuscan, having the dimensions of the several members marked in modules and parts, or feet and inches, as the module in this order also, is subdivided into 12 equal parts. From its strength and heaviness, this order might be ranked below that of the Tuscan, the whole height of the column being only 12 modules 6 parts; but from the general beauty and elegance imparted to it, from a richly ornamental entablature, and neat capital with the plain and graceful fluted column below, it has been ranked as the second order. The ancients gave this order, neither base nor pedestal; but the modern Roman architects gave it both. Vitruvius and Palladio give the whole height of the column at 16 or 17 modules, and in some instances they give it an attic base with a short pedestal of about one fourth of the height of the column, the die of which was square. They also give it an ornamental frieze and projecting astragal below the capital; thus placing this order between the Tuscan and Ionic orders. The shafts of the Grecian Doric and the higher orders, are generally fluted, as shown in the several figures. This order has usually 20 flutes without fillets, one being placed exactly in the centre; the mode of drawing which, has been shown in Figs. 7 and 8 of Plate 8. These two orders, the Tuscan and Doric, as given in Figs. 1 and 2, are much used in mechanical constructions as well as in architecture. Being the plainest of all the orders they admit of being formed, or cast, hollow or solid of the various metals, without flaws or defects, and when used for the nozles, the framing of steam engines, and other machines, can be wholly or partially polished, thus giving elegance and strength in mechanical structures.

Figs. 3, 4, and 5, are also elevations respectively of the Grecian Ionic, Corinthian, and Composite orders, drawn to scales of modules and parts divided off on the outer lines marked, H, of the several figures. The modules of these three orders are subdivided into 18 equal parts, and all the sizes marked in plain figures, as in the former orders in their respective columns of height and projection. These three orders are all shown with the plain attic base,, and without their pedestals, these being usually of the same general proportions as that shown in the Tuscan, viz., one-third of the whole height of their several columns.

The "*inter-columns*" or spaces between the columns arranged at the sides, or in front of buildings, or in colonnades, stone galleries and porticos, vary according to the height of the column, in proportion to its thickness or diameter at the base. Thus Palladio says, that none of the orders but the Tuscan ought to be placed more than three diameters apart. To the Doric he gives three; to the Ionic two and a quarter; to the Corinthian two diameters, and to the Composite one and a half. He states that this last should be considered the minimum, "inter-column," and three and a half the maximum.

115. Each order of architecture comprises three principal parts: the *pedestal*, the *column*, and the *entablature*. In all the orders, the pedestal is usually a third of the whole length of the column in height, and the depth of the entablature in all the orders, the Roman Doric included, is one-fourth of the whole height of the column, the Grecian Doric alone excepted, which has its entablature one-third of the height of the column. The proportion between the diameter and height of the column varies in each order. The height of the Tuscan column is seven times the diameter at the lowest part, *a b*; the Grecian Doric six or six and a fourth times; the Roman Doric, eight times; the Ionic, nine times; the Corinthian and Composite, ten times. The pedestal is frequently altogether dispensed with. All the different parts, in the various orders, bear some proportion to a *module*, which is half the diameter of the lower part of the column, *a b*, Fig. 1. This module may be termed the unit of proportion. It is divided into 12 parts, in the Tuscan and Doric orders; and into 18 parts in the Ionic, Corinthian, and Composite. The whole height of the Tuscan order, without the crown, is 22 modules 2 parts, apportioned as follows:—The column is 14 modules; the pedestal, 4 modules 8 parts; and the entablature, 3 modules 6 parts. The whole height of the Grecian Doric is 16 modules 8 parts, the column being 12 modules 6 parts, and the entablature 4 modules 2 parts. The whole height of the Roman Doric order is 25 modules 4 parts—the column being 16 modules; the pedestal, 5 modules 4 parts; and the entablature, 4 modules. The whole height of the Ionic order is 28 modules 9 parts—the pedestal, 6 modules; the column, 18 modules; and the entablature, 4 modules 9 parts. The whole height of the Corinthian and Composite orders is 31 modules 12 parts—of which 6

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[*] We have adhered to the classification which, from being of more ancient date, is supported by superior authority; but we do not profess, in this work, to decide which carries more reason with it. Reason frequently runs counter to authority. Modern architects say there are only three orders—the first comprising Ancient, Modern, and Tuscan Doric; the second, Greek, Roman, and Modern Ionic; and the third, Corinthian and Composite.

TUSCAN.

Fig. 6.

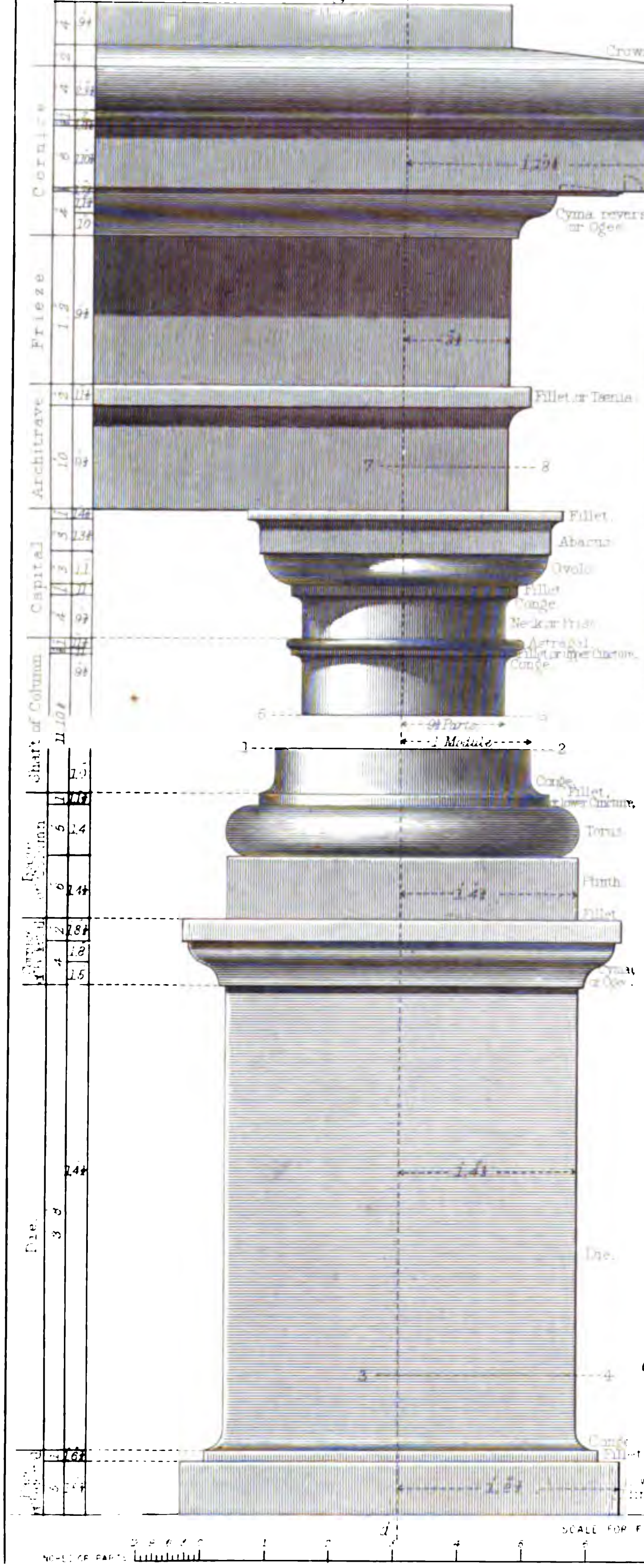


Fig. 10.

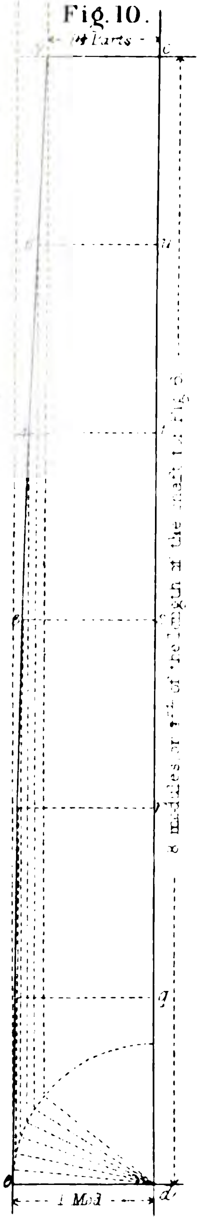
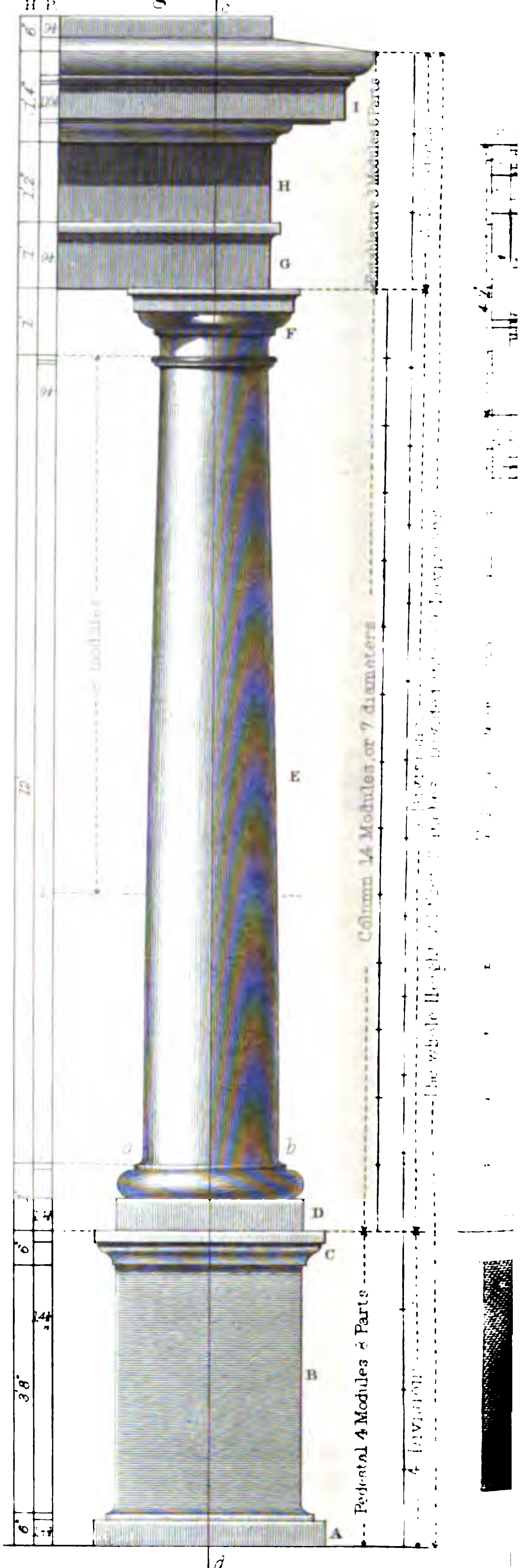


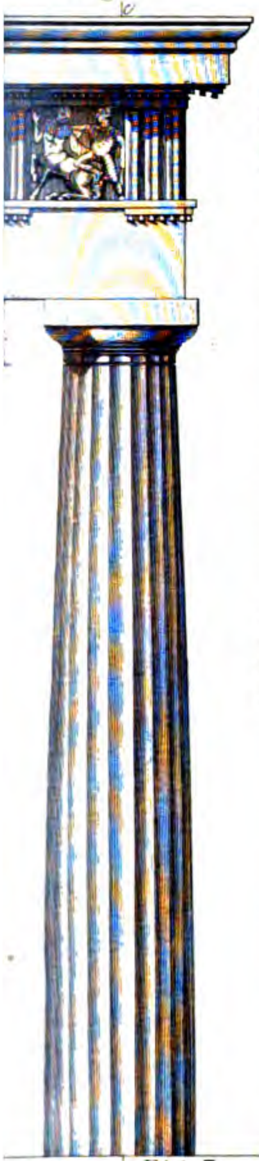
Fig. 1.


H P
A
B
C
D
E
F
G
H
I
a
b
d
Entablature 3 Modules 6 Parts
Column 14 Modules, or 7 diameters
Pedestal 4 Modules 8 Parts
The whole Height

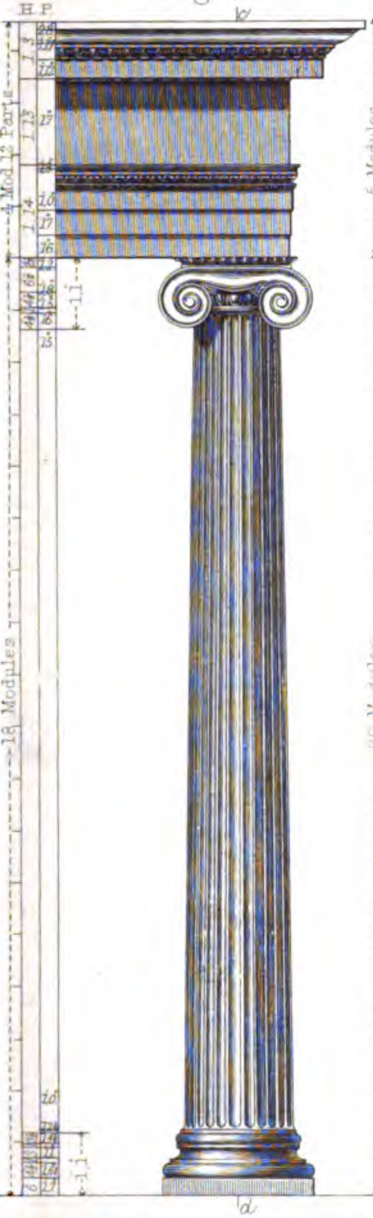
W. Johnson, Patent Offices,
Lincolns Inn Fields & Glasgow

THE FIVE ORDERS OF ARCHITECTURE.

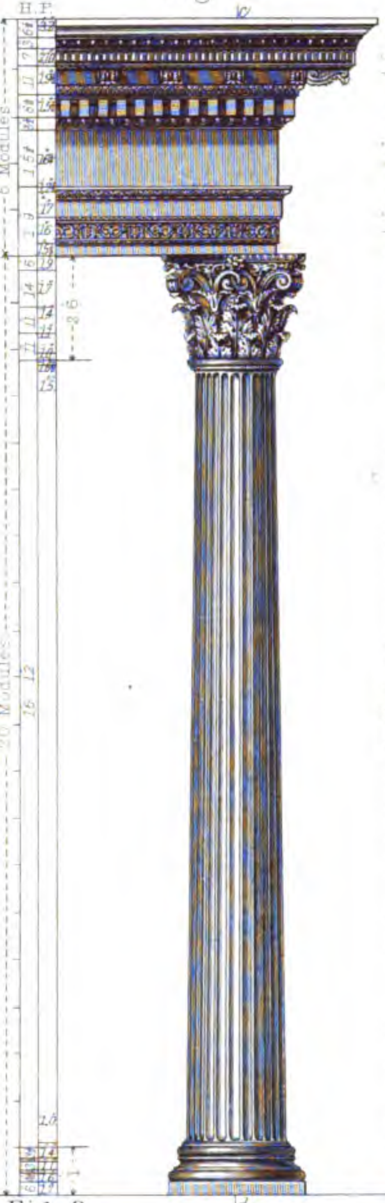
RECIAN DORIC.
Fig. 2.



GRECIAN IONIC.
Fig. 3.



CORINTHIAN.
Fig. 4.



COMPOSITE.
Fig. 5.

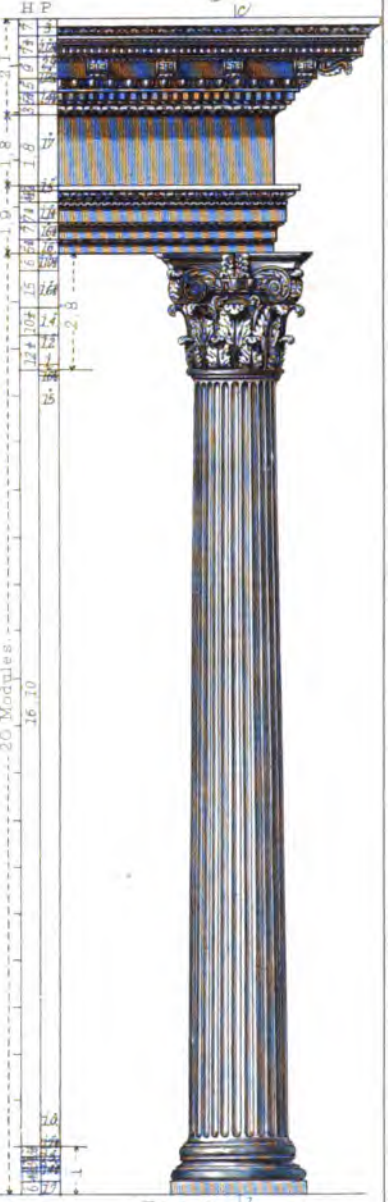


Fig. 7.

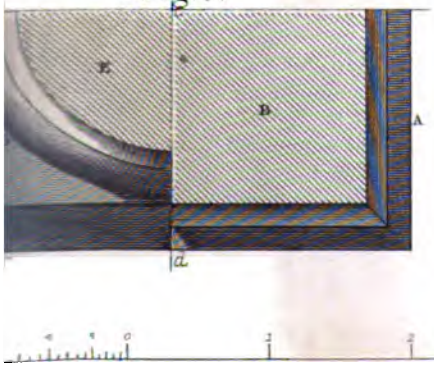


Fig. 8.

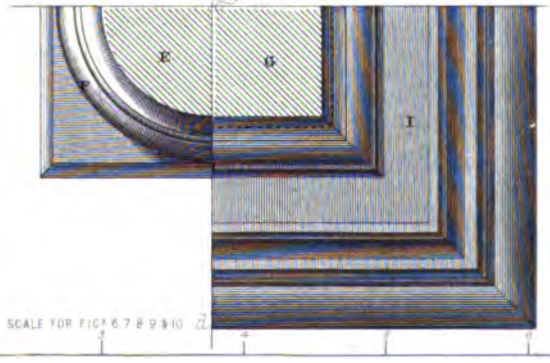
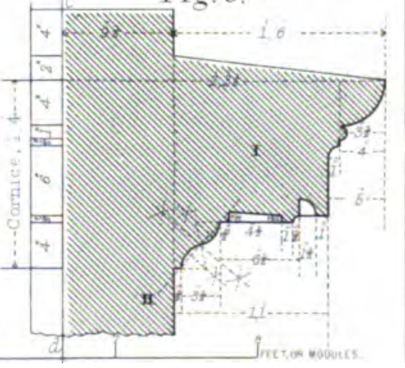


Fig. 9.



F. W. Thorne sc.

modules 12 parts form the pedestal, 20 modules the column, and 5 modules the entablature.

As we do not propose to treat specially of architecture, we have not given details and particulars of all the various orders, but have confined ourselves to the Tuscan, as being the simplest, as well as the one most generally adopted in the construction of machinery. At the end of this Chapter will be found tables of the dimensions of the various components of the Tuscan order, and we also there give a similar table for the Roman Doric order.

OUTLINE OF THE TUSCAN ORDER.

116. The whole height of the pedestal, column, and entablature, being given, the proportions of the different parts may always be determined. Let this height be, for example, 22 feet 2 inches, as marked on the first dotted line to the right of Fig. 1. First, divide it into 19 equal parts or divisions, then take 4 such divisions for the height of the pedestal, 12 for that of the column, and the remaining 3 for the entablature, as marked on the second dotted line, and divided on the sharp line next it. Then, according to the order which it is intended to follow, the height of the column is divided into 7, 8, 9, or 10 equal parts, and the diameter of the lower part of the column will be equal to one of these parts: thus, in the Tuscan order, the diameter, *a b*, Fig. 1, is $\frac{1}{7}$ of the whole height of the columns; the half of this diameter, or the radius o, of the shaft, at *a b*, is the unit of proportional measurement, called the *module*, and with which all the components of the order are measured; it follows then, that in the Tuscan order this module is $\frac{1}{14}$ of the height of the column, in the Doric it is $\frac{1}{16}$, in the Ionic $\frac{1}{18}$, and in the Corinthian and Composite $\frac{1}{20}$. The whole height of the Grecian Doric, having no pedestal, is divided into 16 divisions, 12 being given to the column, and 4 to the entablature.

117. The three members of an order are each subdivided into three divisions. Thus the Pedestal is composed of the Socle, or lower Plinth, A; of the Dado, or Die, B; and Cornice, C: the Column consists of the Base, or Plinth, D; the Shaft, E; and the Capital, F: and in the Entablature are the Architrave, G; the Frieze, H; and the Cornice, L.

118. Before proceeding to delineate these different parts, and the mouldings with which they are ornamented, it is expedient to set off a scale of modules, determined in the manner just stated, the module being, of course, subdivided into 12 equal parts.

To make the mouldings and various details more intelligible, we have drawn the various portions of the order, separately, to a larger scale. Thus Fig. 6 is the elevation of the entablature, capital, base, and pedestal, of the Tuscan order, to a scale of double that of the complete view, Fig. 1, and the scale is thus for the large figures, $\frac{3}{4}$ of an inch to a module. All the numbers indicated on these figures, give the exact measurements of each part and each moulding, so that they may be drawn in perfect accordance with the scales given. It conduces considerably to the symmetry and accuracy of the drawing, to set off all the measurements from the axis or centre line, *c d*. The module being but an arbitrary measurement, it is necessary, in practically carrying out any design to ascertain the different measures in feet and inches, but in this case the trouble is much simplified by making the scales of modules to represent feet also, so that the numbers in the several figures represent modules and parts, and at the same time feet and inches on the scales to which they are respectively drawn, and marked at the foot of the plate. To give a distinct idea as to the degrees of prominence or relief of the various members, as also their height, the sizes are marked in figures, in their respective columns, marked H and P, as well as on parts of the elevation, Fig. 6. In the half horizontal section, Fig. 7, are represented portions of sections in two different planes, the one to the left being at the height of the line, 1, 2, and the other to the right at that of 3, 4. The first shows that the shaft, E, is round, as well as the fillet and the torus, whilst the base, D, and cornice, C, are square: the second section shows, in a similar manner, that the die, B, the lower plinth, A, and its fillet, are square; while the light flat-tint, at 45°, sufficiently indicates the parts in section, which are divided by the centre line, *c d*, to distinguish the sections on the different planes.

Fig. 8 is a fourth section of the top part of the column and entablature, on the lines, 5, 6, and 7, 8, respectively, supposed to be looking from below, or as it were inverted, and showing the shape and configuration of the parts, as in Fig. 7. Fig. 9 is a sectional elevation of part of the entablature to the same scale as Figs. 6, 7, and 8, showing the construction of the cornice, which has all the sizes of height and projection marked in figures, and the same letters of reference in all these figures, for the same parts with the axial line, *c d*, corresponding also in the several figures so as to measure from.

119. The execution of this design offers little or no difficulty; for all the operations required, as well as the parts to which the measurements apply, are carefully indicated. It is, therefore unnecessary to enter into further details, except as far as relates to such parts as involve some peculiarity; the shaft of the column, is an instance.

Referring to the column of the classic orders, whether detached, as was the more ancient practice of the Greeks, or as attached, as was afterwards not uncommon in some of the more modern Roman temples, they all tapered or diminished from the inferior or lower part of the shaft, to the upper or superior part of the neck. In the Tuscan and Grecian Doric, as shown in Figs. 1 and 2, it is customary to make the shaft cylindrical for one-third of the height, that is, of equal diameter throughout that extent: above that point, however, it diminishes gradually in diameter up to the capital.

In the Ionic, Corinthian, and Composite orders, however, the diminution commences from the very bottom of the shaft, at the fillet of the base, and rises in a graceful sweeping line to the narrowest part at the upper cincture or collerett. Vitruvius and others, give rules for this diminution, which decreases accordingly as the length of the columns increase. Thus, shafts of columns of fifteen feet in height should have their inferior diameter at the base divided into six equal parts; five of these parts being taken for the superior diameter at the top under the astragal, gives the diminution for this height of column. Those of twenty feet are to be divided into six and a half parts at their lower diameter, five and a half of these being taken as their superior diameter. Those of thirty feet being divided into seven parts at their lower diameter, are to be reduced at their upper to six of these parts. Those of forty feet to be divided into seven and a half parts at the bottom, and reduced to six and a half parts at the top. While those of fifty feet are to be divided into eight parts at their inferior diameter, and seven of these parts given for the superior diameter. The reasons assigned by him and others why the columns of different heights should thus vary in their diminution, according to their height, is, that the lines or pencils of rays of light coming to the eye from an object which is situated at a considerable elevation, forms a much more acute angle than those coming from one of equal dimensions, but which is less elevated. The taper thus given to the shaft is not regular throughout, being scarcely perceptible at the lower part, and becoming more and more convergent towards the top. Its contour is consequently a curve, instead of a straight line. This curvature constitutes what is termed the *entasis* or swell given to the shaft of the column, and is employed to correct the apparent narrowness of a rectilinear column at the middle. Such defective appearance only takes place when the cylindrical piece, or column, is between a pedestal and an entablature having plane surfaces. A cylinder, or sphere, always seems to occupy less space than a plane surface equal to its greatest section. Thus the outline of a cylinder, or sphere, appears to grow less when it is shaded. Now, where the column is in contact with the plane surface of the pedestal or entablature, it cannot appear less in proportion, the proximity of the latter correcting such appearance, whilst the influence is less felt at the central part, which is furthest from the pedestal and entablature. A true cylinder, therefore, in such a position, appears to be thinner at the middle, and this is corrected by the entasis, or curved contour.

But many authorities consider this a fastidious nicety, and it is frequently disregarded, particularly in designing short thick columns for machinery, and also where the other extreme is reached, and the columns become mere rods.

What may be termed the *mechanical entasis*, is, moreover, employed in beams, levers, and connecting-rods of all descriptions; the object of this convexity, and increased width in the middle part, in such cases, being to obtain strength and rigidity, whilst it undoubtedly adds to the beauty of the outline.

To determine the amount of the *entasis* in the Tuscan column, take the line, *c d*, Fig. 10, which represents two-thirds the height of the shaft, corresponding to that of Fig. 6, and in this order it is 8 modules, on the scale to which this figure is drawn, as marked and shown by dotted lines to the left of the smaller and complete view, Fig. 1. Divide this line, *c d*, into any number of equal parts, as 6, in the points, *q*, *r*, *s*, *t*, *u*, and *c*, and through each of them draw lines at right angles to *c d*. With the point, *d*, as a centre, and a radius, *d e*, equal to one module on the same scale, draw an arc of a circle; next, having made *c v* equal to 9½ parts, as marked in the projection column, P, at the neck or *frise* in Fig. 6, draw through *v*, a line, *v x*, parallel to the axis, *c d*, this parallel line will cut the arc in the point, *x*; divide the arc, *e x*, into six equal parts in the points 1, 2, 3, 4, 5, and *x*; and through these points draw parallels to the axis. These parallels will intersect the horizontal lines drawn through the divisions, *q*, *r*, *s*, *t*, and *u*, of the axis, respectively, in the points, 1^1, 2^1, 3^1, 4^1, and 5^1, a curved line passing through these points gives the *entasis* or proper curve, forming the contour of the shaft. This curve, being symmetrically reproduced on the opposite side of the axis, *c d*, will complete the outline of the shaft.

Another method adopted by some architects is to divide the difference between the diameter at the lower part of the shaft and that of the upper, as at *v*, into the same number of parts as the whole altitude of the shaft, or the part *c*, *d*, which is to be diminished, has been previously subdivided into, which is in this instance, six. Then with a straight line drawn from the first division from the outside perpendicular line (that is the division furthest from *v*, on the line *c*, *v*,) to the point, *e*, at the lower part of the shaft will cut the first horizontal line, *q* 1′, in the first point, 1′, of the curve line, next a line from the second point from the outside, or the fourth from *v*, drawn to the same point, *e*, at the bottom will, in like manner, cut the line, *v* 2′, in the second point, 2′, of the curve line, and in the same manner, the line, from the third point, will give the third point of the curve 3′, and so on till the fifth, or the one next to *v*, will give the last point, 5′, which is required, and the line drawn through the several points will, as in the first instance, give the *entasis* of the shaft. When less swell or entasis is desired, any part of the difference between the inferior and superior diameters from *v*, as a half, for example, taken and divided in the same way as the last, then the first point in this half from the outside as before, will give point 1′, and so on as before, one point after another, till the point next to the last gives point 5′ as before, and the line drawn through all the points will be the curve, but very much flatter and nearer a straight line.

In the entablature and pedestal will be found two similar mouldings, termed *cymatia*; they are both examples of the *cyma reversa*, discussed in Prob. 43, Figs. 15 and 16, Plate 3. The slight peculiarities in their construction, will be easily understood from the enlarged section of the cornice, Fig. 9, in which one of these curves are with the construction shown in dotted lines. The *quarter rounds* and accompanying minor mouldings belonging to the capital and entablature of this order are so simple, that the mode of drawing them will at once be seen from the large views, Figs. 6 and 9.

RULES AND PRACTICAL DATA.

MEASUREMENT OF SOLIDS.

120. The volume or solidity of a body, as already explained, is the extent of space included within its several surfaces. The volume of a solid is determined when it is ascertained what relation it bears to, or how many times it contains, any cube which is adopted as the unit of measurement.

Volume may be measured by cubic yards, feet, or inches, just as we measure surface by square, and length by linear yards, feet, and inches. A cubic foot is $\frac{1}{27}$ of a cubic yard, for—

$$1 \text{ cubic foot} = \tfrac{1}{3} \text{ yard} \times \tfrac{1}{3} \text{ yard} \times \tfrac{1}{3} \text{ yard} = \tfrac{1}{27} \text{ yard};$$

and an inch, or

$$\tfrac{1}{12} \text{ foot} \times \tfrac{1}{12} \text{ foot} \times \tfrac{1}{12} \text{ foot} = \tfrac{1}{1728} \text{ foot}.$$

121. *Parallelopipeds.*—The volume of a parallelopiped is equal to the product of its base multiplied into its height.

Example.—Fig. 4, Plate 7. Let A F = 2 feet, F E = 1·4 feet, and F H = 1·4 feet. Then the base = 1·4 × 1·4 = 1·96, and 1·96 × 2 = 3·92 cubic feet; or more simply, 1·4 × 1·4 × 2 = 3·92 c. ft. Since the dimensions of a *cube* are all equal its volume is expressed by the third *power* of the measure of one of its sides; that is, by the product of one side three times into itself.

Thus the cube, Fig. 1, Plate 7, of which one side measures, say 14 feet, contains 14 × 14 × 14, or $14^3 = 2744$ cubic feet.

The volume of a right prism, whatever be its base, is equal to the product of the base into the height.

TABLE OF SURFACES, AND VOLUMES OF REGULAR POLYHEDRA.

NUMBER OF SIDES.	NAME.	SURFACE.	VOLUME.
4	Tetrahedron,	1·7820508	·1178519
6	Hexahedron, or Cube,	6·0000000	1·0000000
8	Octahedron,	3·4641016	·4714045
12	Dodecahedron,	20·6457788	7·6631189
20	Icosahedron,	8·6602540	2·1816950

122. *Pyramids.*—The volume of a polygonal pyramid is equal to its base multiplied into a third of its height.

Example.—Fig. 7, Plate 7. Let S O, = 2 inches, A B and A D each = 1·4 inches; the cubic contents of the pyramid are—

$$\frac{1{\cdot}4 \times 1{\cdot}4 \times 2}{3} = 1{\cdot}3066 \text{ cubic inches.}$$

Thus the volume of a pyramid is one-third of that of a right prism, having an equal base, and being of the same height.

The volume of a truncated pyramid, with parallel bases, is equal to the product of a third of the height, into the sum of the two bases added to the square root of their product.

Thus, if V represents the volume of a truncated pyramid, of which the height, H, = 3 feet, the lower base, B, = 6 square feet, the upper, B', = 4 square feet; we have—

$$V = \frac{H}{3} \times (B + B' + \sqrt{B\,B'}) =$$

$$\frac{3}{3} \times (6 \text{ s. f.} + 4 \text{ s. f.} + \sqrt{6 \times 4}) = 14{\cdot}898 \text{ sq. feet.}$$

In practice, when there is little difference between the areas of the bases, a close approximation to the volume is obtained by taking the half of the sum of the bases, multiplied into the height. Thus, with the preceding data, we have

$$V = H \times \left(\frac{B + B}{2}\right) = 15 \text{ sq. ft.}$$

123. *Cylinders.*—The cubic contents of any cylinder, as Fig. 13, Plate 7, is equal to the product of the base into the height. Thus, in the case of a cylinder of a circular base, we have $B = \pi R^2$ (75);[*] consequently, the volume, V, = $\pi R^2 \times H$.

First Example.—What is the volume of a cast-iron cylinder, of which the radius, R, = 20 inches, and the length, H, = 108 inches?

$$V = 3{\cdot}1416 \times 20^2 \times 108 = 135{,}717 \text{ cubic inches.}$$

The volume may also be derived from the diameter of the cylinder, in which case we have—

$$V = \frac{\pi D^2}{4} \times H; \text{ or,}$$

$$V = {\cdot}7854 \times 40^2 \times 108 = 135{,}717 \text{ cubic in.}$$

The convex surface of a right cylinder, when developed, is equal to the area of a rectangle, having for base the rectilinear development of the circumference, and for height that of the cylinder. It is therefore obtained by multiplying the circumference into the height, or length. With the data of the preceding case, the convex surface is expressed by the formula—

$$S = 2\pi R \times H, \text{ or } \pi D \times H = 3{\cdot}1416 \times 40 \times 108 = 13{,}571{\cdot}7$$ cubic inches.

The volume of a hollow cylinder is equal to the difference between that of a solid cylinder of the same external radius, and that of one whose radius is equal to the internal radius of the hollow cylinder. Or, it is equal to the product of the sectional area into the height, such area being equal to the difference between two circles of the external and internal radius, respectively.

Example.—It is required to find the volume, V, and the internal surface, S', of a steam-engine cylinder, including its top and bottom flanges in the volume. Let the following be the dimensions:—External diameter, D, = 56 inches: internal diameter, D', = 50 inches; length or height, H, = 120 inches; external projection of the flanges, F, = 5 inches, and their thickness, E, = 4 inches. Then, for the internal surface, we have—

$$S' = 3{\cdot}1416 \times 50 \times 120 = 18{,}850 \text{ sq. in.}$$

For the volume of the body of the cylinder, we have—

$$V' = \frac{\pi 56^3}{4} - \frac{\pi 50^2}{4} \times 120 = ({\cdot}7854 \times 56^2) - ({\cdot}7854 \times 50^2) \times$$

$$120 = 60{,}000 \text{ cubic inches.}$$

And for the additional volume of the flanges—

$$V'' = \frac{\pi (56 + 10)^2}{4} - \frac{\pi 56^3}{4} \times 4 \times 2 = ({\cdot}7854 \times 66^2) -$$

$$({\cdot}7854 \times 56^2) \times 8 = 7666 \text{ cubic inches.}$$

Whence the whole volume—

$$V' + V'' = 67{,}666 \text{ cubic inches.}$$

124. *Cones.*—The cubic content of a cone is equal to the product of its base into a third of its height; or,

$$V = B \times \frac{H}{3}$$

In the right cone, Fig. F, of which the base is circular—

$$V = \pi R^2 \times \frac{H}{3} = \frac{\pi D^2}{4} \times \frac{H}{3};$$

and as π, or 3·1416 ÷ (4 × 3) = ·2618, the formula resolves itself into—

$$V = {\cdot}2618 \times D^2 \times H.$$

Example.—What is the volume of a right cone, of which the height, H, = 24 inches, and the diameter of the base, or D, = 17 inches?

We have—

$$V = {\cdot}2618 \times 17^2 \times 24 = 1816 \text{ cubic inches.}$$

As we shall demonstrate, on a subsequent page, the development of the convex surface of a right cone is equal to the sector of a circle, of which the radius is the generatrix, and the arc the circumference of the base of the cone—consequently, the conical surface is equal to the product of the circumference of the base into the half of the generatrix: whence is derived the following formula:—

$$S = 2\pi R \times \frac{G}{2} = \pi R \times G.$$

With the data of the foregoing example, and allowing the generatrix to be equal to 25½ inches, we have—

$$S = 3{\cdot}1416 \times 8{\cdot}5 \times 25{\cdot}5 = 681 \text{ cubic inches.}$$

125. *Frustum of a cone.*—The volume of a frustum of a cone may be obtained in the same manner as that of the truncated pyramid (123). The convex surface of a truncated cone is equal to the product of half the generatrix of the frustum into the sum of the circumferences of the bases, and is expressed in the following formula:—

$$S = \frac{L}{2} \times 2\pi (R + R') = L \times \pi (R + R').$$

Example.—Let the length, L, of the generatrix of the conic frustum, = 14 inches; the radius, R, of the lower base, = 8·5 inches; the radius, R', of the upper base, = 3·8 inches; then the convex surface—

$$S = 14 \times 3{\cdot}1416 \times (8{\cdot}5 + 3{\cdot}8) = 54 \text{ square inches.}$$

126. *Sphere.*—The volume of a sphere may be ascertained as soon as its radius is known. Its surface is equal to four times

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[*] When we wish to refer the student to any rule or principle already given, we do so by means of the number of the paragraph containing such rule of principle. In the present instance, what is referred to will be found at page 48.

that of a circle of equal diameter. This is expressed by the formula—

$$S = 4\pi R^2 = \pi D^2 = 3{\cdot}1416 \times D^2,$$

or the square of the diameter multiplied by 3·1416.

The volume is equal to the product of the surface into one-third of the radius, as in the formula—

$$V = 4\pi R^2 \times \frac{R}{3} = \frac{4}{3} \times \pi R^3, \text{ or } V = 4{\cdot}188 \times R^3;$$

or, if we employ the diametral ratio—

$$V = \pi D^2 \times \frac{D}{6} = {\cdot}5236 \times D^3.$$

Example.—We require to know what is the surface and the volume of a sphere, of which the diameter measures 25 inches.

The surface—

$$S = 25^2 \times 3{\cdot}1416 = 1963{\cdot}5 \text{ square inches.}$$

The volume—

$$V = {\cdot}5236 \times 25^3 = 8181{\cdot}25 \text{ cubic inches.}$$

To find the radius or diameter of a sphere, of which the volume is known, it is sufficient to invert the preceding operations, the formulas becoming as follows—

$$R^3 = \frac{{}^3V}{4\pi} = \frac{V}{4{\cdot}188};$$

whence,

$$R = \sqrt[3]{\frac{V}{4{\cdot}188}}.$$

Similarly,

$$D^3 = \frac{V}{{\cdot}5236};$$

whence,

$$D = \sqrt[3]{\frac{V}{{\cdot}5236}};$$

which, with the preceding data, gives R = 12·5 inches, and D = 25 inches.

The radius is derived from the surface by means of the following formula:—

$$R^2 = \frac{S}{4 \times 3{\cdot}1416};$$

whence,

$$R = \sqrt{\frac{S}{12{\cdot}5664}}$$

or,

$$D^2 = \frac{S}{3{\cdot}1416};$$

whence,

$$D = \sqrt{\frac{S}{3{\cdot}1416}}.$$

127. ***Spheric sectors, segments, and zones.***—The surface of a zone or spheric segment, is equal to the product of the circumference of a circle of the sphere, into the height of the zone or segment; or,

$$S = 2\pi R \times H.$$

Example.—The height, H, of a spheric segment being 1·5 inches, and the radius, R, of the sphere, 7·5 inches, the surface—

$$S = 2 \times 3{\cdot}1416 \times 7{\cdot}5 \times 1{\cdot}5 = 70{\cdot}686 \text{ sq. inches.}$$

The volume of a spheric sector is equal to the product of the surface of its spherical base, into one-third the radius of the sphere of which it is a portion.

The corresponding formula is therefore—

$$V = 2\pi R \times H \times \frac{R}{3} = \frac{2}{3}\pi \times R^2 H = 2{\cdot}094 \times R^2 \times H.$$

Example.—The volume of the spheric sector, whose spheric base is equal to the surface considered in the previous example, is—

$$V = 2{\cdot}094 \times 7{\cdot}5^2 \times 1{\cdot}5 = 176{\cdot}68 \text{ cubic inches.}$$

The volume of a spheric segment is equal to the product of the arc of the circle of which the chord is radius, into one-sixth of the height of the segment; or,

$$V = \pi r^2 \times \frac{H}{6} = {\cdot}5296 \times r^2 \; H.$$

Example.—Let r = 6·5 inches and H 1·5 inches; then the volume—

$$V = {\cdot}5296 \times 6{\cdot}5^2 \times 1{\cdot}5 = 33{\cdot}56 \text{ cubic inches.}$$

The volume of a spheric ungula is equal to the product of the gore, which is its base, into a third of the radius.

The formula is—

$$V = \frac{2}{3} A \times R^2;$$

where A = the area of the gore.

The volume of a zonic segment is equal to half the sum of its bases, multiplied by its height, *plus* the volume of a sphere of which that height is the diameter; whence the formula—

$$V = \left(\frac{\pi R^2 + \pi R'^2}{2}\right) \times H + \frac{\pi H^3}{6}.$$

128. *Observations.*—The volumes of spheres are proportional to the cubes of their radii, or diameters. Let V = 14·137 cubic inches, and v = 4·188 cubic inches. It will be found that the respective radii are—

$$R = \sqrt[3]{\frac{V}{4{\cdot}188}} = \sqrt[3]{\frac{14{\cdot}137}{4{\cdot}188}} = 1{\cdot}5;$$

and

$$r = \sqrt[3]{\frac{v}{4{\cdot}188}} = \sqrt[3]{\frac{4{\cdot}188}{4{\cdot}188}} = 1;$$

and, consequently, D = 3 and d = 2.

The cubes of these numbers, that is, 27 and 8, have the same ratio to each other as the volumes given; that is to say—

$$27 : 8 :: 14{\cdot}137 : 4{\cdot}188.$$

When of equal height, cylinders are to each other, as well as cones, as the squares of the radii of their bases.

When of equal diameter, these solids are to each other as their heights.

First, then, we have—

$$V = \pi R^2 \times H, \text{ and } v = \pi r^2 \times H;$$

whence,

$$V : v :: R^2 : r^2$$

And secondly,

$$V = \pi R^2 \times H \text{ and } v = \pi R^2 \times h;$$

whence,

$$V : v :: H : h.$$

The volume of a sphere is to that of the circumscribed cylinder as 2 to 3. A sphere is said to be inscribed in a cylinder, when its diameter is equal to the height and diameter of the cylinder.

The volume of an annular torus, or ring, is equal to the product of its section into the mean circumference. We have pointed out (93) that an annular torus is a solid, generated by the revolution of a circle about an axis, situated in the plane of the circle, and at right angles to the plane of revolution.

Let R be the radius of the generating circle, and r the distance of its centre from the axis, we have—

$$V = \pi R^2 \times 2\pi r = 19{\cdot}72 \; R^2 \times r.$$

PROPORTIONAL MEASUREMENTS OF THE VARIOUS PARTS OF AN ENTIRE ORDER.

THE (MODERN) DORIC ORDER.

		Designations of the Members and Mouldings constituting the Order.	Measurements according to Vignoles, in Modules of 12 Parts.				Measurements in Decimals The Module = 1.			
			Amount of Projection from the Axis of the Shaft.	Heights.			Amount of Projection from the Axis of the Shaft.	Heights.		
			M. P.	M. P.	M. P.	M. P.				
ENTABLATURE.	CORNICE,	Reglet,	2 10	1	1 6	4 0	2·833	·083	1·500	4·000
		Cavetto,	2 7	3			2·583	·250		
		Fillet,	2 6½	½			2·542	·042		
		Cymatium,	2 6 2 5	1½			2·500 2·417	·125		
		Corona,	2 4½	4			2·375	·333		
		Fillet,	2 2	½			2·167	·042		
		Mutules,	2 1½	½			2·125	·042		
		Guttæ,	1 3	2½			1·250	·209		
		Fillet,	1 1	½			1·083	·042		
		Cymatium,	1 0½ 11½	2			1·042 ·959	·166		
		Capitals of the Triglyphs,	11	2			·917	·166		
	FRIEZE,		10	1 6	1 6		·833	1·500	1·500	
	ARCHITRAVE,	Tænia,	11½	2	1 0		·959	·167	1·000	
		Facia,	10	10			·833	·833		
COLUMN.	CAPITAL,	Reglet,	1 3½	½	1 0	16 0	1·292	·042	1·000	16·000
		Cymatium,	1 3¼ 1 2¼	1			1·271 1·188	·083		
		Abacus,	1 2	2½			1·167	·209		
		Echinus,	1 1¾	2½			1·146	·209		
		Three Annulets,	11½ 10½	1½			·959 ·875	·124		
		Necking,	10	4			·833	·333		
	SHAFT,	Beading or Astragal,	1 0	1	14 0		1·000	·083	14·000	
		Cincture,	11½	½			·959	·042		
		Shaft Proper,	10 1 0	13 10½			·833 1·000	13·875		
	BASE,	Fillet,	1 1¼	1	1 0		1·104	·083	1·000	
		Beading,	1 2	1			1·167	·083		
		Torus,	1 5	4			1·417	·334		
		Plinth,	1 5	6			1·417	·500		
PEDESTAL.	CORNICE,	Reglet,	1 11	½	6	5 4	1·917	·042	·500	5·333
		Quarter-Round,	1 10⅔	1			1·889	·083		
		Fillet,	1 9⅔	½			1·806	·042		
		Corona,	1 9	2½			1·750	·209		
		Cymatium,	1 6½ 1 5½	1½			1·542 1·459	·124		
	DADO,		1 5	4 0	4 0		1·417	4·000	4·000	
	BASE,	Fillet,	1 6	½	10		1·500	·042	8·333	
		Beading,	1 7	1			1·583	·083		
		Cyma Reversa,	1 7 1 8½	2			1·583 1·708	·166		
		Plinth,	1 9	2½			1·750	·209		
		Sub-Plinth or Socle,	1 9½	4			1·792	·333		
		Total height of the Order,	....			25 4	....			25·333

PROPORTIONAL MEASUREMENTS OF THE VARIOUS PARTS OF AN ENTIRE ORDER.

THE TUSCAN ORDER.

Designations of the Members and Mouldings constituting the order.			Measurements according to Vignoles, in Modules of 12 Parts.				Measurements in Decimals. The Module = 1.			
			Amount of Projection from the Axis of the Shaft.	Heights.			Amount of Projection from the Axis of the Shaft.	Heights.		
			M. P.	M. P.	M. P.	M. P.				
ENTABLATURE.	CORNICE,	Quarter-Round,	2 3½	4	1 4	3 6	2·292	·333	1·333	3·500
		Beading,	2 0	1			2·000	·083		
		Fillet,	1 11½	½			1·959	·042		
		Larmier or Corona,	1 10½	6			1·875	·500		
		Fillet,	1 7½	½			1·625	·042		
		Cymatium,	*1 1½ 10	4			1·125 ·833	·333		
	FRIEZE,		9½	1 2	1 2		·792	1·167	1·167	
	ARCHITRAVE,	Listel,	11½	2	1 0		·959	·167	1·000	
		Facia,	9½	10			·792	·833		
COLUMN.	CAPITAL,	Listel,	1 2½	1	1 0	14 0	1·209	·083	1·000	14·000
		Abacus,	1 1½	3			1·125	·250		
		Echinus, or Quarter-Round,	1 1	3			1·083	·250		
		Fillet,	10½	1			·875	·083		
		Necking,	9½	4			·792	·334		
	SHAFT,	Astragal, Beading,	11	1	12 0		·917	·083	12·000	
		Astragal, Cincture,	10½	½			·875	·042		
		Shaft Proper,	9½ 1 0	11 10½			·792 1·000	11·875		
	BASE,	Fillet,	1 1½	1	1 0		1·125	·083	1·000	
		Torus,	1 4½	5			1·375	·417		
		Plinth,	1 4½	6			1·375	·500		
PEDESTAL.	CORNICE,	Listel,	1 8½	2	6	4 8	1·709	·167	·500	4·667
		Cymatium,	1 8 1 5	4			1·667 1·417	·333		
	DADO,		1 4½	3 8	3 8		1·375	3·667	3·667	
	BASE,	Listel,	1 6½	1	6		1·542	·083	·500	
		Socle or Plinth,	1 8½	5			1·709	·417		
Total height of the Order,						22 2				22·167

With the help of these tables we can easily determine the proper measurement for any member or moulding, in feet and inches, when the height of the whole order is given. For this purpose the given height must be divided by the decimal measurement in the tables for the total given height; the quotient is the measurement of the module proportioned to such height. Then that of any required member is found by multiplying this module into the decimal in the table corresponding to such member.

First Example.—It is required to know what is the diameter of the lower part of the shaft according to the Tuscan order, the height of the entire order being 15 feet.

The height of the entire order being 22·167 when the module $=1$, we have $\frac{22{\cdot}167}{15} = 1{\cdot}4778$ the module, and $1{\cdot}4778 \times 2 = 2{\cdot}9556$ feet, the diameter of the lower part of the shaft.

Second Example.—What is the height of the socle or lower plinth according to the Tuscan order, supposing the module to be 1·4778 feet? We have $1{\cdot}4778 \times {\cdot}417 = {\cdot}616$.

In like manner the dimensions of all the other details may be easily determined according to the Tuscan or Doric order.

---

* When two measurements are given, the first applies to the upper portion, the second to the lower.

COLOURS FO

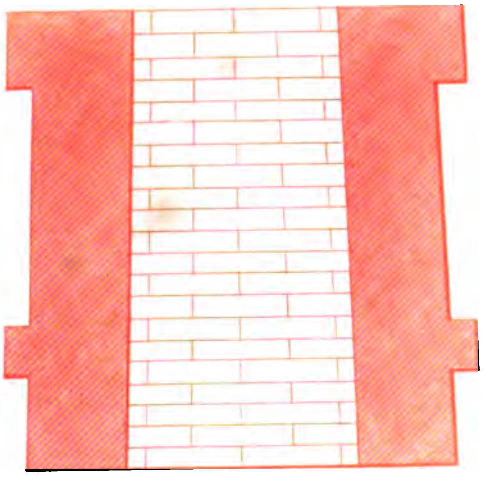
Fig. 1.

STONE.

Fig. 3.

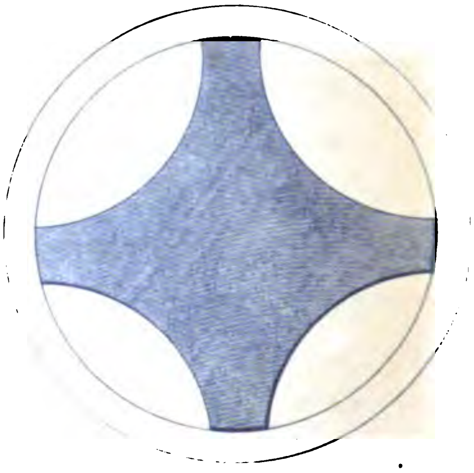
MALLEABLE IRON
OR
STEEL.

Fig. 2.



BRICKWORK

Fig. 4.



CAST IRON.

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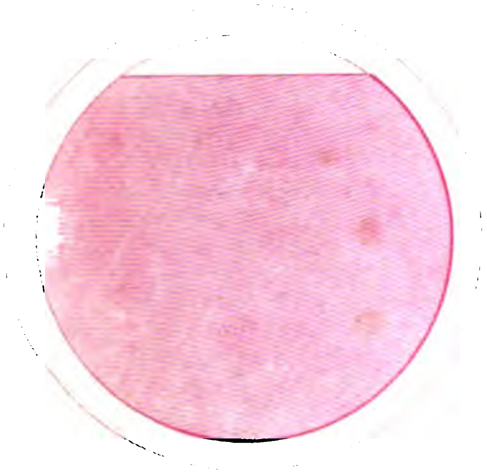
R SECTIONS.

Fig. 5.



COPPER.

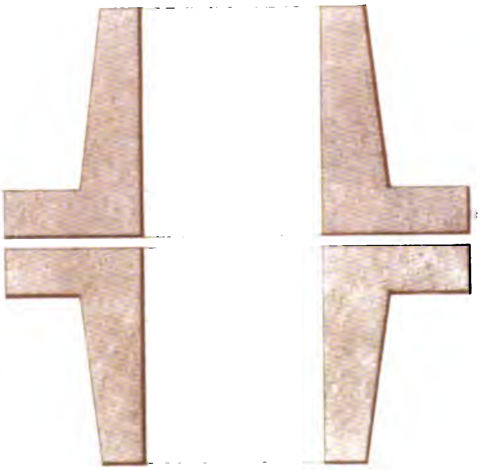
Fig. 7.

WOOD.

Fig. 6.

BRASS.

Fig. 8.



LEATHER.

Murray & Heywood [illegible]

CHAPTER V.

ON COLOURING DRAWINGS.

129. Hitherto we have indicated the sectional portions of objects by means of closely drawn lines to which the name of linear flattinting is given. This is a very tedious process, whilst it demands an amount of artistic skill only obtainable by long practice to enable the draughtsman to produce pleasing and regular effects; and although by varying the strength or closeness of the lines as we have already pointed out (par. 112), it is possible to express approximately the nature of the material, yet the extent of such variation is extremely limited, and the distinction it gives is not sufficiently intelligible for all purposes. If, however, in place of this system, we substitute colours laid on with the brush, we at once obtain a means of rapidly tinting the sectional parts of an object, and also of distinctly pointing out the nature of the materials of which it is composed, however numerous and varied such materials may be. The colours generally adopted in geometrical drawings are conventional—that is, certain colours are understood to indicate particular materials.

In Plate 10 we give examples of the colours applied to the representations of the principal materials in use; such as stone and brick, steel and cast-iron, copper and brass, wood and leather. We propose now to enter into some details of the composition of the various colours given in this plate.

THE COMPOSITION OR MIXTURE OF COLOURS.

PLATE 10.

130. *Stone.*—Fig. 1. This material is represented by a light dull yellow, which is obtained from Roman ochre, with a trifling addition of China ink, warmed with a little carmine.

131. *Brick.*—Fig. 2. A light red is employed for this material, and may be obtained from vermillion, which may sometimes be brightened by the addition of a little carmine. A pigment found in most colour-boxes, and termed *Light Red*, may also be used when great purity and brightness of tint is not wanted. If it is desired to distinguish firebrick from the ordinary kind, since the former is lighter in colour and inclined to yellow, some gamboge must be mixed with the vermillion, the whole being laid on more faintly. In external views it is customary to indicate the outlines of the individual bricks, but in the section of a mass of brickwork this may be dispensed with, except in cases where it is intended to show the disposition or mode of building up. Thus, in furnaces, as also in others tructures, the strength depends greatly on the system of laying the bricks. When vermillion is used in combination with other colours, the colour should be constantly mixed up by the brush—as, from its greater weight, the vermillion has a tendency to sink and separate itself from the others; and if this is overlooked, a varying tint of unpleasing effect will be imparted to the object coloured.

132. *Steel or Wrought Iron.*—Fig. 3. The colour by which these metals are expressed is obtained from pure Prussian blue laid on light—being lighter and perhaps brighter for steel than for wrought-iron. The Prussian blue generally met with in cakes has a considerable inclination to a greenish hue, arising from the gum with which it is made up. This defect may be considerably amended by the addition of a little Chinese white, carmine, or crimson lake—the proportion depending on the taste of the artist.

133. *Cast Iron.*—Fig. 4. Indigo is the colour employed for this metal; the addition of a little carmine improves it. The colours termed *Neutral Tint*, or *Payne's Gray*, are frequently used in place of the above, and need no further mixture. They are not, however, so easy to work with, anddo not produce so equable a tint.

134. *Lead* and *Tin* are represented by similar means, the colour being rendered duller and grayer by the addition of China ink and carmine or lake.

135. *Copper.*—Fig. 5. For this metal, pure carmine or crimson lake is proper. A more exact imitation of the reality may be obtained by the mixture, with either of these colours, of a little China ink or burnt sienna—the carmine or lake, of course, considerably predominating.

136. *Brass or Bronze.*—Fig. 6. These are expressed by an orange colour, the former being the brighter of the two; burnt Roman ochre is the simplest pigment for producing this colour. Where, however, a very bright tint is desired, a mixture should be made of gamboge with a little vermillion—care being taken to keep it constantly agitated, as before recommended. Many draughtsmen use simple gamboge or other yellow.

137. *Wood.*—Fig 7. It will be observable, from preceding examples, that the tints have been chosen with reference to the actual colours of the materials which they are intended to express—carrying out the same principle, we should have a very wide range in the case of wood. The colour generally used, however, is burnt umber or raw sienna; but the depth or strength with which it is laid on, may be considerably varied. It is usual to apply a light shade first, subsequently showing the graining with a darker tint, or perhaps with burnt sienna. These points are susceptible of great variation, and very much must be left to the judgment of the artist.

138. *Leather, Vulcanised India-Rubber, and Gutta Percha.*—Fig. 8. These are all represented by very similar tints. Leather by light, and gutta percha by dark sepia, whilst vulcanised India-rubber may have the addition of a little indigo to that colour.

We may here remark, that if the student is unwilling to obtain an extensive stock of colours, he may content himself with merely a good blue, a yellow, and a red—say Prussian blue, gamboge or yellow ochre, and crimson lake. With these three, after a little practice, he may produce all the various tints he needs; but, of course, with less readiness and facility than if his assortment were larger.

139. *Manipulation of the Colours.*—We have seen by what mixtures each tint may be obtained, and we shall proceed to give a few hints relative to their application. It may be imagined that it is an easy matter to colour a geometrical drawing—that is, simply to lay on the colours; but a little attention to the following observations will not be misplaced, as the student may thereby at once acquire that method which conduces so much to regularity and beauty of effect, and which it might otherwise require some practice to teach.

The cake of colour should never be dipped in the water, as this causes the edges to crack and crumble off, wasting considerable quantities. Instead of this, a few drops of water should be first put in the saucer, or on the plate, and then the required quantity of colour rubbed down, the cake being wetted as little as is absolutely necessary. The strength or depth of the colour is obtained by proportioning the quantity of water, the whole being well mixed, to make the tint and shade equal throughout. Where large surfaces have to be covered by one shade, which it is desired to make a perfectly even flat tint, it is well to produce the required strength by a repetition of very light washes. These washes correct each other's defects, and altogether produce a soft and pleasing effect. This mode should generally be employed by the beginner, as he will thereby more rapidly acquire the art of producing uniform flat tints. A subsequent wash should not be applied until the preceding one is perfectly dry. When the drawing-paper from any cause does not take the colour well, its whole surface should, previous to the application of colour, receive a wash of water, in which a very small quantity of gum-arabic or alum has been dissolved. In proceeding to lay on the colour, care should be taken not to fill the brush too full, whilst at the same time, it must be replenished before its contents are nearly expended, to avoid the difference in tint which would otherwise result. It is also desirable first to try the colour on a separate piece of paper, to be sure that it will produce the desired effect. It is a very common habit with water-colour artists to point the brush, and take off any superfluous colour, by passing it between their lips. This is a very bad and disagreeable habit, and should be altogether shunned. Not only may the colour which is thus taken into the mouth be injurious to health, but it is impossible, if this is done, to produce a fine even shade, for the least quantity of saliva which may be taken up by the brush has the effect of clouding and altogether spoiling the wash of colour on the paper. In place of this uncleanly habit, the artist should have a piece of blotting-paper at his side—the more absorbent the better. By passing the brush over this, any superfluous colour may be taken off, and as fine a point obtained as by any other means. The brush should not be passed more than once, if possible, over the same part of the drawing before it is dry; and when the termination of a large shade is nearly reached, the brush should be almost entirely freed from the colour, otherwise the tint will be left darker at that part. Care should be taken to keep exactly to the outline; and any space contained within definite outlines should be wholly covered at one operation, for if a portion is done, and then allowed to dry, it will be almost impossible to complete the shade without leaving a distinct mark at the junction of the two portions. Finally, to produce a regular and even appearance, the colour should be laid on as *thin* as possible; for the time employed in more frequently replenishing the brush, will be amply repaid by the improved appearance of the work under the artist's hands.

CONTINUATION OF THE STUDY OF PROJECTIONS.

THE USE OF SECTIONS—DETAILS OF MACHINERY.

PLATE 11.

140. We have already shown, when treating of the illustrations in Plate 8, that it is advisable to represent various objects, in section, or as divided or cut through, in order to render their internal arrangement clearly intelligible; and we may now proceed to show with the aid of sundry examples, brought together in Plate 11, that in particular cases sections are indispensable, and even more necessary, than external elevations. It is with this object that, in many of our geometrical drawings, we have given representations of objects, cut or sectioned through their axes or centres, so as to accustom the student to this kind of projection, the importance and utility of which cannot be overrated. Sections are generally taken in this way through the centre of the object, either at right angles or parallel to the vertical or horizontal planes of projection, except when something is required to be shown which could not be delineated in these planes, In such cases sections may be taken on any plane and termed oblique and irregular.

141. *Footstep Bearing.*—Figs. 1 and 2 are the representations, in elevation and plan, of a footstep, formed to receive the lower end of a vertical spindle, or shaft. This footstep consists of several pieces, one contained within the other; and it is evidently impossible to say, from the external views, what their actual entire shape may be, although a part of each is seen in the horizontal projection, Fig. 2. If, however, we suppose the whole to be divided by a vertical plane as dotted through the centre of the plan, Fig. 2, parallel to the vertical plane of projection, Fig. 1, we shall be enabled to form another vertical projection, Fig. 3, showing the internal structure, and this is termed a vertical section, or sectional elevation. This figure shows, first, the thickness of the external cup-piece, or box, A, as also the dimensions of the opening, *a*, which is made in its base, for the introduction of a pin, to raise the footstep proper, B, when necessary; secondly, the thickness and internal depth of the footstep, B, as also the internal vertical grooves, *b*, which serve for the introduction of the key, *c;* thirdly, the form and manner of adjustment of the centre-bit, C, which sustains the foot of the vertical spindle or shaft. This centre-bit, which, of course, should not turn with the spindle-foot, is prevented from doing so by means of the key, *c*, which fits into a cross groove in its under side, the key itself being held firmly by the grooves, *b*, into which its projecting ends are made to fit. Of these details, the cup-piece, A, is of cast-iron, the footstep, B, of gun-metal or brass, the centre-piece, C, of tempered steel, and the small key, *c*, of wrought-iron. Therefore, bearing in mind what has already been said, we may indicate these various materials in the sections, either by line-shading, of different strengths, as in Fig. 3, or by means of colours, corresponding to those employed in plate 10; and we may here remark, that where line-sectioning is used, brass, gun-metal, or bronze, is frequently expressed by a series of lines, which are alternately full and dotted, as shown at Figs. 6 and 12. There are, besides, many ways of varying the effects produced by line-shading. For example, the spaces between the lines may be alternately of different widths, or the lines may be alternately of different strengths. Strictly speaking, Figs. 2 and 3 are all that are necessary for the representation of the object under discussion. The cup-piece, A, however, which is externally cylindrical, has, at four points, diametrically opposite to each other, certain projecting rectangular plane surfaces, *d*, which are provided to receive the thrust of the screws which adjust the footstep accurately in the centre. The width of these facets is shown in the plan, Fig. 2, whilst their depth is obtainable from the elevation, Fig. 1. If, instead of these facets, *d*, being, as they are, tangential to the cylinder, A, they had projected, in the least, at their centres, their depth would necessarily have been given in the section Fig. 3, and in such case the elevation, Fig. 1, might have been altogether dispensed with. Whilst referring to the representation of the projecting facets, in connection with the cylinder, A, we may remark, that when a cylinder is intersected by a plane, which is parallel to its axis, the line of intersection is always a straight line, as seen on each side of Fig. 1, by the junction of the edges of the facets, *d*, with the the cylinder, A, Figs. 1 and 2.

DETAILS OF

IN ELEVATION,P

Fig. 1.

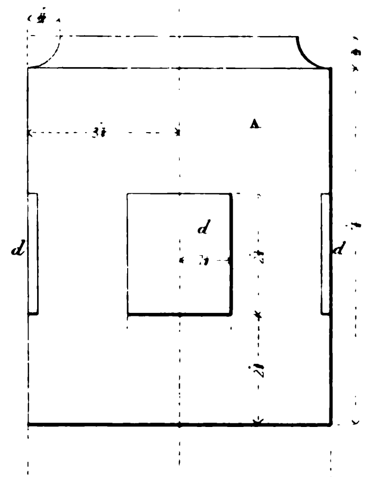


Fig. 4.

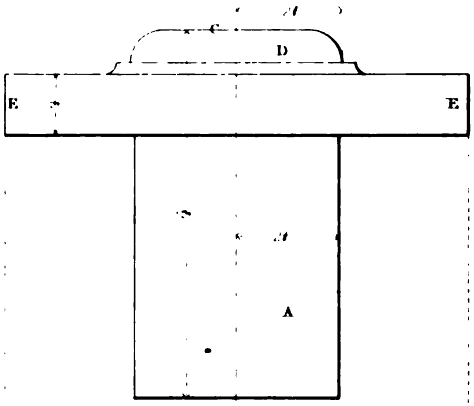


Fig 7

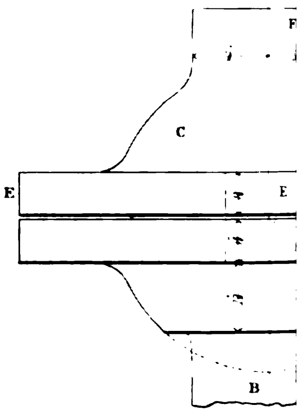


Fig. 2.

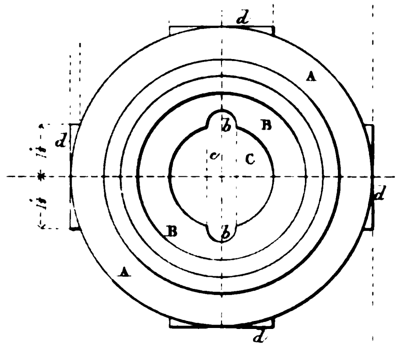


Fig. 5.

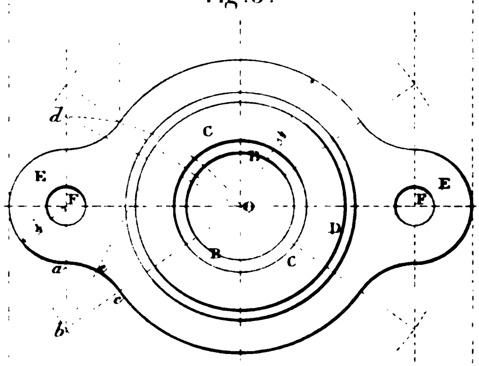


Fig 8.

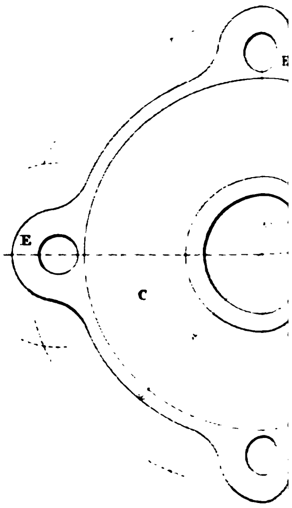


Fig. 3.

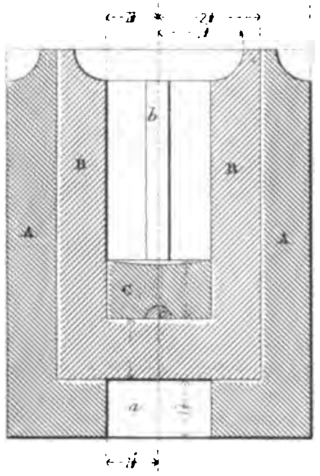


Fig. 6.

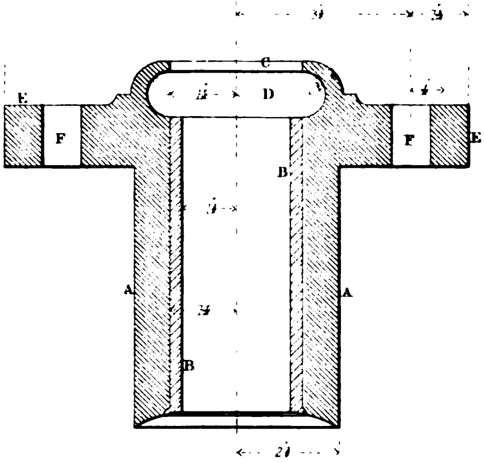
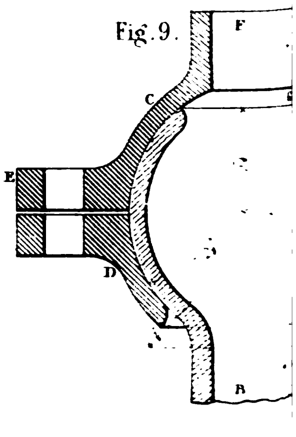
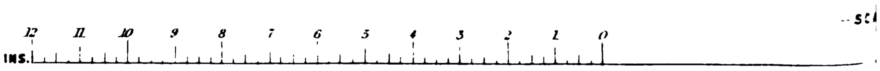


Fig. 9.





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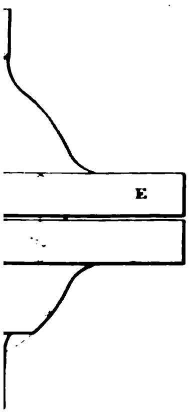
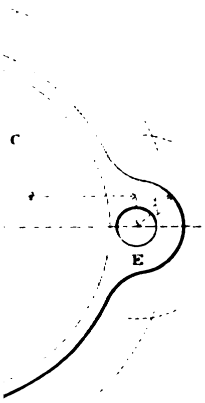
47 Lincoln's Inn Fields & Glasgow

PLATE II.

MACHINERY,

N & SECTION.





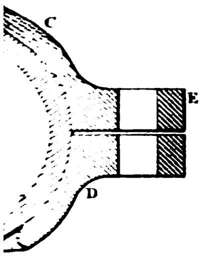


Fig. 10.

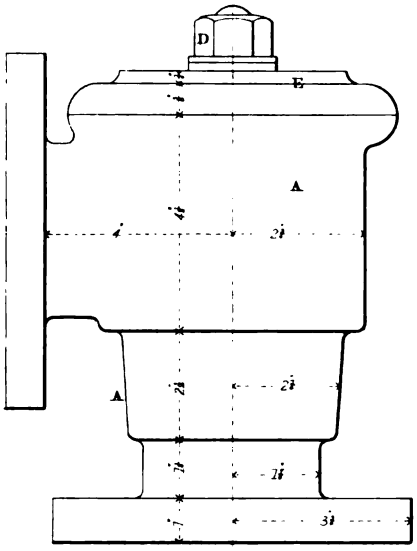


Fig. 16.

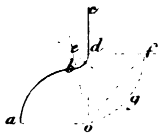


Fig. 13.

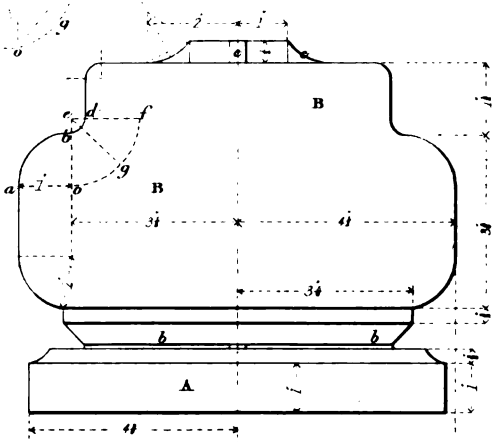


Fig. 11.

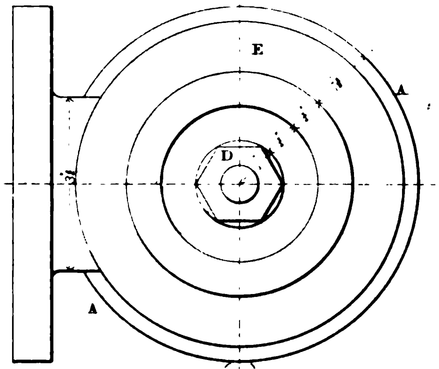


Fig. 14.

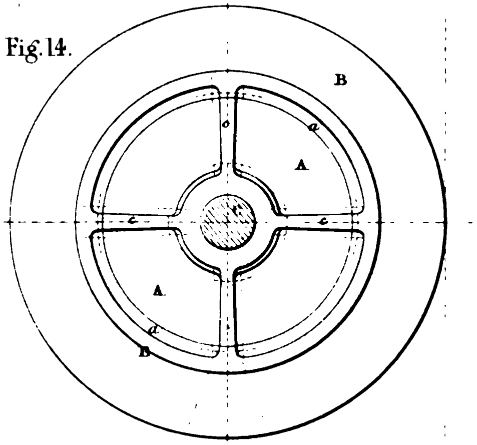


Fig. 12.

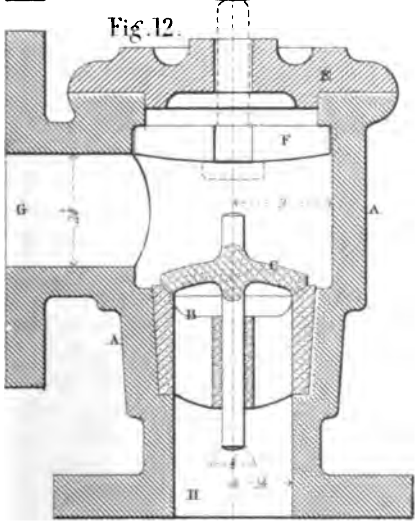
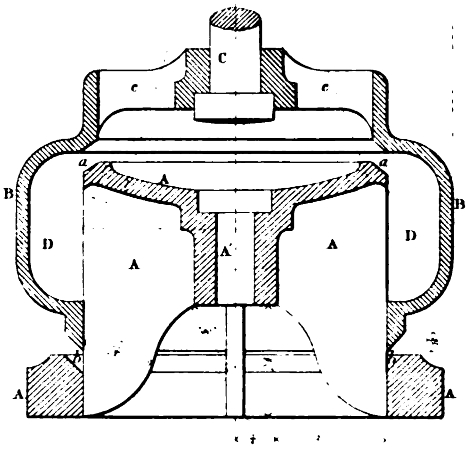


Fig. 15.



142. *Stuffing-box Cover, or Gland.*—In pumps and steam-engine cylinders, the cover is furnished, at the opening through which the piston-rod passes, with a stuffing-box, to prevent leakage. The hemp, or other material used as packing, is contained in an enlargement of the piston-rod passage, and is tightly pressed down by a species of deep annular bush, A, with a flange at the top, which is formed into lugs, E, as represented in elevation and plan, Figs. 4 and 5. In this instance, the necessity of a sectional view is still more obvious than in the case of the footstep already treated of. In the vertical section, Fig. 6, it is shown, that the internal diameter is not uniform throughout, and that there is a bush or deep ring, B, let in through the lower part, A, of the gland. The cylindrical opening, B, of which is only just wide enough to allow the piston-rod to play up and down within it, while the internal diameter of a portion, C, at the very top of the gland, is only a very little larger. The part, D, however, comprised between these two, is much larger in diameter and forms a hollow and annular space, or cup, to hold the oil for lubricating the rod and stuffing-box. It is further discernible in the section, that the flanges or lugs, E, which project on either side of the upper portion of the gland, have each a cylindrical opening, F, throughout their whole depth. These are the holes for the bolts, which force down the gland, and secure it to the corresponding flanges, or lugs, on the stuffing-box. The annular cup, or hollow ring, D, at the upper and internal part of the gland, acts as a reservoir, into which the lubricating oil is first poured, and whence it gradually oozes into the interior. When it is an inverted stuffing-box, the part at C, is made to fit the rod, the same as the part, B, and the oil is admitted into the part, D, by a cock for the purpose. The ring, or bush, B, is forcibly fitted in either from the top or bottom of the gland, and riveted at the opposite end. The lower end or edge of the body part, A, of the gland, is bevelled so as to wedge or force the packing, which is in the socket-part of the stuffing-box beneath it, against the moving-rod, and thus form a steam-tight joint. The bush, B, is generally made of brass, both with a view to lessen the friction, and to its being replaced with facility when worn, without the necessity of renewing the whole gland. The latter is generally made of cast-iron, though the whole is sometimes made of brass or gun-metal.

To complete the outline of the plan, Fig. 5, it is required to draw the arcs, *a e c*, of the lugs, E, gliding into the top flange of the gland. Through the centres, F, of the lugs, draw the perpendicular, *d* F *a b*, to the centre line, F O F; then from the point, *a*, where this perpendicular intersects the outline of the snug, make *a d*, equal to O *c*, the radius of the outline of the flange of the gland (Fig. 9, Plate 3, Problem 40) join *d* O, and bisect it by a perpendicular, which will intersect *d a*, produced in *b*, the centre of the arc required, and *a b* will be its radius, all the other points, *b*, can now be found by taking the radius, O *b*, and O, as a centre, and intersecting the perpendiculars through the centres, F, of the lugs, as shown by dotted lines, in Fig. 5.

143. *Spherical Joint.*—In some cases, a locomotive receives water from its tender by means of pipes which are fitted with spherical joints, as a considerable play is necessary in consequence of the engine and tender not being rigidly connected together; and also to obviate any difficulty of attachment from the pipes in the locomotive not being exactly opposite to those in the tender. This species of joint, represented in elevation and plan, in Figs. 7 and 8, gives a free passage to the water, in whatever position, within certain limits, one part may be with respect to the other. For its construction to be thoroughly understood, the vertical section, Fig. 9, is particularly required. This view, indeed, at once explains the various component parts, consisting—first, of a hollow sphere, A, of the same thickness as the pipe, B, of which it forms the end; and, second, of two hemispherical sockets, C and D, which embrace the ball, A, and which are firmly held together by bolts passing through lugs, E. When this species of joint is used of a small size, as at the junction of a gas chandelier with the ceiling, the two half-sockets are simply screwed together—this method, indeed, being adopted in many locomotives. It must be borne in mind, that our object in this work is simply to instruct the student to represent accurately mechanical and other objects, and for this purpose we employ both precept and example; but such examples do not necessarily refer to the latest and most improved or efficient forms. The half-socket, C, forms part of the continuation, F, of the feed pipe, whilst the half-socket, D, is a detached piece, necessarily moveable, to allow of the introduction of the spherical part, A. This half-socket, D, is partially cut away at the lower part, and does not fit closely to the neck of the ball. This allows the pipe, B, to move to a slight extent from side to side in any direction; and the upper end of the ball, A, is cut away to a corresponding extent, to prevent any diminution of the opening into the pipe, F, when the two portions are thus inclined to each other. The pipe, F, with its half-socket, C, is an example of the combination of a cylinder with a sphere, and gives us occasion to observe, that the intersection formed by the meeting or junction of these surfaces is always a circle in one projection, and a straight line in the other, as shown by the lines, marked G, Fig. 9. The subject of such intersections will be discussed more in detail in reference to Plate 14. The sockets, C and D, are formed with four external lugs, or eye-pieces, E, sometimes termed snugs, for connection by bolts, as before stated. The curved outlines of these lugs, which glide tangentially into that of the body of the socket, give rise to the solution of a problem which may be thus put: *To draw with a given radius an arc tangential to two given arcs.* The solution is thus obtained: with the centres of the sockets and the lugs respectively, and with radii equal to those of the respective given arcs, plus that of the required arc, describe arcs at about the position in which the centres of the required arcs should be; the intersections of these arcs will give the exact centres of the arcs required to be drawn, and the lines joining these centres with the centres of the socket and lugs, give the points of junction of these arcs with the other two, as shown in dotted lines to the right of the lower lug, in Fig. 8. This spherical joint, which requires great accuracy of adjustment of the different parts, is generally cast in brass, being finished by turning and grinding.

144. *Safety Valve and Others.*—To insure, as far as is practically possible, the safe and economical working of steam boilers, they are usually fitted with pressure gauges, level indicators, alarm whistles, and safety-valves. The object of the safety-valve is to give an outlet to the steam as soon as it reaches a greater pressure than it has been determined on, and for which the valve is loaded. Figs. 10, 11, and 12, respectively, represent an elevation, plan, and vertical section of a valve and chest in common use, as safety, suction, and discharge valves. This apparatus consists of two distinct parts; first, the cast-iron valve-chest, A, directly fixed to the top of the boiler by its flanges, or through intermediate branch pipes, the joints being made perfectly steam-tight by means of screw-bolts and layers of canvas and cement; second, the valve-seat, B, and the valve itself, C, which is sometimes cast in iron, but more generally in brass. The valve-piece, C, is cast with a central spindle, which works in a round hole bored in the centre of three arms in

the valve-seat, B, and serves to guide it in its proper place; the openings between these arms forming a passage for the egress of the steam or other fluid. The face, or bearing of the valve-piece, C, or the part by which it rests on the seat, B, consists of a very narrow annular bevelled surface; and the upper edge of the seat, B, is bevelled off internally so that the surfaces on which the valves rest, are in the closest contact, and form a tight joint when shut. When this valve is used as a safety-valve, the upper surface of the valve-piece, C, is furnished with a spindle, which is sometimes carried up through a stuffing-box in the cover and loaded either by a weighted lever, or weights put direct on the spindle outside or inside the chest, A, the weight, being regulated to correspond with the pressure to which it is deemed safe to submit the boiler, so that, when this pressure is exceeded, the valve rises, and the steam blows off as long as relief is necessary. But when this valve is used as a suction or discharge-valve for pumps, as shown, this spindle is very short, and serves to keep the valve, C, from lifting too high, by coming up against the head of the bolt which secures the cover, E, by passing through a cross-bar, F, inside the chest, A, and having a nut, D, outside, or on the top of the cover.

145. *Equilibrium or Double-beat Valves.*—Steam engines of large dimensions, such as those for pumping, employed in Cornwall, as well as marine engines, are often furnished with a species of double-beat or equilibrium valve in place of the ordinary D slide. An example of this description of valve is given in Figs. 13, 14, and 15. It possesses the property of giving a large extent of opening for the passage of the steam, with a very little traverse, and very little power is required to work the valve. The valve here represented consists of a fixed seat, A, of cast-iron or brass, and forming part of, or is secured to the valve chamber; and a bell-shaped valve-piece, B, also in brass, fitted with a rod, C, by means of which it is moved. The contact of the valve with its seat is effected at two places, *a* and *b*, which are formed into accurate conical surfaces, the one, *a*, being internal, and the other, *b*, external. When the valve is closed, these surfaces coincide with similar ones on the seat, and when it is lifted, as in Figs. 13 and 15, two annular openings are simultaneously formed, thus giving a double ingress or egress, as the case may be, to the steam—which enters or issues through the upper opening, *a*, through the central part, or large annular space, D, formed in the valve-piece, B. The rod or spindle, C, of the valve, B, is fixed to a centre-eye cast in one with the valve-piece, and connected to it by four arms, *c*. The seat is similarly constructed, with four arms, or deep feathers, having an annular ring at the base, the top edge of which is bevelled and ground to fit the lower edge of the valve, B, and when lifted, forms the lower opening at *b*. Through the hole, A^1, in the centre at the top end, which is composed of a solid hollow disc, or concave surface, there is passed a bolt, which, with a nut and washer, screws the valve-seat steam-tight, to the steam-chest in which they are placed. The external contour of this valve presents a series of undulations, involving the following problems in their delineation:—*To draw the curved junction of the body of the valve with the upper cylindrical part.* This is similar to the one treated of in reference to Figs. 9, or 9 A, Plate 3, Problem 40, and may be easily drawn with the assistance of the enlarged detail, Fig. 16, Plate 11. Let *o* be the centre of the given arc, *a b*, of the body part, and *d* the point of contact in the line, *c d*, forming the outline of the upper cylindrical part, through which the required circle is to be drawn, tangentially to them. Through *d*, draw the perpendicular, *e d f*, making *d f* equal to *o a*, the radius of the given arc, join *f o*, and bisect it by the perpendicular, *g e*, which will cut the line, *f d*, produced in the point, *e*, the centre of the required circle, and *e d*, or *e b*, will be its radius. Next, for the junction of the arms, *c*, with the more elevated boss, we require: *To draw an arc tangent to a given straight line, and passing through a given point.* The solution of this is extremely simple, we have merely to erect a perpendicular on the top line, through the given point of contact in it, and a perpendicular bisecting the line joining this point of contact, and the one in the boss, will cut it in the centre of the required circle; this construction is dotted in at the top to the left of Fig. 13.

The central leaves or feathers of the seat, A, Fig. 15, are finished off at the lower part by an ogee, drawn according to Problem 42, already discussed in reference to Figs. 13 and 14, of Plate 3.

The student will now see the imperative necessity of internal views, or sections for, the perfect intelligibility of the construction and action of various pieces of mechanism. With reference also to the examples collected together in this plate, a little consideration will show that the internal formation could not generally be sufficiently indicated by dotted lines; for, besides the complication and confusion that would result from such a method, many such lines would confound themselves with full ones representing some external outline.

We have not thought it necessary to enter more into detail respecting the modes of constructing the various outlines, being persuaded that the dotted indications we have given, along with the references to former problems, will be quite sufficient for the student who has advanced thus far, the more so since the requisite operations bear great resemblance to those treated of in reference to Plate 3.

SIMPLE APPLICATIONS.

SPINDLES, SHAFTS, COUPLINGS, WOODEN PATTERNS.

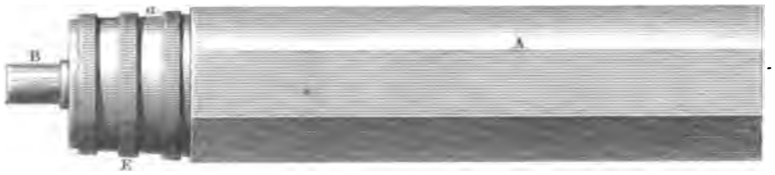
PLATE 12.

146. For the transmission of mechanical action, under the form of rotatory or partial rotatory motion, the parts of machinery technically known as shafts or spindles, of wrought and cast-iron and wood, are used. Shafts of the latter kind, namely, cast-iron and wood, are employed chiefly in hydraulic motors, water and wind mills, and in all machines where a great strain has to be transmitted, rendering considerable bulk necessary. Of these two kinds, wooden shafts, being more economical, have been preferred in some cases, particularly when the length is great, since they will better sustain severe shocks. Wrought-iron shafts are employed for the transmission of motion in factories and workshops, and for the main paddle-shafts of steam-vessels. Wrought-iron has the advantage of being less brittle than cast, and of possessing greater tenacity and elasticity, and is preferable in situations where they are liable to irregular strain or sudden shocks.

147. *Wooden Shafts.*—Figs. 1, 4, 5, and 6, represent different projections of a wooden shaft, such as is used for a water-wheel. Fig. 4 shows, on one side, a lateral elevation of the shaft, furnished with its iron ferules or collars, and its spindle; and at the same time, on the other side, a vertical section, passing through the centre of the shaft, giving the ferules in section, but supposing the gudgeon or central spindle, with its feathers, to be in external elevation, Generally, in longitudinal sections of objects enclosing one or more pieces, the innermost or central piece should not be sectioned, unless it has some internal peculiarity—the object of a section being to show and explain such peculiarity where it exists, and being quite unnecessary where the object is simply solid. In the same manner, it is not worth while sectioning the various

SHAFTS &

Fig.1.



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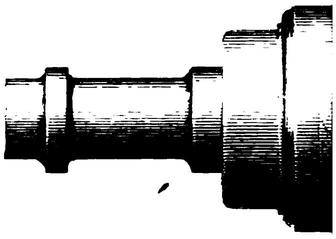
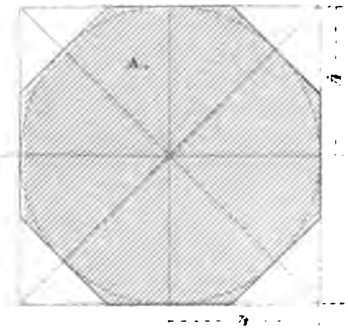


Fig.5.



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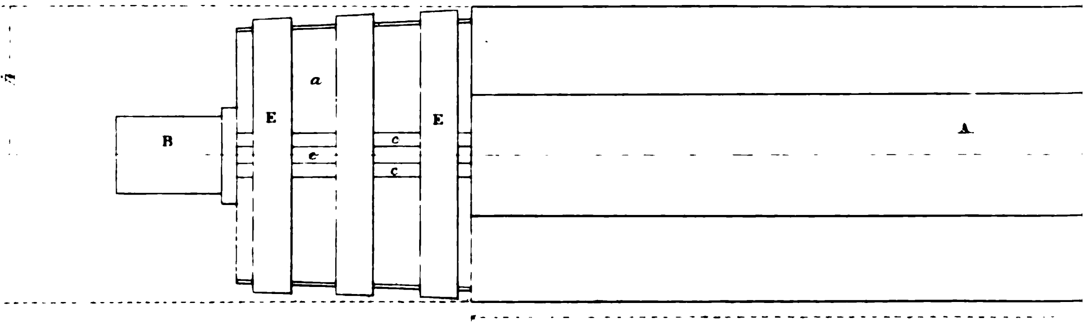
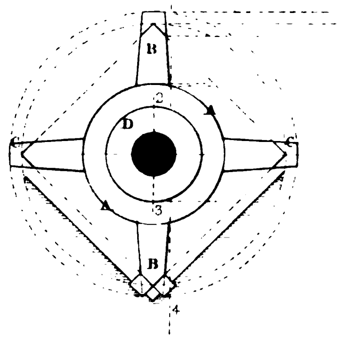


Fig.8.



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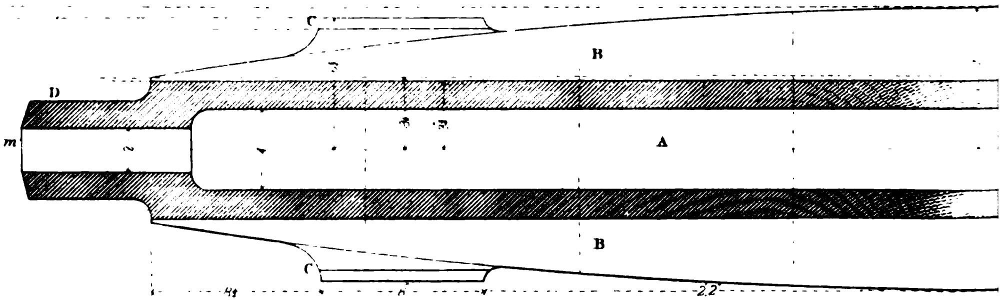


Fig.10.

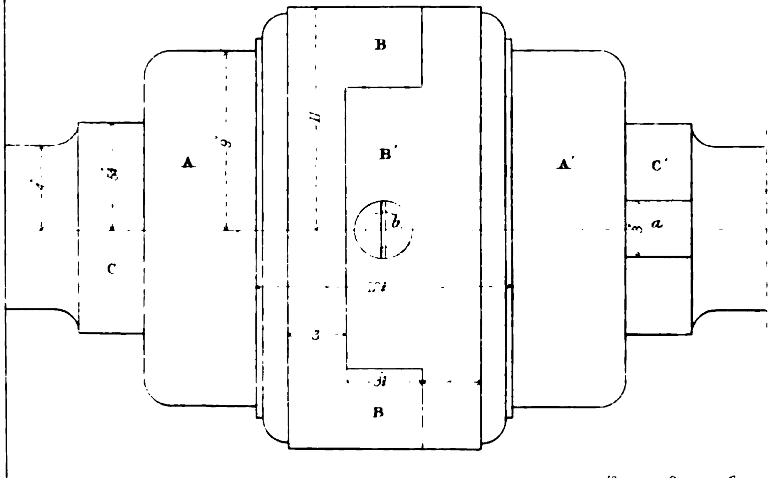
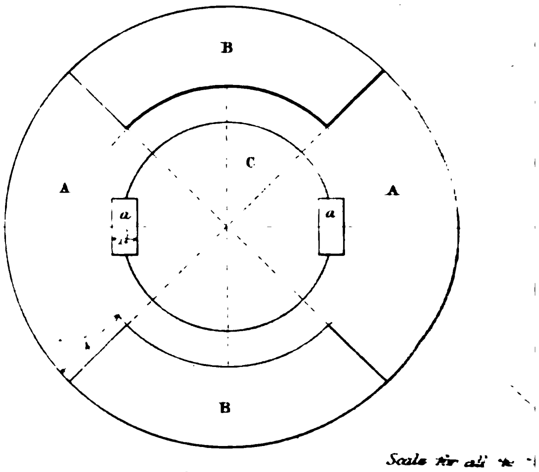


Fig.11.



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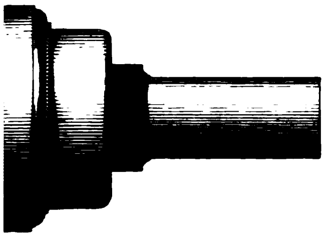
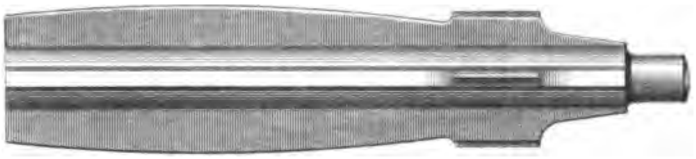


Fig.3.



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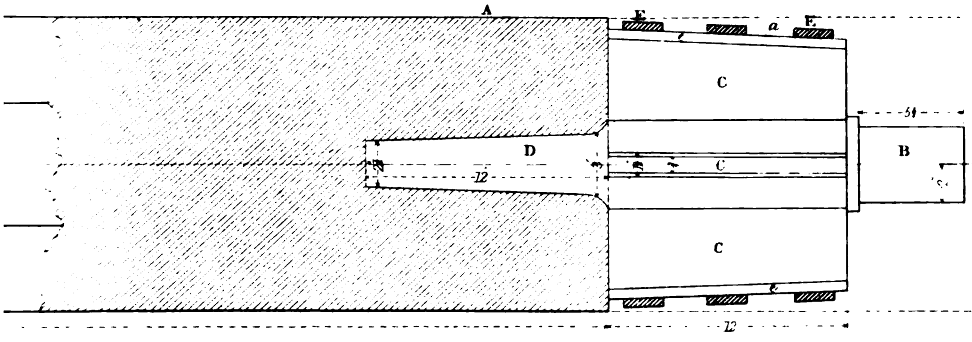
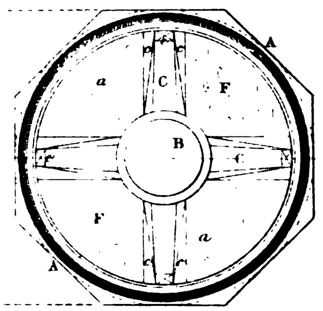


Fig.6.



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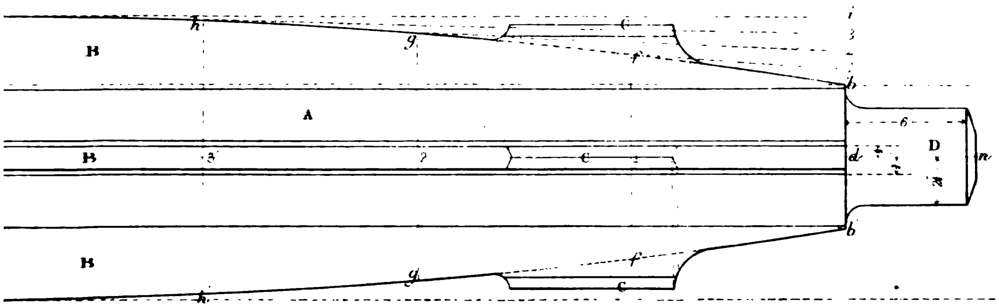


Fig.9.

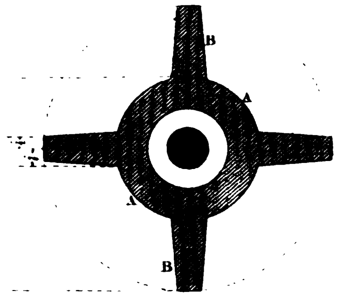


Fig.12.

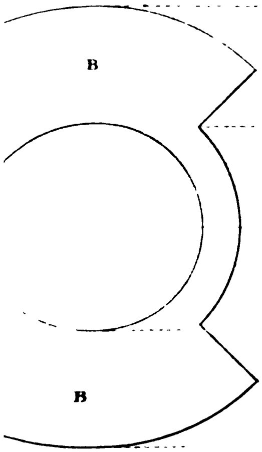


Fig.13.

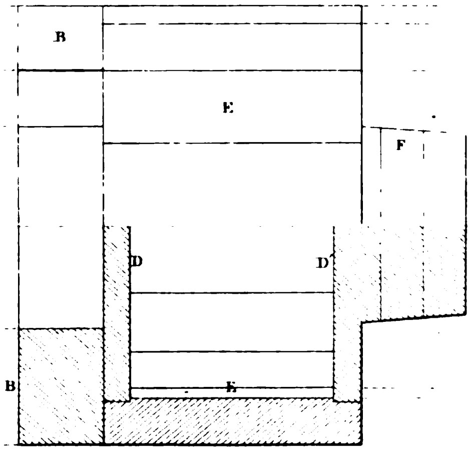
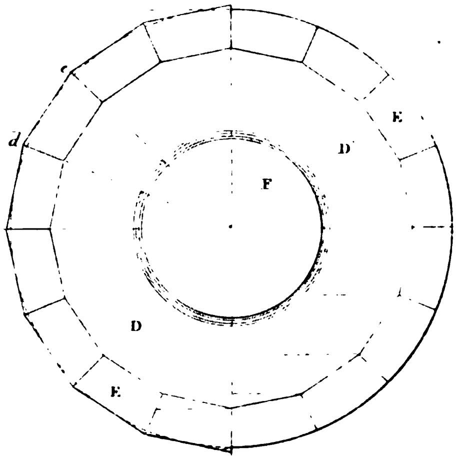


Fig.14.



t 1 2 & 3, which are respectively half of Figs 4,10 & 7.

3 4 5 6 7 FEET.

minutiæ of machinery such as bolts, and nuts, simple cylindrical shafts, rods, and screws, unless these are constructed with some internal peculiarity.

Fig. 5 is a transverse section through the middle of the shaft, and merely shows that it is solid, and that it has the external contour of a regular octagon. Fig. 6 is an end view of the same shaft, showing the fitting of the spindle, with its feathers, into the socket and grooves, formed in the end of the shaft to receive them, and the binding of the whole together by the ferules or hoops. These views are what are required to determine all the various parts of the shaft. It consists, in fact, of a long prismatic beam of oak, A, of an octagonal section, and of which the extremities, *a*, are rounded, and slightly conical. The spindles, B, which are let into the ends, are each cast with four feathers, C, and a long tail-piece, D, uniting and strengthening them. Some engineers construct the spindles without the additional tail-piece, D. Though this simplifies the thing considerably, it is an arrangement which does not possess so much strength as when the spindle is longer. The beam-ends are turned out and grooved to receive these spindles, the grooves for the feathers being made rather wider than the feathers themselves. When the spindles are introduced into the sockets, at *a*, thus formed for them, the whole are bound together by means of the iron hoops, E, which are forced on whilst hot. After this, hard wooden wedges, *c* and *e*, are jammed in on each side, and at the edges of the feathers, thus tightening and solidifying the whole mass. In addition to this, iron spikes, F, are sometimes hammered in, to jam up the fibres of the wood still closer. Fig. 1, which is a shaded and finished elevation of one end of the shaft, gives an accurate idea of its appearance when complete and ready for adjustment in its bearings.

148. *Cast-iron Shaft.*—There are several kinds of cast-iron shafts. Some are cast hollow, others are quite solid, and cylindrical or prismatical in cross section. Such as are intended to sustain very great strains, are generally strengthened by the addition of feathers, which project more towards the middle. These feathers give great rigidity to the piece. A shaft of this description is represented in Fig. 7, half being in external elevation, and half in section through the irregular line, 1, 2, 3, 4, in Fig. 8, which is an end view of the same shaft, and Fig. 9 a tranverse section through the line, $a\ a^1$, in Fig. 7. In practice, it is not considered absolutely necessary that a section should follow a straight line. Frequently a much greater amount of explanation may be given in one view, by supposing the object sectioned by portions of planes at different parts, and solid and easily comprehended portions are generally shown in elevation, as the feathers of the shaft, in the present instance, or the spokes of a spur-wheel or pulley. The shaft under consideration is such a one as is employed for hydraulic motors. The body, A, is cylindrical and hollow, and it is cast with four feathers, B, disposed at right angles to each other, and of an external parabolic outline, so as to present an equal resistance to torsion and flexure throughout. Near the extremities of these feathers, four projections are cast, for the attachment of the bosses of the water-wheel. These projections are formed with facets, so as to form the corners of a circumscribing square, as shown in Fig. 8 at the lower part; and they are planed to receive the keys, by which they are fixed and adjusted to the bosses or naves, which are grooved at the proper places to receive them. The spindles, or journals, D, which terminate the shaft at each end, are cast with it, and are afterwards finished by turning. The shaft thus consists of only one piece or casting.

149. Although we have already shown the mode of drawing a parabola, in Plate 5, the outline of the shaft feathers affords a practical exemplification, which it will be useful to illustrate. We here also give the method generally adopted (because of its simplicity) when the curve is a very slow or obtuse one, such as is given to the feathers of shafts, beams, side-levers, connecting-rods, and similar castings. It is understood, in these cases, that two points in the curve are given; the one, *a*, Fig. 7, being at the summit, and at the same time in the middle of the piece, and the other, *b*, situated at the extremity. In the present instance, we suppose the heights, $a\,c$ and $b\,d$, from the axial line, $m\,n$, of the shaft to be given. This line, $m\,n$, may also be taken as the centre line of a beam, or connecting-rod. After having drawn through the point, *b*, a line, as dotted in, parallel to the axis, divide the perpendicular from *a* to this line, into any number of equal parts, and transfer these divisions to the line, $b\,i$, the prolongation of the line, $d\,b$; then draw lines from the points, 1, 2, 3, to the summit, *a*. Further, divide also the length, $c\,d$, into the same number of equal parts as the perpendicular, and through the divisions, 1^1, 2^1, 3^1, draw other perpendiculars, the respective points, *f*, *g*, *h*, of intersection of these with the lines already drawn, will be points in the required curve. As the lower feather is an exact counterpart of the upper one, the perpendiculars may be prolonged downwards, and corresponding distances, as $1^1\,f^1$; $2^1\,g^1$; and $3^1\,h^1$; set off on them. To draw, also, the half of each feather to the left, it is merely necessary to erect perpendiculars of corresponding lengths, at corresponding points in the axis. A different mode of drawing this curve is sometimes adopted; namely the one which we have already given in Plate 9, for the entasis of the Tuscan column. As, however, it does not possess the advantages of the true parabolic form, and as the curve becomes too sudden towards the extremities, we think the mode given in Plate 12 is to be preferred.

Fig. 3 represents a portion of the shaft just discussed, shaded and finished, the lines running in different directions, the better to distinguish the flat from the round surfaces.

150. *Shaft Coupling.*—In extensive factories, and other works, where considerable lengths of shafting are necessary, they have to be constructed in several pieces, and coupled together. These couplings are generally of cast-iron, and formed of one or more pieces, according to their size. One form consists of a species of cylindrical socket, accurately turned internally, which receives the ends of the two shafts to be connected, these being scarfed or halved into each other, so as to be bound well together, and revolve like one continuous piece. According to another form, two sockets are employed, of increased diameter at the part where they meet, and formed at this part into quadrant-shaped clutches, gearing with each other. The coupling represented in side elevation, in Fig. 10, is of this kind; and in front or end elevation, as separated in Fig. 11.

This coupling was designed for an eight-inch shaft, of which the diameter at the collars, or swell for the coupling, was 10½ inches. The socket, A, of the coupling, is adjusted on the end of the first part of the shaft, C. The other socket, A¹, is similarly adjusted on the end of the other part, C¹, of the shaft. These two socket-pieces gear with each other, when brought together, by means of the projections or clutches, B and B¹, concentric portions, however, being adapted to fit the one into the other, to insure the coincidence of their centres. In some cases a small hole is bored in the centre and end of each shaft, into which a pin is inserted, to insure the firm and true adjustment of both shafts in the same axial line. Fig. 11, which is a front view of the socket-piece, A, shows the exact shape and dimensions of these projecting clutches, each occupying a quadrant of the circle on the face of the socket-piece.

Those of the second piece, A^1, are precisely the same, occupying, however, the intervals of those on A, so that the two may fit exactly into each other, as shown in Fig. 10. The perfect and accurate union of these coupling-pieces with the two portions of shafting is obtained, in the first place, by means of two keys, *a*, diametrically opposite to each other, and let half into the shaft, and half into the socket-piece; and secondly, by screws, *b*, one of which is visible in Fig. 10. The keys, *a*, are for the purpose of fixing the coupling-pieces to the two portions of shafting, making them solid therewith; and the screws, *b*, prevent the longitudinal separation of the two halves of the coupling. This is also sometimes done by bolts passed through the enlarged part of the sockets, parallel to the axis in the centre of the clutches, B, and the recess for receiving them.

THE MODE OF CONSTRUCTING A WOODEN MODEL OR PATTERN OF A COUPLING.

151. After the design for any piece of mechanism which it is proposed to cast has been decided on, it is generally necessary to construct a model or pattern in wood, by which to form the moulds for the casting. The proper formation of such a pattern is no easy matter, and requires considerable skill on the part of the pattern-maker, as also a knowledge of the kind of wood most appropriate, and of the various precautions needed to insure success when the mould comes to be prepared.

It is customary to construct the patterns of deal, because of its cheapness. Sometimes, however, plane-tree or sycamore or oak is used; and for small patterns, and such as require great precision, mahogany, box, or walnut. Whatever kind of wood is used, it should be perfectly dry, and well seasoned. The pattern is made solid, or hollow, and built up, according to the dimensions of the object. For a drum, for instance, a column of any considerable width, a steam-engine cylinder, or for a coupling of large size, such as that represented in Figs. 12, 13, and 14, the pattern is generally hollow, for economizing the wood, and reducing the weight of the piece. Also, if built up, there is less risk of warping or alteration of form, from changes of temperature. In Fig. 13, the pattern of a coupling-piece is represented, partly in external side elevation, and partly in longitudinal section, being cut by a vertical plane, passing through the axis. Fig. 12 is a front elevation, showing the projecting clutches, B. It is easy to see from these views, that the pattern is formed of two boards, D D^1, round the circumferences of which are fitted a series of staves, E, secured to the boards by screws. The wood for these staves is first cut up into pieces of the required thickness, and the sides are then bevelled off, to coincide with the radii, as at *d* and *e*, Fig. 14. They are then fitted to the boards, D D^1, and at this stage present the appearance of the left hand portion of Fig. 14. The drum is afterwards put into the lathe, and the circumference is reduced to a cylindrical surface, like the right-hand portion of the same figure.

On one of the ends, D, of this drum, is fixed the projecting clutch-piece, B, which has been previously cut out of a board of greater thickness, so as to present the outline as shown in Fig. 12. On the opposite end, D^1, of the drum, are fixed several discs, or thicknesses, of wood, F, which are turned down to a diameter proportionate to the central socket which it is intended to form in the coupling-piece. After having been turned where necessary, the pattern is treated with sand-paper to make the surface as smooth as possible, and to prevent the adherence of the loam of which the mould is formed. Such patterns, particularly when of small size, are moreover coated with black-lead, well rubbed in, to give a polish and hardness to the surface. The diameter of the core-piece, F, is less than that of the shaft to which the coupling is to be fitted, so as to leave some margin in the casting for turning and grinding down the socket to the exact dimensions. The core itself, which gives form to the socket, is a cylinder of loam placed in the centre of the mould, fitted into the recess formed for that purpose by the piece, F. As in the present example, the core being very short, the mould would be constructed on end, and would not require further support; but where a core is very long, or placed in a horizontal position, it requires to be supported at both ends, and, further, to be strengthened by wires or rods passing through its centre. For this reason, a core-piece, as F, is only attached to one end of the drum. It will be observed that this is slightly conical; the drum is so also, but to a less extent; the core itself, however, is quite cylindrical.

152. DRAW, *or Taper, and* SHRINK, *or Allowance for Contraction.*—In order that the pattern may be lifted from the mould without bringing away portions of the sides, it is necessary to form its sides with a slight taper, or *draw*, as it is technically called. For example, the diameter of the core-piece, F, as well as that of the drum itself, must be less at the lower extremity, or at the part first introduced into the mould, than at the opposite extremity. A very slight difference of diameter is sufficient for the purpose.

Cast-iron, as is the case with most of the metals used in engineering, is of less bulk, when cold than when in a state of fusion, and because of this contraction, it is necessary to make the patterns of somewhat larger dimensions than the casting is to be when finished. It follows that when the pieces to be cast have afterwards to be planed, turned, ground, or grooved, it is necessary to bear in mind, in constructing the wooden pattern, not only the after reduction due to the contraction, or, as it is termed, the *shrink*, of the metal, but also that which is occasioned by the reducing processes involved in finishing the article. In general, *grey* iron requires an allowance for shrink of from 1 to 1$\frac{1}{10}$ per cent.; *white* iron, however, requires a much larger allowance. The allowance to be made for the reduction caused by the finishing processes, depends entirely on their nature.

When, with a view to avoid the expense of constructing a pattern, the mould is formed from the actual object which is to be reproduced or multiplied, the mould-makers obtain the necessary margin by shifting the model slightly during the formation of the mould. This, of course, can only be done with advantage when the piece is not of intricate shape.

ELEMENTARY APPLICATIONS.

PLATE 13.

RAILS AND CHAIRS FOR RAILWAYS.

153. In the majority of British railways, the two malleable iron rails on which the trains run are placed at a parallel distance apart of 4 feet 8½ inches, termed the "narrow gauge;" but in some instances they are placed at the distance of 7 feet, termed the "broad gauge;" and they are usually rolled in lengths of 15, 18, and sometimes 21 feet. Rails are of various shapes; but the one we describe and give an example of, is that which is in most general use, and is termed the double T headed rail. These rails are supported by pieces of cast-iron called "chairs," which are fastened down to sleepers lying across the line of parallel rails at intervals of 3 feet, thus giving 6, 7, or 8 sleepers, as the case may be, to every pair of rails. The sleepers are 5 inches thick, and have a bearing surface of 9 feet long

RAILS & RAI

Fig. 1.

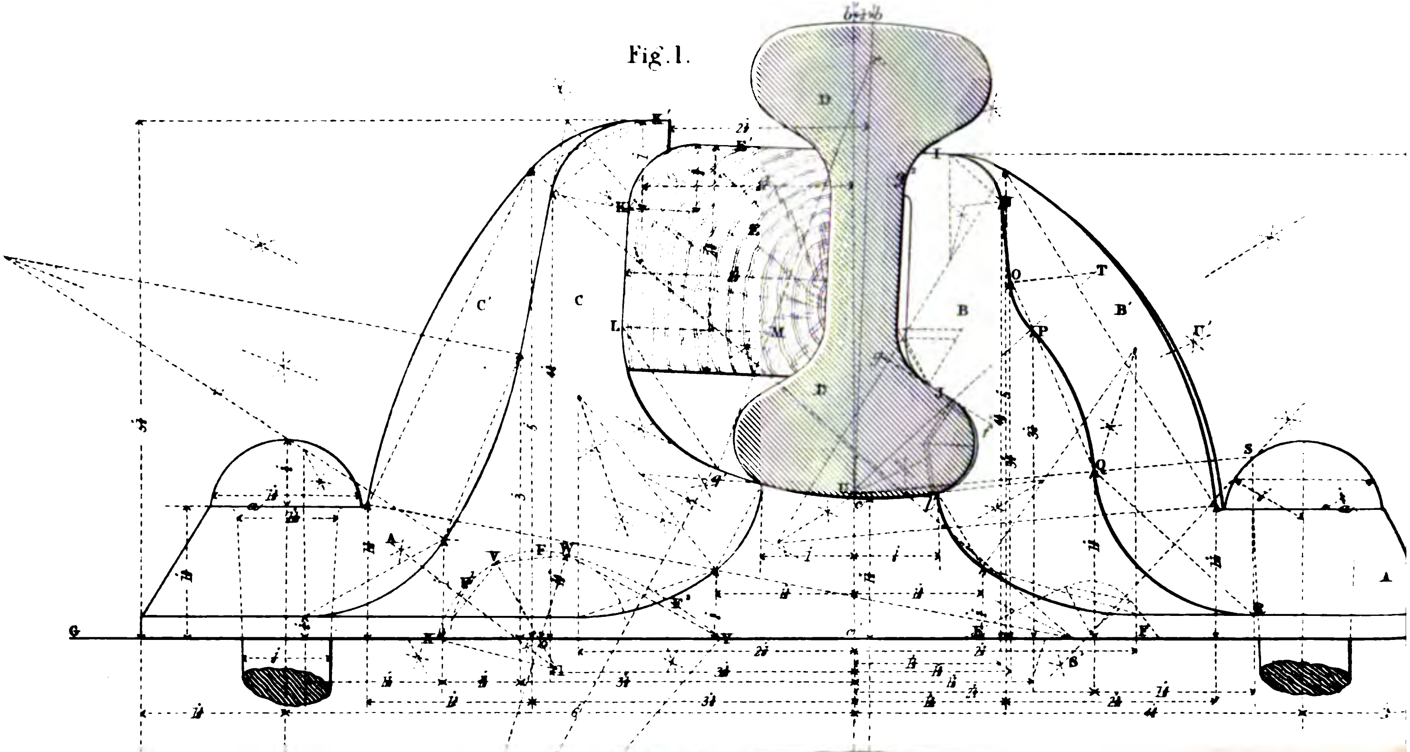
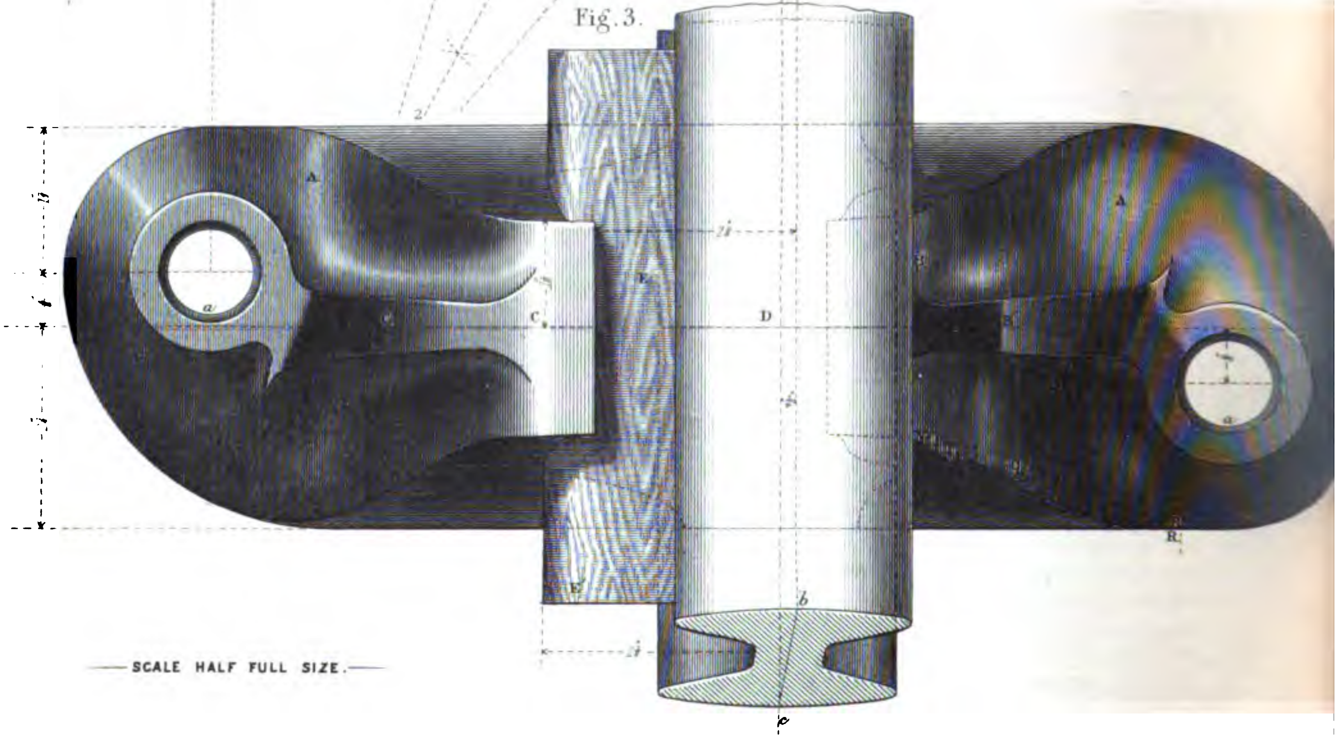


Fig. 3.



SCALE HALF FULL SIZE.

W. Johnson. Patent Offices.
47 Lincoln's Inn Fields & Glasgow

WAY CHAIRS.

Fig. 4.

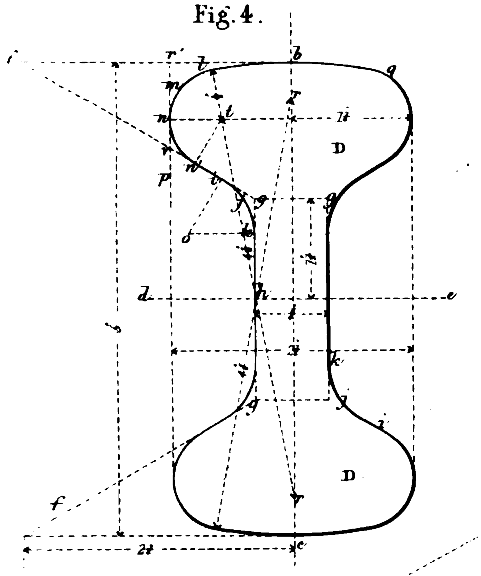


Fig. 2.

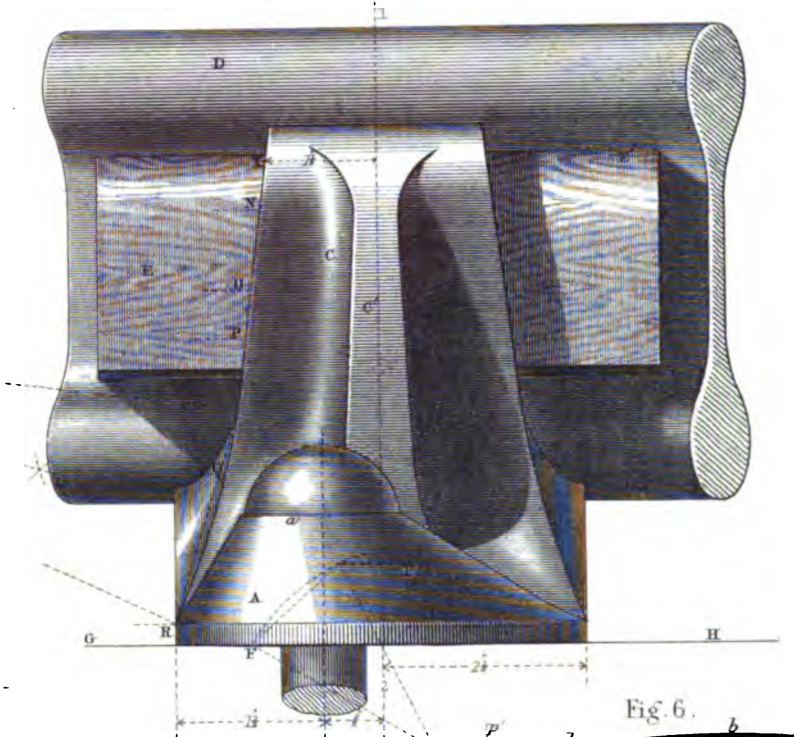


Fig. 5.

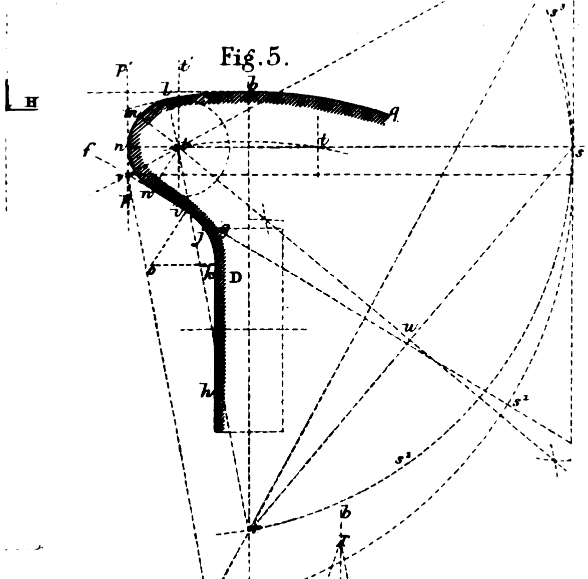


Fig. 7.

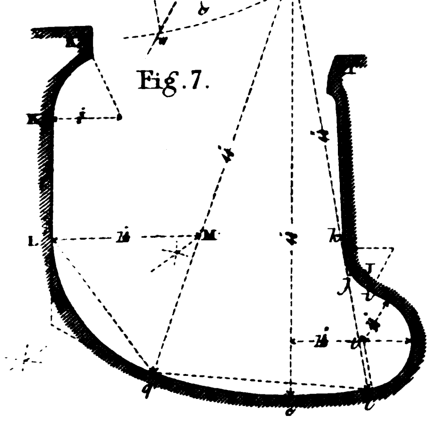
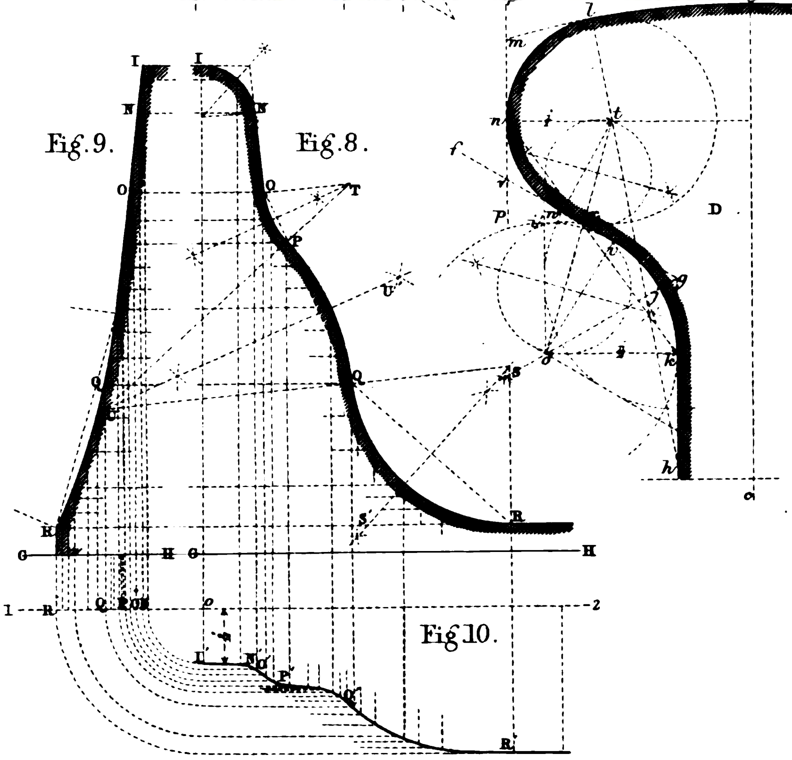


Fig. 9.

Fig. 8.

Fig. 6.

Fig 10.



Mackay & Kirkwood Glasgow

by 10 inches in width, and are generally sunk into the earth, or ballast of broken stones, to their upper surface. In the best lines of railway, this ballast, or top surface of the roadway, is always well drained. The chairs at the junction of the rails are usually about double the length, and sometimes of greater width also, so as to render the rectilinear junction of the rails most secure and perfect.

In Plate 13, we give details of the most approved rail and chair now in general use, drawn to one half of their natural size—all the sizes necessary for the drawing of the same being marked in plain figures of feet, inches, and parts of inches, at their respective parts on the drawing. There are many different forms of rails and chairs; but as the rules and modes of drawing all of them are pretty much alike, we think the example here given will make the subject sufficiently understood, and give the young beginner both a liking and facility for putting in practice the problems which have already been given, and in combining them so as to produce the simple forms and figures of this plate, so that he may subsequently be enabled to comprehend the more abstruse ones, which will enable him to draw complete plans and complicated figures. Figs. 1 and 2 of this plate are end and side elevations, at right angles to each other, of a railway chair, with a portion of the rail which it supports. Fig. 3 is a plan of the same, corresponding to Fig. 1; and Fig. 4 is an end view of the rail detached, so as to avoid confusion in showing the methods of drawing the rail and chair. This chair, which is made of cast-iron, and is designed with the view of combining solidity and strength with economy of material, consists of a flat base, A, 4¼ inches in length by 13¾ inches in width, over the snugs, which have two holes, *a*, one in each of them, set diagonally, as seen in the plan, Fig. 3. These holes have a slight taper towards the top side, so as to allow of the bolts or spike-heads being stronger, and to have a wedging effect in tightening and holding down the chair to the sleepers, into which they are driven. The advantage of putting the bolts or nails on the angle, as stated, is, that the two nails of the same chair do not enter the wooden sleeper in the same line of fibres, and therefore has much less tendency to split it. The chair has also two jaws, B and C, standing up in a vertical direction from the base, A, having a kind of dove-tail recess between them for holding the rail, D, and wooden key, or wedge piece, E, and are each strengthened on the outside by a central feather or bracket, B^1 and C^1, running out along the sole as far as the head of the nails will admit. The jaw, B, is made rather shorter than C, and nearly fits the contour of the rail, while the latter fits the outside of the key, E, which, being nearly symmetrical, can, on an emergency, be reversed upside down; but as the jaw, C, and key, E, have a slight taper towards the right hand side, the key cannot be reversed end for end; that is, when standing on the outside of the rail, from which side the keys are always driven. Thus it will be seen that it is a chair and rail on the left hand side of a line of railway we have shown, and the head, or thick part of the key, is that marked, E^1. When the keys are removed, the recesses in the chairs are large enough to allow of the removal of the old, and replacement of new rails, without disturbing the chairs, which have been previously set to the proper gauge. Below each of the jaws, B and C, and their feathers, at the thick part formed by their junction with the sole, A, there are two hollows formed, for economy and lightness, also to prevent the shrinking of the metal, and consequent imperfection of the casting. These hollows, as well as the holes through which the bolts pass, are shown by dotted lines in Figs. 1 and 2; a system which is frequently adopted in instances like this, in order to dispense with a cross section, where the parts are few and simple, and in this way the number of figures are reduced without sacrificing clearness of detail.

154. In commencing to draw the end view of the chair and rail, Fig. 1, the student lays off the horizontal base line, G H, and vertical centre line, $b^1\ c^1$, in the usual way, then with the given thickness of the sole draw a line parallel to the base line, and this will give the proper top surface at *a*, of the snugs, where the nails pass through. On the centre line, $b^1\ c^1$, set off $c^1\ c$, equal to the given thickness of the sole at the part under the rail, and from the point, *c*, draw the centre line, *b c*, of the rail, lying at an angle as regards the vertical line, $b^1\ c^1$, with an inclination inwards of one in twenty, or a ¼ of an inch on the whole height of the rail, which is 5 inches. At this point the rail, D, must now be drawn, since it is from it that the other parts of the chair take their form and character. At Fig. 4, we have shown a vertical end view of the rail detached, so as to show the method of drawing it distinct from the chair, and in the construction, to illustrate some of the former problems, which will be of use for drawing other parts of the chair further on. It presents a figure which is symmetrical both with reference to the vertical centre line, *b c*, and also to the horizontal centre line, *d e*, this permits of its being turned when the top side is slightly worn, or a flaw discovered on the top surface, which may do no harm on the under side, or prevent its being firmly secured in the chair. This rail is considered one of the strongest forms that can be adopted for the iron it contains, which is 75 lbs. per yard, its outline is composed of straight lines and arcs, which are geometrically and evenly joined into each other, forming the pleasing, sturdy figure of an I, but it is usually termed the double T headed rail, marked thus "I." The rail itself is shown in sharp outline, Fig. 4, and the more obvious operations necessary for drawing it, which would only be in pencil, are shown in dotted lines, with the various sizes and lengths of radii marked in plain figures, and are for the most part a repetition and combination of the problems treated of in Chapter III., on Linear Drawing. In the detached Figs. 5 and 6 are a few of the more abstruse problems necessary for drawing the rail, Fig. 4, one of which requires and gives rise to another practical illustration of Problem 38, Fig. 3 or 4, Plate 3—which is, to draw an arc, *i j k*, tangent to the straight lines, *f g* and *g h*, the radius, *o k*, being given equal to $\frac{11}{16}$ of an inch, or one of the points of contact, *i*, *k*, being given. This problem meets with an application at *i j k*, Fig. 4, and is here shown detached and to a larger scale, to make it more distinctly understood.

155. In Fig. 5, we have an example of the Problem 40 or 41, which requires that an arc, *l m n*, be drawn tangent to a straight line, $p\ p^1$, and to a given arc, *q b l*, the point of contact, *n*, or the radius, *n t*, being given; in the first instance, as in Fig. 9 or 11, Plate 3, make *n s*, perpendicular to $p^1\ n\ p$, and equal to the radius, *b r*, of the given arc, *q b l*; join *s r*, and bisect it by the perpendicular, *t u*, the point, *t*, of intersection with the line, *n s*, is the centre of the required arc, and *n t*, its radius. In the second instance let the radius, *n t*, be given equal to ½ an inch, then draw $t\ t^1$ parallel to $p\ p^1$, Problem 4, with a perpendicular distance between them of ½ an inch, then from *r*, the centre of the given arc, *q b l*, with a radius, *r t*, equal to *r b*, the radius of the given arc, minus the given radius, *n t*; describe the arc, *t t*, and it will cut the line, $t\ t^1$, in *t*, the centre of the required arc, *n m l*, Problem 41, Fig. 11, Plate 3; join *r t*, and produce it to cut the given arc in *l*, which will be the point of junction of the two arcs. Otherwise let it be required to draw an arc, $n^1\ n\ m\ l$, tangent to the same given arc, *q b l*, and to two given lines, making an angle with each other, $P\ P^1$ and *f g*. Bisect the angle, $p^1\ v\ g$,

contained by these lines, Problem 38, Fig. 5, Plate 3, by the line, $v\ v^1$, and the centre of the required arc must fall on a point in this line. Then, by Problem 40, Fig. 8, Plate 3, on the line $v\ f$, produced with v as a centre, describe an arc, $s\ s^1\ w$, having its radius, $v\ s^1$, equal to $r\ b$, the radius of the given arc; tangent to this arc, and parallel to the other given line, $p\ p^1$, draw the line, $s\ v^1$, intersecting the line, $v\ v^1$, in v^1; join v^1 and r, the centre of the given arc, producing it till it intersects the arc, $s\ s^1\ w$, in w, join $v\ w$, and parallel to it through the centre, r, draw $r\ t$, till it intersects $v\ v^1$ in the point, t, which will be the centre of the required arc, $l\ m\ n$, which can now be drawn with a radius equal to a perpendicular let fall on either of the given lines from t; or to the part, $t\ l$, of $r\ t$, produced till it cuts the given arc. When this construction is correctly executed, an arc drawn with the point, t, as a centre, and $t\ r$, as radius, will touch the line, $s\ v^1$, tangentially; that is, the line, $s\ v^1$, which was made tangent to the arc, $s\ s^1\ w$, is also tangent to the arc, $s^2\ s^2\ r$.

156. Let the two circles, or arcs, $i\ j\ k$ and $l\ m\ n\ n^1$, be each drawn respectively tangents to the two given points, k and n, in the lines, $g\ h$ and $p^1\ p$, then another rather nice problem would be required, viz.:—To draw the tangent, $f\ g$, to the two given circles, $i\ j\ k$ and $l\ m\ n\ n^1$, having $o\ k$ and $t\ n$ respectively as their radius, Fig. 6. This problem is an example of Problem 9, Fig. 10, Plate 1, amplified. It is first required to find a common point, x, on the line $t\ o$, which joins the centres of both circles; to effect this, we draw through the centres, t and o, any two diameters or radii, $t\ n$ and $o\ k$, parallel to each other, and join their opposite extremities, $n\ k$, it will cut the line, $t\ o$, in x, the common point required, which now reduces the problem to the drawing of a tangent, Problem 9, to a circle, from a given point without it. Thus, bisect $x\ o$, and draw the circle, $o\ i\ x'i^1$, cutting the given circle, $i\ j\ k$, in i and i^1; join $x\ i$, and produce it in the direction of g; then, proceeding in the same way, by drawing a circle having $x\ t$, as its diameter, cutting the circle $l\ m\ n\ n_1$ in n^1, join $x\ n^1$, and produce it in the direction of f. Then $f\ g$ is in the same straight line, and is the common tangent in this instance to the two given circles. The application of this problem is frequently required in drawing machinery; but in general practice, it is found correct enough to lay a straight-edge tangent to both circles "by the eye," or to a point and the one circle, as the case may be, and then draw the line through the point, and touching the circumference of the circles. The end section of the rail having been drawn in its proper position, Fig. 1, to the two given centre lines, $b\ c$, $d\ e$, as shown.

157. We now proceed to draw the jaw, B, and its feather, B^1, the height and thickness of the several parts, marked on the drawing in inches and parts of inches, being given; this jaw bears hard on the rail for a little distance down from the top, I, at g, and again at the part J, near the bottom, corresponding to one of the tangential lines, $f\ g$; between these two points, down the side of the centre web of the rail, there is a narrow space formed by a parallel line, which forms the inner side of the jaw, and from which an arc corresponding to $k\ j\ i$, Figs. 4 and 7, is drawn tangent to it, and to the line, $g\ f$, at J. At this part, and tangent to this line and to the arc, $l^1\ c\ q^1$, Figs. 1 and 7, there is an arc drawn corresponding to the arc, $n^1\ n\ m\ l$, but somewhat smaller, so as to allow a small space at this part from J to l; so that when the wedge, E, on the other side of the rail, presses it over and downwards on the bearing parts at I J, and the lower surface, $l^1\ c\ q^1$, there is a wedging action produced between this surface, which fits the lower surface of the rail, and the part at J, which holds the rail firmly in its place. The inner surface of the jaw, C, is formed by a line, K L, parallel to the centre line, $b\ c$, of the rail, and by an arc L q^1, tangent to this line and the arc, $q^1\ c\ l^1$, drawn from one of the centres, r, of the rail, Figs. 4 and 7. This recalls, Problem 40, the point of contact, L, the point of contact, q^1, or the radius, L M, being given; at the top, K, it has a small fillet piece formed by an arc, Problem 38, tangent to the top of the wedge, E, and the line, K L, a point of contact or radius being given. This fillet-piece projects inwards for the purpose of holding down the wooden key or wedge firmly in position. The inner contour of the recess between the two jaws for holding the rail is now completed, and the method of drawing it will be understood from the outline shown detached at Fig. 7, all the centres of the arcs being shown with their several cords and radii in dotted lines, cutting the arcs and straight lines at the points where they respectively join each other tangentially. The outline, I N O P Q R, of the outside of the jaw, B, and its feather, B^1, as well as that of C and C^1, have their several cords and radii "dotted in" in a similar manner, and are all further examples of the problems given in Chapter III., and those just illustrated at Figs. 4, 5, 6, and 7, according as the several points in the curve lines are given, or their radii. The points, I, N, O, P, Q, and R, being found, by squaring up the horizontal measurements given along the base till they intersect the corresponding perpendicular measurements, and by joining these points as cords, and bisecting them by perpendiculars, the centres of the respective arcs will be found. Thus the perpendicular, S S^1, bisecting the cord, Q R, Figs. 1 and 8 cuts the perpendicular, R S, in S, the centre of the required arc. The arcs, I N, O P, and most of the others in the outer contour of the jaws and their strengthening feathers, are found in the same way; but in the centre of both, there is the arc, P Q, sweeping into two other arcs, O P and Q R, the centre of which may be found by Problem 44, Fig. 18, Plate 3, or by bisecting the cord, P Q, by the perpendicular, U U^1, which will cut either of the lines, T P, or S Q, produced in U, which will be the centre of the required arc; or if both the points of contact, P and Q, are given, then T P and S Q will cut each other in U, the centre of the required arc.

158. The hollow part under the chair at F, is another example of the same problem, showing the three arcs forming the curve sweeping into each other in the same direction, and having all their centres on the same side of the line, as shown in dotted lines at the lower part of the jaw, C, to the left of Fig. 1. The height and radius of the top or centre arc, F, being given, and two points, V and W, taken in it through which the other two arcs, F^1, F^2, are required to glide tangentially into it, join these points, V and W, with o, the centre of the given arc, F, and produce them indefinitely through o. Then if the radii of the other two arcs are given, it is only necessary to make these lines respectively equal to the two given radii, thus make V 1, equal to the radius of the arc, F^1, required; 1, will be its centre, and in the same way a point, 2, can be found in the line, W o, produced, which will be the centre of the arc, F^2. If, however, it is other two points, X and Y, in the base line, G H, that are given, through which the two arcs, F^1, F^2 are required to pass, then join, V X, and W Y, and bisect them by perpendiculars which will respectively intersect, V o, and W o, produced in the points, 1 and 2, the centres of the required arcs, F^1, F^2, respectively. Another view of this hollow part at right angles to Fig. 1, is shown in dotted lines in Fig. 2, and is drawn in the same way as the other, but having the two side arcs symmetrical, as regards the centre arc and centre line, 1, 2, Fig. 2.

159. In proceeding to draw the plan, Fig. 3, we may observe, that a third view can nearly all be projected from the two former when

they are, as in this case, at right angles to each other. So that having drawn the centre line, 1, 2, in the proper position required in the plan, and parallel to the base line, G H, Fig. 1, and corresponding to the centre line, 1, 2. In Fig. 2, the widths of the several parts are here "squared down" from Fig. 1, and the several lengths of these parts, respectively, taken from the centre line, Fig. 2, and transferred to their corresponding parts from the centre line, Fig. 3, will give the general outline of the plan. All the operations being rendered evident from the important points of the several parts being marked by the same letters of reference in all the figures, and several of the corresponding measurements also, marked off from the centre lines. Although all the lines and points in the plan of the chair, Fig. 3, are found in the way described, we have given a separate projection of the irregular curved line, or edge, I^1 N^1 O^1 P^1 Q^1 R^1, on the horizontal plane, formed by the intersection of the two irregular surfaces, I N O P Q R, at right angles to each other, in the end and side views, Figs. 1 and 2. For this purpose we have reproduced this portion in Figs. 8, 9, and 10, the two first being vertical sections, or outlines, of these surfaces, at right angles to each other, and corresponding respectively to Figs. 1 and 2; and the last, or Fig. 10, is the horizontal projection of the curve line formed by these two surfaces, and corresponds to the plan, Fig. 3. To project Fig. 10, from Figs. 8 and 9, we first draw any line, 1, 2, Fig. 10, parallel to the base line, G H, Figs. 8 and 9; this line is generally taken as the centre line of the object being drawn, and, consequently, will correspond to the same centre line in the other figures, and thus be the best line to measure distances from, being, moreover, symmetrical to many parts of the object. We next square over the several points, I, N, O, P, &c., in Fig. 8, to the corresponding points, I, N, O, P, &c., in Fig. 9, from these points let fall perpendiculars to the line, 1, 2, and carry them round parallel to it by concentric quarter circles, drawn from a point, *o*, in this line, they will cut, or intersect perpendiculars let fall from the corresponding points, I, N, O, P, &c., in Fig. 8, respectively, in the points I^1, N^1, O^1, P^1, &c., Fig. 10. A mixed line drawn through these points is the horizontal projection of the irregular curved edge formed by the intersection of the two surfaces represented by the letters I, N, O, P, Q, R, in Figs. 1 and 2, and required to complete the plan of the chair, Fig. 3. The position of the first point, I^1, in the line I G, produced in Figs. 8 and 10, is arbitrary, as regards its distance from the line, 1, 2; but as here shown the line 1, 2, was drawn at half of the distance of the centre line from the point, I^1, in Figs. 2 and 3, so that as this distance in these figures is $1\frac{1}{8}$ inches, the radius, I O, of the quarter circle, which gave the first point, was $\frac{9}{16}$ of an inch on the scale to which these figures are drawn. The other circles being all drawn concentric, and from the same centre, *o*, as the first, necessarily give all the other points N,^{1} O^1, P^1, &c., in their proper relative position to the point, I^1, it being determined.

It will be observed that the larger portions, P, Q, and Q, R, of the curved surface, Fig. 8, are divided arbitrarily into several parts, the points thus taken at random being squared over, and determined in the same way as the main points already described; but to prevent confusion, these are only indicated by short dotted lines at points from whence they are taken, and to which they are carried. The more numerous the points which are taken, the more correct will be the projection of the curve line. There are two curves similar to the one just discussed, required to form the plan of the jaw, B, and two others formed by the edges of the jaw, C, which, though somewhat different, are determined in a similar way to the preceding.

We may here remark, that we have given explanatory diagrams of most of the sweeps or combinations of curves, both that the student may be well exercised in many of the problems already discussed, and also with a view of collecting, in one plate, several of the difficulties which more frequently meet the draughtsman in the course of his practice. In such objects as that chosen for exemplification, very little of the nicety here carried out is observed, and the curves are generally obtained by measurements with callipers from the object itself, or are formed of arcs, or curves, determined by a few points and drawn by the eye, or a pair of compasses, through these points.

The rails are not adjusted in their chairs vertically, but are inclined slightly towards each other, in such a manner that their centre lines, *c b*, form a slight angle with the vertical, $c\,b^1$: this inclination is given to counteract any tendency that the carriages may have to run off the rails, as is the case more particularly in curves, from the effort made by the wheels to run in a straight line. The expedient of laying the outer or convex rail at a level slightly higher than the other, is also resorted to in quick curves, for the like purpose of keeping the trains on the line.

CHAPTER VI.

THE INTERSECTION AND DEVELOPMENT OF SURFACES, WITH APPLICATIONS.

160. Nowhere is descriptive geometry more useful, in its application to the industrial arts, than in the determination of the lines of intersection, or junction of the various solids, whether the intersection be that of two similar solids with each other, as a cylinder with a cylinder; or of dissimilar ones, as a cylinder with a sphere or a cone. With the aid of this branch of geometry, we can determine, in the most exact manner, the proportions of all the curves, both of double and of single curvature, which may be produced by the intersections of surfaces of revolution, the constructive or generative data of which are known.

The applications of forms in which such curves occur are exceedingly numerous; they abound in the works of the brazier, the tinsmith, the joiner, the carpenter, the builder, and the architect, as well as in all descriptions of machinery. This treatise would, indeed, be incomplete, were we not to render the delineation of these curves quite familiar to the student. Intimately connected also with this branch of the subject, is the development of curved surfaces; that is, the determination of the dimensions of such surfaces when extended in a plane, so that the workman may be able to cut out such pieces with the certainty of their taking the form, and fitting the place assigned for them.

The study of projections, moreover, comprises the methods of delineation of such curves as helices, spirals, and serpentines, which frequently occur in mechanical and architectural construction.

THE INTERSECTIONS AND DEVELOPMENT OF CYLINDERS AND CONES.

PLATE 14.

PIPES AND BOILERS.

161. The intersections of cylindrical or conical surfaces may be curves of either single or double curvature. A curve is said to possess a *double curvature*, when it is not wholly situate in one plane. The problem to be considered in representing the lines of intersection consists in the determination in succession of the projections of several points in the curve, and the completion, by tracing the projections of the entire curves through the points thus found. The principle generally to be followed in these cases is, to imagine two cylinders, for instance, to be cut by one and the same plane—their intersection in that plane being a point in the line sought. Thus, to obtain a point in the line of intersection of two right cylinders, A and B, represented in Figs. 1 and 2, draw any plane, $c\ d$, parallel to the axes of both cylinders, Fig. 2. This plane cuts the vertical cylinder, A, as projected horizontally in the points, e, f, and as projected vertically through the lines, $e^1\ e^2$, and $f^1\ f^2$. The horizontal cylinder, B, is cut in $c\ d$, in the horizontal projection, and in $c^1\ d^1$, in the vertical projection, which latter is obtained as follows:—The semi-base, $g\ i$, of the cylinder, B, is drawn in plan as in Fig. 3; then prolonging $d\ c$, until it cuts this plan in c^2, it will give the distance, $c^2\ h$, of the cutting plane from the axis of the cylinder; this distance is then transferred to $i\ c^1$, Fig. 1, and through c^1 the line, $c^1\ d^1$, is drawn as required. The points, e^2, f^2, of intersection of this line, $c^1\ d^1$, with the lines, $e^1\ e^2$, $f^1\ f^2$, are points in the intersecting line respectively on each side of the vertical cylinder, A.

By repeating this operation with another plane, as $m\ n$, parallel to the first, other two points will obviously be found, as l^2 and o^2. The extreme points, $a^1\ k^1$, are naturally determined by the intersection of the outlines of the cylinders. As for the point, b^1, it is found by means of the imaginary plane, $g\ p$, tangential to the vertical cylinder, A, and also to the horizontal one, B, when, as in this case, the two are of equal diameter.

As many points having been found as constitute a sufficient guide, according to the scale or size of the drawing, and the proportion between the intersecting solids, there remains but to unite them into one continuous line and thereby the delineation of the curve of intersection is completed. It will be observed that, in Fig. 1, the vertical projection of this curve is a straight line, as $a^1\ b^1$, or $b^1\ k^1$, these two being at right angles to each other; this results from the fact, that the cylinders, A and B, chosen for this illustration are of equal diameter, and have their axes situate in the same plane, and at right angles to each other, in such a manner that the curves of intersection, which are elliptical, lie in a plane at right angles to the plane of the vertical projection. It follows, then, that this peculiarity being known, all that is necessary in similar cases—that is, when two equal right cylinders, having their axes in the same plane, cut each other—is simply to draw lines from the extreme points, a^1 and k^1, to the summit, b^1, the projection of the point of intersection of their axes—the operations above described being, in such cases, altogether dispensed with.

162. When the cylinders are of unequal diameters, the curve of intersection becomes one of double curvature, notwithstanding that the axes of the cylinders may lie in the same plane. Thus, Figs. 4 and 5, which represent two intersecting cylinders, A and B, of different diameters, show that the curve of intersection, $a^1\ b^1\ k^1$, drawn according to the method before given, is one of double curvature, becoming the more flattened at b^1, as the difference between the diameters of the cylinders is greater. To render it quite plain that the operation is the same, we have indicated the various points obtained by the same letters which mark the corresponding points in Figs. 1 and 2. We must further remark, that in Figs. 4 and 5, the curve is determined with the assistance of two elevations, taken at right angles to each other; whilst, in Figs. 1 and 2, an elevation and a plan were employed, similarly, at right angles to each other.

We show the application of this curve in Figs. 6 and 7, which represent a steam-engine boiler, C, seen partly in elevation and partly in section. The tubular piece, D, which is a species of manhole, is supposed to be cylindrical, and is attached to the body of the boiler by means of a flange, which gives rise to the external intersectional curves, $a\ b$, $c\ d$, and the internal one, $e\ f$.

THE INTERSECTION OF A CONE WITH A SPHERE.

163. Whenever a cone is cut by a plane parallel to its base, the section presents an outline similar to that of the base; consequently when the cone under consideration is a right cone with a circular base, all such parallel sections are circles. Thus in Figs. 8 and 9, representing a right cone, $A^1\ B^1\ S^1$, the plane, $a^1\ b^1$, parallel to its base, $A^1\ B^1$, cuts the cone so as to present a circle, of which the diameter is exactly contained within the extreme generating lines, $A^1\ S^1$, $B^1\ S^1$. If, then, with the centre, S, and radius, $a\ S = a^1\ b^1 \div 2$, we describe a circle, it will be the outline of that part of the cone intercepted by the cutting plane.

The section of a sphere, C, by any plane whatever, is also a circle. When this intersecting plane, $a^1\ b^1$, for instance, is perpendicular to the plane of projection, it is necessarily projected as a straight line, as in Fig. 8, and as a circle, as in Fig. 9, in the plane of projection to which it is parallel. It follows, from these properties, that we have a very simple method for determining the curve of intersection of a cone with a sphere, whatever may be the relative position of their axes. This method consists in supposing a series of parallel planes to cut both the cone and the sphere, so as to produce circular sections of both—the intersections of the outlines of which will consequently be points in the curve sought, as indicated in Fig. 9.

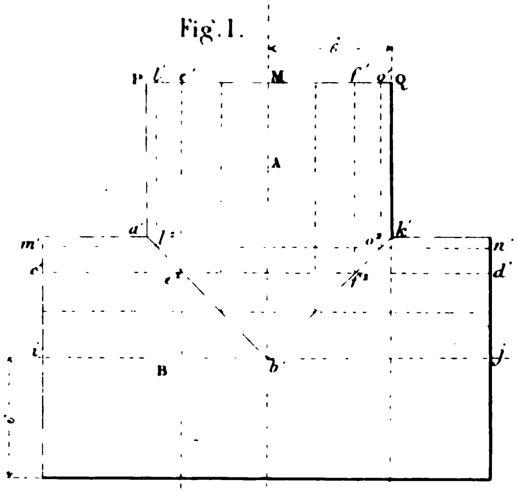
The intersection, $a^1\ b^1$, is a circle, the diameter of which is limited by the extreme generating lines, $S^1\ A^1$, $S^1\ B^1$, of the cone, where they encounter the great circle of the sphere, C. The same method holds good when the cone is cut by any plane, $a^1\ g^1$, inclined to the base, the outline of the section being in this case always an ellipse, which is projected in the plan, Fig. 9, by the line, $a\ i\ n\ g\ n\ i$, the resultant of the various intersections in the planes adopted in the construction and obtainment of the curve, which, in the horizontal projection, when the axis of the cone is vertical to the plane of projection, is also always an ellipse, having its minor, or conjugate axis, not in the line, $i\ i$, through the centre of the cone, but in the line, $n\ n$, Fig. 9, in the centre between the two points, a, g; in this instance, however, the ellipse is almost a circle.

The same occurs with the intersection of a cone, $A^1\ B^1\ S^1$, with a cylinder, $a^1\ b^1\ d\ f$; and when their axes lie in the same straight line, the intersection, $a^1\ b^1$, is also a circle, the diameter of which is equal to that of the cylinder.

164. If their axes are parallel, though not in the same straight line, the intersection of these two surfaces becomes a curve of double curvature, which may be determined either according to the mode

EXAMPLES OF THE INTERSEC

Fig. 1.



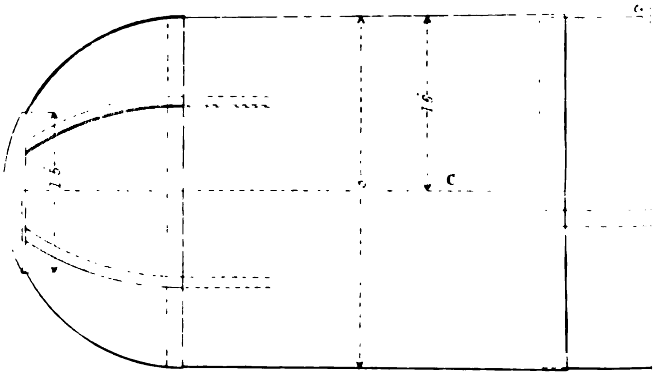


Fig. 3.

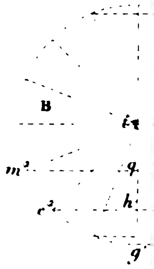


Fig. 2.

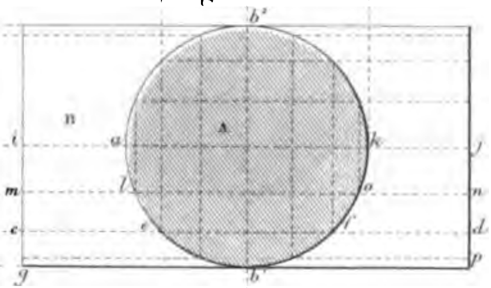


Fig. 4.

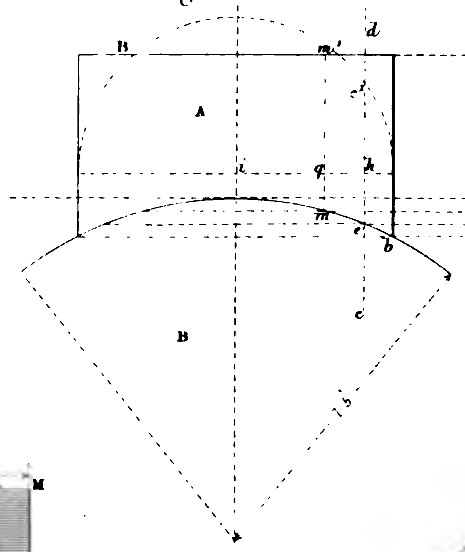
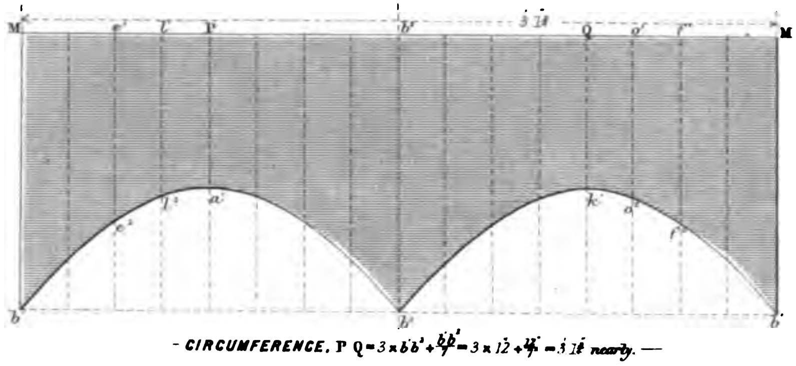
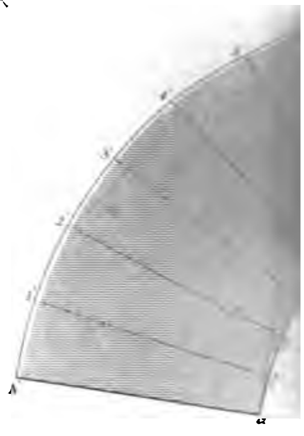


Fig. 10.



- CIRCUMFERENCE. P Q $= 3 \times b'b^2 + \frac{b'b^2}{7} = 3 \times 12 + \frac{12}{7} = 3'\,1\frac{3}{4}''$ nearly. —



—SCALE FOR FIGS 1, 2, 3, 4, 5, 8, 9, 10 & 11.—

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ION OF CYLINDERS & CONES.

Fig.6.

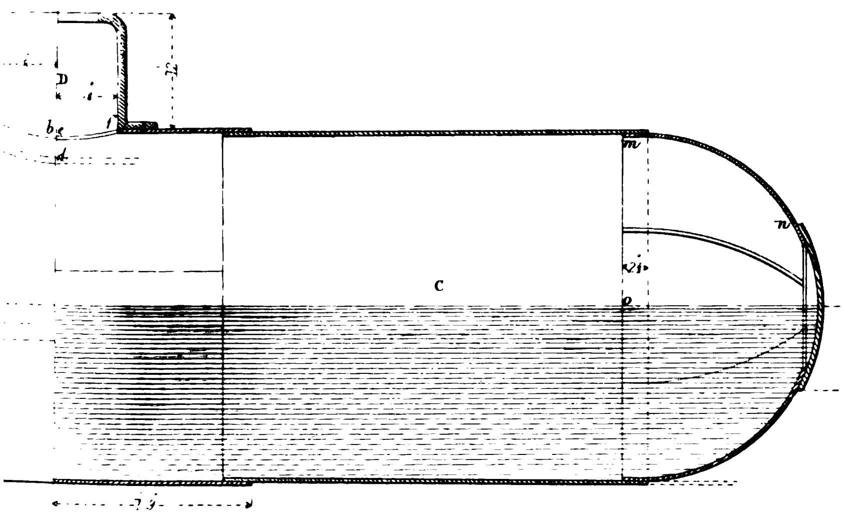
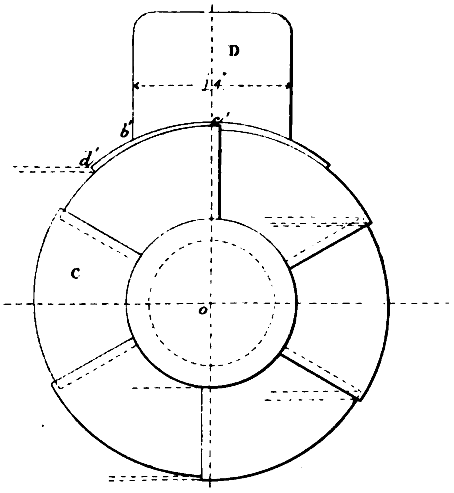


Fig.7.



ig.5.

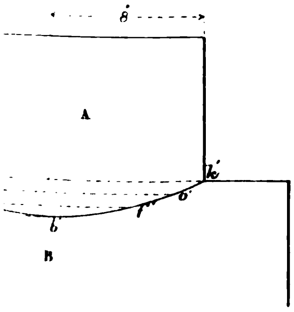


Fig.12.

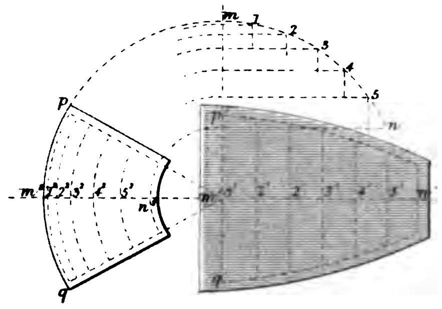


Fig.8.

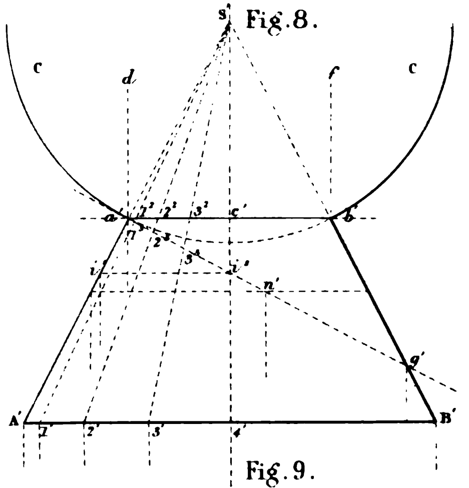


Fig.11.

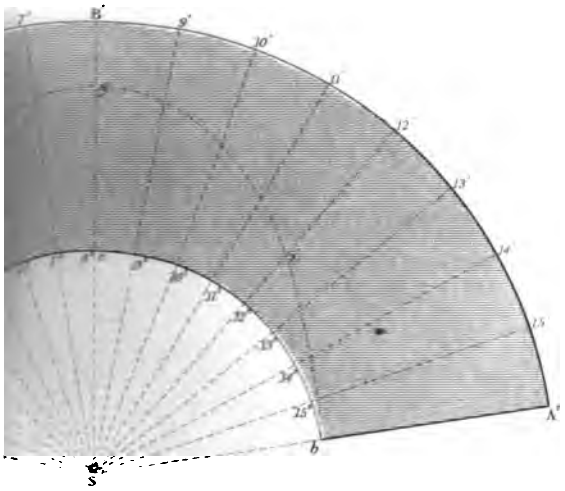
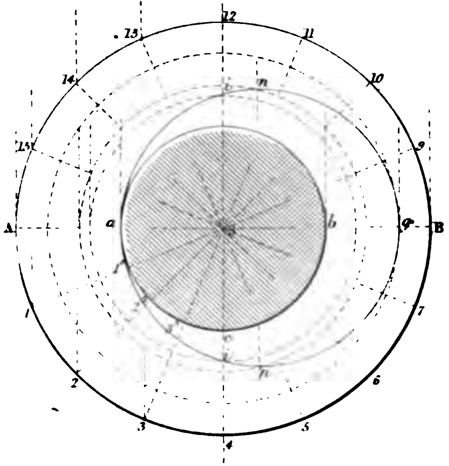


Fig.9.



— SCALE FOR FIGs 6.7 & 12. —

[illegible]

adopted in reference to Figs. 1 to 5, or by supposing a series of planes to cut the cone parallel to its base, and consequently at right angles to the generating lines of the cylinder; by this means circular sections will be produced, those of the cylinder being always the same, but those of the cone varying according to the distance of the planes from its apex. The points of intersection of the various circles representing, respectively, sections of the cone and cylinder, will be points in the curve of intersection sought.

DEVELOPMENTS.

165. By this term is meant the unrolling, extending, or flattening out upon a plane, of any curved surface, in order to ascertain its exact superficial measurement.

The more generally used surfaces or forms which are capable of development in this manner, are—the cylinder, the cone, prisms, pyramids, and the *frusta*, or fragments of these solids.

Tin and copper-smiths and boiler-makers, who operate upon metals which come into their hands in the form of thin sheets, have continually to transform these sheets into objects which are analogous in form to these solids.

To do their work with skill and exactitude, and not by mere guess, and also to avoid the cutting of the material to waste, they should make plans of the whole or part of the object as finished, so that they may calculate the exact development of the surface, both as to form and size, and cut it at once from the sheet of metal with all possible precision.

THE DEVELOPMENT OF THE CYLINDER.

166. Taking Fig. 1, which we have on a former occasion considered as a couple of solid cylinders, to represent, in the present case, two pipes or hollow cylinders formed of thin sheet metal, let us endeavour to ascertain what should be the shape and size of the pieces of flat metal, of which these two cylinders should be formed. It is to be observed, in the first place, that the rectification or development into a straight line of a circle, is equal to its diameter multiplied by 3·1416, &c.; whence the development of the base, P Q, of the right vertical cylinder, A, Fig. 1, of which the diameter measures 12 inches, is obviously equal to 12 × 3·1416 = 37·699 inches.

This length, then, 3′, 1¾″, is set off on the line, M M, Fig. 10, and the circumference having been divided into a number of equal parts, as was done to obtain the curve of intersection of the two cylinders in Fig. 2; the line, M M, is divided into the like number of equal divisions. Through each of these points of division, perpendiculars are erected upon the line, M M, representing so many generatrix lines corresponding to those of the cylinder, A, Fig. 1; and for the sake of greater intelligibility, we have marked the corresponding lines by the same letters. Next, on each of these are set off distances, M b^1, $e^1\ e^3$, $l^1\ l^2$, P a^1, $b^2\ b^2$, Q k^1, $o^1\ o^2$, $f^1 f^2$, and M b^1, equal to the respective distances in Fig. 1. By these means are obtained the points, b^1, e^2, l^2, a^2, b^3, in Fig. 10, through which the curve passes which forms the contour of intersection corresponding to that portion of the semi-cylinder, $b^1\ a\ b^2$, Fig. 2, which is intersected by the horizontal cylinder, B; and as the other half of the cylinder is precisely the same, the curve has simply to be repeated, as shown in Fig. 10.

It is unnecessary to detail the method of finding the development of the horizontal cylinder, B, as it is identical in principle with that just explained.

It may be gathered from the above example, that the principle generally to be followed in obtaining the development of cylindrical surfaces, is, first, to unfold it in a direction at right angles to one of its generating lines, or in the direction the generatrix takes in the construction of the solid, and then to set off from the straight line thus produced, at equal distances apart, any number of distances previously obtained from the projections of the outline or line of intersection when the cylinder is joined to another, or of its section when cut by any plane. The curve of this outline is finally obtained by tracing a line through the extremities of the generating lines drawn perpendicular to the base.

THE DEVELOPMENT OF THE CONE.

167. As in the case of the cylinder, so likewise, in order to find the development of the cone, do we unfold it, as it were, in the direction of motion of its generatrix. Now, as all the generating lines of a right cone are equal, and converge to one point—the apex, it follows that, when the conical surface is developed upon a plane, these generating lines will form a radii of a portion of a circle; consequently, if with one of the generating lines, as a radius, we describe a circle, and cut off as much of the circumference as shall be equal to that of the base of the developed cone, we shall obtain a sector of a circle equal in area to the lateral surface of the cone, as developed upon a plane.

Fig. 11 represents the development of the frustrum, or truncated cone, $a^1\ b^1$, $A^1\ B^1$, as projected in Fig. 8, and of which the apex would be S^1, were the cone entire. The operation is as follows:—

We shall suppose the cone to be developed in the direction taken by the generatrix, $S^1\ A^1$, Fig 8; therefore with a radius equal to $S^1\ A^1$ and with the centre, S^1, describe the fragment of a circle, $A^1\ B^1\ A^2$, Fig. 11.

Having divided the circular base, A B, Fig. 9, of the cone into some arbitrary number of equal parts, say 16, and having drawn the respective generating lines, 1 S, 2 S, 3 S, &c., set off on the arc, $A^1\ B^1\ A^2$, Fig. 11, an equal number of arcs, each equal to the respective arcs obtained by the subdivision of the circle, A B, Fig. 9. From the points thus obtained, $1^1\ 2^1\ 3^1$, &c., Fig. 11, draw the radii, $1^1 S^1$, $2^1\ S^1$, $3^1\ S^1$, &c., representing generating lines corresponding to those projected in Fig. 9.

By this operation we obtain the development of the entire cone, and find that it produces the figure $S^1\ A^1\ B^1\ A^2$, Fig. 11, the circular perimeter of which is equal to that of the base of the cone itself. The cone, however, under consideration, is divided by a plane, $a^1\ b^1$, Fig. 8, parallel to its base, which reduces it to a frustum of a cone; the development of the conical surface of which is equal to the space contained between the arc, $A^1\ B^1\ A^2$, corresponding to the base of the cone, as just determined, and the arc, *a c b*, of lesser radius, drawn with the same centre, S^1, and with a radius equal to the generatrix, $S^1\ a^1$, of the portion of the cone taken away.

The development, then, of the truncated cone, is the fragment of an annular space, distinguished in Fig. 11 by a flat tinted shade.

168. In the case of a truncated cone, of which the dividing plane is inclined to the base, as $a^1\ g^1$, Fig. 8, instead of being parallel therewith, or if the cone is joined to a cylindrical or spherical body, and the line of intersection is curved in any way, the development of this edge of the conical surface will no longer take the form of the arc, *a c b*, Fig. 11. Its true representation will be obtained by setting off, on the several radii, Fig. 11, lengths corresponding to the respective generatrices, as intercepted by the plane, or curve of intersection. In order to obtain the lengths of the respective generating lines, which can be done from the vertical projection, Fig. 8, each intermediate one, as $4^1\ i^2$, &c., must be squared

across to an extreme one, as at A^1 i^1, and indicated by the horizontal lines; this will give the exact length of each—being otherwise, as projected, considerably fore-shortened. Thus, the division of the cone by the inclined plane, a^1 g^1, Fig. 8, produces an ellipse, the development of which in Fig. 11, takes the form of the curve, *a i g i b*.

In the construction of the boiler, represented in Figs. 6 and 7, which is formed of several pieces of sheet metal, we shall find extensive applications of the principles just discussed. It must be borne in mind that, in calculating the development, or the size and shape of the component pieces, an allowance must be made for the *lap* of the pieces over each other, for the purpose of joining them together, as indicated in the drawing.

169. In cylindrical steam boilers, the extremities are generally constructed in the shape of hemispheres—this form offering the greatest resistance to internal or external pressure.

As the sphere cannot be developed upon a plane, these hemispherical ends cannot be made in a single piece, unless cast or forged. In practice this difficulty is overcome, by forming this portion in from 5 to 8 gores, according to the size of the boiler, these being surmounted by a central cap-piece. After being cut out, these several pieces are hammered to give them the necessary sphericity.

In Fig. 12, we give a method of approximately determining the development of one of such gores; this consists in drawing with the centre, o^1, an arc, *m n*, corresponding to the radius of the hemisphere. On this arc, from *m* to *n*, set off the circular length of the gore; set off, also the length, *p q*, corresponding to one of the six divisions, as seen in the end view, Fig. 7. On the arc, *m n*, Fig. 12, mark an arbitrary number of points, at equal distances asunder, as 1, 2, 3, 4, 5; draw through these horizontal lines, cutting the vertical, *m* o^1, thereby giving the various radii, o^1 1^2, o^1 2^2, o^1 3^1, &c., with which arcs are drawn as indicated; the rectification, or development, of these arcs, contained between the radii, *p* o^1, *q* o^1, are then obtained, and transferred to perpendiculars drawn through the points, 1^1, 2^1, 3^1, &c., on the line, o^1 n^1, which is the rectification, or development, of the arc, *m n*, with its series of divisions. Thus, from the arc, *p q*, is obtained the line, p^1 q^1, and similarly the entire figure, p^1 n^1 q^1, which is an approximation to the surface of the gore, as supposed to be flattened out. The necessary allowance for lap is superadded, as shown by the flat tinting in the drawing, Fig. 12. The gore cut to this outline in sheet metal is then hammered to a proper form upon a mandril, or anvil, with a spherical surface.

THE DELINEATION AND DEVELOPMENT OF HELICES, SCREWS, AND SERPENTINES.

PLATE 15.

HELICES.

170. That curve is called a cylindrical helix, which is generated by a point travelling round a cylinder, having, at the same time, a motion in the direction of the length of the cylinder—this longitudinal motion bearing some regular prescribed proportion to the circular or angular motion. The distance between any two points in any straight line parallel to the axis of the cylinder, is called the *pitch*—in other words, the longitudinal distance traversed by the generating point during one revolution.

This definition at once suggests a method of drawing the lateral projection of this curve, when the two projections of the cylinder and the pitch are known. This method consists in dividing the circumference of the base of the cylinder into any number of equal parts, and drawing parallels to the axis through the points of division projected on the vertical plane; at the same time a portion of the axis, equal to the pitch, must be divided into the like number of equal parts, and as many lines must be drawn perpendicular to the axis. The intersections of the corresponding lines of each set will be points in the curve.

Let A^1 and A, Figs. 1 and 2, be the vertical and horizontal projections of a right cylinder, and the line, a^1 a^2 the length of the pitch of the helix, generated by the traverse, as already defined, of the point projected in *a* and a^1.

The circle described with the radius, *a o*, and representing the base of the cylinder, is divided into 12 equal parts, starting from the point, *a*. Through each of the points thus obtained, a vertical line is drawn. The pitch, a^1 a^2, is similarly divided into 12 equal parts, and a corresponding number of horizontal lines are drawn to cut the vertical ones in the points, 1^1, 2^1, 3^1, &c.; these points are next connected by the continuous line, a^1, 1^1, 3^1, 6^1, 9^1, a^2, which forms the vertical or lateral projection of the helix.

Half of this curve is indicated by a sharp full line, as being on the front surface, *a*, 3, 6, of the cylinder, whilst the other half is in dotted lines, representing the portion of the curve which is on the other side, *a*, 9, 6, Fig. 2.

The number of divisions of the circumference of the cylinder is a matter of indifference as regards the accurate delineation of the curve, and we may therefore select a number that calls for the simplest operations—an even number, for example, as 6, 8, or 12; and in the present instance, wherein the starting point, *a*, lies in the horizontal diameter, *a* 6, of the base, it will be observed that two points occur in the same vertical line, as 2, 10, Fig. 2, which gives the points, 2^1, 10^1, in the vertical projection, Fig. 1.

The operations will be similar, if the given starting point be diametrically opposite to a^1, as b^1, the pitch, b^1 b^2, being equal to a^1 a^2.

171. The conical helix is different from the cylindrical helix, simply in that it is described on the surface of a cone instead of on that of a cylinder, and the operation consequently differs but slightly from the last. In this case, the horizontal and vertical projections of the cone are given, and also the pitch. Fig. 3 is the vertical projection of a truncated cone, C, and it has been placed above Figs. 1 and 2, so as to square up the several divisions in its bases, a^1 b^1, and c^1 d^1, which are represented in the plan, Fig. 2, by the concentric circles described with the respective radii, *o a*, and *o c*. The outer circle having been divided as already shown, radii are drawn to the centre, *o*, from all the points of division, 1, 2, 3, &c., which cut the inner circle in the points, *e*, *f*, *g*, &c. These points are then projected upon the upper base, c^1 d^1, in Fig. 3, those on the outer circle being similarly projected on the lower base, a^1 b^1; the respective points in each base are next joined, thus forming a series of generatrices of the cone, as 1^2 e^2, 2^2 f^2, o^1 o^2, &c., which would all converge in the apex, if the cone were complete. These lines are cut by horizontals drawn through a corresponding number of divisions in the length of the given pitch, a^1 c^1, and the points of intersection thus obtained lie in the curve which it is required to draw. The horizontal projection of the curve is then obtained, by letting fall from the points of intersection last obtained a series of verticals, which cut the respective radii in the plan, Fig. 2. This produces a species of spiral, or volute, *a*, e^3, f^3, g^3, h^3, i^3, &c. By following out the same principles, helices may

HELICES

Fig. 3.

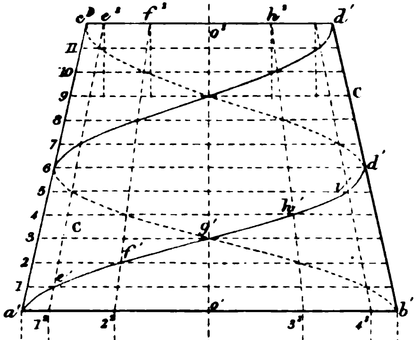


Fig. 5.

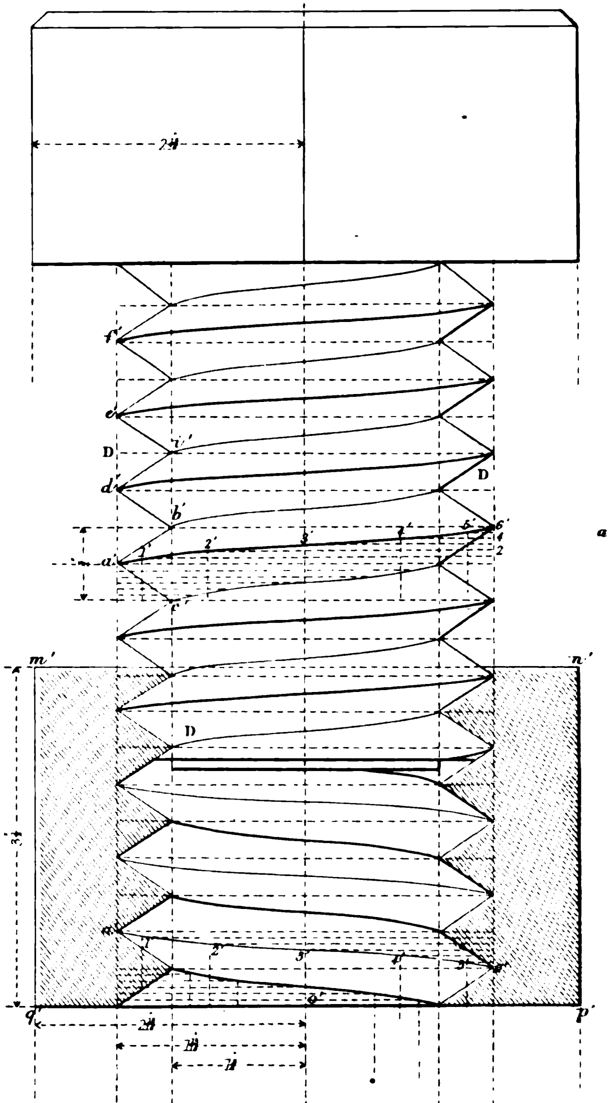


Fig. 13.

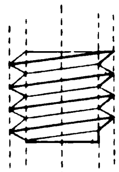


Fig. 14.

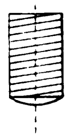


Fig. 7.

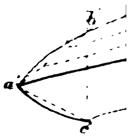
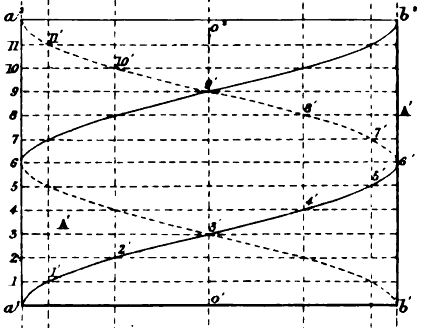


Fig. 1.



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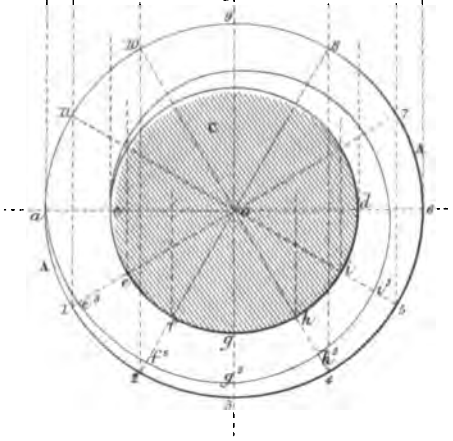
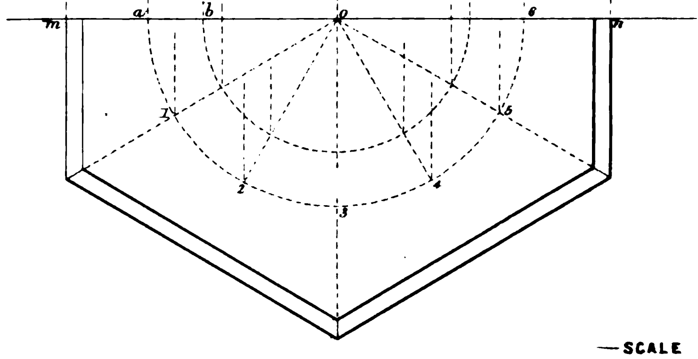


Fig. 6.



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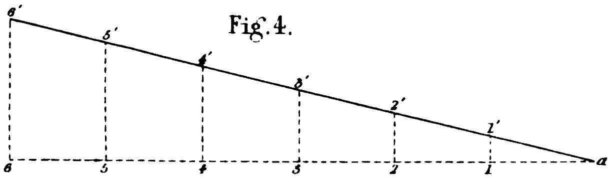

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SCREWS.

Fig. 8.

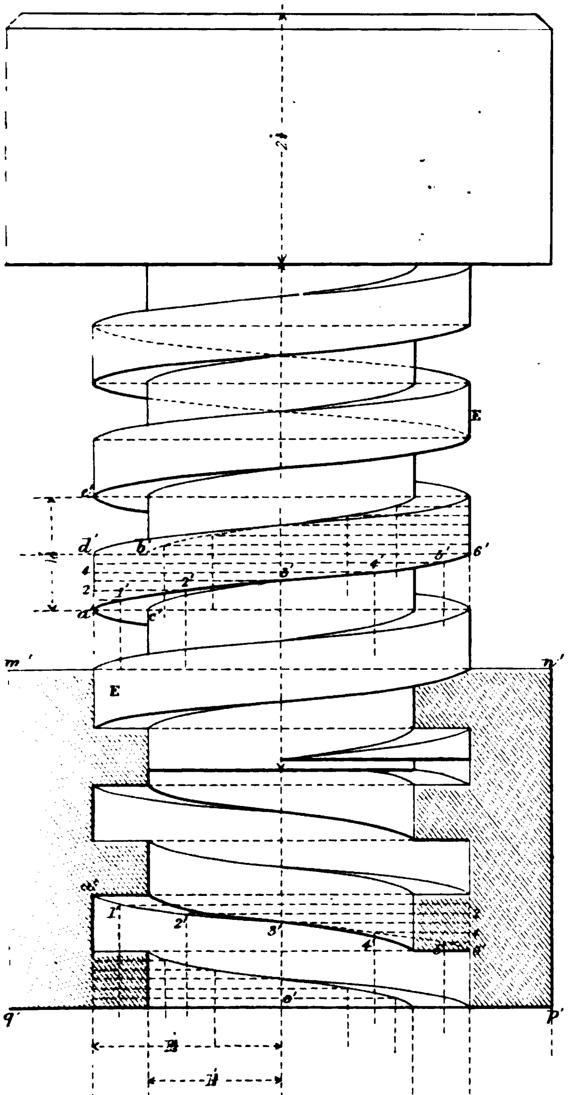


Fig. 9.

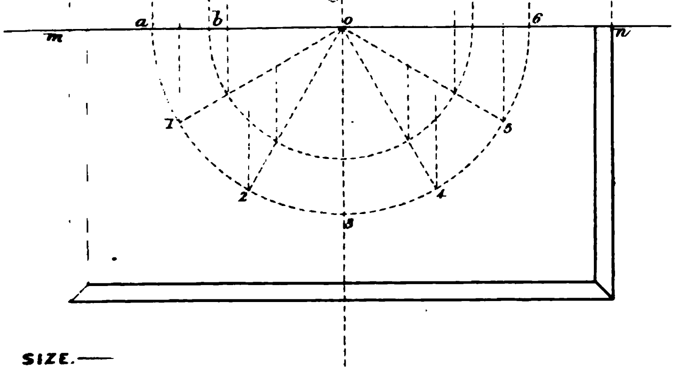


Fig. 15.

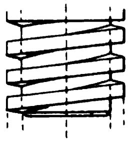


Fig. 16.

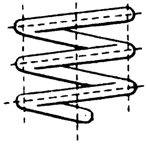


Fig. 10.

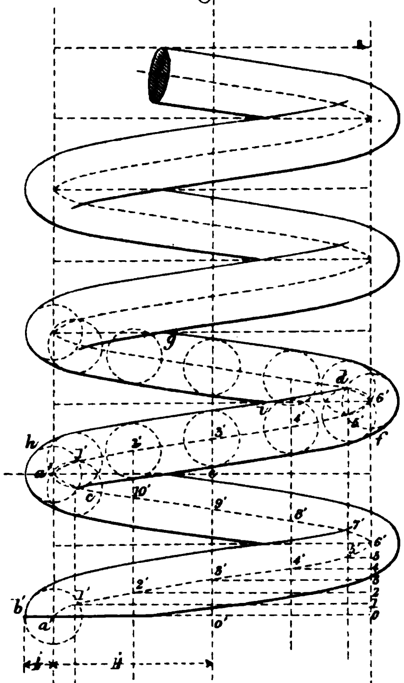
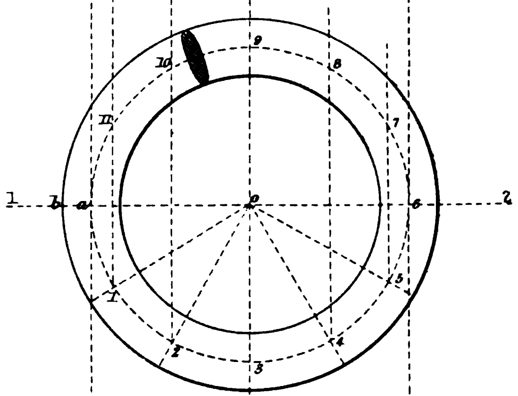


Fig. 11.



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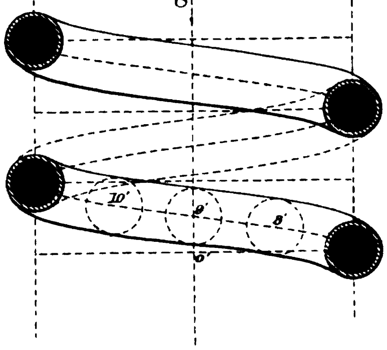

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be represented as lying upon spheres, or any other surfaces of revolution.

THE DEVELOPMENT OF THE HELIX.

172. It will be recollected that a cylinder, and also a cone, are capable of being developed upon a plane surface, and that the base of either, when rectified, or converted into a straight line, is equal to the diameter multiplied by 3·1416. Let, then, *a* 6, Fig. 4, be a portion of the development of the base of the cylinder, A, Figs. 1 and 2; to obtain the development of the helix drawn upon this cylinder, we must first divide it off into lengths, corresponding and equal to the arcs obtained by the division of the circle, of which *a o* is the radius. On each of the divisions thus obtained, as 1, 2, 3, &c., we then erect perpendiculars, making them equal respectively to the distances from the starting point, *a*, of the several divisions of the pitch. The extremities of these perpendiculars, as 1^1, 2^1, 3^1, &c., will be found to lie in the same straight line, *a* 6^1, which consequently represents the development of a portion of the helix. The development of a helix of one revolution is a straight line, forming the hypothenuse of a right-angled triangle, the base of which is equal to the circumference of the cylinder, and the height to the pitch of the helix.

Several helices drawn upon the same cylinder, and having the same pitch, or a helix which makes several revolutions about a cylinder, is represented by a series of parallel curves, the distance between which, measured on any line parallel to the axis, is always equal to the pitch.

The development of the conical helix may be obtained by means of an operation analogous to that employed for the development of the cone, Fig. 11, Plate 14; and in this case the result will be a curve, instead of a straight line.

We meet with numerous applications of the helical curve in the arts, for all kinds of screws, staircases, and serpentines.

SCREWS.

173. Screws are employed in machinery, and in mechanical combinations, either for securing various pieces to each other, so as to produce contact pressure, or for communicating motion. Screws are formed with triangular, square, or rounded threads, or fillets.

A screw is said to have a triangular thread, when it is generated by a triangle, the three angles of which describe helices about the same given axis, situate in the same place as the triangle. Figs. 5 and 6 represent respectively the vertical and horizontal projections of a triangular-threaded screw, such as would be generated by the helical movement of the triangle, $a^1\ b^1\ c^1$, of which the apex, a^1, is situate on a cylinder of a radius equal to *a o*, and of which the other angles, b^1, c^1, are both situate on the internal cylinder, having the radius, *b o*, which is called the core of the screw, and is concentric with the first. The difference *a b*, between the radii, *a o* and *b o*, indicates the depth of the thread.

When, as in the case taken for illustration, the screw is single-threaded, the pitch is equal to the distance between the two points, b^1 and c^1; that is, to the base of the triangle. The screw is one of 2, 3, 4, or 5 threads, according as the pitch is equal to 2, 3, 4, or 5 times the base of the generating triangle. From what has already been discussed, the method of delineating the triangular-threaded screw will be easily comprehended; for all that is necessary is to draw the helices, generated by the three angular points, in the manner shown in reference to Figs. 1 and 2. We have, notwithstanding, given the entire operation for one semi-revolution of the thread, in Figs. 5 and 6. When one of the curves, as $a^1\ 3^1\ 6^1$, is obtained, it is repeated as many times as there are revolutions of the thread on the length of the screw. To do this with facility, and without repeating the entire operation, it is customary to cut out a pattern of the curve in hard card-board, or, by preference, in veneer wood; then setting this pattern to the points of division, $d^1\ e^1\ f^1$, previously set off, the curves are easily drawn parallel to one another. The curve, formed by the base of the triangle, $b^1\ c^1$, or inner helical curves, may be found and repeated in the same manner as the curve just described, formed by the outer edge, or apex of the triangle, $a^1\ 3^1\ 6^1$.

It must be observed that, to complete the outline of the screw, these curves forming the top and bottom of the thread require to be joined by the short lines, $b^1\ d^1$, $d^1\ i^1$; which, though shown in straight lines in Fig. 5, should, if it is wished to be precise, be shown by lines slightly curved and tangential to the curves passing through the points, a^1 and b^1, as in the detached view, Fig. 7. These curves are the result of a series of helices, traced by the component points of the lines forming the sides, $a^1\ b^1$, $a^1\ c^1$, of the generating triangle. In practice, this nicety is disregarded, and simple straight lines are employed.

174. A screw is termed square-threaded when the cross section of the thread is a square or a rectangle, the parallel sides of which lie in right concentric cylinders, and the angles or corners of which describe helices about the axes of these cylinders. Figs. 8 and 9 represent projections of a square-threaded screw—the thread being generated by the square, $a^1\ c^1\ b^1\ d^1$. The horizontal side, $a^1\ c^1$, determines the depth. The height, $a^1\ d^1$, marks the width of the thread, and $d^1\ e^1$, is the width of the interval, which is generally equal to that of the thread.

When the screw is single-threaded, the pitch, $a^1\ e^1$, is equal to the sum of the widths of the thread and of the interval, or, in the case before us, to twice the side of the square. Of course, when the pitch is equal to 2, 3, or 4 times $a^1\ e^1$, the screw is 2, 3, or 4-threaded, in all cases having as many intervals as threads. The operations called for in delineating the screw of a single square thread, are fully indicated upon the figures. The delineation of a screw of several threads does not possess any additional points of difficulty.

INTERNAL SCREWS.

175. An internal or female screw is one cut on the inner face of a hollowed body. Such a screw is intended to work in conjunction with an ordinary external screw, in such a manner that the helical grooves of one receive the helical elevations of the other. When the body having an internal screw is small, it is called a nut. In order to show the helical fillets or threads of the internal screw, it is necessary to represent it as cut by a plane passing through its axis; it is in this manner that we have represented the nuts, m^1, n^1, p^1, q^1, in Figs. 5 and 8, the first having a triangular thread fitting to and embracing the screw, D, which is represented as just introduced into it; the other has a square thread similarly adapted to the screw, E.

It follows, from these nuts being represented in section, that we only see the half of each corresponding to the posterior portion of their respective screws, D and E; and, in consequence of this, the helical curves are inclined in the opposite direction to those representing the anterior portions of the screws.

Those screws are distinguished as right-handed screws, of which the thread ascends in passing from left to right, as in the screws, D, E. Left-handed screws are those in which the thread descends in passing from left to right.

SERPENTINES.

176. Serpentine is the name given in practice to a pipe or tube bent to the helicoidal form; but, in geometry, it is the term given to the solid generated by a sphere, the centre of which traces a helicoidal path.

This form is often employed, whether hollow, as for pipes, such as the worm of a distilling apparatus, or solid, as for metal springs.

The first thing to be done in delineating this solid, is to determine the helix traced by the centre of the generating sphere, its pitch, and the radius of the cylinder on which it runs, being given. The helix having been drawn, a series of circles are described with the radius of the sphere, and with various points of the curves as centres; curves drawn tangential to these circles, will then form the outline of the object. Figs. 10 and 11, represent the elevation and plan of a serpentine formed in this manner. The circle drawn with the radius, *a o*, is the base of the cylinder, on which lies the helix generated by the moving point projected vertically in a^1. Next is given the radius, a^1 b^1, of the generating sphere, and the pitch, a^1 a^2, of the helix. This helix is then projected in the manner indicated on the drawing, and already described, by the curve, a^1 1^1, 2^1, 6^1, 9^1, and a^2; it may be continued indefinitely, according to the number of convolutions desired. With different points of this curve as centres, and with the radius, a^1 b^1, are then described a series of circles pretty near to each other, and two curves are drawn tangential to these, as shown on Fig. 10.

In going over this figure with ink, it is of importance to limit these curves to the portion of the outline, which is quite apparent or distinct; thus, for the anterior portion, *a* 1, 2, 3, 6, Fig. 11, of the serpentine, the lower curve, *c e f g*, ends at the point of contact, *c*, with the circle whose centre is a^1, whilst the upper one, *h i d*, ends in the point, *d*, on the circle described with the centre, 6^1. The posterior portions of these curves are limited by the points, *g* and *i*, where the bend of the serpentine goes behind, and is hid by the anterior portion.

The horizontal projection of the serpentine is always comprised within two concentric circles, the distance asunder of which is equal to the diameter of the generating sphere, as in Fig. 11.

Fig. 12 represents a sectional elevation of a tubular serpentine, which is supposed to be divided by a plane, 1, 2, in Fig. 11, passing through its axis. It is, consequently, the posterior portions that are visible, and they are inclined upwards from right to left; the section at the same time shows the thickness of the tube or pipe.

In the arts we see serpentines, both solid and hollow, in the form of conic or other helices; of this kind are the springs employed in moderator lamps, and distillery worms are sometimes made in this way.

177. *Observation.*—The curves representing the outline of screws and serpentines, the rigorously exact delineation of which we have just explained, are considerably modified when these objects come to be represented on a very small scale; thus the triangular-threaded screw may be represented, as shown in Fig. 13, by a series of straight parallel, instead of curved, lines—these being inclined from side to side to the extent of half the pitch. These lines should be limited by two parallels to the axis on each side, marking the amount of relief of the thread. When the scale of the drawing is still smaller, and greater simplicity desirable, the draughtsman is content with a series of parallel lines, as in Fig. 14, limited by a single line on each side parallel to the axis.

For the square-threaded screw, the helical curves may similarly be replaced by straight lines, as in Fig. 15. The same also applies to the serpentine, as shown in Fig. 16.

APPLICATION OF THE HELIX.

THE CONSTRUCTION OF A STAIRCASE.

PLATE 16.

178. Staircases which afford a means of communication between the floors of houses, are frequently constructed in some part or other in the form of a helix. The cage, or space set apart for the staircase, may be rectangular, circular, or elliptic.

Figs. 1 and 2 represent the elevation and plan of a staircase, the recess, shaft, or cage, *a b c d*, of which is rectangular; this space being provided for the construction of the main frame of the stair, with its steps and balustrade, and with a central space left for the admission of light from above. In the case of a cylindrical cage, the curve with which the steps rise is helical from bottom to top; but in a staircase within a rectangular cage, the steps rise for some distance in a straight line, and only take the helical curve at the part forming the junction between the rectilinear portions running up alternate sides of the rectangle. Stairs are sometimes made without this curved part, a simple platform, or "resting-place," connecting the two side portions.

For the division of the steps, we take the line, *e f g h i*, passing through the centre of their width, and taking exactly the direction it is intended to give the stairs. The first or lowest step, A, is generally of stone, and is larger and wider than the others.

For the stairs, as for the helix, the pitch or height, say from the basement to the floor above, is divided into as many equal parts as it is wished to have steps. The centre line, *e f g h i*, is also divided into a like number of equal parts. As a general rule, the number of steps should be such, that the height of each does not exceed 8 inches. The larger the staircase is, the more may this height be reduced, in some instances, as low as 5 and 6 inches. The width of steps vary much, but they should never be less than 9 inches at the centre.

If, for example, in the given height of 12′ 3″. we wish to make 21 steps, we must divide this height into 21 equal parts, which in this instance will give 7 inches to each step; draw a series of horizontal lines from each division, which will represent the horizontal top surfaces of the steps.

For those steps which lie parallel to each other, it is simply requisite to erect verticals upon the points of division on the centre line in the plan. The points of intersection of these with the horizontals above, as 1, 2, 3, 4, Fig. 1, indicate the edges of these steps. For the turning steps, however, or winders, as those steps are called which are not parallel to each other, a particular operation is necessary, termed the balancing of the steps, the object of which is to make the steps as nearly equal in width as possible, without, at the same time, making them very narrow on the inner edges, or rendering the curve too sharp or sudden. Where the stairs are narrow, as in the case we have illustrated, the balancing should commence a step or two before reaching the curved portions. This balancing may be obtained in the following manner:—A part of the rectilinear portion, *p l*, equal to three steps, is developed, and then a part of the curved portion, *l m n*, equal to three more steps. On the vertical, *t q*, Fig. 3, set off the heights of the first

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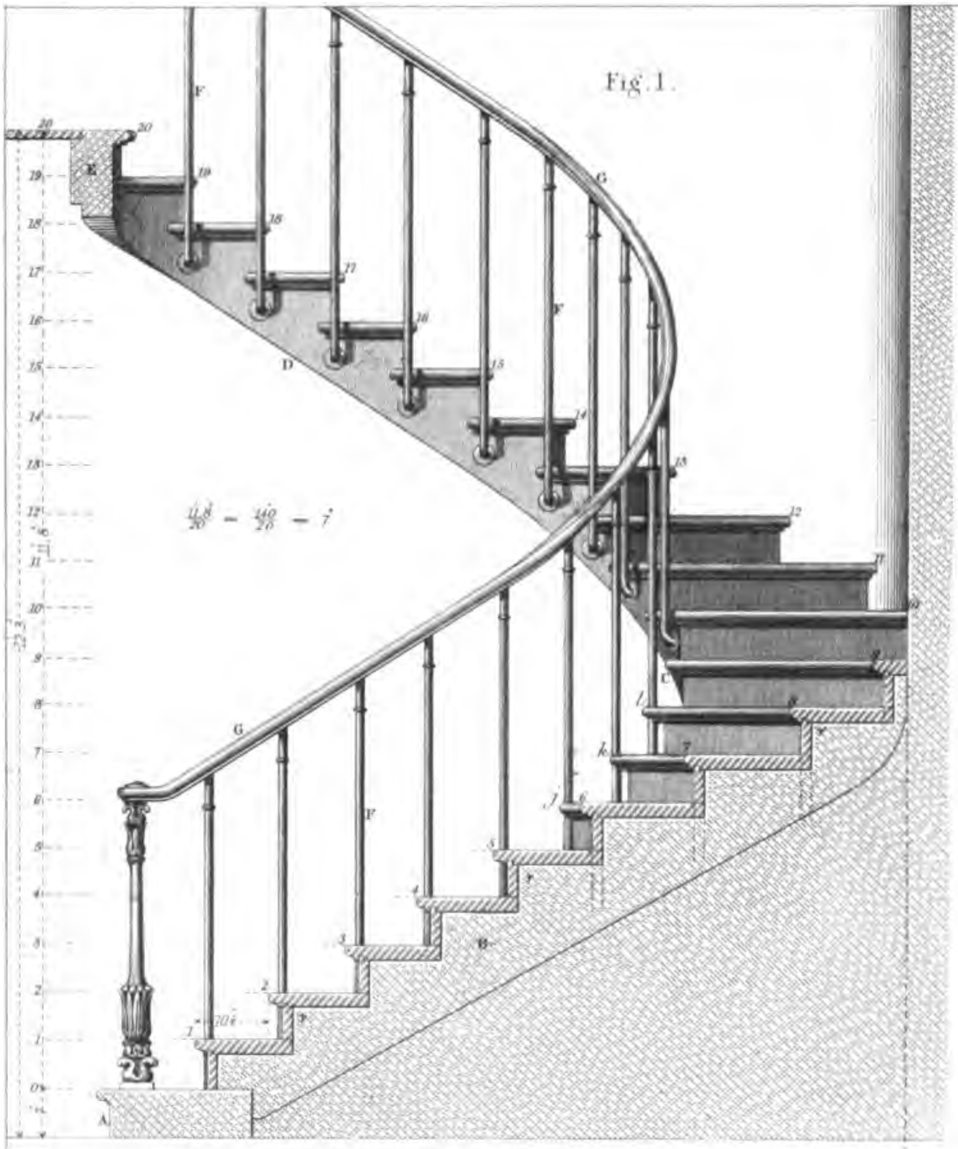
Fig. 1.



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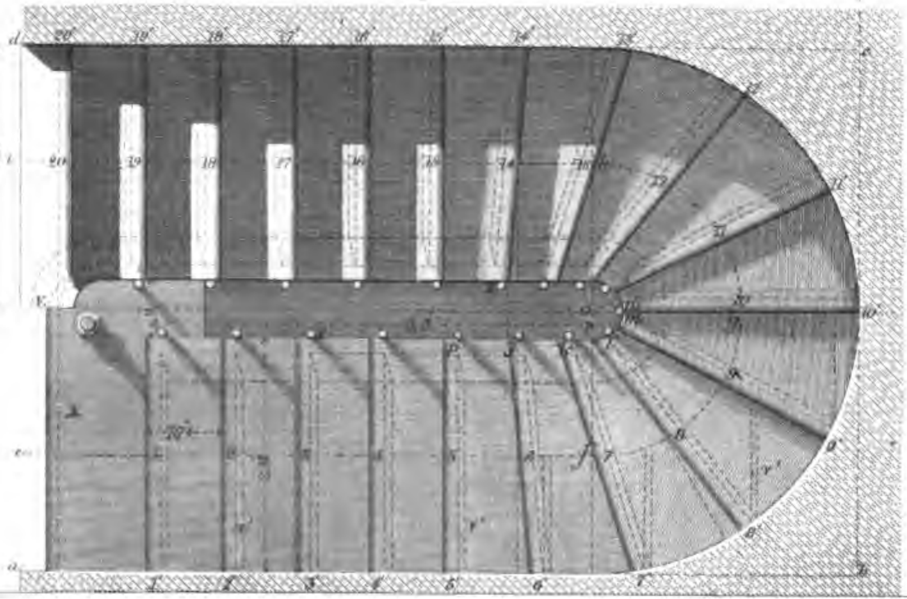

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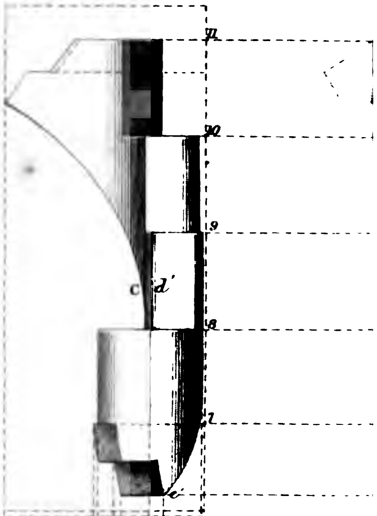

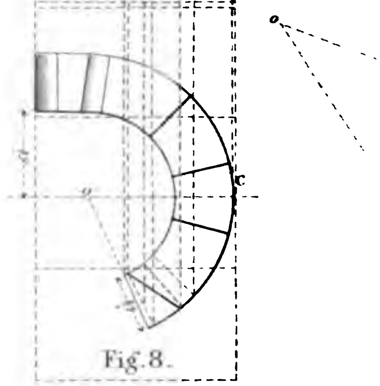

Fig. 8.

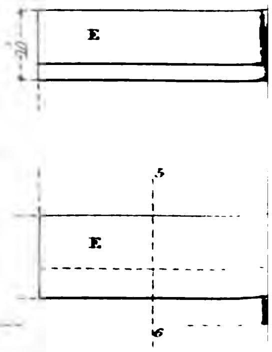

Fig. 14.

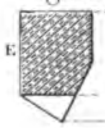

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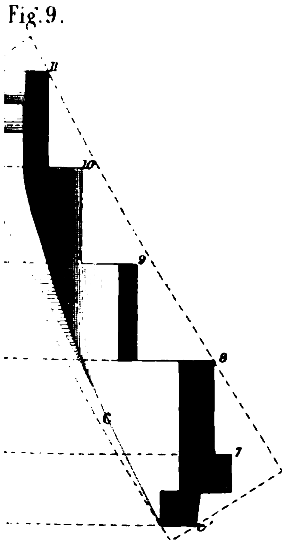
Fig. 9.



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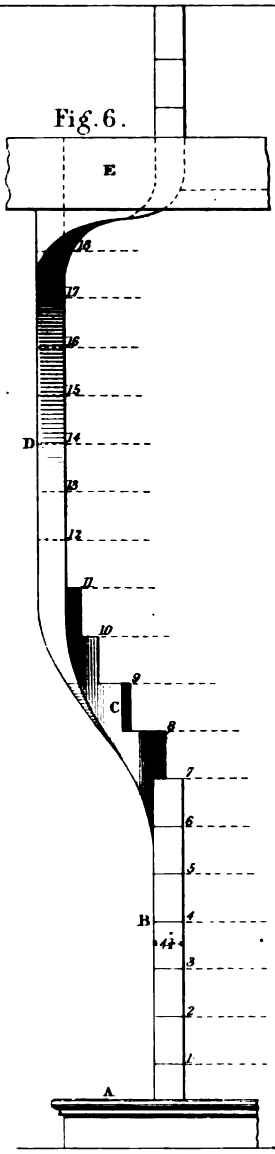
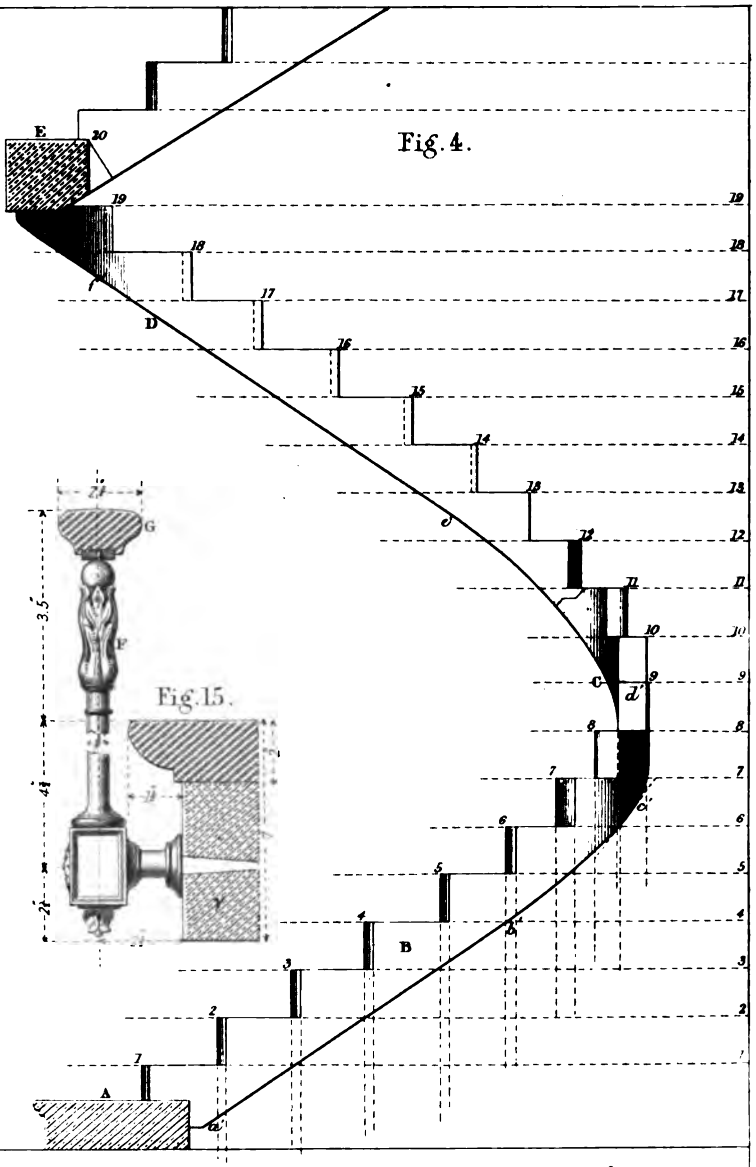


Fig. 4.

Fig. 15.



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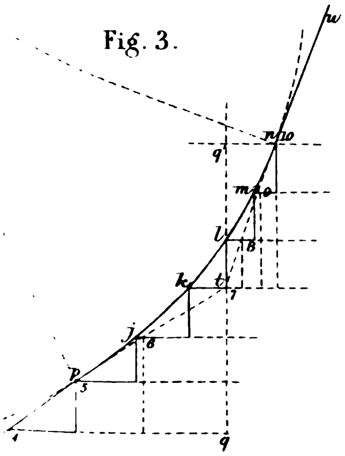

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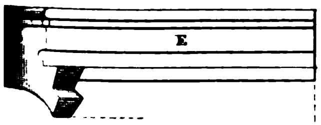

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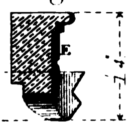

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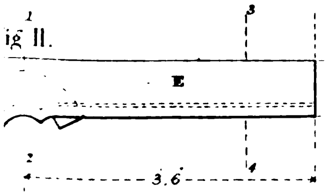

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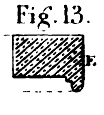

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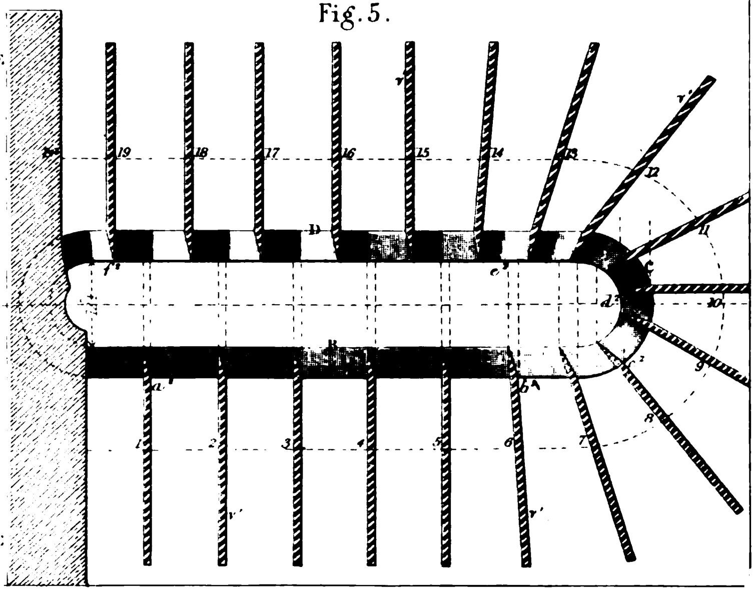

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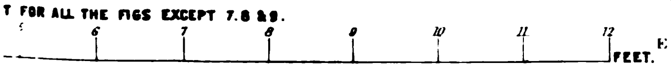

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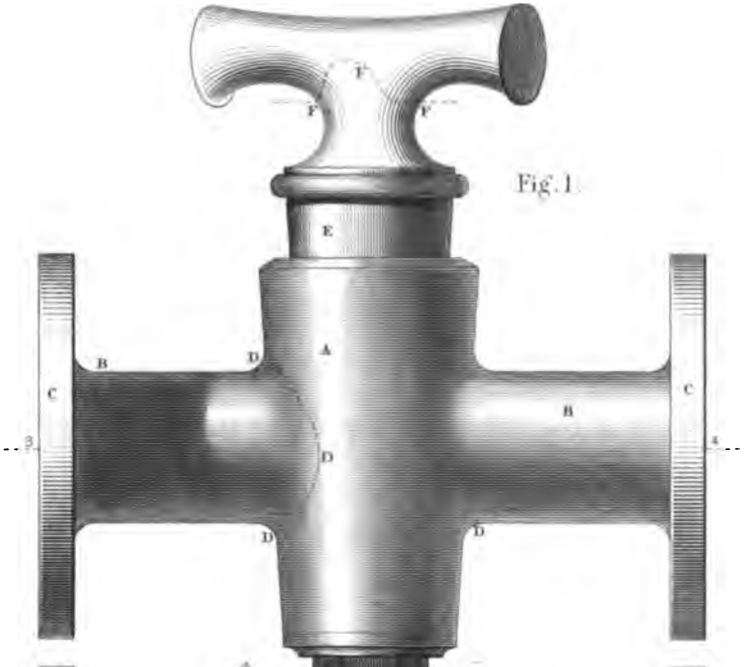

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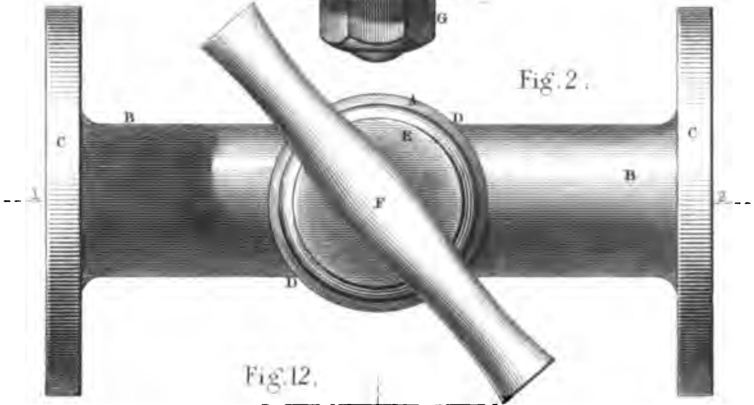

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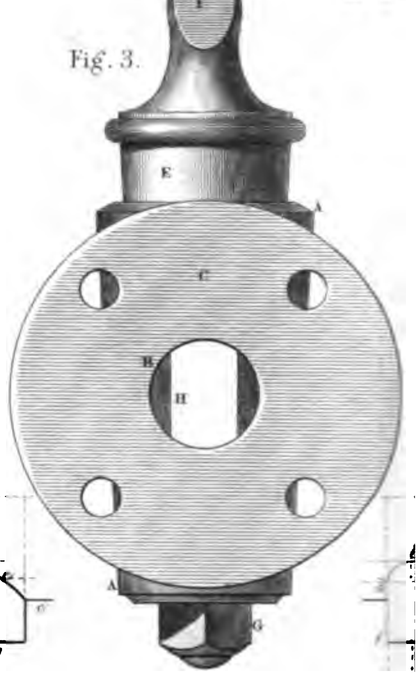

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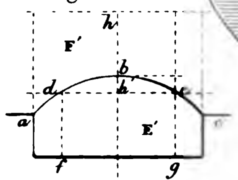

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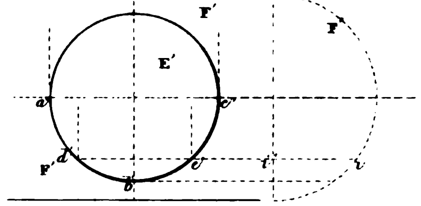

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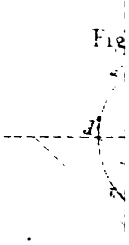

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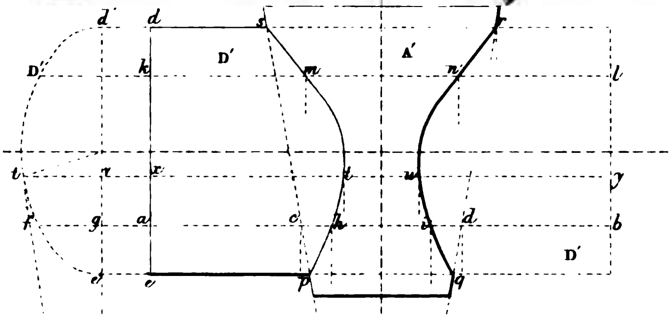

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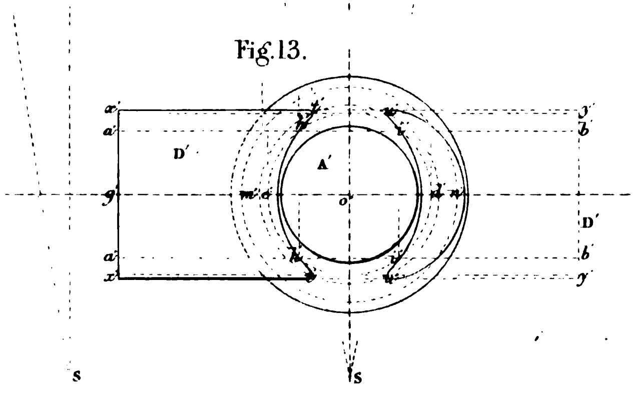

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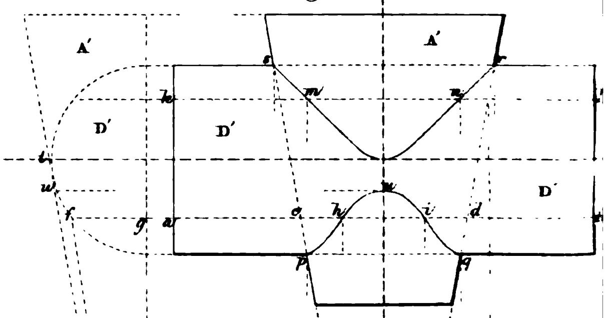

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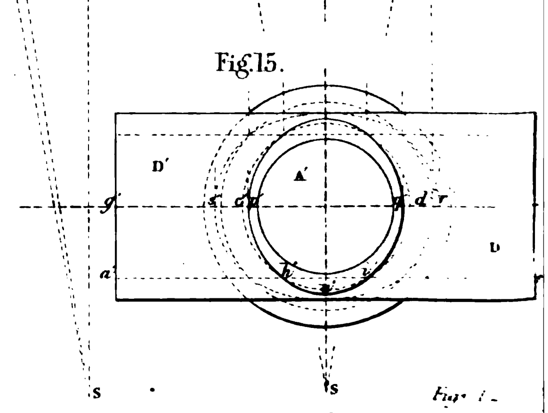

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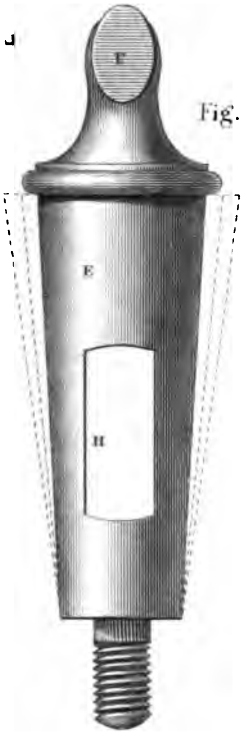

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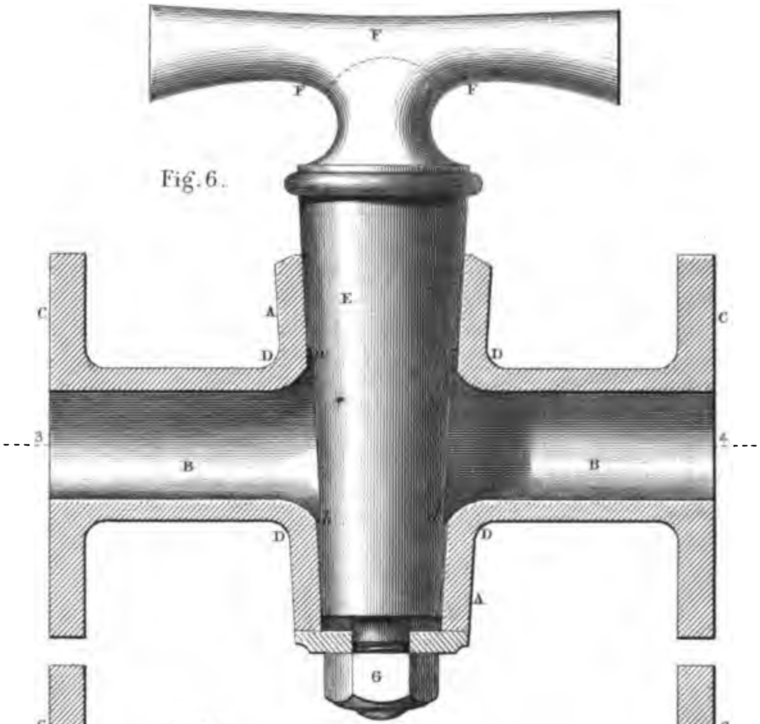
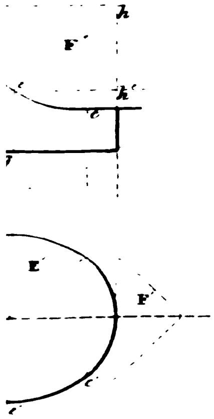


Fig. 6.

Fig. 10.



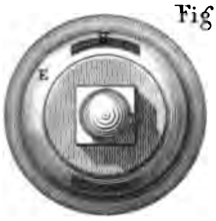


Fig. 5.

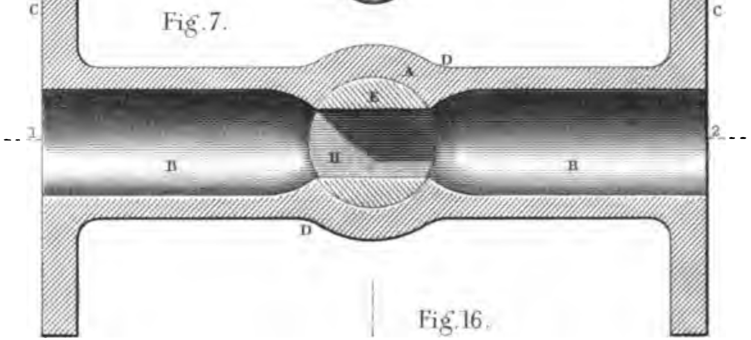


Fig. 7.

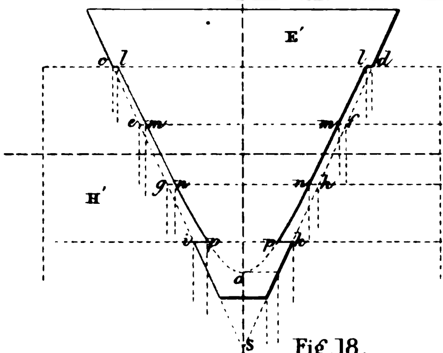


Fig. 16.

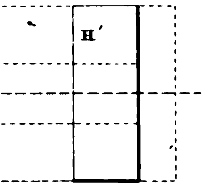


Fig. 17.

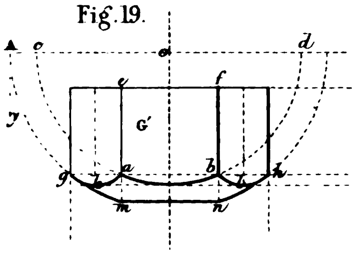


Fig. 19.

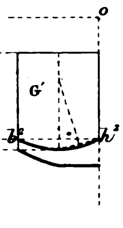


Fig. 21.

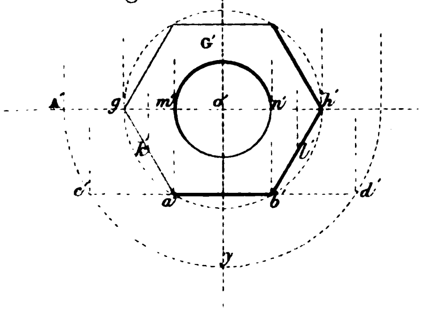


Fig. 20.

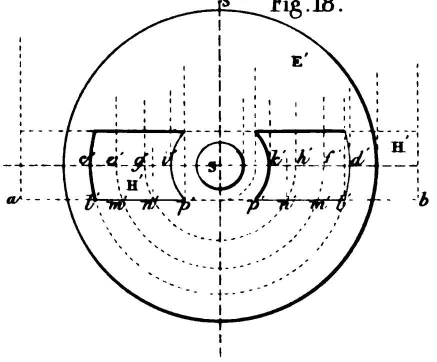


Fig. 18.

three steps; and through the point, q, draw the horizontal, $q\,4$, representing the development of the widths of the steps in a straight line. Also, on the prolongation, $t\,q^1$, of the vertical, $t\,q$, set off the heights of other three steps. Through the point, q^1, draw a horizontal, and make $q^1\,10$, equal to the arc, $l\,m\,n$, in the plan, Fig. 2, as rectified. The straight line, $t\,10$, will then be the development corresponding to the curve of the framepiece. At the latter point, n, erect a perpendicular on this line, and at the point, 5, a perpendicular to the straight line, $t\,4$. The point of intersection, o, of these two perpendiculars, gives the centre of the arc, $p\,k\,n$, which is drawn tangentially to these lines. Then, through each point of division on the vertical, $q\,q^1$, draw horizontals, meeting this curve in the points, $j\,k\,l\,m$, through which draw parallels to $q\,q^1$. Then transfer the respective distances, $j\,6$, $k\,t$, $l\,8$, $m\,9$, comprised between the arc and the two straight lines, $t\,4$, and $t\,n$, on the line of the framepiece, $p\,k\,n$, in the plan, Fig. 2, as at $j\,k$, $k\,l$, $l\,m$, $m\,n$. Next, draw straight lines through the points, $j\,k$, $l\,m$, and through the respective points of division, 6, 7, 8, 9, and 10, on the centre line, which will give the proper inclination for the steps as balanced. The second half of the curved portion is obviously precisely the same as the first in plan, and may easily be copied from it.

Having thus determined the position of the steps in the horizontal projection, they must next be projected on the vertical plane, by means of a series of verticals, which cut the respective horizontals drawn through the points of division, 1, 2, 3, 4. As in Fig. 1, the anterior wall, $a\,b$, of the recess or shaft being supposed to be cut away by the line, $a\,6^1\,10^1$, in the plan, to show the outer edges of the steps, which are determined by erecting verticals on the corresponding points, 6^1, 7^1, 8^1, 9^1, &c.

The perpendicular portions, v and v^1, of the steps, which are overhung by the horizontal portions, and consequently invisible in the plan, Fig. 2, are, nevertheless, indicated there in dotted lines, parallel to the edges of the steps. To render them quite distinct, however, and at the same time to show the manner in which they are fitted into the framepiece, we have represented them in Fig. 5, without the actual steps, supposing them to be cut in succession, horizontally, through the centres.

The framepiece is the principal piece in the staircase. It is situated in the centre of the cage, and sustains each step, and, consequently, must be constructed very accurately, for upon it, in a great measure, depends the strength and solidity of the staircase. For a staircase of proportions like those of the one represented in the plate, the framepiece is generally made of oak, in three pieces; the middle piece, C, corresponding to the curved portion, whilst the other two, B and D, joined to that one, form the rectilinear portions. A special set of diagrams is necessary, to determine the shape and proportions of the various parts of this framepiece. The method here to be followed is, in the first place, to draw the joints, by which the vertical portions of the steps are attached to the framepiece. These can easily be obtained by squaring them up from Figs. 5 to Fig. 4, in which last are the horizontal division lines, corresponding to Fig. 1. It will be observed, that the joints referred to are bevelled off, so as not to be apparent externally. The faces on the framepiece are seen on Fig. 4, at the parts, B, C, and the method of obtaining them is sufficiently indicated by the dotted lines. The framepiece has a certain regular depth throughout, and is cut on the upper side, to suit the form of the steps, and below, to the curvilinear outline, $a^1\,b^1\,c^1\,d^1\,e^1\,f^1$, which is nothing but the combination of a helix with a couple of straight lines. These straight lines $a^1\,b^1$, and $e^1\,f^1$ are of course parallel to the curve passing through the edges of the steps, 1, 3, 5, 13, 16, 19. The curved part, $b^1\,c^1$, which corresponds to the anterior face, $b^2\,c^2$, of the framepiece, is drawn in precisely the same manner as a common cylindrical helix. It is the same with the part, $d^1\,e^1$, which corrresponds to the interior portion, $d^2\,e^2$. If, in order to better indicate the space occupied by the framepiece, it is wished to construct an end view of it, this may be done, as in Fig. 6, from the data furnished by Figs. 4 and 5.

In Figs. 7, 8, and 9, we have given, on a larger scale, the different views of the curved part, C, of the framepiece, so as to show its construction more plainly, as well as the form of the joint connecting the three parts of the framepiece together. Each of these figures is inscribed in a rectangle, indicated by dotted lines, and representing the rectangular parallelopiped, in which the piece may be said to be contained. To strengthen the junction of this piece with the portions, B and D, they are connected by iron straps or binding-pieces let in, or by bolts passing through, the entire thickness of the wood.

Figs. 10 and 11 represent in elevation and plan, the details of the landing-stage, which forms the top step of one flight of the stairs, and is on a level with the upper floor. It is with this piece that the upper portion, D, of the framepiece is connected, by a joint similar to that uniting the other portions. Fig. 12, is a section, through the centre of this step-piece, through the line, 1, 2, in the plan; whilst Fig. 13, is another section, through the line, 3, 4; and Fig. 14, a third, through the line, 5, 6. The form, dimensions, and joint of this piece, are all fully indicated in this series of figures.

The shaft of the staircase, or the open space left in the centre of the cage, is partially occupied by a balustrade, formed by a number of rods of iron or wood, attached at their lower extremities to the framepiece, as shown in Fig. 15, and united above by a flat bar of iron, surmounted by a hand-rail, G, of polished furniture-wood, of the form given in Plate 3, Fig. 7. The position of the rods as given in the plan, Fig. 1, is sufficient for the determination of their vertical projection or elevation.

THE INTERSECTION OF SURFACES.

APPLICATION TO STOPCOCKS.

PLATE 17.

179. We have already had before us several examples of intersections of surfaces, as in pipes and boilers, and we shall now proceed to the consideration of some others. Such intersections are to be found in the construction of stopcocks; for which reason we take one of the most common of these contrivances as an illustration.

A stopcock is a mechanical arrangement, the function of which is to establish, or interrupt, at pleasure the passage of gases or liquids through pipes. It consists of two distinct parts, one called the cock, and the other, adjusted and moveable in the first, termed the key or plug.

Stopcocks are generally made of brass, composition-metal, or cast-iron, and the cock is formed with or without flanges, for attachement to vessels or piping. The key is generally conical, so as to fit better in its seat. The degree of taper given to the key varies from 1 inch in 12, to 1 in 7 on the radius, according to their size and other circumstances; also in accordance with opinions of the different constructors. We have shown in dotted lines, Fig. 4, various degrees of taper, to be adopted according as greater tightness or greater facility of movement is required. The part of the cock which receives the key is, of course, turned out to a corresponding conical surface. This portion of the cock is connected to the tubular portions by shoulders, of a slightly elliptical contour. A stopcock of this description is represented, in elevation and plan, in Figs. 1 and 2.

In these figures will be seen the conical part, A, which embraces the key, the cylindrical portions, B, united to the former by the shoulders, D, and terminated by the flanges, C.

The conical key, E, adjusted in the cock, is surmounted by a handle, F, by means of which it is turned. The key is retained in the cock by a nut, G, working on a screw, formed on the lower projecting end of the key. Fig. 3, represents an end view of this cock, and Fig. 4, is a vertical projection of the key alone. Fig. 5, is a view of the key, looking on the lower end. Fig. 6, is a vertical section through the line, 1, 2, in Fig. 2; and Fig. 7 is a horizontal section through the line, 3, 4, Fig. 1.

It will be easy to see, from these various views, that, in order to represent the stopcock with exactitude, it is necessary to find, the projection of the intersection of the elliptical shoulder, D, of the round pipe, B, with the external conical surface of the central part, A, of the cock. This intersection is indicated in the vertical projection, Fig. 1, by the curved dotted line, D D D; but in small cocks such as the one here shown, the sharp outline of this intersection is usually taken away by rounding the one surface gently into the other, as shown by the shading in these figures.

Another similar intersection is formed by the junction of the cylindrical surface of the cross-handle, F, at right angles, with the cylindrical part at the top of the key, E. This curved line, for the same reason as in the first instance, is also indicated by dotted lines, F F F, in Figs. 1 and 6, the first having the handle, F, turned at an angle as regards the plane of projection, and in the latter case laying parallel to it, and consequently requiring two different cases, or examples of projection. We have, also to determine the intersection with the external surface of the key of the rectangular opening, H, provided for the passage of fluids; and also the intersection of a prism with a sphere, which occurs in the shape of the nut, G, which secures the key in its place. The various operations here called for are indicated on the figures, which we shall proceed to explain.

Figs. 8 and 9, show the geometrical construction of the intersection of the horizontal cylinder, F^1, of the handle, with the vertical cylinder, E^1, of the key. The curve is obviously obtained according to the method already described in reference to Figs. 1 and 2, or 4 and 5, Plate 14. We have, however, repeated the operations, the examples affording a variation from that previously given, and though round cylinders are here given instead of the elliptical ones shown in the key, Figs. 1, 3, and 6, yet still the construction and method would be the same in both cases, only substituting the one form of cylinder for the other.

180. When the horizontal cylinder, F^1, Figs. 10 and 11, becomes inclined to the vertical plane, its curve of intersection with the vertical cylinder, E^1, assumes a different appearance as projected in this plane. Its construction, however, is precisely the same, as follows:—To obtain any point in the curve, we proceed just as in the preceding example, drawing the vertical plane, $d^1\ e^1$, parallel to the axes of the cylinders, this plane cutting the vertical cylinder through the lines, *d f* and *e g*. This same plane cuts the horizontal cylinder, as projected in plan, at $d^1\ e^1$, whence the vertical projection is obtained, after drawing the semi-cylindrical end view, as in Fig. 9. The distance, $i\ i^1$, being set off on $h\ h^1$, the horizontal line, *d e*, drawn through the point, h^1, represents the intersection of the plane with the horizontal cylinder, $h\ h^1$, being, of course, measured from its axis. It will be seen that the line, *d e*, is cut by the vertical lines, *d f* and *e g*, in the points, *d*, *e*, which lie in the curve sought; and the same construction will apply to every other point in the curve, *d b e c*.

181. Figs. 12 and 13 represent the intersection of an elliptical cylinder with a right cone of circular base, corresponding, say, to the external conical surface, A^1, of the cock, at its junction with the elliptical shoulders as at D^1. Fig. 13 is the plan, as looking on the cock from below, showing the horizontal projection of the intersecting curves.

The solution of the problem requiring the determination of these curves consists in applying a method already given—namely, in taking any horizontal plane which cuts the cone, so as to present a circular section on the one hand, and the cylinder in two straight lines on the other—the points of intersection of these straight lines with the circle representing the section of the cone. Thus, by drawing the plane, *c d*, Fig. 12, we obtain a circle of the diameter, *c d*, which is projected horizontally, as with the centre, o^1, Fig. 13; we have also two generatrices of the cylinder, both projected in the vertical plane in the line, *a b*, and in the horizontal plane in the two lines, a^1, b^1. Having drawn the semi-base of the cylinder, *d e*, as at $d^1 f e^1$, to the left of Fig. 12, and having taken the distance, *f g*, and set it off, in Fig. 13, on each side of the axis, as from g^1 to a^1, we thereby obtain the two generatrices, $a^1\ b^1$, $a^r\ b^1$, which cut the circle of the diameter, $c^1\ d^1$, in the four points, h^1, i^1, which are squared across to, and projected in, the vertical plane in the points, *h*, *i*. In the same manner we obtain any other points, as *m*, *n*; *k l* being the plane taken for this purpose. The extreme points of the curves are obtained in a very simple and obvious manner, *p*, *q*, and *s*, *r*, being the points of intersection of the extreme generatrices, or the outlines of the cone and the extremes of the longer axis of the ellipse. With regard to the points, *t*, *u*, which form the apices of the two curves, their position may be obtained from the diagram, to the left of Fig. 12, by drawing from the point, S, which would be the apex of the entire cone, a tangent, S *t*, to the base, $d^1 f e^1$, of the cylinder, and projecting the point of contact, *t*, in the line, *x y*, representing a plane cutting the cone, which must be projected in the horizontal plane. Then, making $g^1\ x^1$, Fig. 13, equal to *t r*, Fig. 12, and drawing horizontals through x^1, their intersections, $t^1\ u^1$, with the circular section of the cone, will be the points sought, which are accordingly squared over to Fig. 12. The operations just described are similar, it will be observed, to those employed in obtaining the intersection of two cylinders.

If, in the case of the cone and cylinder, the latter had been one of circular instead of elliptical base, as is frequently the case, still the construction, as a little consideration will show, must be precisely the same, and the resulting curves would be analogous—that is, when the diameter of the cylinder is less than that of the cone at the part where it meets the lowest generatrix of the cylinder; the curves, however, assume a different appearance when the diameter of the cylinder exceeds this, as is shown in Figs. 14 and 15. In this case the intersections are represented by the curves, *s t r* and *p u q*; the method of obtaining these is fully indicated on the diagrams.

182. The opening or slot, H, cut through the key of the stopcock, is generally rectangular, rather than circular, or similar to the tubular portions of the cock. The object of adopting this shape is to make the key as small as possible, and yet afford the required extent of passage. This rectangular opening gives rise, in Fig. 6, to the intersectional curves, *a b*, *c d*, which are portions of the

hyperbola, resulting from the section made by a plane, cutting the cone parallel to its axis. The operations whereby they are determined are indicated in Figs. 16, 17, and 18.

To render the character of the curve more apparent, we have, in these figures, supposed the generatrices of the cone to make a much greater angle with the axis than in any of the former figures. The line, *a b*, represents the vertical plane in which the curve of intersection lies. It is evident that, if we delineate a series of horizontal planes, as *c d*, *e f*, *g h*, *i k*, Fig. 16, we shall obtain a corresponding series of circles in the horizontal projection, these circles cutting the plane, *a b*, in the points, l^1, m^1, n^1, p^1, &c. These points are squared over to the vertical projection, Fig. 16, giving the points, *l*, *m*, *n*, *p;* and the apex, *o*, of the curve is obtained, by drawing in the plan, Fig. 18, with the centre, S, a circle tangent to the plane, *a b*, and then projecting this on the vertical plane, Fig. 16, as shown. From these diagrams, it is easy to see that the opening, H, will be partly visible when the key is seen from below, as in Fig. 25.

183. Figs. 19 and 20 represent the vertical and horizontal projections of the nut, G, which secures and adjusts the key in its socket, to a larger scale than in Figs. 1 and 3. This nut, G^1, in these figures, is hexagonal, being terminated by a portion of a sphere, the centre of which lies in the axis of the prism. Each of the facets of the prism cuts the surface of the sphere, so as to present at their intersection portions of equal circles, which should be determined in lateral projection. The diameter of the sphere is generally three or four times that of the circle circumscribing the nut, but, to render the curves more distinct, we have adopted a smaller proportion in the case under examination. The sphere, *y*, is represented by two circles of the radius, *o* A; and the nut by an hexagonal prism, the axis of which passes through the centre of the sphere. The anterior facet, a^1 b^1, of this nut, cuts the sphere, so as to show a circle of the diameter, c^1 d^1. This circle, projected vertically on Fig. 19, cuts the straight lines, *a e* and *b f*, of the prism, in the points, *a* and *b;* and the portion of the circle comprised between these two points, consequently, represents the intersection of this facet with the sphere. The other two facets, a^1 g^1 and b^1 h^1, which are inclined to the vertical plane, also cut the sphere, so as to produce, at their intersection with the surface, arcs of equal radii with that of the facet, *a b*. From their inclination, these arcs become slightly elliptical, being comprised, on the one hand, between the points, *a* and *g*, and *b*, *h*, on the other. The summits of these ellipses are obtained by drawing horizontal lines tangential to the arc, *a b*, and cutting it in the points, *k*, *l*, by perpendiculars drawn through the middle of the lateral facets. In practice, it is quite sufficient to describe circular arcs, passing through the points, *g*, *k*, *a*, and *b*, *l*, *h*.

We have already seen, in reference to Plate 14, that the intersection of a right cylinder with a sphere, through the centre of which its axis passes, gives a circle projected laterally as a straight line. Thus, the opening, o^1, which passes through the nut, being cylindrical, produces, by its intersection with the sphere, a circle of the diameter, m^1 n^1, in the plan, projected vertically in the straight line, *m n*.

Fig. 21 indicates the analogous operations required to determine the same intersections when the nut, G, is seen with one of the angles in the centre, and only two facets visible, as represented in Fig. 3.

The elliptic curve, b^2 l^2 h^2, corresponding to the one, *b l h*, must obviously be comprised between the same two horizontal lines passing through these points, and an arc is drawn through them as before.

We may here observe, that the proficient draughtsman will, doubtless, deem it unnecessary, except in extraordinary cases, to enter into such minute details of construction for the various intersectional curves as those we have discussed, being guided simply by his own judgment, and the appearance presented by different experimental proportions. Younger draughtsmen, however, will find that some practice in obtaining the exact representation of the various curves, according to the modes here given and rules laid down, will be of great advantage to them—enabling them, from possessing a thorough theoretical knowledge of the relations of the various forms of solids to each other, to approach much nearer truth, when, in process of time, they relinquish the aid of such constructive guides.

CHAPTER VII.

THE STUDY AND CONSTRUCTION OF TOOTHED GEAR.

184. Toothed gear is a mechanical expedient, universally employed for the transmission of motion. It is met with of all proportions, from the minute movements of the watch, to the gigantic fittings of manufacturing workshops. Toothed gear is generally constructed with a view to the following principle of action—that the lateral acting-surfaces develop the same arc during the same duration of contact, whilst their angular velocities vary inversely as their diameters. By the angular velocity of any body, turning about a centre, is meant the angle passed through by the body in a unit of time; whilst the real or linear velocity of any point is the space passed through by this point, whether the direction of motion be rectilinear or circular. Thus, various points on a crank, taken at different distances from the centre of the shaft, have all the same angular velocity, whilst their actual velocity differs considerably, because of their respective distances from the centre. It is the same with a pendulum, which vibrates through an angle, or has an angular motion about its centre of suspension. The angular velocity of a body is greater, as the angle passed through in the same time is greater. Two points may have the same angular velocity, although the space passed through by each may be very different. Thus, all the points in the pendulum are affected with an equal angular motion, whilst their actual velocities, or the course traversed by each, varies as the distance from the centre of motion.

This description of gear consists of a series of projections, or teeth, regularly arranged on straight, cylindrical, or conical surfaces, termed *webs*, and disposed so as to act on each other during a limited time.

In order, however, that the gearing action may take place in a regular and even manner, it is indispensably necessary that the surfaces of the teeth should bear upon each other tangentially, throughout the entire duration of their contact; and for this purpose, far from being arbitrarily designed, their form should be determined with the utmost geometrical exactitude, for on their figure entirely depends their accurate and easy working. It is, therefore, obviously incumbent on the student to give particular attention to the delineation of these teeth.

Designs and calculations as to the teeth of wheels have reference to what is called the *pitch surface*, as will be seen more clearly hereafter. This is an ideal smooth surface, half-way between the bottoms of the grooves and the summits of the teeth. The figures of the teeth are regulated by the principle, that the teeth of a pair of wheels shall give the same velocity, by their sliding contact, which the ideal smooth pitch surfaces would give by their rolling contact.

The curves generally adopted in practice for the outline of teeth, are the involute, cycloid, and epicycloid.

It will be useful to investigate the nature and construction of these curves, both on account of their application to the teeth of wheels, and also because of their employment in several other mechanical contrivances.*

INVOLUTE, CYCLOID, AND EPICYCLOID.

PLATES 18 AND 19.

INVOLUTE.

FIGURE 1.—PLATE 18.

185. When a thread is unwound from the circumference of a circle, and is kept uniformly extended, its extremity will describe the curve known as the involute.

To make this clearer, we may wind a thread a few times round the edge of a fixed circular disc, and attach a pencil to its free extremity. Now, if we cause the pencil to make a line on a sheet of paper as we unwind the thread, which must be kept on the stretch, it will describe the curve in question.

This definition serves as a basis for obtaining the geometrical delineation of the involute. Let A B C be the given circle of the radius, A O, and A the extremity of the thread wound upon it. Starting from the point, A, mark off, at equal distances apart, several points, as *a*, *b*, *c*, *d*, so near to each other, that the intervening arcs may be taken for straight lines without sensible error. Through each of these points draw tangents to the circle, or perpendiculars to the corresponding radii; and on these tangents set off distances, equal to the rectifications of the respective arcs, A *a*, A *b*, A *c*, &c.; by which means are obtained the points, a^1, b^1, c^1, &c., and the curve passing through these points is a portion of the involute. By continuing the development or unwinding of the thread, the curve may be extended to a series of convolutions, increasing more and more in radius, and becoming a species of spiral. After one complete evolution of the circumference, the shortest distance between two consecutive convolutions is always the same, and equal to the development or rectification of the circumference of the generating circle, which forms the nucleus of the curve.

The points, *a*, *b*, *c*, being taken at equal distances apart on the circumference, the tangents are respectively double, triple, &c., that of the first, A *a*; and if, as we directed, these points are sufficiently near to each other, the curve may be drawn, with closely approximate accuracy, by describing a succession of arcs, having these tangents for radii. Thus, with the point, *a*, as a centre, and radius, $a\ a^1$, the first arc, A a^1, is drawn; and with the centre, *b*, and radius, $b\ b^1$, the second arc, $a^1\ b^1$, in like manner; and similarly with the rest.

We shall show the application of the involute to toothed gear, worm wheels, and also for *cams* and *eccentrics*.

CYCLOID.

FIGURE 2.—PLATE 18.

186. When a circular disc is rolled upon a plane surface in a rectilinear direction, any point in the circumference of this disc generates the curve called the *cycloid*. Thus, any point taken on the outside of a locomotive wheel in motion, describes as many repetitions of the curve as the wheel makes revolutions.

In order that the curve may be perfect and true throughout, it is necessary that the motion should take place without any sliding upon the plane; in other words, the length of the straight line forming the path of the disc should be equal to the portion of the circumference which, during motion, has been applied to, or in contact with, the plane throughout that length.

It is required to delineate the cycloid generated by the point, A, of a circle of the given radius, A O, and rolling upon a given straight line, B C.

There are several methods of solving this problem.

1*st Solution.*—Set off on the circumference of the flat-tinted circle, A D, starting from the point, A, a number of equal distances, A a^1, $a^1\ b^1$, &c., so small that the arcs so divided may be taken as straight lines. Set off the same distances, A *a*, *a b*, &c., a like number of times from A, along the straight line, A C, and at the points, *a*, *b*, *c*, *d*, erect perpendiculars, cutting the line, O O^1, generated by the centre of the rolling circle, and parallel to the given straight line, B C. In this way are obtained the points of intersection, *o*, o^1, o^2, which are the centres of the circle when in the positions corresponding to the points of contact, *a*, *b*, *c*, *d*. With each of the former points as centres, describe portions of circles, $a\ a^2$, $b\ b^1$, $c\ c^2$ on each of these arcs tangent to the line, B C, and from the latter points of contact successively set off the lengths of the arcs, A a^1, A b^1, A c^1, &c., from *a* to a^2, *b* to b^2, and from *c* to c^2, and so on throughout. The curve, A $a^2\ b^2\ c^2\ d^2\ e^2$ passing through the points thus obtained, is the cycloid required.

2*d Solution.*—The points in this curve may also be obtained by drawing horizontal lines through the points of division, $a^1\ b^1\ c^1$, of the original circle, and then intersecting these by the arcs drawn with the respective centres, *o*, o^1, o^2.

3*d Solution.*—In place of drawing arcs of circles with the various centres, as indicated on the right-hand side of Fig. 2, the curve may be obtained by setting off successively from the vertical, A O, on the horizontals, as before drawn, distances equal to those respectively contained between the original circle and the perpendiculars through the several corresponding positions of the centre; thus, the distances, $e\ e^1$, $f\ f^1$, $g\ g^1$, $h\ h^1$, &c., are set off from 1 to b^2, 2 to c^2, 3 to d^2, &c.

To avoid confusion, we have constructed the diagram referring to this last solution to the left-hand side of Fig. 2, which shows a portion of a second cycloid similar to the first.

When the generating circle has made half a revolution, the summit of the curve is reached, as at D^1, the point corresponding to the diameter, A D. The length, A C, of the given straight line, is obviously equal to the rectification of the semi-circumference of the generating circle, whose radius is A O.

By continuing the construction, a complete curve may be obtained, having equal and symmetrical portions on either side of the vertical, C D, and having for its base a line double the length of A C, and consequently equal to the rectification of the entire circumference of the generating circle.

The cycloid is the curve generally given to the teeth of wheel gear and endless screws.

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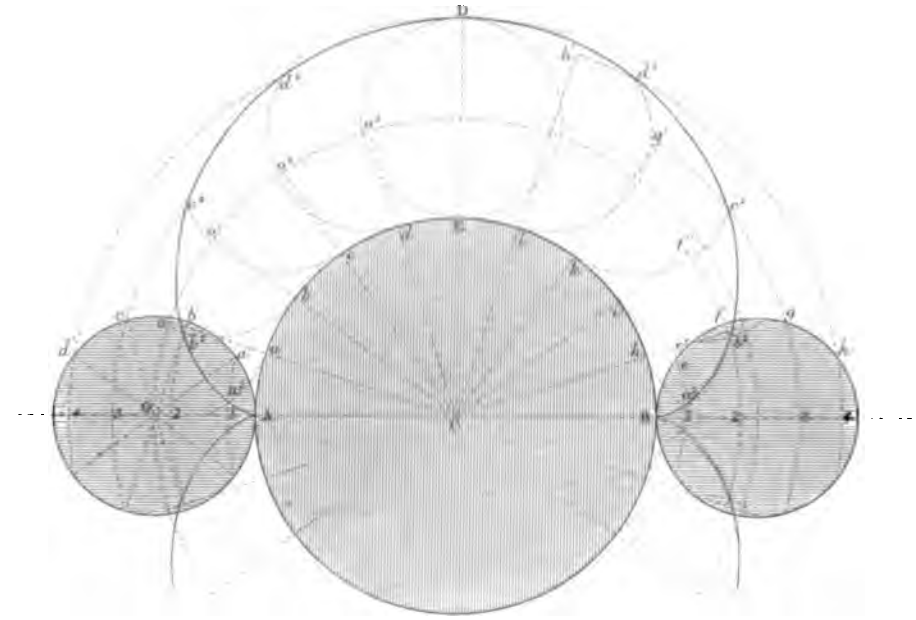
* The mathematical and mechanical student cannot have a better text-book in his hands for this and other branches of applied mechanics, than Professor Macquorn Rankine's Manual.

SPUR

Fig. 1.



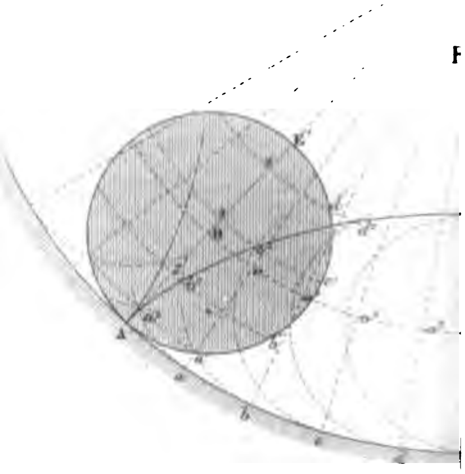


Fig. 4.

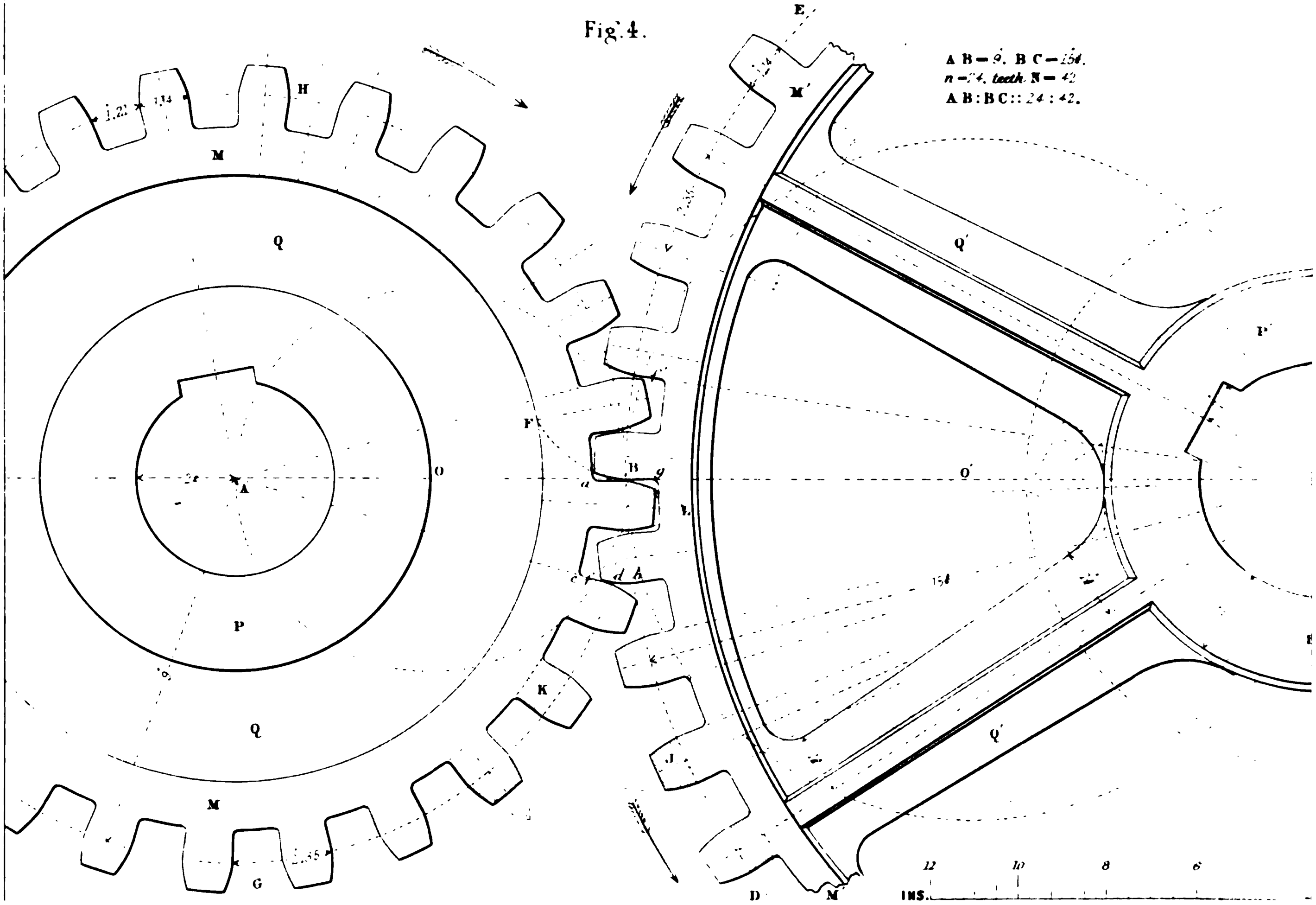


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PLATE 19.

EARING.

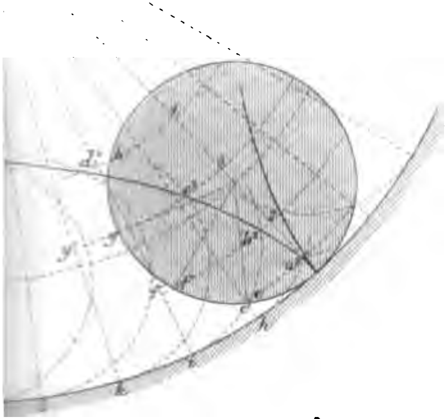


Fig. 3.

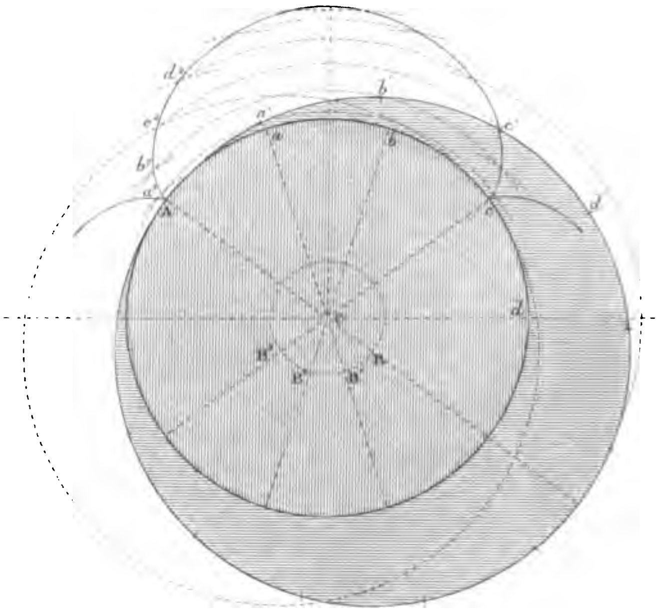
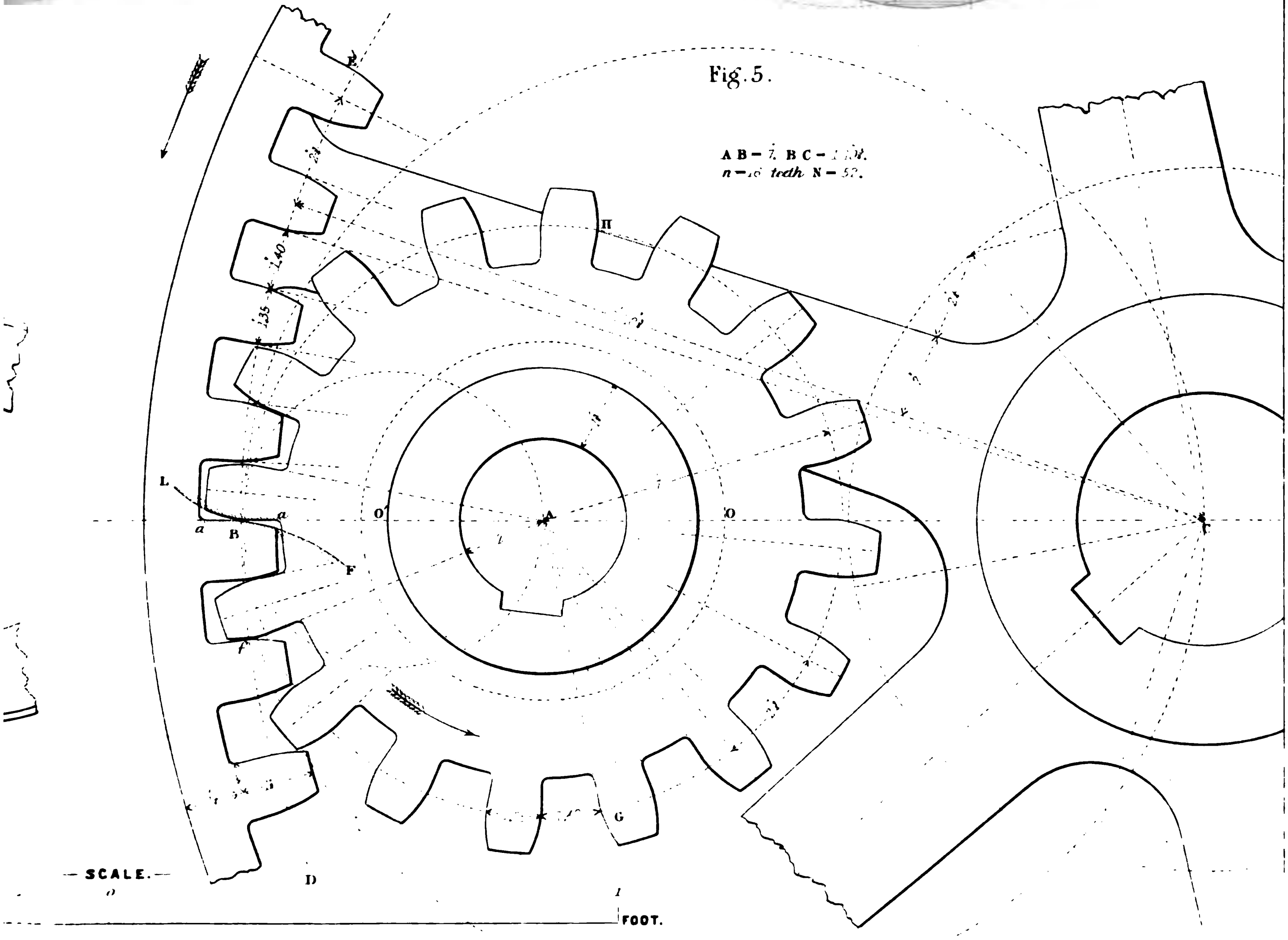
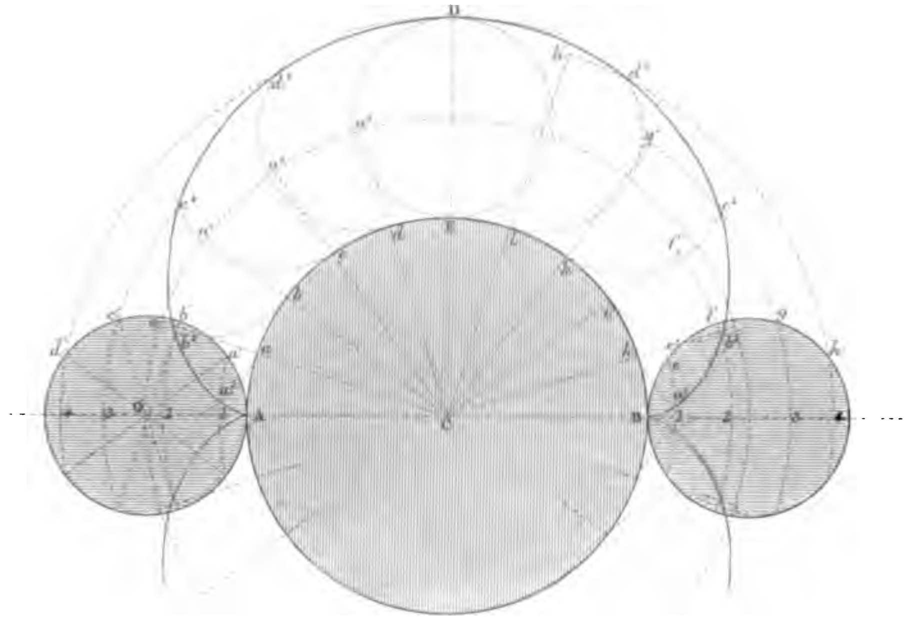


Fig. 5.



SPUR

Fig. 1.



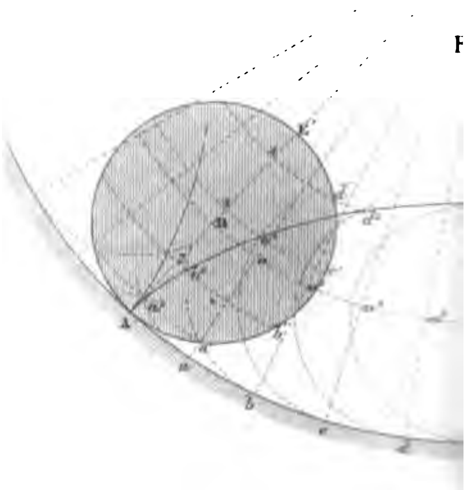
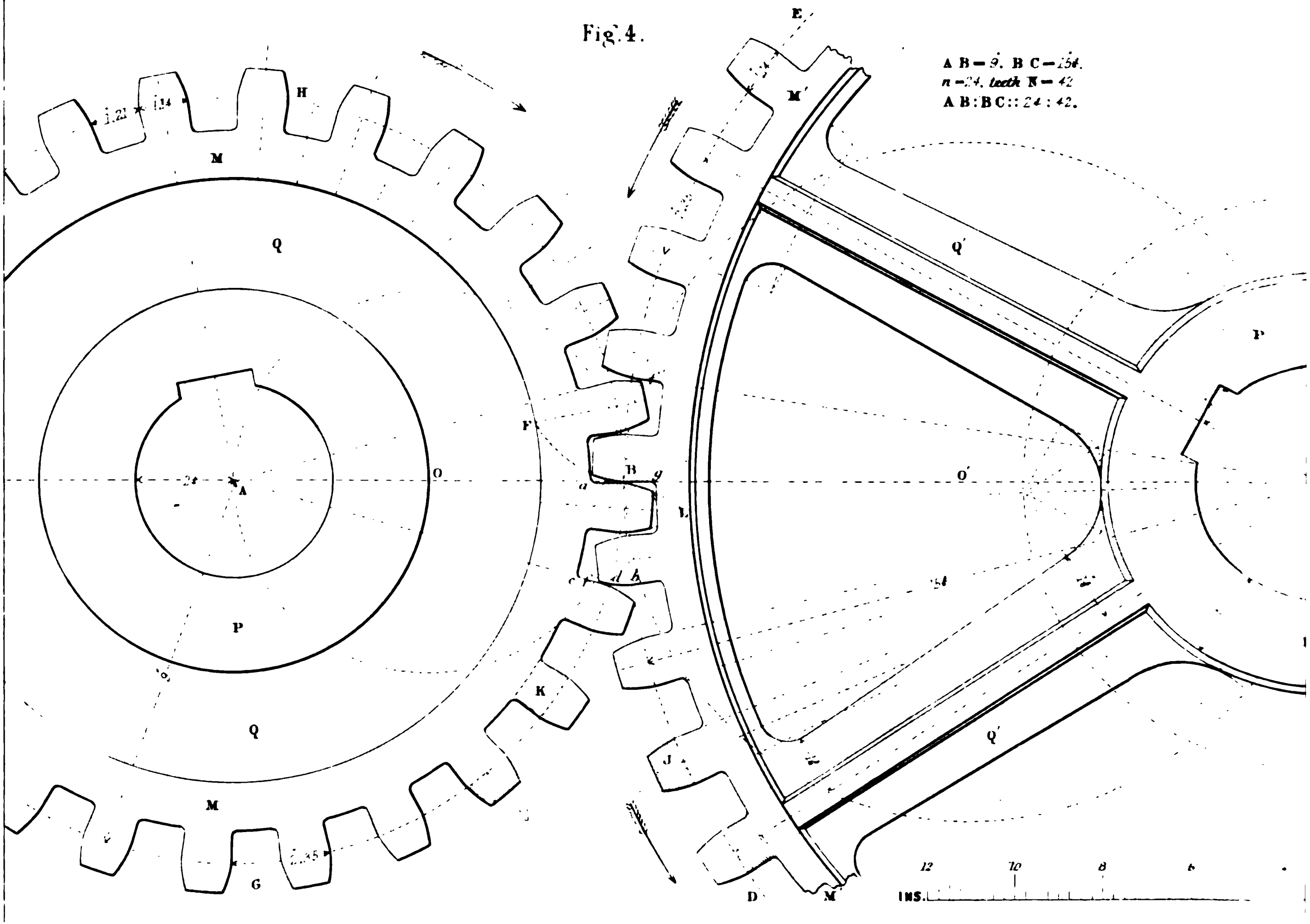


Fig. 4.



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EARING.

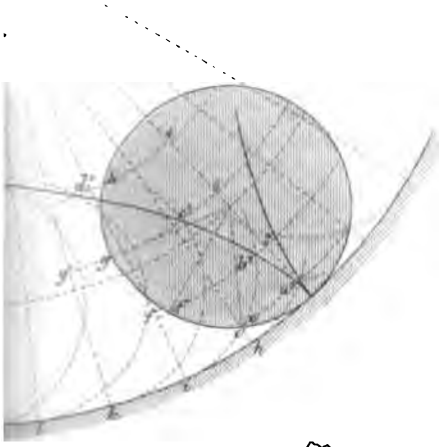


Fig. 3.

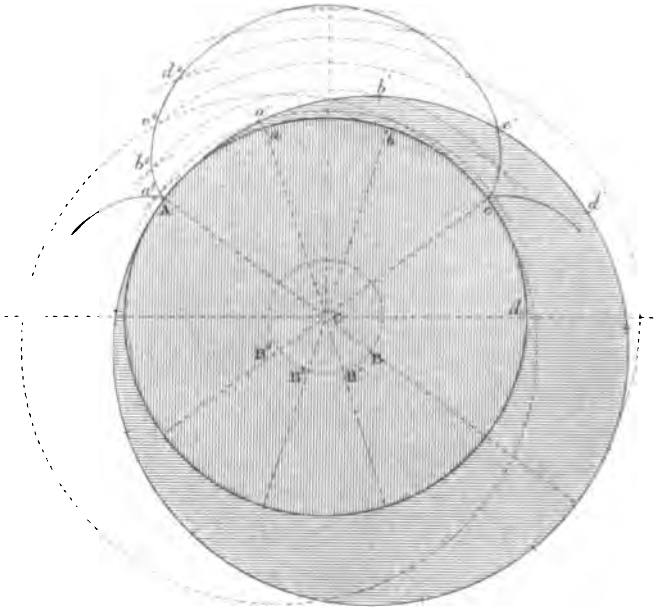
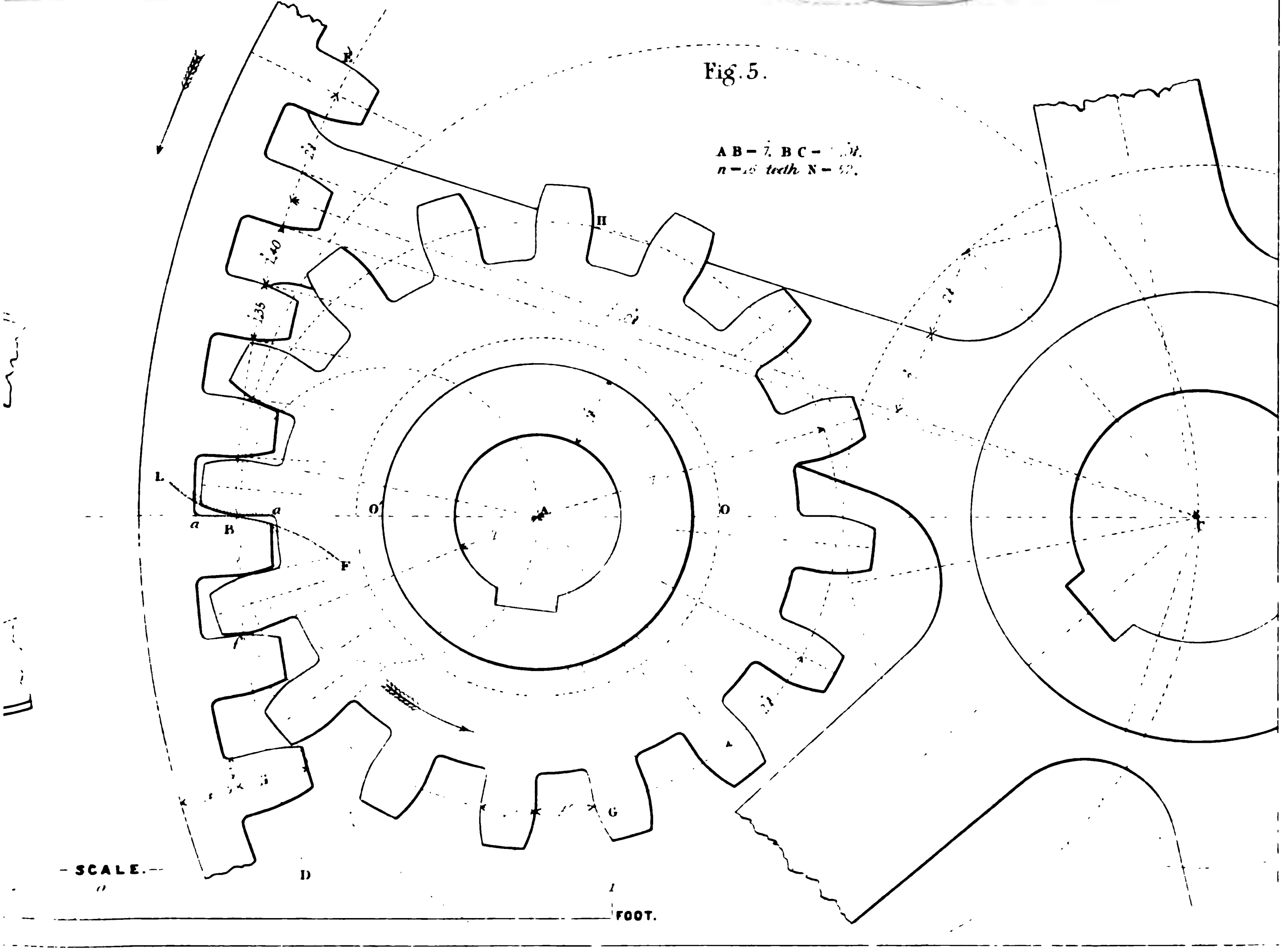


Fig. 5.



EXTERNAL EPICYCLOID.

FIGURE 1.—PLATE 19.

187. The epicycloid only differs from the cycloid, in that the generating circle, instead of rolling along in a straight line, does so around a second circle, which is fixed. When the two circles are in the same plane, the point taken generates a *right* or *cylindrical* epicycloid; when the two circles are situate in different planes, but maintaining a uniform angle to each other, the generated curve becomes a *spheric* epicycloid; in this case the generating circle is supposed to revolve about a fixed centre, at the same time rolling along the circumference of the stationary circle.

1st Solution.—For the delineation of the right epicycloid, the modes of construction to be adopted are analogous to that given for the cycloid. Thus, let A O be the radius of the generating circle, and A C the radius of the fixed circle; divide the former into a number of equal parts in the points, a^1, b^1, c^1, d^1, &c., and on the latter divide off as many arcs equal to the arcs of the former, starting from A, as at *a*, *b*, *c*, *d*, &c. Through these latter points of division draw radii, C *a*, C *b*, C *c*, and prolong them so as to cut a circle, the radius of which is C O; this circle being generated by the centre of the moving one during its rotation about the stationary one; in this way are obtained the points, *o*, o^1, o^2, o^3, which are the successive positions of the centre of the generating circle, as during its rotation it is successively in contact at the points, *a*, *b*, *c*, *d*, of the fixed circle. Then, with these points, *o*, o^1, o^2, &c., as centres, describe the several arcs of equal radii with the generating circle, making them severally equal to the corresponding arcs, A a^1, A b^1, A c^1, as from *a* to a^2, *b* to b^2, *c* to c^2, &c. The curve passing through the points, a^2, b^2, c^2, &c., is the epicyloid required.

2d Solution.—The points of this curve may also be determined by drawing, with the centre, C, arcs passing through the points of division, a^1, b^1, c^1, d^1, and cutting the arcs described with the various centres, *o*, o^1, o^2, o^3, in a^2, b^2, c^2, d^2, which are so many points in the epicycloid.

3d Solution.—The curve may also be delineated by transferring the distance between the points, *e*, *f*, *g*, &c., of the generating circle in its original position, and the radii, C *h*, C *i*, C *k*, produced, passing through the different points of contact on the stationary circle, measured upon the arcs described with the centre, C, to the same arcs, but so that the extremities of the whole may lie in the prolongation of the radius, C B. Thus the distances, *e* e^1, *f* f^1, *g* g^1, &c., are set off, from 1 to a^2, 2 to b^2, 3 to c^2, &c. The diagram referring to this construction forms the right-hand portion of Fig. 1.

When the generating circle has made an entire revolution, the curve obtained is a complete epicycloid, A D B, comprising two equal and symmetrical portions, one on each side of the line, C D, which is, in this instance, equal to two diameters of the moving circle.

EXTERNAL EPICYCLOID DESCRIBED BY A CIRCLE ROLLING ABOUT A FIXED CIRCLE INSIDE IT.

FIGURE 3.—PLATE 19.

188. For this diagram, which is analogous to the preceding one, the radii of the circles are given, C A being that of the fixed circle, and B A that of the moving one. Divide the first circle into any number of equal parts, in the points, *a*, *b*, *c*, *d*, &c., and divide off, on the larger circle of the radius, B A, a like number of arcs, equal to those on the other circle, as from A to a^1, a^1 to b^1, &c. Then, with the point C as a centre, and with the radius, B C, describe a circle, cutting the radii, C A, C *a*, C *b*, C *c*, produced through C, in the points, B^1, B^1, B^2, B^3, and with each of the last as centres, and with the radius, A B, describe arcs, which will be tangents to the fixed circle, at the different points of contact, *a*, *b*, *c*, in succession. Then, with the centre, C, describe arcs, passing successively through the points, a^1, b^1, c^1, d^1, on the moving circles, as in its first position. These last will cut the arcs tangential to the given circle, in the points, a^2, b^2, c^2, d^2, and the curve passing through these points is the epicycloid sought.

The other two methods given, of drawing the common epicycloid, are also applicable to this last case.

INTERNAL EPICYCLOID.

FIGURE 2.—PLATE 19.

189. The epicycloid is termed internal, when the generating circle rolls along the concave side of the circumference of a fixed circle.

Let C A be the radius of the fixed circle, and B A that of the generating circle. We commence, as before, by dividing the moving circle into a certain number of equal parts, and then dividing the fixed circle correspondingly, so that the arcs thus obtained in each may be equal. We then proceed as in the case of the external epicycloid, according to whichever of the three solutions we propose adopting, all being alike applicable. The operations are fully indicated in Fig. 2, and the same distinguishing letters are employed as in Fig. 1.

When the generating circle is equal to half of the fixed circle, the epicycloid generated by a point in the circumference is a straight line, equal to the diameter of the fixed circle. Thus, in Fig. 3, Plate 18, the epicycloid generated by the point, A, of the moving circle of the radius, A C, after a semi-revolution, coincides exactly with the diameter, A B.

If, with circles of the same proportions as those in Fig. 3, Plate 18, we take a point, D, outside the generating circle, but preserving a constant distance from it, the epicycloid generated by it will be the ellipse, D F E G, having for its transverse axis the line, D E, equal to the diameter, A B, of the fixed circle, augmented by twice the distance, D A, of the point, D, from its extremity; and for its conjugate axis, the line, G F, equal to twice the same distance, D A, alone. If it is wished to determine this curve according to its properties as an epicycloid, and without having recourse to the methods given in reference to Plate 5, and proper to the ellipse, it may be done by adding the distance, A D, to that of the radius, C A, in each successive position occupied by the generating circle during its rotation. If the generating point be taken inside the moving circle, the curve produced will also be an ellipse.

The epicycloid is the curve most employed for the form of the teeth, whether of external or internal, spur or bevel wheels.

Toothed gearing may be divided generally into two classes, namely, right, cylindrical, or "spur" wheels, and conical, angular, or "bevel" wheels. In the first are comprehended the action of a rack or pinion, that of a worm or tangent-screw with a worm wheel, and that of two wheels. We may remark, that in all these wheels the teeth are so formed and arranged as to act equally well whichever wheel of a couple be the driver, and in whatever direction the motion takes place.

THE DELINEATION OF A RACK AND PINION IN GEAR.

FIGURE 4.—PLATE 18.

190. A rack is a straight and rigid rod or bar, formed with teeth on one side, so as to take into or gear with the teeth of a right wheel, generally of a small diameter, and in this case termed a pinion. Such a rack is represented at A B in this figure.

In proceeding to construct this design, as well as for all kinds of toothed gear, it is necessary to have determined beforehand the thickness, *a b*, of the teeth, as this dimension varies according to the power or strain to be transmitted; and rules and tables, for this purpose, will be found at the end of the chapter.

When the rack and pinion are made of the same metal, the thickness of the teeth should be the same in both. The spaces or intervals between the teeth ought also to be equal in such cases. Theoretically speaking, the intervals should be equal to the thickness of the teeth; but in practice, they are made a little wider, to admit of freer action.

191. The *pitch* of the teeth comprises the width of the tooth and that of the interval. In a wheel this pitch is measured upon a circle of a given radius, termed the *primitive*, or *pitch*, circle, and in the rack on a straight line tangent to the pitch circle of the pinion, and also called the *primitive*, or *pitch*, *line*.

192. Let O C be the radius of the pitch circle of a pinion gearing with a rack, of which the pitch line is A B. We propose, in the first place, to determine the curve of the teeth of the pinion, so as to gear with and drive the rack, and we shall subsequently determine the curve of the teeth of the rack, enabling it to gear with and drive the pinion.

The operations consist in traversing the straight line, A C B, tangentially to the pitch circle, O C; during this movement the point, C, will generate an involute, C D, which may be drawn in the manner indicated in Fig. 1, a construction which is further repeated at a^1 d^1, &c., on one of the teeth of the pinion, Fig. 4.

This curve possesses this property, that if the teeth are formed to it, and the pinion be turned on its axis, the point of contact, C, will always be in the straight line, A B, traversing this line at precisely the same velocity as the pinion at that distance from the centre, that is, at the pitch centre; consequently, we divide this pitch circle into as many equal parts as there are to be teeth and intervals in the pinion, and at each of the points of division repeat the involute curve, C D, which will, of course, fulfil the same conditions at the various positions; then, each of these divisions rectified is set off on the pitch line, A B, of the rack, as many times as is necessary. For each tooth the curves are placed symmetrically with reference to the radius which passes through their centres, as indicated at O d^1, so that the pinion may act equally well when turning in one direction as in the other.

193. Since the teeth cannot have an indefinite length, they may be limited as far as is compatible with the following considerations:—The tooth of the wheel, which is the driver, should not relinquish contact with the one upon which it acts, until the tooth immediately succeeding it has taken up its original position, which, in the working of two wheels, corresponds to the line joining the centres, and in that of a pinion and rack, to the radius, O C, perpendicular to the pitch line, A B.

Thus, supposing the pinion to move in the direction indicated by the arrow, the tooth, E, which is acting on the tooth, H, of the rack, should continue to impel it until the following tooth, G, shall have taken its place, when it will itself have taken the place of the tooth, F, having made the tooth, H, of the rack traverse to I. It will be observed that the curved part of the tooth is in contact at the point, *c*, on the pitch line, A B; the tooth might be cut away at this point, but in practice, in order that the pinion teeth may act through a somewhat greater interval, and to avoid the play resulting from wear, they are truncated at a little beyond this point, *c*, a circle being described with the centre, O, cutting the curves of all the teeth at equal distances from the centre.

To allow of the passage of the curved portion of the teeth of the pinion, the rack must be grooved out, so as to present bearing surfaces, which are determined simply by the perpendiculars, $b^2 f$, $c\ d$, $b^1 g$, to the pitch line, A B, and passing through the points of division already set out on this line.

These perpendiculars, at the same time, form the sides or flanks of the rack teeth.

Rigorously speaking, the depth of the intervals on the rack should be limited by the straight line, *m n*, tangential to the external circle of the pinion; but, to prevent the friction of the teeth against the bottom, it is preferable to augment the depth of the hollows by a small quantity; joining the sides of the teeth with the bottom by small quadrants, which, avoiding sharp angles, gives greater strength to the teeth.

194. As in practice, toothed gear is constructed so as to drive, or be driven, indifferently, we require yet—to complete the design under consideration—to give to the teeth of the rack such curvature as is necessary to enable them to drive the pinion with which they are in gear in their turn, always fulfilling the conditions of a regular and uniform motion, both of the rack at its pitch line, and of the pinion at its pitch circle.

With a view to the determination of this curve, we may remark, that if, with the radius, O C, as a diameter, we describe the circle, O L C, *i*, and cause it to roll along the straight line, A B, the point of contact, C, will generate a cycloid, C K, which may be constructed according to the modes indicated in Fig. 2.

If the same circle is made to roll along the interior of the pitch circle, G C J, of the pinion, the same point, C, will generate a right epicycloid, coinciding with the radius, O C, as has been seen in reference to Fig. 3.

Then, if we give to the teeth of the rack the curve, C K, and to the flanks of the pinion teeth the straight line, C O, the arrangement will exactly fulfil the condition sought; that is to say, that, in impelling the pinion teeth from right to left, the curve, C K, of the rack teeth will constantly apply itself to the straight line, O C, being always tangential to it.

For example, suppose the curve, C K, to be traversed to the position, C^1 L, the radius, O C, will then be in the position, O L; then, if from the point, L, the straight line, L C, be drawn, the angle, O L C, will be a right angle; that is to say, the line, O L, will be perpendicular to L C, and, consequently, tangential to the curve, L C^1, in the point, L. If, therefore, the motion of the rack is regular and uniform, that of the pinion will be equally so. The same curve, C K, is drawn at each of the points of division of the pitch line of the rack, as was already done for the teeth of the pinion.

To find the proper length to give to the teeth, all that is necessary is to place, in the generating circle, O L C, a chord, L C, equal to twice C b^2, and through the point, L, thereby obtained, to draw a straight line, M N, parallel to A B. If, through all the points of division in the pitch circle of the pinion, are drawn radii converging in the centre, O, they will give the flanks of the teeth, as *i j*, *k l*, &c., which are limited by a circle described with the centre, O, and tangential to the straight line, M N; for the same reason as that assigned in the case of the rack, however, the spaces between the teeth are made a little deeper, and the sides of the teeth are joined to the bottoms by quarter circles, the circle in which the bottoms lie being described with a radius somewhat less than that of the circle last drawn.

As it would be a tedious process to repeat the operations for determining the curves in the case of each individual tooth, it is a convenient plan to cut a piece of card or thin wood to the curve, so as to form a pattern or template, by the application of which to each of the points of division, the sides of the teeth may be drawn, care being taken to make the two sides of each perfectly symmetrical with reference to the centre line of the tooth.

Even the labour of making a template or pattern is often dispensed with, and, in place of the curve, a simple circular arc is employed for the side of the tooth, the arc being of such a radius as to approximate as near the true curve as possible. With this view the arc should be tangential to the side of the tooth, and passing through the external corner. Thus, supposing it is wished to substitute an arc for the true curve of the rack teeth, such as *o r*, of the tooth, P, since this arc has to pass through the point, *r*, corresponding to L, and obtained by making r^1 *r* equal to L *q*, and to be a tangent at *o*, to the vertical, *o p*, draw the chord, *o r*, and bisect it by the perpendicular, *s t*, and its point of intersection, *s*, with the pitch line, A B, will be the centre of the required arc, and the sides of all the teeth may afterwards be drawn with the same radius, care being taken to keep the centres in the line, A B.

An analogous operation will give the proportions of the arc, substituting the curve of the pinion teeth.

THE GEARING OF A WORM WITH A WORM-WHEEL.

FIGURES 5 AND 6.—PLATE 18.

195. This system of gear is constructed on the same principles as that of a rack and pinion, which system requires that, in the first place, the worm and worm-wheel be supposed to be sectioned by a plane passing through the axis of the former, and at right angles to that of the latter. The representation of this section becomes analogous to the diagram, Fig. 4; that is to say, the pitch circle, G C J, of the worm-wheel being given, and also the straight pitch line, A B, of the worm tangential to this circle and parallel to the axis of the worm, the involute curve, C D, is sought for the teeth of the wheel, and the cycloid, C K, for those of the worm. The lengths of these curves are limited, as in the preceding example, and when the whole is complete, an outline will be produced similar to the sectioned portions of Fig. 6. It is in this manner that the gearing of the worm and worm-wheel is made to depend upon the same principles as that of a rack and pinion, and the same method may be employed in construction in determining the outline of the teeth, as we have shown.

To represent the worm and worm-wheel geometrically in external elevation, instead of a section of the teeth alone, it is necessary to know the diameter and pitch of the worm on the one hand, and the thickness of the worm-wheel, Fig. 5, on the other.

Let $M^1 A^1$ be the distance of the pitch line, A B, from the axis, M M, of the worm, and *a b* the width of the wheel. When the worm is single-threaded (Par. 174), the pitch of the helix is the same as that of the teeth, and, therefore, the thickness of a tooth, added to the width of an interval. In this case, each revolution of the worm turns the wheel to the extent of one tooth, and this is the arrangement represented in the figures. If the worm, however, is double or triple-threaded, its helical pitch will be correspondingly two or three times the pitch of the teeth; and in this case, each revolution will turn the wheel to the extent of two or three teeth.

The worm-wheel being of a certain thickness, and requiring to gear with the convolutions of the worm, must necessarily have its teeth inclined to correspond with the obliquity of the worm-thread. It is further to be observed, that the sides of the wheel-teeth being simply tangential to the worm-thread, contact cannot, rigorously speaking, take place in more than one point of each tooth and convolution. This point constantly changes with the motion, but always lies in the plane, $O^1 M^1$, of the section.

In delineating the convolutions of the worm-thread, helicies have to be drawn passing through the external corners, *d*, *e*, and internal corners, *f*, *g*. We have repeated these points to the left-hand side of Fig. 6, where the required operations are fully indicated, in connection with the projection, Fig. 5, and in accordance with the principles already explained (Par. 170). The corresponding points in the two Figs. 5 and 6, are distinguished by the same letters and numbers.

196. In order to represent an external elevation, of the teeth of the worm-wheel, it is requisite to develop (Par. 172), a portion of the cylindrical surface generated by the revolution of the pitch-line, A B, about the axis of the worm, and containing the portion, A *i k l m*, for example, of the helix, described by the central point of contact, A. To obtain this, make the line, $E^1 A^1$, Fig. 7, equal to the semi-circumference, A^1 *m* E^2, rectified. At the point, E^1, erect the perpendicular, $C^1 E^1$, and make it equal to C E, Fig. 6, or half the pitch, and join C^1, A^1, whereby will be obtained the actual inclination of the worm-thread. On each side of the point, *m*, on $E^1 A^1$, mark distances, $m\,a^1$ and $m\,b^1$, equal to $m^1\,a$ and $m^1\,b$, Fig. 5, and through these points draw parallels to $C^1 E^1$, and the portion, *p q*, of the enclosed line comprised within them, will serve to determine the width and inclination of the teeth of the worm-wheel. Through the points, *p*, *r*, draw *p t* and *r s* parallel to $E^1 A^1$, and mark off the distances, *t s* and *s q*, which are equal on the pitch circle of the wheel, Fig. 6, and from *s* to *t* and *q*, after having drawn through the points, *s*, but only in faint pencil or dotted lines, the contours of the teeth as sectioned at F and G^1. It is then sufficient to repeat these outlines through the points, *t* and *q*, limiting their length by the same internal and external circles.

Finally, the edge view of the worm-wheel, Fig. 5, being the lateral projection of the teeth, is determined by squaring across the points, *u*, *v*, *x*, to u^1, v^1, x^1, which give the interiors of the teeth; and the points, u^2, v^2, x^2, being squared over to u^3, v^3, x^3, give their exterior edges.

Worm-wheels are sometimes constructed with the form of the teeth concave, and concentric with the axis of the worm, with the view of their being in contact with the convolutions of the worm-thread throughout a certain extent, in place of only touching at single points.

This arrangement, which requires a particular operation for its construction, is generally adopted when great precision is required, and when it is wished to avoid, as much as possible, any play between the teeth and the worm-thread during the transmission of motion.

CYLINDRICAL OR SPUR GEARING.

PLATE 19.

THE EXTERNAL DELINEATION OF TWO SPUR-WHEELS IN GEAR.

FIGURE 4.

197. Spur-toothed wheels are such as have their teeth parallel, and lying upon a cylindrical surface or web. When a couple of such wheels are of unequal size, the smaller one is generally called a pinion, and the larger one a spur-wheel. Two wheels, which are intended to gear together, cannot work satisfactorily in concert, unless their radii or pitch circles are exactly proportional to the number of teeth contained by each. Consequently, in order to construct designs for couples of toothed wheels, it is necessary to know—the number of teeth of each, and the radius of one or other of them; or the radii or diameters of both, and the number of teeth of one; or the distance between their centres, and the radius or number of teeth of one; or finally, the number of revolutions of each in the same time, and the distance between their centres, or the radius and number of teeth of one of them.

If we assume the following data, $A B = 72$-eighths of an inch, and $B C = 126$, these being the respective radii of the pitch circles of two right wheels, and $n = 24$, the number of teeth of the pinion—we at once ascertain the number of teeth, N, of the spur-wheel, by the following proportional formula:—

$$A B : B C : : n : N, \text{ or } 72 : 126 : : 24 : N = 42.$$

Then describe the pitch circles of the radii, A B and B C, and divide them respectively into 24 and 42 equal parts, thereby obtaining the pitch, or the central point of each tooth, which is exactly the same on both pitch circles. Next subdivide the pitch into four equal parts, to obtain the centres of the intervals, and at the same time, the points through which the flanks of the teeth pass. If, with the line, A B, on the line of the centres, A C, as a diameter, we describe a circle, the centre of which is at O, and suppose this circle to roll round the pitch circle, D B E, of the spur-wheel, the point, B, at present in contact will generate an epicycloid, B F, as shown previously in reference to Fig. 1; and this curve is the one proper to give to the side of the teeth of the spur-wheel, and it is accordingly repeated symmetrically on each side of the several teeth as shown in the diagram. If, further, we suppose the same circle of the radius, O B, to roll round the interior of the pitch circle, G B H, of the pinion, we shall obtain the internal epicycloid (sometimes called hypocycloid), B O, as already explained in reference to Fig. 3, Plate 18, and a portion, B *a*, of this, forms the flank of the pinion tooth.

Supposing the curve, B F, to form a part of the wheel, turning about the centre, C, in the direction of the arrow, I, it will fulfil the condition of impelling the flank, B *a*, which forms part of the pinion, so as to turn about the centre, A, in the like uniformity. In other words, the space passed through by the point, B, on the pitch circle, G B H, will be exactly the same as that passed through by the same point, B, considered as belonging to the spur-wheel, on the pitch circle, E B D.

198. In proceeding to determine the length to be given to the tooth, it is first to be observed that the epicycloidal curve should be sufficiently long to bear upon the side of the tooth, through an extent of circumferential movement equal to the length of the pitch from the line of centres; that is to say, until the flank, at present in the position, B *a*, shall have arrived to the position *c d*. At this moment, it will be observed that the curve, B F, has reached the position, *b f*, and is in contact with the flank of the pinion tooth in the point, *f*, on the circumference of the generating circle of the radius, A O. It will thus be obvious that the point, *f*, may be obtained by simply cutting off on the generating circle, an arc, B *f*, equal to the length of the pitch. Through this point, *f*, describe a circle having C for its centre, and it will cut all the teeth at the proper length.

The depth of the intervals is theoretically determined by describing, with the centre, A, a circle tangential to the first; but in practice, as it is necessary to leave a slight space between the ends of the teeth and the bottoms of the intervals into which these work, the circle in question is described with a somewhat smaller radius, as A *a*.

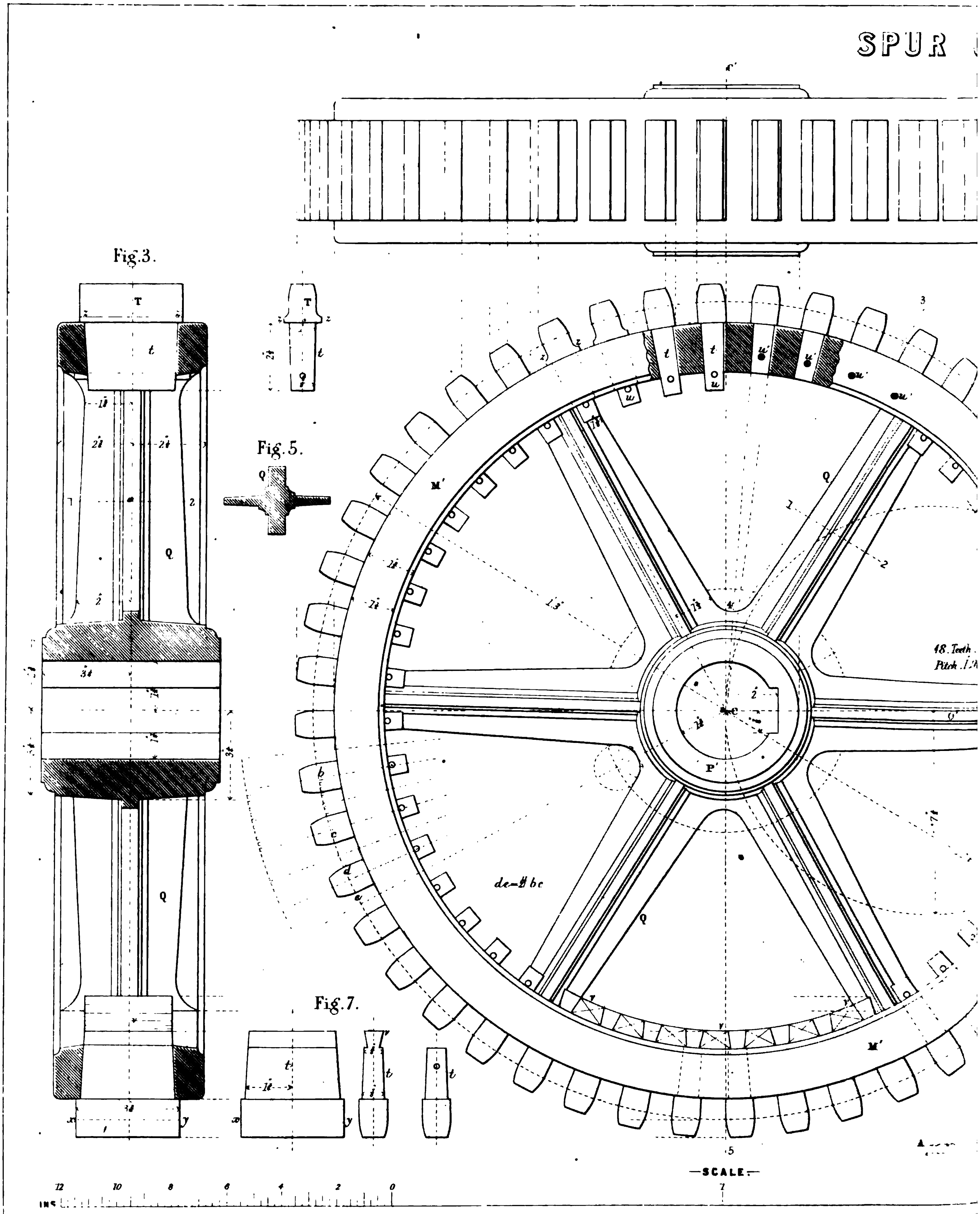
199. Hence it is manifest, on the supposition that the spur-wheel is intended always to be the driver, without being driven at any time by the pinion, that the teeth of the spur-wheel would only require to be of the form indicated at J, and those of the pinion, like the portion of a tooth, K, slightly tinted for the sake of distinction; but generally, and for obvious reasons, all spur gear is so constructed as to act reciprocally, and equally well, whichever be the driver, and we must, therefore, shape the teeth of the pinion, so that it may, in its turn, perform that function.

With this view, describe a circle with the centre, O^1, of the radius, B C, taken as a diameter; and suppose this circle to roll round the pitch circle, H B G, of the pinion, the point, B, at present in contact, will generate the epicycloid, B L, which is the proper curve to be given to the teeth of the pinion. The same point, B, considered as on the spur-wheel, will, as we have seen, generate a straight line, $B^1 O^1$, when rolling in the same manner round the interior of the circle, E B D, and this line forms the flank of the tooth of the spur-wheel. The operation proceeds in the same manner as for the pinion, the length of the teeth of which is determined by making the arc, $B f^1$, equal to the length of the pitch and describing, with the centre, A, a circle passing through the point, f^1.

The depth of the intervals of the spur-wheel is, in like manner, limited by a circle described with the centre, C, and radius, C *g*, which is somewhat short of being a tangent to the external circle of the pinion, so as to allow a little play to the teeth in their passage as already explained. In this manner are obtained the complete forms of the teeth, which are regular, symmetrical, and similar to each other, and satisfy the conditions of reciprocally acting gearing.

In the graphic operations here described, we have supposed the intervals between the teeth to be exactly equal in width to the teeth themselves; but as, in practice, it is necessary to allow of some play between the teeth, in order that they may work into each other with facility, this object is attained by reducing the thickness of the teeth a little; and in the drawing, when the scale is not very large, it will be sufficient to delineate the ink lines just within the thickness of the pencil lines. Where it is wished to be more precise, this allowance may be calculated at about $\frac{1}{15}$th or $\frac{1}{20}$th of the pitch. To give strength to the teeth, the interior angles of the intervals are rounded, as shown at each tooth in Fig. 4.

When the pinion is but of small diameter, the web, M, which carries the teeth, is cast solid with the boss, the interval being filled up with a disc; but when the wheel is larger, as in the case of the spur-wheel, the web, M^1, is attached to the boss, P^1, by arms, Q, which are strengthened by feathers, rounded in at the angles, as represented in Fig. 4.



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EARING.

.1.

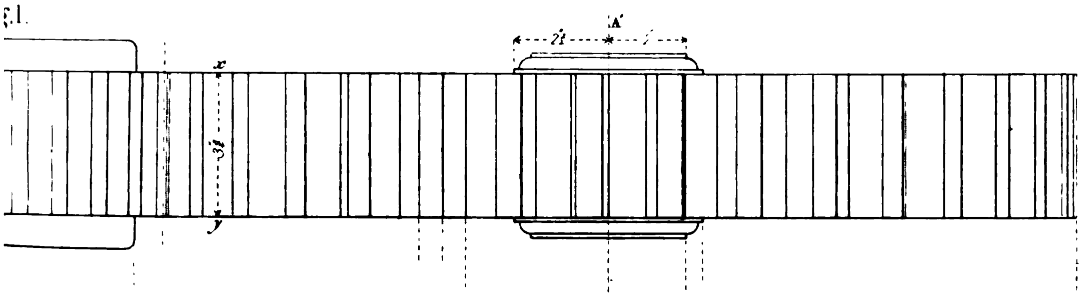


Fig.2.

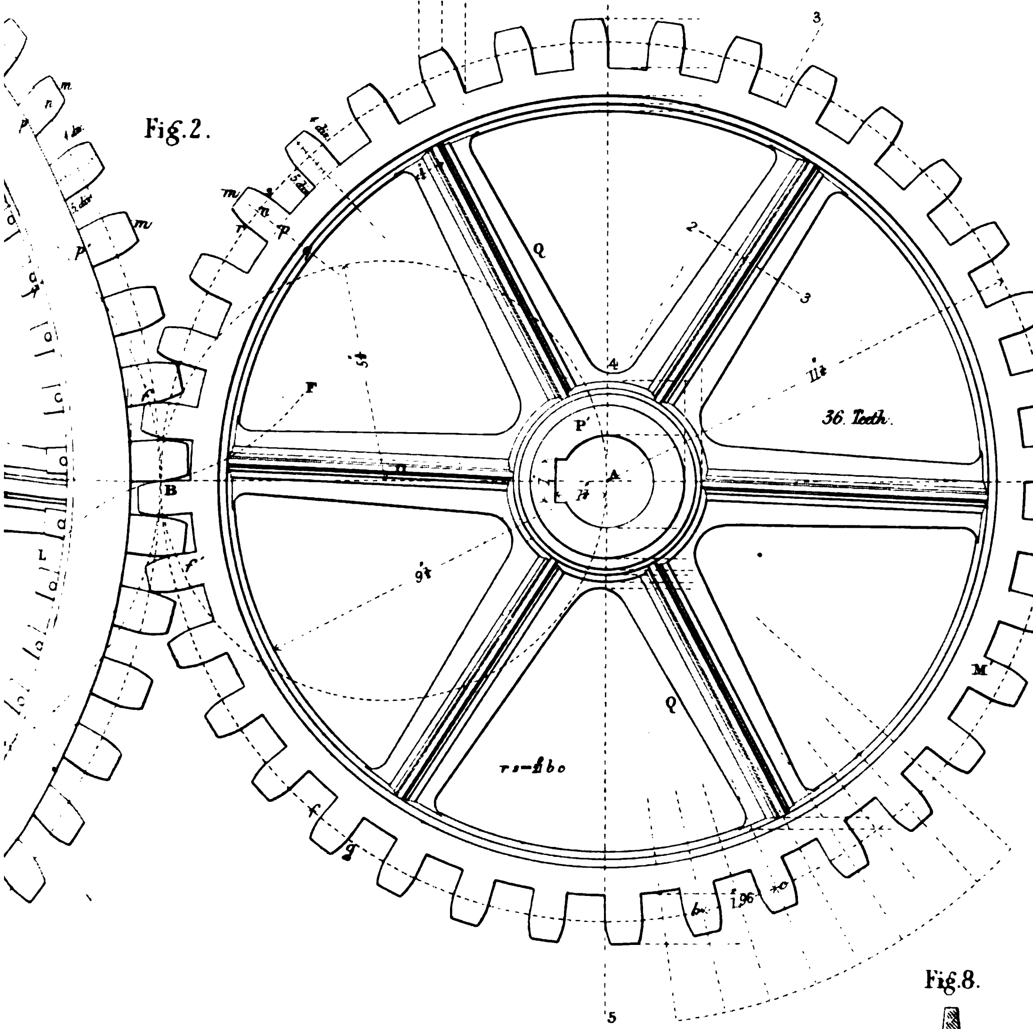


Fig.6.

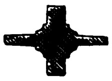


Fig. 4.

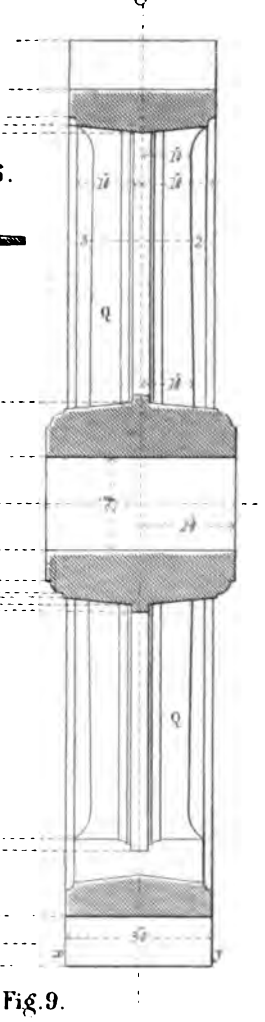


Fig.8.

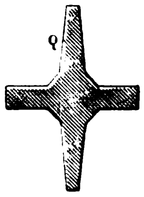


Fig.9.

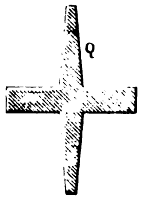
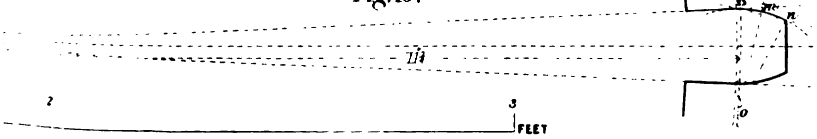


Fig.10.



DELINEATION OF A COUPLE OF WHEELS GEARING INTERNALLY.

Figure 5.—Plate 19.

200. The principles observed in determining the relative numbers of the teeth, with reference to the example just discussed, apply in like manner to the case before us; that is, such numbers must be in the exact ratios of the diameters of the pitch circles. The curvature of the teeth is also determinable by means of the same operations, modified to suit the different positions of the parts with respect to each other. Thus the curve, B L, of the pinion tooth, is generated by the rolling round the pitch circle, G B H, of the circle described with the centre, O, and radius, O B, equal to the half of B C, the radius of the pitch circle, D B E, of the larger wheel. This is an application of the operations explained in reference to Fig. 3. The flanks, B *a*, or the sides of the teeth, are obtained by simply drawing radii, or lines converging in the point, C.

In the same manner, the curve, B F, of the teeth of the large wheel, is generated by rolling along the interior of its pitch circle, D B E, a circle described from the centre, O^1, and radius, B O^1, equal to half the radius, B A, of the pitch circle, G B H, of the pinion. These curves being obtained, the outlines of the teeth are completed in the manner explained in reference to Fig. 4. It may, however, be observed that, in the diagram, Fig. 5, though the teeth might be cut off by a circle passing through the point, *f*, and described with the centre, A, they are prolonged beyond that, so that the teeth remain longer in contact, and a greater number of teeth are, consequently, engaged at one time, allowing the strain to be distributed over a greater number of points. It is the fact of the curvatures of the two lines of teeth being in the same direction, which admits of a greater number of teeth being engaged at once, without that increase of friction, and other disadvantages, which would result from such an arrangement with wheels of the ordinary description, such as those shown in Fig. 4.

THE PRACTICAL DELINEATION OF A COUPLE OF SPUR-WHEELS.

PLATE 20.

201. In the cases treated of in the preceding sections, which comprehend the general principles involved in rack and wheel gearing, we have assumed that the rack and pinion, or pinion and spur-wheel, are constructed of the same material, and in this case the thickness of the teeth is the same in any two working together. It very often happens, however, in actual construction, that one of the two has wooden, and the other cast-iron teeth, or of other dissimilar material. When this is the case, the thickness of the one description must necessarily be greater than that of the other, to compensate for the difference in the strength of the materials. The pitch, however, will still be the same for both wheels; for, since the intervals on one wheel correspond to the teeth on the other, a tooth and an interval on one must obviously be equal to an interval and a tooth on the other. A couple of wheels of this description are represented in elevation and plan in Figs. 1 and 2.

We here assume the wheels to be in the ratio to one another of 3 : 4; whence, giving the pinion 36 teeth, the spur-wheel must have 48. After dividing the pitch circle of the spur-wheel, drawn with the radius, C B, into 96 equal parts, the points of division representing the centres of the teeth and of the intervals, and the pitch circle of the pinion drawn with the radius, A B, likewise, into 72 equal parts—with the centres, O and O^1, describe the circles which generate the epicycloidal curves, B F and B L. Take $\frac{11}{21}$ of the pitch, *b c*, for the thickness of the wooden tooth, *d e*, and $\frac{9}{21}$ for that of the cast-iron tooth, allowing the remaining $\frac{1}{21}$ for the play in working. Next draw a series of radii, to indicate the flanks of the teeth, both of the pinion and spur-wheel, and at the point of their junction with the pitch circle, draw the curved portion of each, with the aid of a small pattern or template, cut to the curves, B L and B F; and finally litmit the lengths of the teeth and the depths of the hollows in the manner already pointed out, in reference to Plate 19.

As draughtsmen are generally satisfied with representing the epicycloidal curves by arcs of circles which almost coincide with them, and nearly fulfil the same conditions, such arcs must be tangential to the radial sides of the teeth at their points of intersection with the pitch circle. They are determined in the following manner:—Let Fig. 10 represent one of the pinion teeth, drawn to a larger scale. Through the point of contact, B, draw a tangent, B *o*, to the pitch circle; then bisect the chord, B *n*, which passes through the extremities of the curve, by a perpendicular, which will cut the tangent, B *o*, in the point, *o*. This is the centre of the arc, B *m n*, which very nearly coincides with the epicycloidal curve. The same arc is repeated for each side of all the teeth of the pinion, the radius, B *o*, being preserved throughout. An analogous operation determines the radius of the arc to be substituted for the curve in the teeth of the spur-wheel.

It is generally advisable to make wooden teeth about three-fourths as long as the pitch, and cast-iron teeth about two-thirds as long. In no case, however, should the lengths of the teeth in the two wheels geared together be less than those obtained by calculation, and determined by the points, f, f^1, situated on the circles described with the centres, O O^1, by which the epicycloids are generated. The ratio of the curved external portion, *n m*, of the tooth to the flank, *n p*, is 4 : 5. In other words, the whole height or length of the tooth being divided into 9 equal parts, 4 of these are to be taken for the length of the curved portion, and 5 for the rectilinear flanks. When the teeth are of cast-iron, the thickness, *p q*, of the web should be equal to the thickness, *r s*, of the tooth. Sometimes it is made only ¾ths of this; but in that case it is strengthened by a feather on the interior.

For wooden-toothed wheels, since it is necessary that the tenon, *t*, of the tooth be firmly secured, the web is made of a thickness, p^1 q^1, often double that of the tooth. The tenons of the teeth must be adjusted very carefully and accurately in the web. They are made with a slight taper, and are secured by iron pins or wires on the inner side of the web, or through the centre of it as at u and u^1, passing through them, or by a series of wooden keys or wedges, *v*, driven in between them, and forming strong dove-tail joints. These modes of fixing the teeth are shown at different parts in Fig. 2, and more in detail in Fig. 7. There is a third modification, which also possesses some advantages. We have represented it at T, Fig. 3; it will be seen that it consists in forming the teeth with a couple of shoulders, *z*, which allow of the tenons, *t*, being made much stronger, and also reduces the weight of metal, two objects of great importance.

The width, *x y*, of the teeth is equal to two or three times their pitch. In wheels entirely of cast-iron, the web is of the same width as the teeth; but it is much broader when the teeth are of wood, for it requires to be mortised, to receive the tenons of the teeth, and should have a width equal to that of the teeth, plus an amount equal to once and a half or twice their thickness. We have already mentioned, that in wheels of moderate size, the web, M^1, is attached to the boss, P^1, by arms, Q. The number of these arms varies, 4, 6, or 8, being used according to the diameter. In the present case the wheels have six arms; this number, amongst other reasons, being more particularly convenient, because the number of teeth are divisible by 6. Whence it follows, that the feathers which strengthen the arms on either side of the wheel, can be made to lie between two of the teeth, at each of the six points of attachment to the web.

The feathers are joined to the body of each arm by cavetto or concave quarter-round mouldings, with or without fillets, as indicated in Figs. 5 and 6, which represent sections of the arms taken through the lines 1, 2; 2, 3, respectively in Figs. 2, 3, and 4.

At other times the feathers are united to the body of the arm by plain chamfer portions, as shown in Fig. 8; or, even more simply still, and without filling up the angle formed, as in Fig. 9, the feathers being united, as it were, to the body of the arm without any additional moulding.

In all cases, however, these feathers are made with a taper, being thicker at their point of union with the body, and gradually decreasing in thickness outwardly.

Figs. 3 and 4 represent cross sections of the wheels, taken through the irregular line, 3, 4, 5, of the wheel and pinion respectively in Fig. 2. We may observe, in reference to these sections, that at the upper part of each the plane of section is supposed to be parallel to the arm, or the arm is, as it were, turned so as to be parallel to the plane, C^1 C, or A^1 A, Fig. 2, in order that it may be projected in the sectional view without foreshortening. At the lower parts of these views, however, the arms are projected, as in the oblique position represented in Fig. 2.

In this description of drawings, these oblique projections are generally dispensed with, and are, indeed, avoided, as they do not readily give the exact measurements of the parts represented.

The operations indicated on the figures complete the general design of Plate 20, whether of the plan, elevation, or sections.

THE DELINEATION AND CONSTRUCTION OF WOODEN PATTERNS FOR TOOTHED WHEELS.

PLATE 21.

SPUR-WHEEL PATTERNS.

202. If, as we have already endeavoured to impress upon the student, great care is required in the construction of wooden patterns in general, above all is this care and extreme accuracy called for in the execution of the patterns of toothed wheels, because of the great exactitude absolutely needed in the proportions of the various parts—as that, for example, between their diameters and numbers of teeth.

The pattern-maker must make allowance, not only for the shrinking of the cast-iron, but also for the quantity of metal to be taken away by turning and finishing afterwards. Moreover, the pattern, which is necessarily in many pieces, must be joined together so strongly and solidly, that it may not run the risk of changing its shape during the construction of the mould.

For wooden-toothed wheels, the web requires to be made with a number of openings or mortices to receive the tenons of the teeth. But in place of producing these mortises on the wooden patterns—which system, besides weakening it, would render the formation of the mould much more difficult—small projections corresponding to the teeth are fixed externally to the web. These projections or "prints" as they are sometimes termed, form sockets in the mould, in which the actual loam cores are fixed, which form the mortises when the piece is cast.

Bearing in mind these various considerations, we may proceed to the construction of the patterns for two spur-wheels, similar to those we have last described, and represented in Plate 20.

PATTERN OF THE PINION.

203. Figs. 1 and 2 show a vertical section and a half plan of the wooden pattern of the pinion. It is composed of many principal pieces—namely, the rim, web, or crown, as it is variously termed, and its teeth; the boss, with its core-pieces; and the arms, or spokes, with their feathers. We shall proceed to examine these various parts in succession.

WEB OR CROWN.

The pattern-maker takes planks of the requisite thickness, varying from ½ an inch to an inch, and cuts out of it a series of arcs, A, of a uniform radius, corresponding to that to be given to the pinion, with the addition of the allowance for shrinkage and loss from finishing. These arcs are built up like brickwork, the joints of one layer, or series, being opposite to solid portions of the contiguous layers, as shown in Fig. 3. This arrangement prevents the liability to warp or change the form, from variation in the humidity of the atmosphere, as would be the case were the crown made of a single piece.

The rim or web being thus, so far finished and glued together, the joints being quite dry, it is then put into a lathe, and there turned quite true, both externally and internally. The two surfaces are here made perfectly parallel, and the whole is reduced to the exact dimensions determined on, and shown upon a large working drawing of the actual size, previously prepared, generally by the pattern-maker himself.

At this stage, the external surface of the rim or crown is divided off by lines, showing the position of the teeth, which are then sometimes simply screwed or nailed on. It is, however, much preferable, and conduces very much to the solidity of the wheel, to cut out grooves of a trifling depth on the periphery, into which the teeth are fixed, being formed with a dovetail for that purpose, as shown at B, in Fig. 2. Sometimes the dovetail pieces are made separate, and afterwards fixed to the teeth as shown at B^1.

BOSS.

The boss or eye is made in two pieces, each one solid block of wood, D, except when the wheel is of a large size, in which case the boss requires to be built up of several pieces.

These blocks are each turned separately to the exact dimensions given in the plans, and they secure between them the thickness of the body part of the arms, termed also the face arms.

ARMS OR SPOKES.

The face arm, C, Fig. 4, is also cut out of planks of a uniform thickness, being formed not only to the external contour of that part of the arm which is afterwards the only part visible in the casting, but also comprising, above and beyond this, the projections by which, in the pattern, it is attached to the boss on the one hand, and to the rim or crown on the other. The extremity, *a*, of the boss end of the arm is in the form of a sector, corresponding to a sixth part of the circle of the boss, the pinion having six arms; the lateral facets, *b*, of this part are grooved out, to receive small

SPUR-WHEE

Fig. 1.

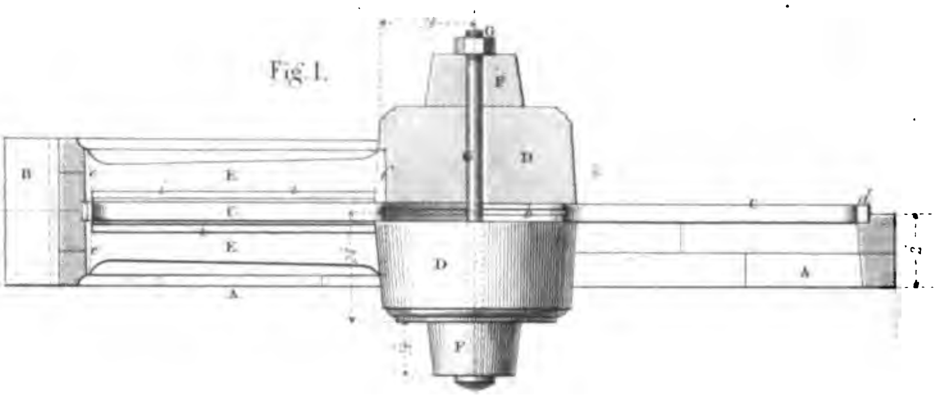


Fig. 10.

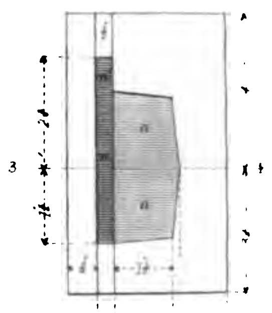


Fig. 2.

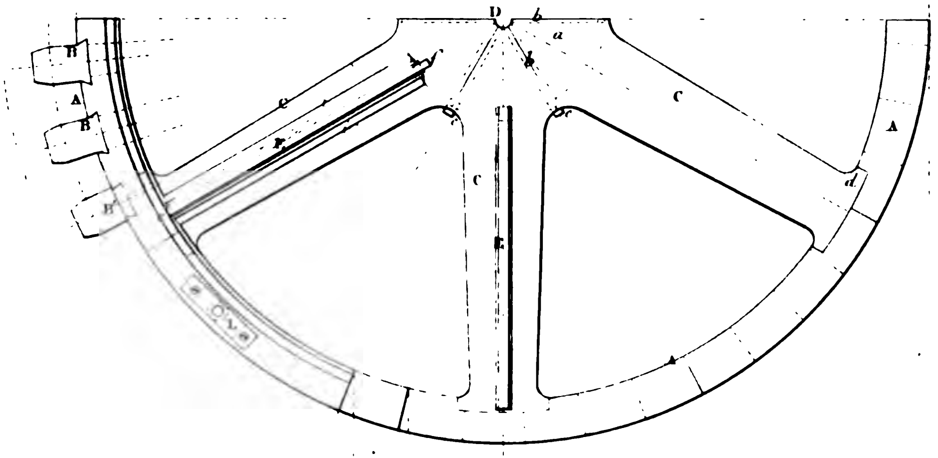


Fig. 11.

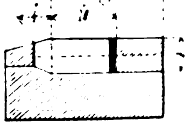


Fig. 12.

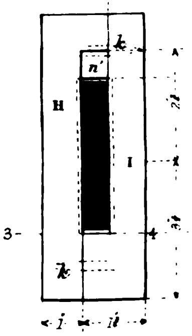


Fig 13

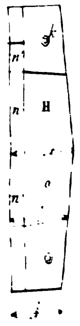


Fig. 3.

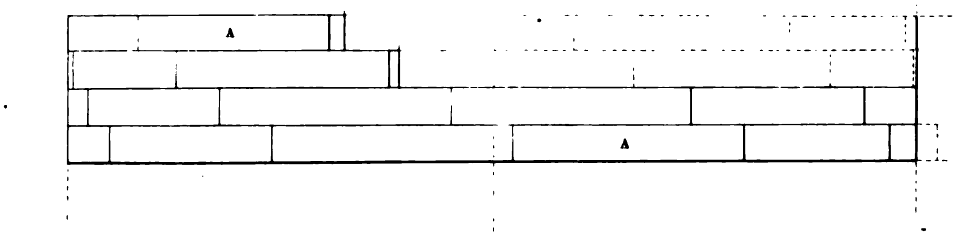
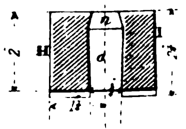


Fig. 14.



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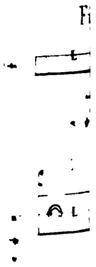


Fig. 5.

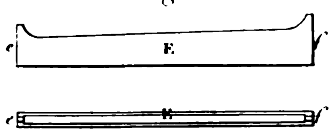
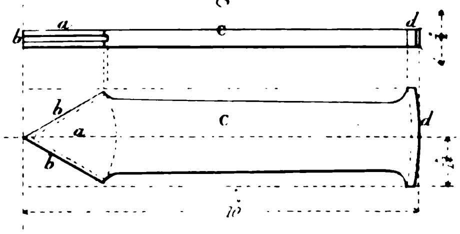
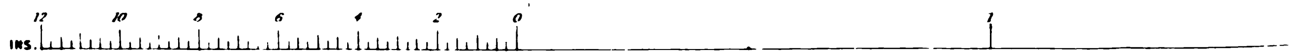


Fig. 4.



—SCALE:—



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PATTERNS.

Fig. 7.

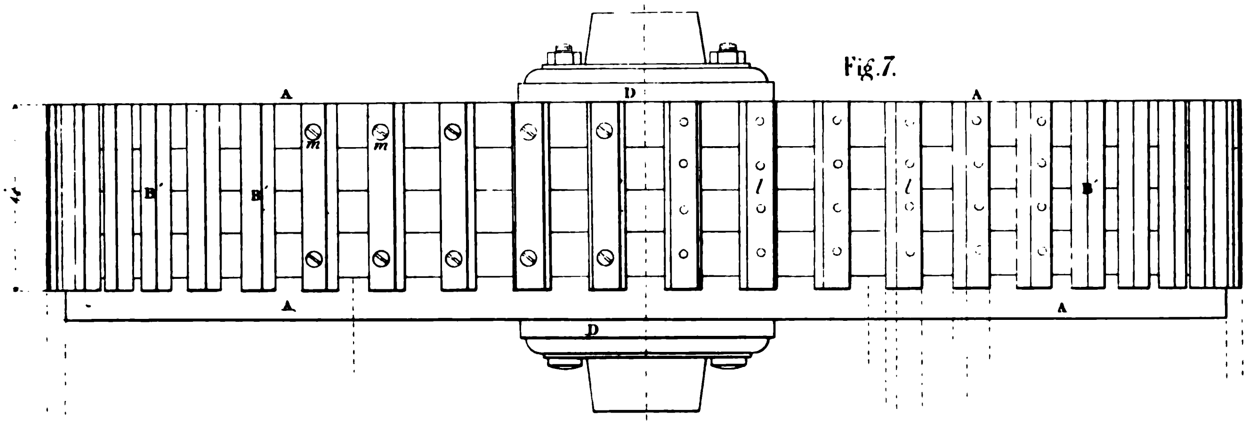


Fig. 8.

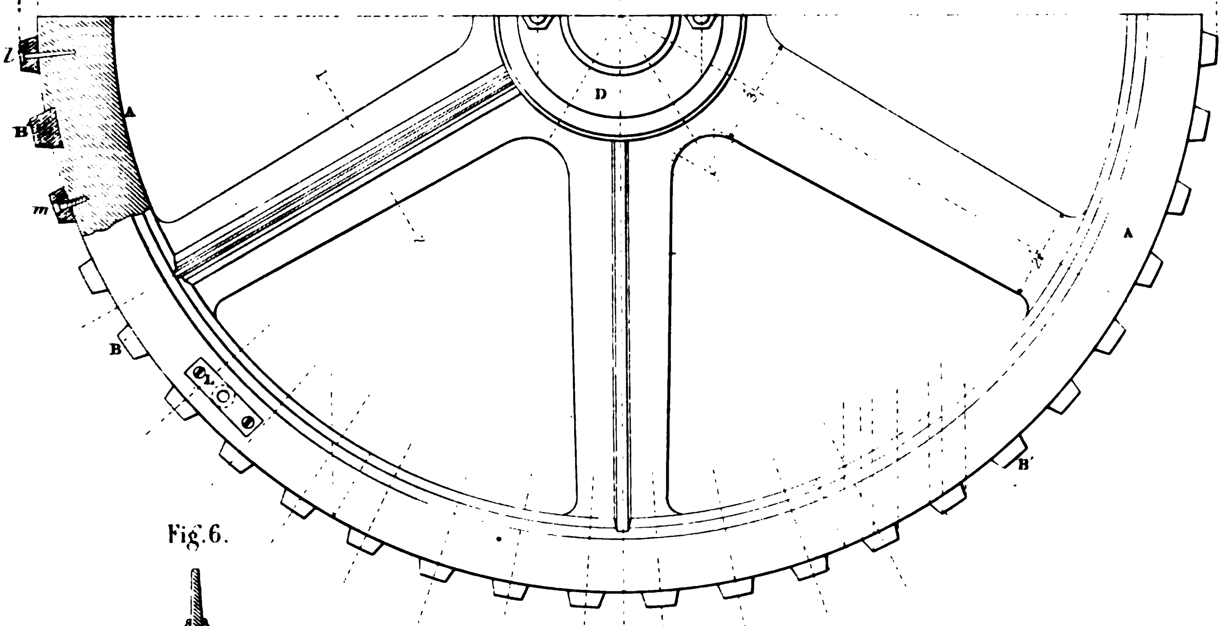


Fig. 6.

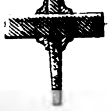
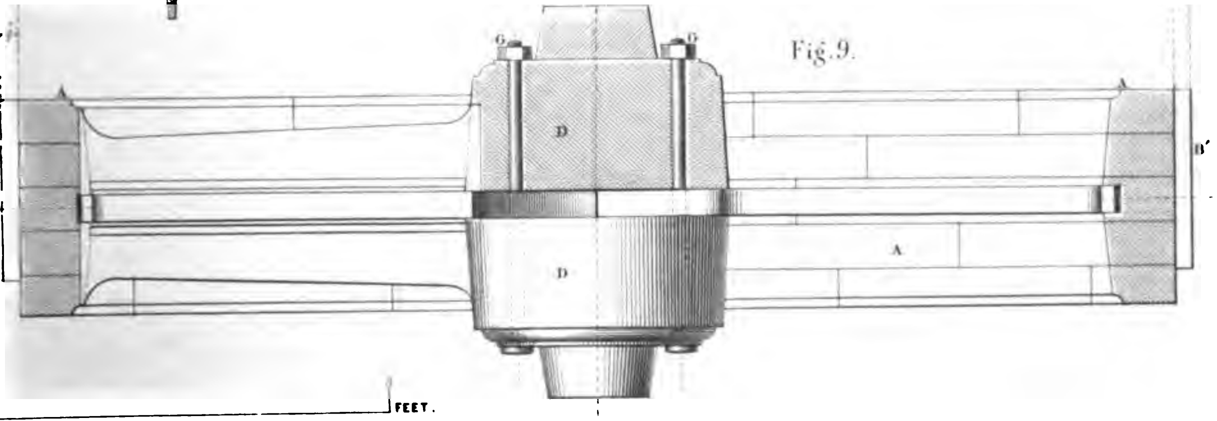


Fig. 9.



FEET.

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tongue-pieces, or keys, *c*, Fig. 2, so as to form a strong joint, when glued together. The other extremity, *d*, of the arm is cut circularly, to the form of the crown, or web, into which it is fitted, penetrating to a slight extent, the crown being previously formed with a socket to receive it.

Next, the feather arms, E, have to be attached to the body or face arms, C. These feathers, E, are each cut out in separate pieces, to the shape indicated in Fig. 5: they have supplementary projections, *e* and *f*, at their opposite extremities, whereby they are fixed into the web and boss. When all these feathers are in their place, and the arms glued into the rim or web, the two portions, D, D, of the boss are fixed to them, the grooves for the reception of the ends of the feathers being glued, as well as the other parts, to give greater solidity. Finally, the boss is surmounted by the conical projecting pieces, F, F, which serve to produce in the mould the cavities, or sockets, which retain the loam core in position, the core being provided to produce the eye of the wheel, into which the shaft is fitted.

To give compactness and strength to the whole, a bolt, G, is passed through the centre; and this method of securing permits of the core projections, F, F, being changed for larger or smaller ones, if desired, without having to pull the entire wheel to pieces. If, to add to the elegance of the shape of the wheel, it is wished to ornament the arms with mouldings, as at *i*, these are applied at the angles or junction of the feathers with the body of the arm. These are simply glued or nailed on. The sectional view, Fig. 6, taken on the line 1, 2, Fig. 8, shows the form and position of these mouldings.

It is to be observed that, in wheels of a moderate size, when cast-iron teeth are to work on cast-iron, they are at once cast to the exact shape, and the pattern is constructed accordingly; but it is almost always indispensable, where cast-iron and wooden teeth have to work together, to finish and reduce the former after being cast; and the projections, B, on the pattern answering to them, must consequently be made of larger proportions every way, to provide for the quantity of metal taken away in the finishing process.

PATTERN OF THE WOODEN-TOOTHED SPUR WHEEL.

204. Figs. 7, 8, and 9 represent, in elevation, plan, and vertical section, the wooden pattern of the spur-wheel, which gears with the pinion just described. It consists, like the pinion, of the crown or web, the boss, and the arms; these various parts are designated by letters corresponding to those employed in the preceding example, and they are constructed exactly in the same manner.

There is, however, an essential difference in the exterior of the rim or web, A.—In place of this carrying the projections, B, cut to the shape of the teeth, and such as will actually be produced on the casting, it has other projections, B^1, of a simpler form, intended to produce in the mould the sockets for receiving core-pieces which form the mortises in the casting, to receive the tenons of the wooden teeth. These projections are let into the web, or simply applied thereto, and fixed by nails, as at *l*, or by screws, as at *m*, the latter method being preferable, as it has the advantage of permitting the number of teeth to be changed without injury to themselves or to the web. In the wooden pattern, the projections, B^1, are carried up to the top edge, or face of the web, on that side which is to be at the top of the mould, so as to allow of the more accurate adjustment and dropping in of the cores, and also to facilitate the removal of the pattern from the mould. These core-pieces, however, are so formed as to make the mortises no wider than is necessary, and to leave a sufficient thickness of metal for the strength of the crown, or web, as already pointed out in reference to Plate 20.

CORE-MOULDS.

205. The core-pieces or prints for the mortises should not only be placed at equal distances apart throughout the circumference of the web, but they must all also be of precisely the same form and dimensions throughout, so that the cores may fit their proper places in the mould exactly, and thus make the mortises of the wheel when cast perfectly true and equal. With this view, the cores are all formed in a small mould or box, made of wood or iron for that purpose, and termed a "core-box:" there are several methods of forming these, but we will only describe the two most common, namely, the one which forms the core on its side, and the one which forms it on edge. Thus, Fig. 10 represents a face view, and Fig. 11 a horizontal section, through the line 3, 4; in Fig. 10, of one form of core-mould, consisting of a single piece. The portion, *n*, of the cavity corresponds to the projecting core-piece, B^1, outside the rim, or web, and the portion marked *o*, to the mortise, or hollow socket, in the web: this last has the same section as the web in the width of the cut-out part. The moulder fills the cavity of the core-mould with loam, previously prepared, and after pressing it well in, levels it off with a straight-edged doctor or scraper; he finally inverts the mould, thus releasing the core complete. The operation is repeated as many times as their are teeth; and when the cores are all dried in a stove, or on a hot plate they are placed with great care in the mould, their supplementary projections, n^1, being let into the sockets formed to receive them—thereby insuring the accuracy of their adjustment.

Figs. 12, 13, and 14, show another construction of wooden core-mould, formed in two separate pieces, H and I. These have between them the cavity, *n o*, corresponding to that in the one just described. In this last case, the surface of the core, which requires to be levelled off with a scraper, is only at one of the extreme edges, instead of on the lateral faces, as in the other, and the cores are released by separating the two pieces, H, I, which are rendered capable of accurate adjustment to each other, by means of pins, *k*. The parts, n^1, are sometimes dispensed with, and filled in with green sand, after all the cores are properly placed in the mould.

To return to the wheel itself: when it is of very large dimensions, the blocks, D, of the boss, are secured together by two or more bolts, G, in place of one.

The mould for the wheel, is usually in two pieces, the lower part or "drag," being let into the ground in the moulding shop; the upper frame, or top part is moveable, and it will be obvious that very great care is required, both in lifting this off the pattern, and also in withdrawing the pattern from the drag, so as not to injure the regularity and sharpness of the impression; and for this purpose, sufficient "draw" or taper must be given to the various parts, as the crown and its core-prints, the boss, and the feathers on the arms, as already pointed out.

When the patterns are heavy, two or more screw-staples, or "draw-plates," L, Figs. 2, 8, and 15, of iron or brass, are countersunk into the crown, and into these, draw-handles are screwed, by which the pattern is lifted out of the mould.

Note that in Figs. 2, 8, and 9, are combined, in single views, several different projections, to avoid repetition of the diagrams, and to simplify the whole drawing, and bring it into a small space. This system is very much used in drawings, or plans, made for actual construction. These irregularities are not numerous, and will be seen at once, such as dotting in parts which would not be seen on the views in which they are shown, and also taking the sections on different planes in the same view, as in Figs. 1 and 9, the lines of which are, however, dotted in the plans corresponding.

RULES AND PRACTICAL DATA.

TOOTHED GEARING.

206. It has been already laid down as a fundamental rule, that in order to work well, all toothed wheels coupled together must have the same ratio between the numbers of their teeth as between their diameters.

It follows from this principle, that when we know the radii of the pitch circles of two wheels, and the number of teeth of one of them, we can determine that of the other, and reciprocally.

Thus, putting N, to represent the number of teeth of a wheel of the radius, R; and n, to represent the number of teeth of a wheel of the radius, r, we have the direct proportionals, $N : n :: R : r$; whence we can, at any time, ascertain any one of the terms when the other three are known.

First Example.—Let the radius of the pitch circle of a spur-wheel be 12 inches, and the number of teeth on it 75, what should be the number of teeth on a pinion gearing with it, the radius of the pitch circle of which is 8 inches?

We have

$$75 : n :: 12 : 8; \text{ whence}$$

$$n = \frac{75 \times 8}{12} = 50 \text{ teeth.}$$

Second Example.—Let 75 and 50, respectively, be the number of the teeth of a spur wheel and pinion, and 12 inches the radius of the pitch circle of the former, the radius of the pitch circle of the latter may be found by means of the proportion—

$$75 : 50 :: 12 : r; \text{ whence,}$$

$$r = \frac{50 \times 12}{75} = 8 \text{ inches.}$$

207. The velocities of rotation, or the numbers of revolutions of the shafts of a spur-wheel and pinion in gear with each other, are in the inverse ratio of the respective diameters, radii, or numbers of teeth of the two wheels.

Consequently, putting v, to represent the velocity of rotation of the pinion shaft, the radius of the pitch circle of which equals r, and the number of the teeth, n, and putting V, to represent the velocity of the spur wheel shaft, of which the pitch circle radius equals R, and number of teeth, N, we have the inverted proportions—

$$v : V :: r : R,$$

and

$$v : V :: n : N.$$

In either of these proportions, we can determine, as in the former example, any one term when the three others are known.

First Example.—A spur wheel, the pitch circle radius of which is 10 inches, has a velocity of 25 revolutions per minute; what is the pitch circle radius of a pinion to gear with it, and make 60 revolutions in the same time? By the inverse proportion,

$$25 : 60 :: r : 10;$$

whence,

$$r = \frac{25 \times 10}{60} = 4\tfrac{1}{6} \text{ inches,}$$

the pitch circle radius of the pinion.

A spur-wheel has 60 teeth, and is required to run at 25 revolutions per minute, and at the same time to drive a pinion at the rate of 75 revolutions per minute, what should be the number of teeth of the latter?

Here,

$$57 : 25 :: 60 : n;$$

whence,

$$n = \frac{25 \times 60}{75} = 20,$$

the number of teeth the pinion must have.

These principles apply also to pulleys or drums put in communication with one another by cords or belts, and known as belt-gearing.

Sometimes, in systems of geared spur-wheels, all that is known is the distance apart of their centres, the number of teeth which they are to carry, or the number of their revolutions in the same time. In this case we have, on the one hand, an inverse proportion between the distance of their centres, the sum of their revolutions, and between their respective radii and revolutions; and, on the other hand, a direct proportion between the distance of the centres, the sum of the teeth on both wheels, and their respective radii, or the number of teeth of each.

Let D be the distance apart of the centres of a spur-wheel and pinion of the respective radii, R, r, and number of teeth, N, n, or the reciprocal velocities, V and v; we have first the following inverse proportion,

$$D : V + v :: R : V;$$

and, secondly, the direct proportion,

$$D : N + n :: N : R.$$

First Example.—Let 45 inches be the distance between the centres of a spur-wheel and pinion, the former of which is to make 15½ revolutions per minute to the other's 22, what should be their respective radii?

We have first,

$$45 : 22 + 15{\cdot}5 :: R : 22;$$

whence,

$$R = \frac{45 \times 22}{22 + 15{\cdot}5} = 26{\cdot}4 \text{ inches,}$$

and

$$45 : 22 + 15{\cdot}5 :: r : 15{\cdot}5;$$

whence,

$$r = \frac{45 \times 15{\cdot}5}{22 + 15{\cdot}5} = 18{\cdot}6 \text{ inches.}$$

When the pitch circle radius of one of the wheels is ascertained, it is evidently unnecessary to search for the other radius by means of the second proportion, for it is sufficient to subtract the one found from the sum of both; thus,

$$45 - 26{\cdot}4 = 18{\cdot}6; \text{ or,}$$

$$45 - 18{\cdot}6 = 26{\cdot}4.$$

Second Example.—The distance, D, between the two centres being known = 45 inches, and one wheel carrying 44 teeth and the other 31, what are their respective radii?

We have here, in the first place,

$$45 : 44 + 31 :: R : 44;$$

whence,

$$R = \frac{45 \times 44}{31 + 44} = 26{\cdot}4,$$

and

$$45 : 31 + 44 :: r : 31;$$

whence,

$$r = \frac{45 \times 31}{31 + 44} = 18{\cdot}6,$$

or, more simply,

$$r = 45 - 26{\cdot}4 = 18{\cdot}6 \text{ inches.}$$

In like manner, the respective radii of a spur-wheel and pinion, to gear together, may be determined geometrically, when the distance between their centres is known, as well as the numbers of revolutions of each, by the following rule:—

Divide the distance into as many equal parts as there are of any measure contained exactly in the sum of the velocities, such measure being also contained exactly any number of times in each of the velocities alone. Then, for the pinion radius, take as many of these measures as are contained in the lesser velocity, and for the radius of the spur-wheel, the remainder of them.

Example.—Let 16 inches be the distance between the centres of a spur-wheel and pinion which make 6 and 4 revolutions respectively, or any equi-multiples or equi-submultiples of these, as 12 and 8, or 3 and 2. Divide the distance into 10 equal parts, and take 4 of these for the pinion radius, and 6 for the spur-wheel radius.

This rule is of very simple application when the ratios of the numbers of revolutions are whole numbers, such as 1 : 4, or 2 : 5; for all that is necessary is to add the two together, to divide the distance between the centres to correspond, and to take the respective numbers of measures for each wheel.

The following table will be of great assistance in the solution of various problems connected with systems of gearing, when of the number of teeth, the pitch, and the radius, any two are known.

TABLE FOR CALCULATING THE NUMBERS OF TEETH AND DIAMETERS OF SPUR GEAR, FROM THE PITCH, OR *VICE VERSA.*

Number.	Coefficient.	Number.	Coefficient.	Number.	Coefficient.	Number.	Coefficient.	Number.	Coefficient.
10	3·183	39	12·414	68	21·644	97	30·875	126	40·106
11	3·501	40	12·732	69	21·963	98	31·193	127	40·424
12	3·820	41	13·050	70	22·281	99	31·512	128	40·742
13	4·138	42	13·369	71	22·599	100	31·830	129	41·061
14	4·456	43	13·687	72	22·917	101	32·148	130	41·379
15	4·774	44	14·005	73	23·236	102	32·467	131	41·697
16	5·093	45	14·323	74	23·554	103	32·785	132	42·016
17	5·411	46	14·642	75	23·872	104	33·103	133	42·334
18	5·729	47	14·960	76	24·191	105	33·421	134	42·652
19	6·048	48	15·278	77	24·509	106	33·740	135	42·970
20	6·366	49	15·597	78	24·827	107	34·058	136	43·289
21	6·684	50	15·915	79	25·146	108	34·376	137	43·607
22	7·002	51	16·233	80	25·464	109	34·695	138	43·925
23	7·321	52	16·552	81	25·782	110	35·013	139	44·244
24	7·639	53	16·870	82	26·100	111	35·331	140	44·562
25	7·957	54	17·188	83	26·419	112	35·650	141	44·880
26	8·276	55	17·506	84	26·737	113	35·968	142	45·199
27	8·594	56	17·825	85	27·055	114	36·286	143	45·517
28	8·912	57	18·143	86	27·374	115	36·604	144	45·835
29	9·231	58	18·461	87	27·692	116	36·923	145	46·153
30	9·549	59	18·780	88	28·010	117	37 241	146	46·472
31	9·867	60	19·098	89	28·329	118	37·559	147	46·790
32	10·186	61	19·416	90	28·647	119	37·878	148	47·108
33	10·504	62	19·734	91	28·965	120	38·196	149	47·427
34	10·822	63	20·053	92	29·284	121	38·514	150	47·745
35	11·140	64	20·371	93	29·602	122	38·833	151	48·063
36	11·459	65	20·689	94	29·920	123	39·151	152	48 382
37	11·777	66	21·008	95	30·238	124	39·469	153	48·700
38	12·095	67	21·326	96	30·557	125	39·788	154	49·020

RULES CONNECTED WITH THE PRECEDING TABLE.

I. To find the diameter of a spur-wheel, when the number and pitch of the teeth are known.

Multiply the coefficient in the table, corresponding to the number of teeth, by the given pitch of the teeth in feet, inches, or other measure, and the product will be the diameter in feet, inches, &c., to correspond.

First Example.—What is the diameter of the pitch line of a spur-wheel, of 63 teeth, having a pitch of 1½ inches?

Opposite the number 63, in the table, we find the coefficient, 20·053. Then—

$$20{\cdot}053 \times 1{\cdot}5 = 30{\cdot}08 \text{ inches,}$$

the diameter of the spur-wheel.

Second Example.—What are the diameters of the pitch lines of two wheels, of 41 and 150 teeth respectively, the pitch of their teeth being ¾ inch?

On the one hand we have

$$13{\cdot}05 \times {\cdot}75 = 9{\cdot}7875 \text{ inches.}$$

the diameter of the pinion of 41 teeth; and on the other,

$$47{\cdot}745 \times {\cdot}75 = 35{\cdot}8 \text{ inches,}$$

the diameter of the spur-wheel of 150 teeth.

II. To find the pitch of the teeth of a spur-wheel, when the diameter of the pitch line and number of the teeth are known.

Divide the given diameter by the coefficient in the table corresponding to the number of the teeth, and the quotient will be the pitch of the teeth sought.

First Example.—What is the pitch of a wheel of 30·08 inches diameter, and of 63 teeth?

Here—

$$30{\cdot}08 : 20{\cdot}053 = 1{\cdot}5 \text{ inch},$$

the pitch required.

Second Example.—It is required to construct a spur-wheel, of 126 teeth, to work with the preceding, what must be its diameter?

Here—

$$1{\cdot}5 \times 40{\cdot}106 = 60{\cdot}159 \text{ inches}.$$

the diameter of a wheel of 126 teeth and of the same pitch.

III. To find the number of teeth of a wheel, when the pitch of the teeth and diameter of the pitch line are known.

Divide the given diameter by the given pitch of the teeth, the number in the table corresponding to the quotient will be the number of teeth sought.

If the quotient is not in the table, take the number corresponding to that nearest to it.

First Example.—The diameter of a spur-wheel is 30·08 inches, and the pitch of the teeth is 1·5 inch, what number of teeth should the wheel have?

$$30{\cdot}08 : 1{\cdot}5 = 20{\cdot}53\,;$$

which quotient corresponds to 63 teeth.

Second Example.—What should be the number of teeth of a pinion, the diameter of which is 875 millimètres, and which is intended to gear with a rack, of which the pitch is 25 millimètres?

$$875 : 25 = 35.$$

The number most nearly corresponding to this is 110, the number of teeth to be given to the pinion.

ANGULAR AND CIRCUMFERENTIAL VELOCITY OF WHEELS.

208. When it is known what is the angular velocity of the shaft of a fly-wheel, spur-wheel, or pulley, the circumferential velocity may be found by the following rule:—

Multiply the circumference by the number of revolutions per minute and the product will give the space passed through in the same time; and this product being divided by 60, will give the velocity of the circumference per second.

Example.—Let the diameter of a wheel be 4 feet, and the number of its revolutions per minute 20, what is the velocity at the circumference?

The circumference of the wheel $= 4 \times 3{\cdot}1416 = 12{\cdot}5664$; then

$$12{\cdot}5664 \times 20 = 251{\cdot}328 \text{ feet},$$

the space passed through per minute by any point in the circumference; and

$$\frac{251{\cdot}328}{60} = 4{\cdot}2,$$

the velocity in feet per second.

When the velocity at the circumference is known, the angular velocity, or the number of turns in a given time may be ascertained by the following rule:—

Divide the circumferential velocity by the circumference, and the quotient will be the angular velocity, or number of revolutions in the given time.

In the preceding case, 4·2 feet being the circumferential velocity per second, and 4 feet the diameter, we have

$$\frac{4{\cdot}2}{4 \times 3{\cdot}1416} = {\cdot}334,$$

the angular velocity per second; and

$${\cdot}334 \times 60 = 20,$$

the number of revolutions per minute.

In practice, it is easy to ascertain the velocity of a wheel, the motion of which is uniform. With this view, a point is marked with chalk on the rim of the wheel, and note is taken of how often this point passes a fixed point of observation in a given time; then this number of revolutions is multiplied by the circumference described by the marked point, and the product divided by the duration of the observation expressed in seconds. The result will be the velocity of the circumference of the wheel. Every other point on the wheel will have a different velocity, proportioned to its distance from the centre of motion.

Example.—A wheel, 2 feet in diameter, having, according to observation, made 75 revolutions per minute, what is its circumferential velocity (per second)?

$$V = \frac{75 \times 3{\cdot}14 \times 2}{60} = 7{\cdot}83 \text{ feet},$$

circumferential velocity of the wheel.

Reciprocally, when the circumferential velocity (per second) is known, the number of revolutions per minute is found by means of the formula—

$$N = \frac{V \times 60}{3{\cdot}14 \times D};$$

or with the data of the preceding case,

$$N = \frac{7{\cdot}83 \times 60}{3{\cdot}14 \times 2} = 75 \text{ revolutions per minute.}$$

When several spur-wheels or pulleys are placed on the same shaft, the circumferential velocity of every one of them is found in the same manner, by multiplying the number of revolutions by the respective circumferences, and dividing the products by 60.

Example.—Three wheels or pulleys, *a*, *b*, *c*, are fixed on one shaft; the radius of the pulley, *a*, is equal to 1·1 feet; that of the pulley, *b*, 1·6 feet; and that of the pulley, *c*, 2·15 feet; and the shaft makes 12 turns per minute,—what is the circumferential velocity of these three pulleys?

For the pulley, *a*, we have—

$$V^a = \frac{6{\cdot}28 \times 1{\cdot}1 \times 12}{60} = 1{\cdot}38 \text{ feet per second.}$$

for the pulley, *b*—

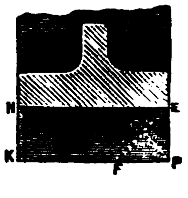
$$V^b = \frac{6{\cdot}28 \times 1{\cdot}6 \times 12}{60} = 2 \text{ feet.}$$

and for the pulley, *c*—

$$V^c = \frac{6{\cdot}28 \times 2{\cdot}15 \times 12}{60} = 2{\cdot}7 \text{ feet.}$$

DIMENSIONS OF GEARING.

209. In designing tooth-gearing of all descriptions, it is necessary to determine—first, the strength and dimensions of the teeth; second, the dimensions of the web which carries the teeth; and, third, the dimensions of the arms.



210. "The teeth of wheels," says Professor Rankine in his excellent *Manual* of applied mechanics, "are made sufficiently strong, to provide against an action analogous to combined twisting and bending, which may arise from the whole force transmitted by a pair of wheels happening to act on one corner of one tooth. Let the shaded part in the annexed figure represent a portion of a cross section of the rim of a wheel and let E H K P be the face of a tooth, on one corner of which, P, acts the force represented by that letter. Conceive any sectional plane, E F, to intersect the tooth from the side, E P, to the crest, P K, and let P G be perpendicular to that plane.

Let h be the thickness of the tooth, und let $EF = b$, $PG = l$. Then the moment of flexure at the section, EF, is Pl, and the greatest stress produced by that moment of flexure at that section, is

$$p = \frac{6\,Pl}{6\,h^2},$$

which is a maximum, when the angle, PEF, = 45°, and $b = 2\,l$, having then the value

$$f = \frac{3\,P}{h^3};$$

consequently, the proper thickness for the tooth is given by the equation,

$$h = \sqrt{\frac{3\,P}{f}}.$$

This formula is Tredgold's, according to whom the proper value for the greatest working stress, f, is 4500 lbs. per square inch, when the teeth are of cast-iron."

211. The following table will be found useful in calculating the thickness and pitch of the teeth of spur-wheels with reference to the power:—

TABLE *of Dimensions of the Teeth of Wheels, that will safely transmit a given Number of Horses' Power at a given Velocity.*

Pitches in Inches.	Thickness of Teeth in Inches.	Horse Power at 3 Feet per Second.	Horse Power at 4 Feet per Second.	Horse Power at 5 Feet per Second.	Horse Power at 7 Feet per Second.	Horse Power at 11 Feet per Second.
4	1·9	19·	25·5	32·	45·	70·5
3½	1·6	14·75	19·5	24·5	34·25	54·
3	1·4	11·	14·5	18·	2·5	39·5
2½	1·2	7·5	10·	12·5	17·5	27·5
2	·95	4·75	6·25	8·	11·	17·25
1¾	·83	3·5	5·	6·25	8·5	13·5
1½	·71	2·75	3·5	4·5	6·25	10·
1¼	·59	2·	2·5	3·125	4·4	6·8
1⅛	·53	1·5	2·25	2·5	3·5	5·5
1	·48	1·2	1·6	2·	2·8	4·4
⅞	·41	1·	1·4	1·75	2·5	3·8
¾	·36	·7	·9	1·125	1·5	2·5
⅝	·33	·5	·625	·75	1·	1·7
½	·24	·3	·4	·5	·7	1·1
⅜	·18	·2	·25	·3	·4	·6
¼	·12	·0225	·1	·125	·175	·275

The breadth of the teeth may be found by dividing the horse-power by the velocity of the wheel at the pitch circle per second in feet, and doubling the quotient. Thus the breadth of the teeth for a first mover from a sixteen-horse engine, the velocity of the wheel being eight feet per second, will be

$$\frac{16}{8} = 2 \times 2 = 4 \text{ inches.}$$

Another mode of calculating the breadth when the pitch is given, is so to adjust the former that there shall be one square inch in the teeth for each 160 lbs. of force transmitted by the teeth. The interval between two teeth measured on the pitch circle is usually made about one fifth wider than the thickness of the tooth on the same circle; so that, if we suppose the whole space occupied by a tooth, and the adjoining interval to be divided into eleven parts, five of those parts will be occupied by the thickness of the tooth, and six by the adjoining interval. About one tenth of the pitch is allowed, as clearance, between the points of the teeth and the bottoms of the spaces between the teeth of the other wheel.

DIMENSIONS OF THE WEB.

212. The width of the web is ordinarily equal to that of the teeth when the whole is of cast-iron. Nevertheless, in some cases—as, for example, where very great irregularities in the pressure and speed, and reiterated shocks have to be borne in the heavy machinery in engine shops—the web is made wider than the teeth, projecting also on either side of the teeth, so that these are wholly or partially imbedded, which increases their power of resistance very considerably. These lateral webs are generally each made of about half the thickness of the tooth.

The thickness of the web, or crown, is never made less than three-fourths that of the tooth, and very frequently it is further strengthened by an internal feather, as already mentioned.

When the teeth are of wood, the web is much thicker, to give sufficient hold to the tenons of the teeth (Par. 201); it is generally made about 1·5 to 2· times the thickness of the tooth.

NUMBER AND DIMENSIONS OF THE ARMS.

213. The number of arms, or spokes, which a spur-wheel ought to have, has not, up to the present time, been precisely and scientifically determined. According to general experience, up to a diameter of about three feet, four arms are sufficient; from three to six or seven feet, six are necessary and sufficient; beyond eight feet, eight arms are used; and for sixteen feet, ten are given: it is seldom this last number is exceeded, except for wheels of extraordinary dimensions.

The section of the arms of the wheel is always in the form of a cross, the stronger portion of which lies in the plane of the circumferential strain, whether these arms are cast in one piece with the boss and the crown, as is the case with wheels of small diameter—that is, of such as have not a greater radius than six or seven feet; or whether they are cast in separate pieces, and afterwards fitted together. The thicker part, then, of the arm must be strong enough to bear the circumferential strain. Experience has shown, that when a spur-wheel is in motion, and acted upon by a considerable force, this strain has a tendency to make the arms assume a twisted shape, and produce on them a lateral inflexion. It is to obviate and prevent this, that the arms are strengthened by feathers.

The power acts with greatest effect near the boss of the wheel, so that it is necessary to make them wider at this part than near the crown, so as to approximate to the form which presents an equal resistance throughout. This will be observed in the figures in Plate 20. The boss must have such a thickness as will allow of the wheels being solidly fixed on the shaft. A thickness of five inches may be considered a maximum for the bosses of moderately-sized wheels. The dimensions of the arms should be in proportion to the width of the web or crown, their thickness being ordinarily about one third that of the crown or web. This proportion is a good one for wheels under six or seven feet in diameter. For larger sizes, one-fourth the width of the web is considered sufficient.

The lateral feathers should have, at the very most, only the thickness of the arm. Generally, the width of the arm near the web is made about two-thirds of its width near the boss.

WOODEN PATTERNS.

214. When a casting has not to be turned, or otherwise reduced, about 1 per cent. must be allowed in the dimensions of the pattern, and if the piece has to be turned, a little more than this. It is, however, impossible to give any rule in this last case, as the allowance to be made depends entirely upon the nature and destination of the piece. The larger the piece, the greater should be the per centage given.

No piece can be cast with mathematical precision. Sometimes the pattern loses its true shape, and lines, which have been made perfectly straight or circular, become twisted, notwithstanding that every precaution has been taken in perfecting it; and sometimes in lifting it from the loam, the moulder is forced to move it laterally, to some slight extent, so that the casting becomes larger at one part, or twisted at another. Moreover, the metal does not shrink equally at all parts. With regard to the last-mentioned source of error, it has often been found that the diameter of a wheel, measured through the line of the arms, is sensibly less than as measured across the centres of the spaces between the arms. This difference is indeed so great, that in wheels of 10 to 15 feet diameter, it reaches an eighth or a sixth of an inch.

It is manifest, that all these considerations must be borne in mind when constructing wooden patterns for castings; otherwise, errors of considerable magnitude will arise.

CHAPTER VIII.

CONTINUATION OF THE STUDY OF TOOTHED GEAR.

CONICAL OR BEVEL GEARING.

215. Cylindrical or spur-wheels are only capable of transmitting motion between shafts which are parallel to each other. When the shafts are inclined, or form any angle with each other, the wheels require to be made conical, and are then called bevel-wheels.

In order that this description of gear may be capable of working well and regularly, and of transmitting considerable power when needed, as with spur gear, it is essential that the shafts or axes of any pair working together be situated in the same plane; in this case, the axes will meet in a point which is the apex common to the two wheels.

Formerly, when it was required to transmit power through shafts intersecting each other at right angles, a species of lantern-wheel was employed for one of the wheels, consisting of a couple of discs with cylindrical bars for teeth, passing from one to the other parallel to the axis; and the wheel to gear with this one was formed with similar teeth, also parallel with the axis, but projecting up from a single disc or ring. This form of gearing is still to be found in old mills; but it is very defective, and very inconvenient when any speed is required.

Sometimes, as for particular kinds of spinning machinery—the cotton-spinner's fly or roving-frame, for example—bevel-wheels are used, in which the axes are not situated in the same plane; these are termed "skew bevels," and the pitch surface is hyperboloidal. This kind of wheel does not work well, and is seldom employed, except where the size is very small, or where a small power only has to be transmitted; the peculiar form of their teeth, also render them very difficult to construct. Their use is so limited, that further details respecting them are uncalled for. Indeed, they ought rather to be avoided, since there are very few cases in which common bevel wheels cannot be substituted for them with advantage.

The teeth of bevel-wheels are made of wood or metal similarly to four-wheel teeth, and their geometrical forms are determined on the same principles.

DESIGN FOR A PAIR OF BEVEL-WHEELS IN GEAR.

PLATE 22.

216. We propose, in the present example, to give the larger wheel wooden teeth, and the smaller ones cast-iron ones, as was done with the pair of spur-wheels last described.

Let A B and A C, Figs. 1 and 2, be the axes of the two wheels assumed here to be at right angles to each other; though we must observe that what follows will apply equally well to the construction of a couple of wheels, the axes of which make any angle with each other, acute or obtuse.

It is, in the first place, necessary to determine the position the pitch circles should occupy on their respective axes. With this view, on any point, B, taken on the axis, A B, erect a perpendicular, B D, and make it equal to the radius of the smaller wheel, and through the extremity, D, draw a line, D L, parallel to this axis; in the same way, at any point, E, taken on the axis, A C, erect the perpendicular, E F, equal to the radius of the larger wheel, and through the extremity, F, draw F H parallel to A C. The point of intersection, G, of these two lines, F H and D L, is the point of contact of the two pitch circles, the radii of which are G I and G K. Make I H and K L, respectively, equal to the radii, and join the points, H G L, to the common apex, A, thereby determining what are termed the "pitch" cones, A H G, and A G L, of the two wheels, the generating straight line, A G, being the line of contact of the two cones. These pitch cones possess the same properties as the pitch circles, or, more correctly, pitch cylinders, of spur-wheels; that is to say, their rotative velocity is in the inverse ratio of their diameters, and their diameters are proportional to the respective numbers of their teeth.

The proportions of the pitch cones being thus obtained, with the centres, O and O^1, Figs. 2 and 3, taken on the prolongation of the given axes, describe the pitch circles, A H^1 I^1 and G^1 K^1 L^1. Divide these circles into as many equal parts as there should be teeth; that is to say, in the present case, 24 and 48, respectively, which operation will give the pitch; each part is then bisected to obtain the centres of the teeth and of the intervals, and on each side of the centre lines are set off the demi-widths of the teeth, regard being had to the difference to be made between the wooden and cast-iron teeth, as already explained (Par. 201.)

The external contours of the teeth will be situated in cones, the generating lines of which are perpendicular to those of the pitch cones; they are obtained by drawing through the point of contact, G, on the line, A G, a perpendicular, B C, meeting the axis of the smaller wheel in B, and that of the larger one in C; the points, B and C, are the apices of the two cones, B H G and C G L.

If these last-mentioned cones be developed upon a plane, it will be easy to draw upon it the exact forms of the teeth. Now, we have seen (Par. 167), that the development of a cone on a plane surface takes the form of a sector of a circle, which has for radius the generatrix of the cone, and for arc the development of the base of the cone. As it is unnecessary to develop the entire cone in the present case, it is sufficient to describe with any point, B^1 Fig. 4,

BEVEL

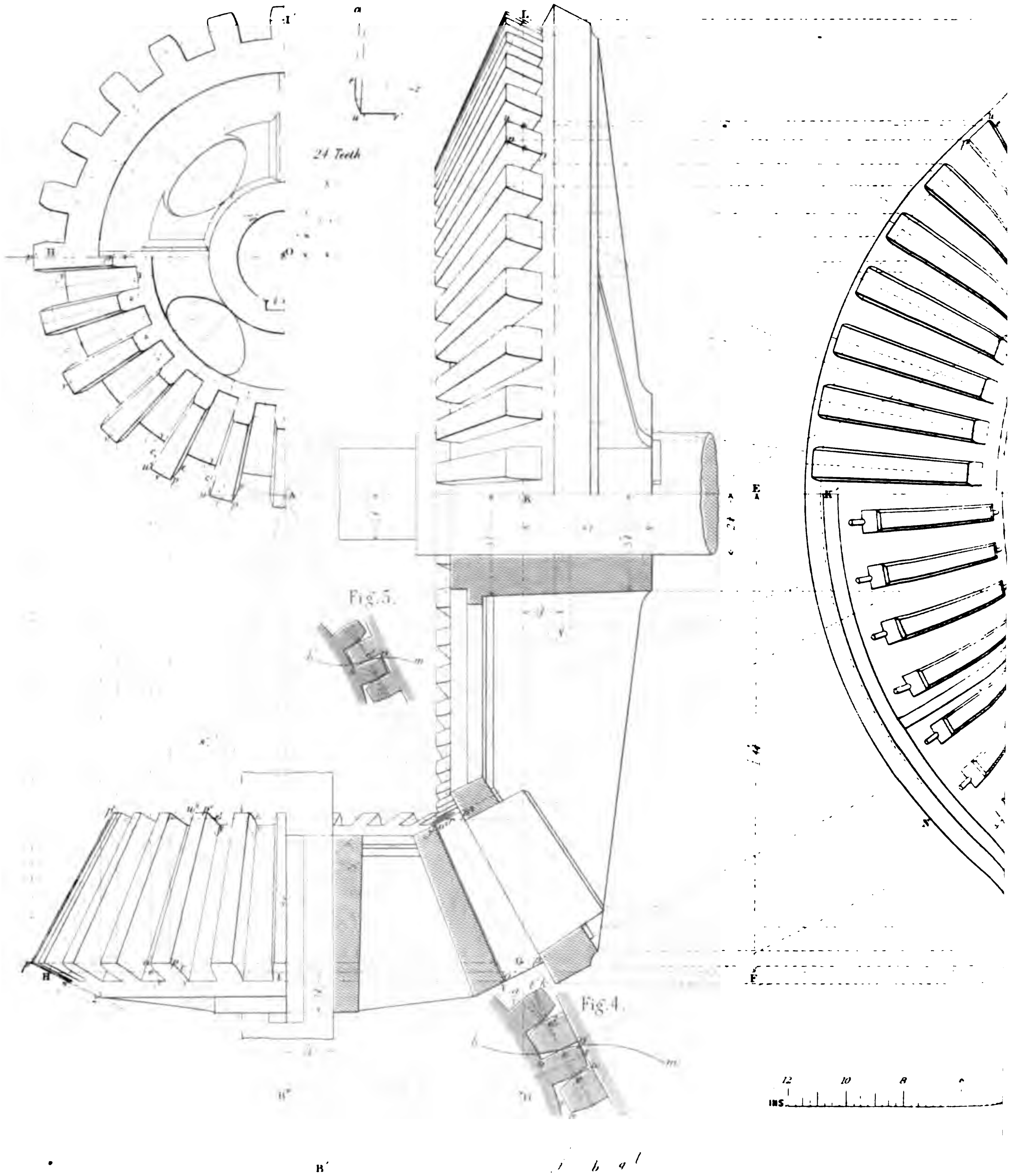


Fig. 3. Fig. 6. Fig. 1. Fig. 5. Fig. 4.

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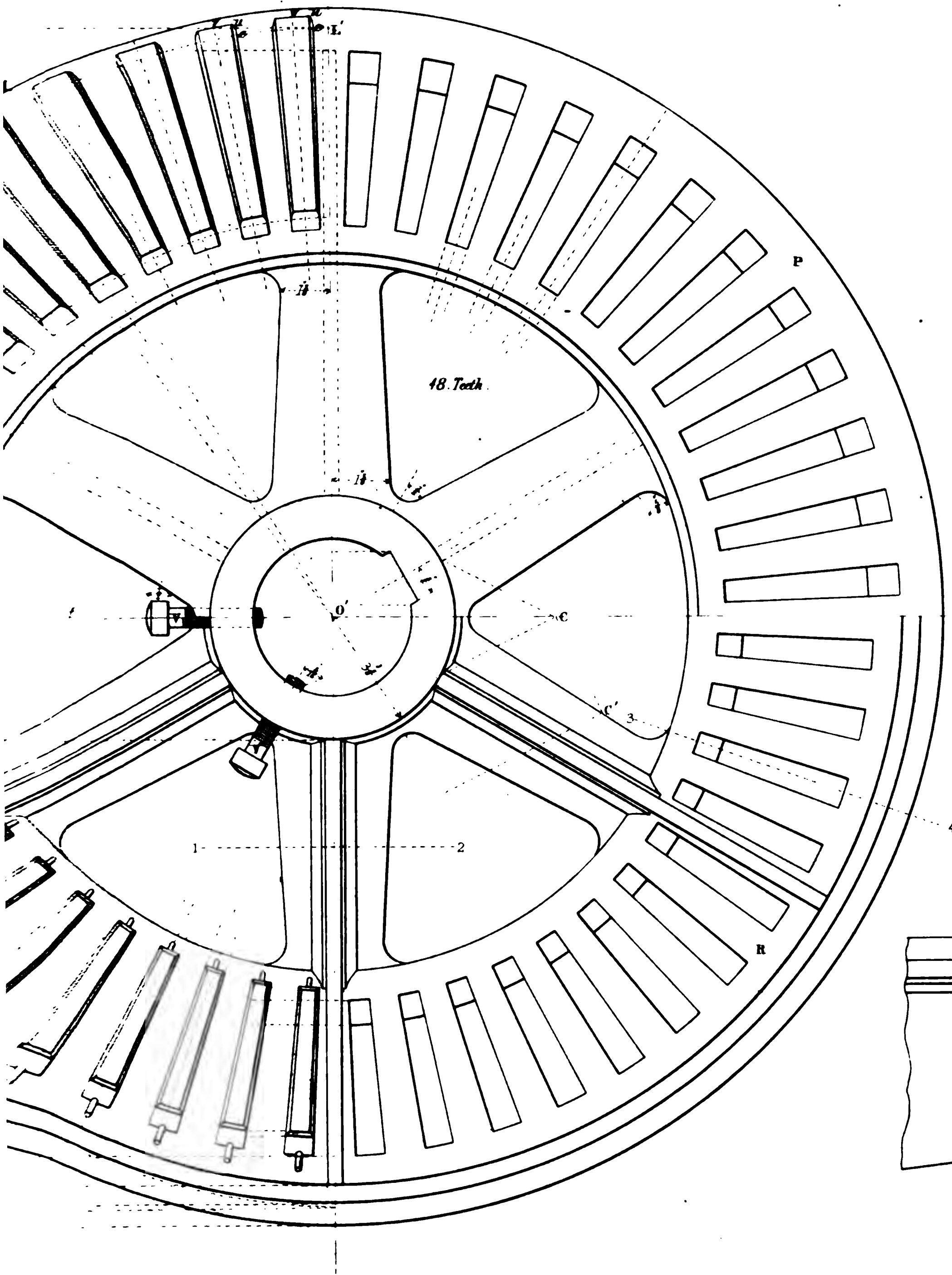
EARING.

Fig. 2.



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2 0 1 FOOT.

Fig. 7.

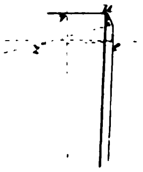


Fig. 8.

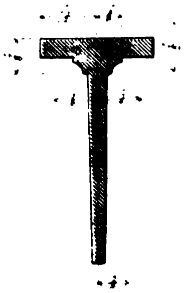


Fig. 9.

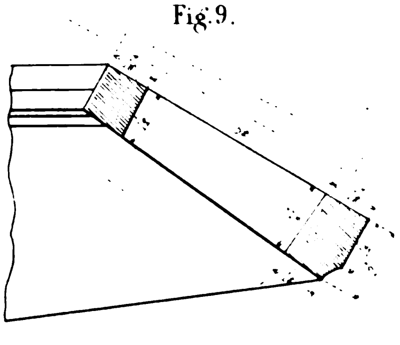
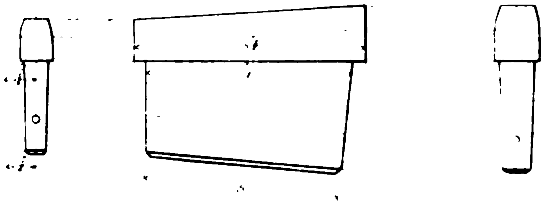


Fig. 10.



with a radius equal to B G, an arc, *a c b*, on which, starting from the point, *c*, are divided off distances—one, *c e*, equal to the thickness of the tooth of the smaller wheel, Fig. 3, and the other *c d*, to that of the tooth of the larger wheel, Fig. 2. The same operation is performed for the larger wheel; that is, with the point C^1, situated on the prolongation of B^1 *c*, and with a radius equal to C G, describe the arc, *f c g*, on which are measured the distances, respectively, equal to the former ones, *c e*, and *c d*.

This done, the outlines of the teeth are obtained by means of precisely the same operations as those explained in reference to the spur-wheels. Thus, on the radius, B^1 *c*, considered as a diameter, describe a circle, *i c j*, which, in rolling round the circle, *f c g*, considered as the pitch circle of the larger wheel, determines the epicycloid, *c h*, which gives the curvature of the teeth of the larger wheel; in the same manner, the circle, *k c l*, described on the radius, *c* C^1, as a diameter, and rolling round the circle, *a c b*, gives the epicycloid, *c m*, which is taken for the curve of the teeth of the smaller wheel. After having repeated these curves symmetrically on each side of the teeth, these are limited by drawing chords in the generating circles from the point, *c*, each equal to the pitch of the teeth, as *c n*, *c k*, and then with C^1 and B^1 as centres, describe circles passing, one just outside the point, *n*, and the other just outside the point, *k*; and to indicate the line of the web, describe a second couple of circles, nearly tangents to the preceding. Then project the points, *o* and *p*, which indicate the depth and extremities of the teeth, over to the line, B C, in o^1 and p^1; through these last draw straight lines to the apex, A, which will represent the extreme generating lines of the teeth, as in vertical section.

As all the teeth converge in one point, it is obvious that the contour of the inner ends of the teeth cannot be the same as that of the outer ends; the difference is the greater, according as the width, G *r*, on the generating line of contact is itself greater, in proportion to the entire cone, and to the greater or less angle formed by the extreme generatrices. In other respects, this contour is determined in the same manner as the first. Thus, through the point, *r*, is drawn the straight line, *s t*, perpendicular to A G, which cuts the two axes, and gives the proportions of the two cones, on the surface of which lie the contours of the inner ends of the teeth. Continuing the operation as above, portions of the cones are developed, arcs being described with the points, s^1 and t^1, as centres, and radii, equal to *r s* and *r t*. The diagram, Fig, 5, which is analogous to Fig. 4, fully explains what further is to be done.

What has been said, so far, has referred only to one tooth of each wheel. In proceeding with the execution of the design, after cutting out templates to the form of the teeth as obtained by means of them, the outline is repeated, as often as is necessary, on the external cones, the generating lines of which are B G and G C, for the outer ends of the teeth, and on the internal cones, the generating lines of which are *r s* and *r t*, for the outlines of the inner ends of the teeth. At the same time, and in order that the operation may be performed with regularity, a series of lines should be drawn through the points, *o*, *p*, of the two wheels lying on the surface of the external cones, A H G, A G L, and uniting at the apex, A, by means of a "false square," of a form analogous to that represented at X, in Fig. 3, Plate 23, for the smaller wheel, and like that represented at T, Fig. 4, of the same Plate, for the larger wheel.

The forms of the teeth being thus obtained, the partial section, Fig. 1, of the two wheels is drawn, the radii of the shafts being given, as well as the thickness of the bosses and webs, the proportions employed in the present example being indicated on the drawings. It will be observed that those teeth which are of wood are adjusted in the web of the larger wheel, in the same manner as in the spur or cylindrical wheels, the forms of the tenons being modified, so that their sides all incline to the common apex, A. The sections, together with the developments, Figs. 4 and 5, are sufficient for the purposes of construction, as all the required measurements can be obtained from them; but when it is desired to produce a complete external elevation of the two wheels, it will be necessary to find the projections of the teeth and other parts. With this view, the teeth are first actually drawn upon the planes of projection parallel to the bases of the wheels, as shown in Figs. 2 and 3. It will be recollected, that divisions have already been made on the pitch circles, A H^1 I^1, and L^1 K^1 G^1, indicating the centre lines, as well of the teeth as of the intervals, and marking the positions of the flanks; and, consequently, all that remains is to draw the external outlines and the curved portions. For the smaller wheel, the operation consists in projecting to p^1 in Fig. 3, the point, *p*, Fig 1, which limits the lower and outer edge of the tooth, and in describing with the centre, O, and radius, O p^1, a circle limiting the whole of the teeth externally, and corresponding to the section of the cone in which the points, *p* p^1, Fig. 1, lie. In this circle, also, terminate the curves of the outer portions of the teeth, and their exact points of intersection are obtained by measuring on each side of the centre lines, v^1 O, distances, v^1 u^1, v^1 p^1, equal to the corresponding distances, *v u*, *v p*, in Fig. 4. Then, through the points, u^1 p^1, draw a series of lines, converging to the centre, O; and through the points, *e*, *e*, found in a similar manner, draw similarly converging lines, indicating the inner angles of the intervals. Further, find the circular arc to represent the epicycloidal curve, passing through the points, u^1, p^1, and tangential at the same time to the lines, O *e*, at the points, *e*, *e*.

The mode of doing this is shown in Fig. 6: it consists in drawing through the point, *e*, a line, *e z*, at right angles to the radius, O *e*, and in bisecting the chord, e^1 u^1, by a perpendicular cutting, *e z* in *z*, which will be the centre of the required arc. Arcs of the same radius, *e z*, are employed for the curves of all the teeth on the smaller wheel, and the outline of these is completed by determining, in a similar manner, the arcs for the corresponding curved portions of the inner ends of the teeth, after having projected and drawn circles through the points corresponding to *r* and p^2, of Fig. 1. Finally, the lines of the web between the teeth—that is, the bottom lines of the intervals—are drawn, the projections of the points, *y* and y^1, being found, and circles described with the centre, O, passing through them. It will be observed that, on a portion of Fig. 3, is represented a view of a quarter of the lower and inner side of the wheel, whilst the other portion of the figure is an external view, showing the teeth as in plan; in the former case, the outline resembles that of a spur-wheel, for, as it is the larger ends of the teeth and web on which we are looking, the narrower and inclined portions are hid behind.

The lateral projection of the teeth of the small wheel, Fig. 1, is obtained, first, by successively projecting or squaring over, from the plan, Fig. 3, the points, *e*, *e*, to the pitch line, G H; and secondly, by similarly squaring over the points, *p*, *p*, to the external line, *p* p^1. Through the points, *e*, *e*, draw a series of lines converging at the apex, B, and representing the flanks of the teeth, and limited by the line, *y y*; then draw curves tangential to these flanks at the points, *u*, *p*, making them pass through the extreme points, *e*, *e*. Where the scale of the drawing is very large, and it is wished to be particularly precise in delineating these curves, points intermediate between u, p, and e, e, may be obtained by describing intermediate circles in Fig. 3, representing sections of the cone, projected in straight lines in Fig. 1, over to which are projected the points of intersection of the curves, with the circles in Fig. 3. Through the points, u, p, draw straight lines, converging in the apex, A, and find the lateral projection of the inner ends of the teeth, supposing planes to pass through the points, r, y^1, and p^2; these points in the circular projection of the planes, Fig. 3, being squared over to the corresponding rectilinear projections in Fig. 1. The inner ends of the teeth are then completed by drawing the flanks, $e^1\ y^1$, all converging in the apex, s, and joined by arcs passing through the points, e^1, p^1.

The upper left-hand quadrant, M, of Fig. 2, is a face view of the teeth of the larger bevel-wheel, with wooden teeth, the whole being drawn in the same manner, as Fig. 3. The method of finding the centre, Z, of the arc, which is substituted for the curved portion of each tooth, is shown in Fig. 7. From this view, Fig. 2, are obtained the various points required to produce the lateral projection of the teeth in Fig. 1. The operations are precisely the same as those just described in reference to Fig. 3, and the smaller wheel; the same distinguishing letters are also used to point out the similarity.

The same Figure (2) also comprehends at N a second quadrant of the wheel, drawn as seen from the under side, so as to show a face view of the tenons of the wooden teeth, the sides of which all converge in the point, o^1. A third quadrant, P, gives a view of the outer side of the web or crown, the teeth being supposed to be removed, so that the mortises are seen. The last quadrant, B, gives a back view of the web, also without the teeth.

Fig. 8 is a section of one of the arms of the larger bevel-wheel, made through the line, 1—2, in Fig. 2. Fig. 9 is a section of the web made through the line, 3—4, Fig. 2 passing through the centre of the mortise; and Fig. 10, comprehends a lateral projection and two end views of one of the wooden teeth.

Bevel, as well as spur-wheels, are fixed on their shafts by means of keys, and pressure screws, V, are often added to insure their perfect adjustment centrally.

THE CONSTRUCTION OF WOODEN PATTERNS FOR A PAIR OF BEVEL-WHEELS.

PLATE 23.

217. The observations we have already made with reference to the patterns of spur-wheels, are evidently equally applicable to the construction of patterns for bevel-wheels; still, at the same time, the difference in the form of the latter calls for further details, more especially apertaining to them.

PATTERN OF THE SMALLER BEVEL-WHEEL.

218. Figs. 1 and 2 represent the two projections of the pattern of the smaller of the two wheels in the preceding Plate. Fig. 3 is a vertical section through the line, 1—2, of Fig. 2, showing on one side the layers of wood put roughly together, and intended to form the crown; and on the other, a view of the same as finished, with the arm and its feathers.

It will be seen from these figures that the crown is built up in the same manner as that of the pinion in Plate 21; the layers of wood are, however, in steps, increasing in diameter downwards, so as to give the required conical form when turned. When these pieces are glued together, the whole is turned externally and internally in such a manner as to conform exactly to the full-sized drawing, previously made on a board planed smooth for the purpose. "Squares," also, should be made from the drawings, to serve as guides in producing the correct conical inclination.

After turning the top face, $b^1\ b^1$, perpendicular to the axis of the cone, the pattern-maker proceeds to turn the external conical surface, $a^1\ b^1$, of the web or crown. As a guide in doing this, he takes a "false square," T, Fig. 4, of which one side, $b\ b$, corresponds to the plane face, $b^1\ b^1$, and the other, $a\ b$, to the inclination of the conical, generatrix, $a^1\ b^1$: it is very easy with this to take off just as much of the wood as is necessary, without the liability of going too far. It is also necessary to determine the inclination of the generatrix, B a^1, of the outer cone, perpendicular, it will be recollected, to the contact generatrix, G r, by means of the square, X, Fig. 3, the side, $a\ b$, of which is applied exactly to the conical surface, $a^1\ b^1$, and the side, $a\ c$, then gives the inclination of the conical surface, $a^1\ c^1$; a similar square, made from the full-sized drawing, to suit the angle, $a^1\ b^1\ s$, and reversed, end for end, will give the inclination of the internal conical surface, $b^1\ d^1$, the generatrix of this, the smaller cone, being $s\ r$, parallel to B G, that of the larger one.

Finally, the thickness at $a^1\ c^1$ and $b^1\ d^1$, is measured on the wooden web, so as to obtain the proportions of the internal conical surface, $c^1\ d^1$, to be turned out in a similar manner.

The pattern being so far advanced, the external conical surface is divided into as many equal parts as there are to be teeth and intervals, and, with the assistance of the "false square," T, lines which represent generatrices of the cone, are drawn through the points of division to indicate the positions of the teeth or of the grooves to receive them.

Each tooth is cut out separately according to the full-size drawing made, as already mentioned, which, besides containing the vertical section, Fig. 3, should also show the exact form of each end of the tooth, B, and of the dovetail joint attaching them to the web. Fig. 5 shows a portion of this drawing for the larger ends of the teeth.

Mortises have now to be cut in the crown to receive the ends of the arms, C, and their feathers, E. As the wheel under consideration is of very small diameter, the number of arms is limited to four; these arms are so placed inside the crown that the feathers, are all on one side, and towards the wider end of the cone. Their attachment to the web is by means of a circular groove or mortise, seen at $e^1\ f^1$ Fig. 2, and at $g^1\ d^1$, Fig. 3, and they are united at the centre to each other and to the boss, in the same manner as the arms of the pinion, described in reference to Plate 21. The arms are not placed in the middle of the boss, as in the spur-wheel and pinion, but are simply applied to the base of the boss, which may, consequently, be of a single piece; and the feathers are let into a groove extending their whole length, and are fixed into the boss and crown at either extremity. The boss is slightly coned, so as to give the "draw" necessary in the construction of the mould. Its outer edges are indicated by the lines, $m\ n$, $m\ n$, whilst the other lines, $o\ p$, which are, on the contrary, parallel to the axis, show the depth of the grooves cut to receive the feather arms, E. These last, as shown in the section, Fig. 10, and taken on the line, 3, 4, Figs. 2 and 3, are thickest near the face arms.

The core pieces, F, are added on either end of the boss, and the whole is held firmly together by means of a central bolt.

PATTERN OF THE LARGER BEVEL WHEEL.

219. Figs. 6 and 7 represent the elevation and half plan of the pattern of the larger bevel wheel, with wooden teeth, represented in Plate 22. Fig. 8 is a vertical section through the axis of the

BEVEL-WHEE

Fig. 1.

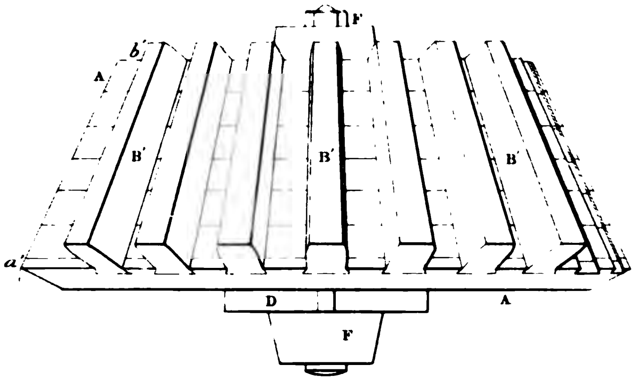


Fig. 2.

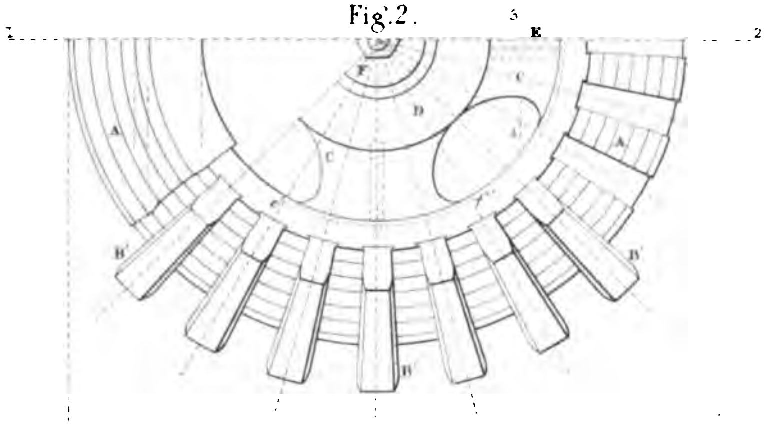
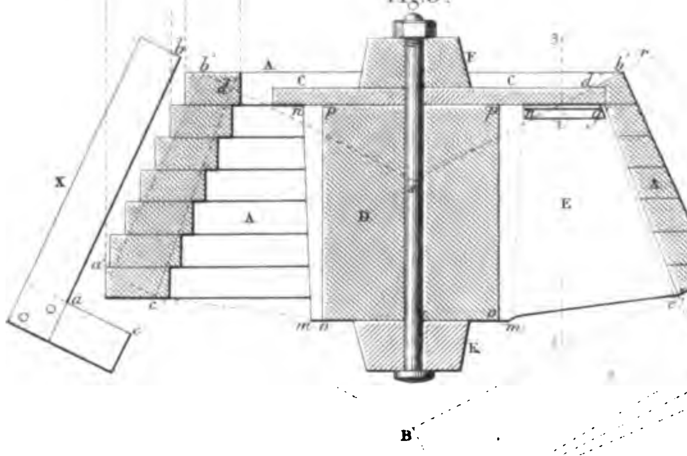


Fig. 3.



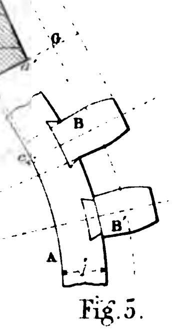


Fig. 5.

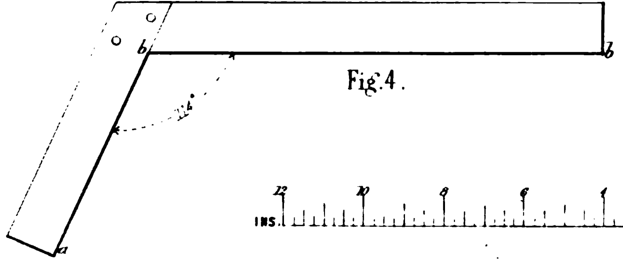


Fig. 4.

Fig. 10.

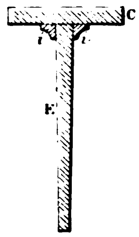


Fig. 11.

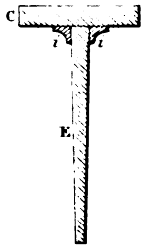


Fig. 12.

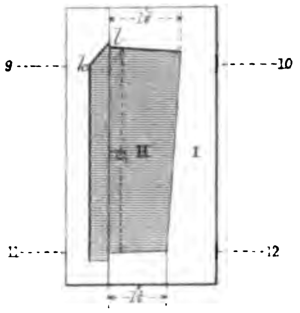


Fig. 13.

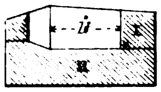
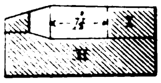
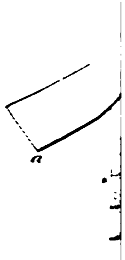


Fig. 14.





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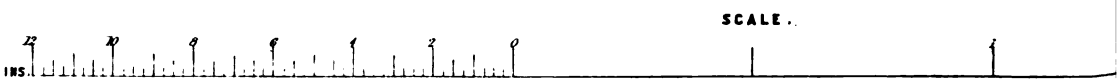

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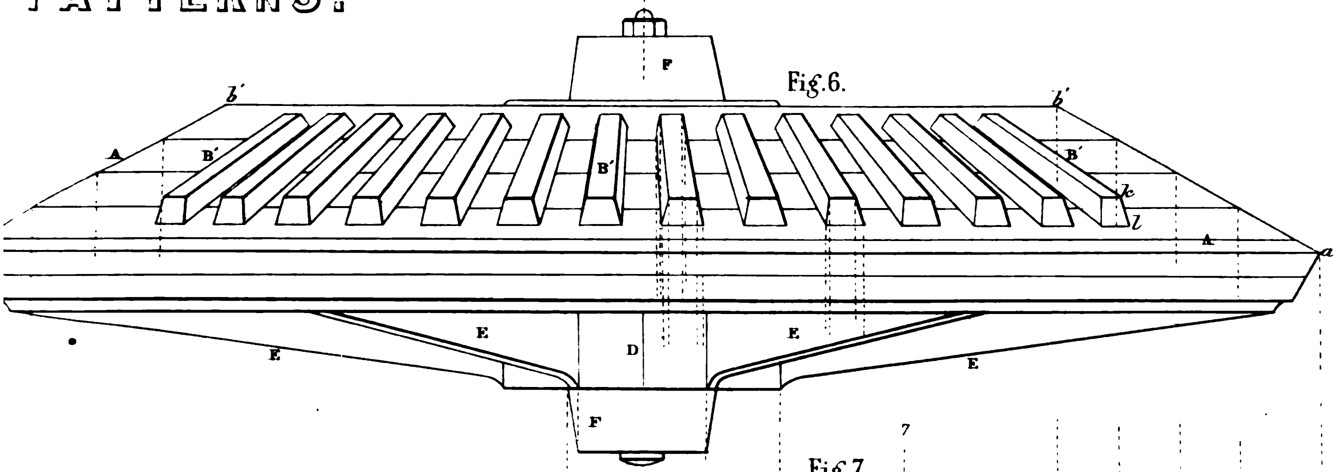
Fig. 6.

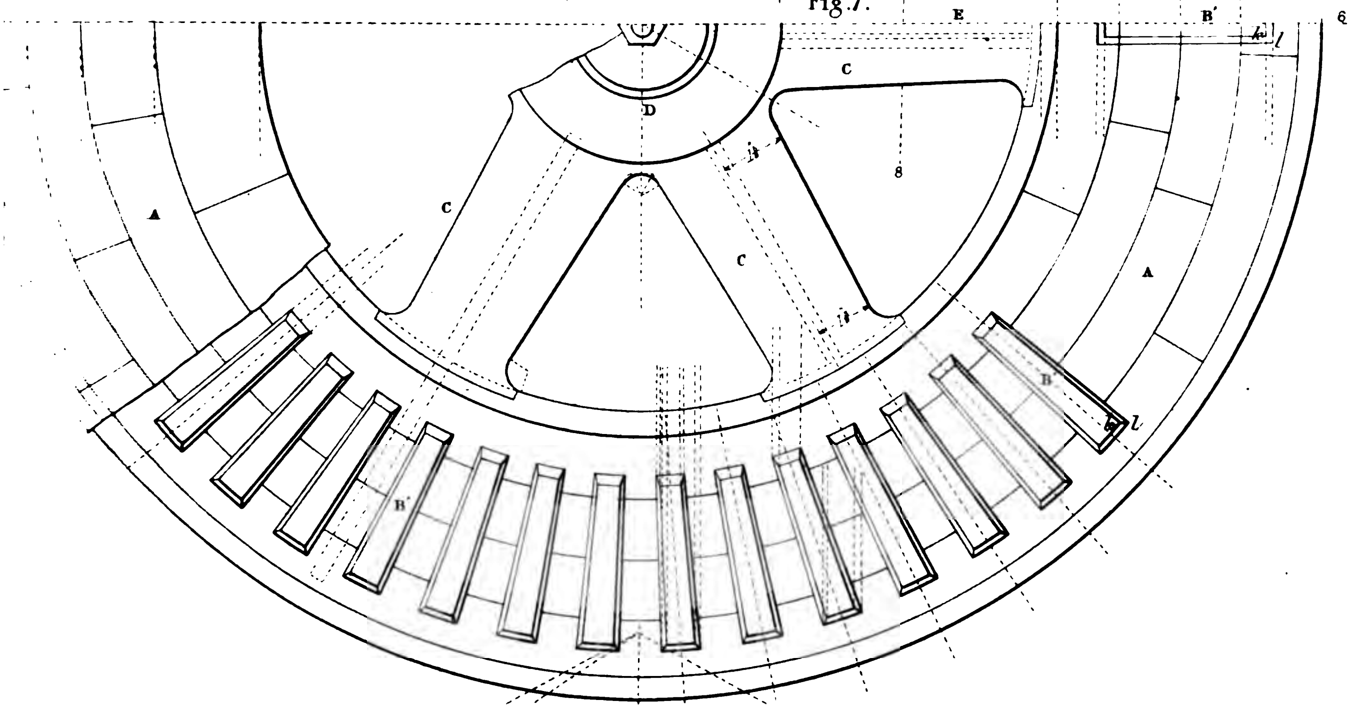
Fig. 7.

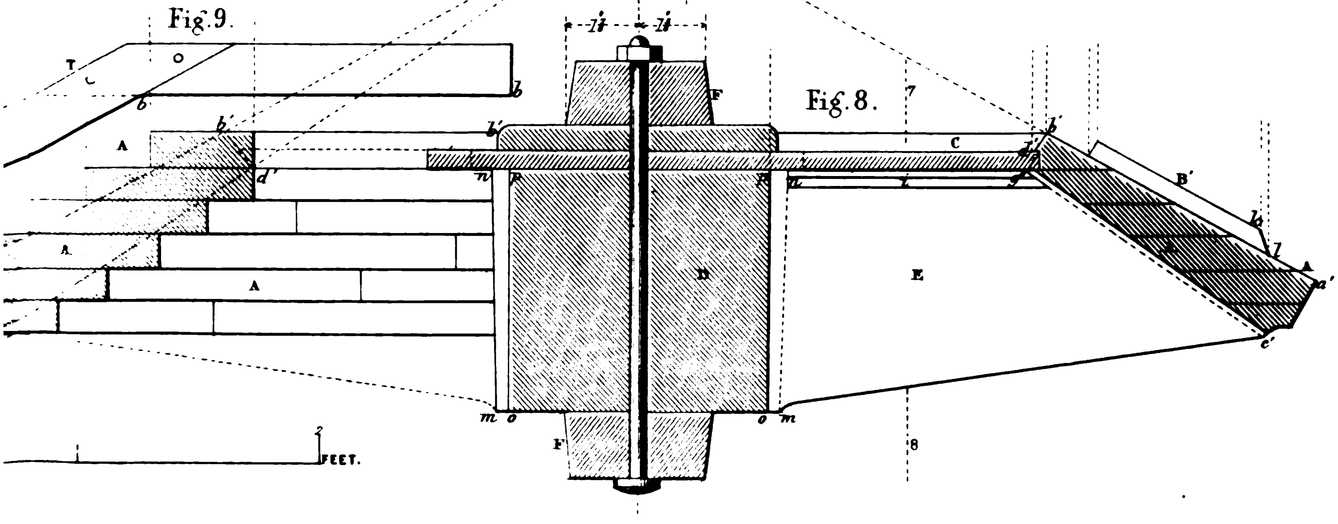
Fig. 9.

Fig. 8.

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Fig. 2.

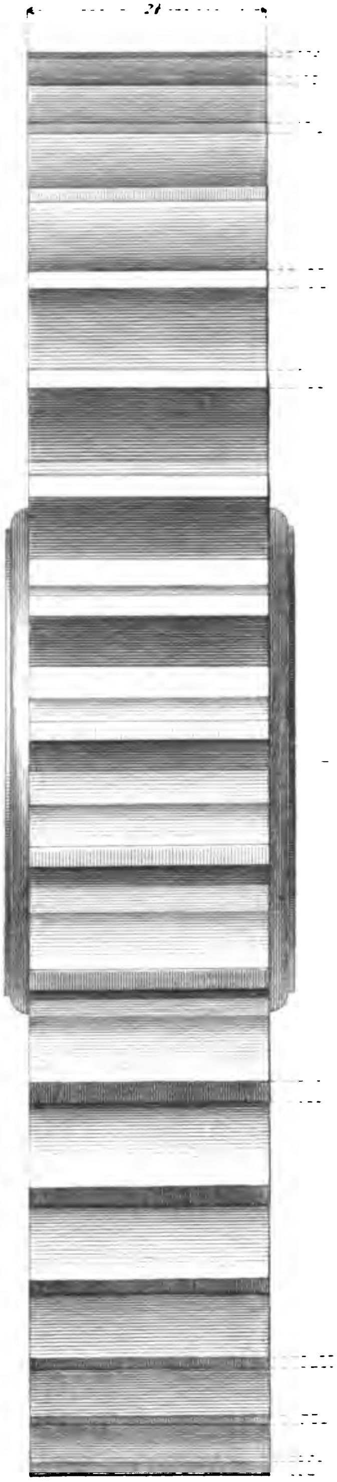


Fig. 1.

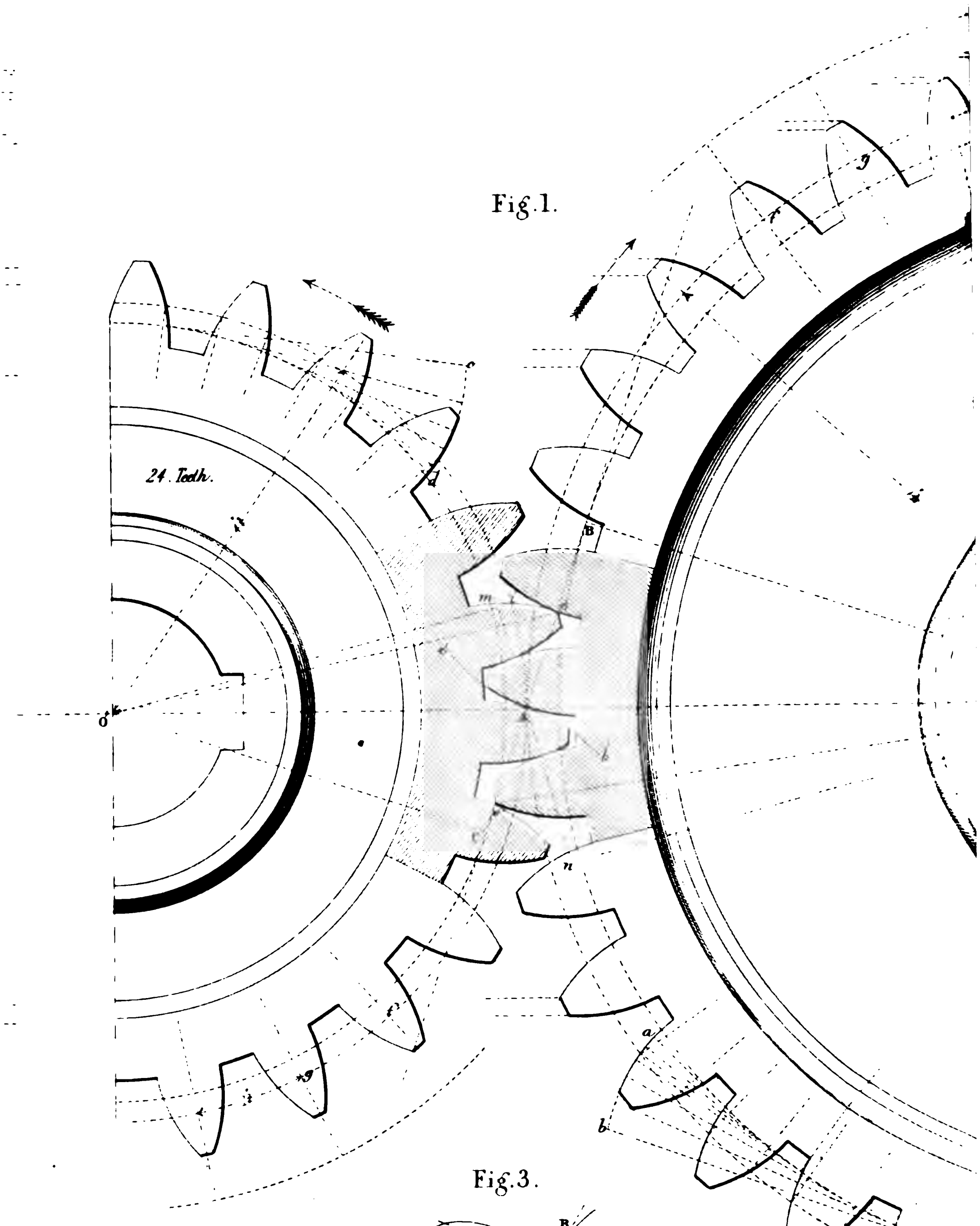
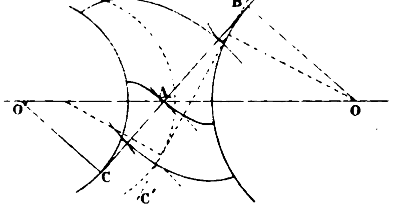


Fig. 3.



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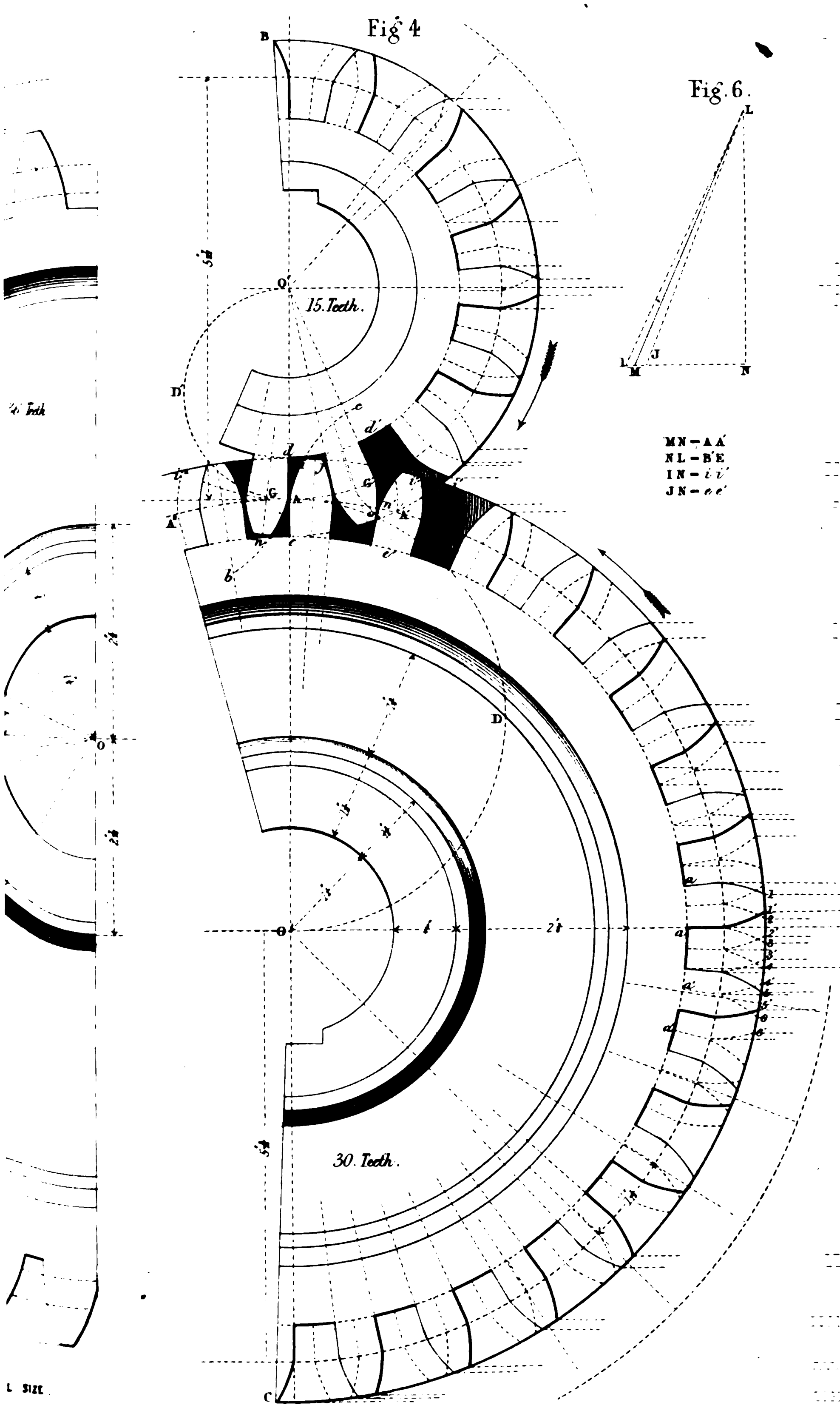
, TEETHED WHEELS.

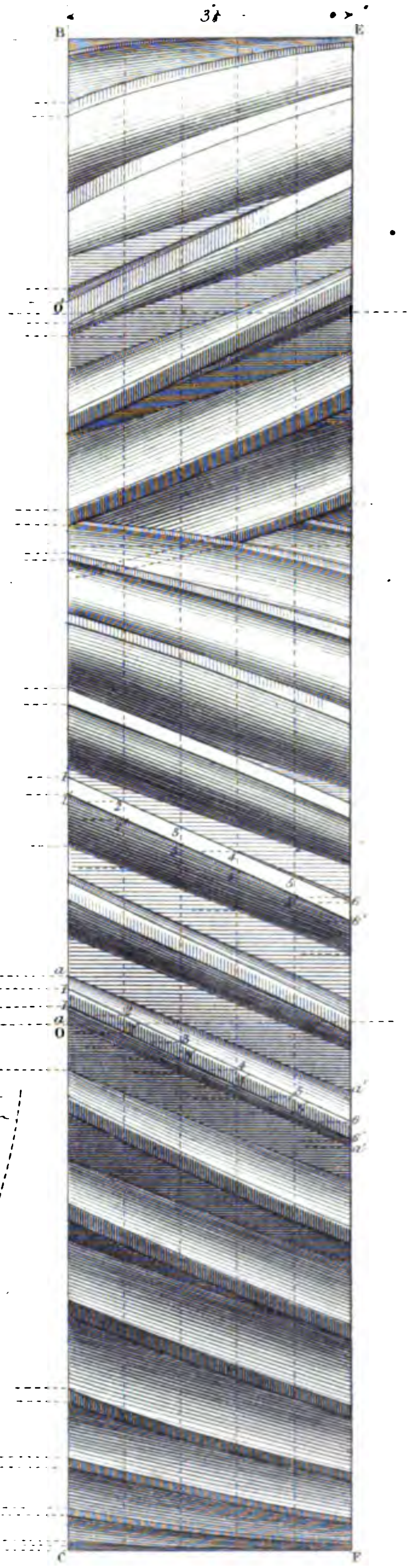
Fig. 4

Fig. 6.



MN = AA'
NL = B'E
IN = ii'
JN = ee'

Fig. 5.



[illegible]

wheel, taken on the line 5, 6, Fig. 7; showing on one side the arrangement of the pieces of wood built up upon one another, and forming the crown or web, A, and on the other side, the same piece, turned and finished, attached by the arm, C, to the boss, D.

Fig. 9 represents the false square, T, employed as a guide for giving the proper inclination to the external conical surface, a^1 b^1, of the crown.

Fig. 11 is a transverse section of one of the arms, or spokes, taken through the line, 7, 8; in Figs. 7 and 8.

Whatever explanations are called for regarding the construction of the crown, A, the arms, C, and the boss, D, as well as the uniting of these parts with each other, have already been given in reference to preceding examples. We have distinguished all the corresponding parts and working lines by the same letters.

The only difference between this last and the preceding example, consists in the disposition of the tooth pieces, or prints, B^1, placed on the outside of the crown, to form the sockets in the mould for receiving the core pieces for the mortises, into which the wooden teeth are to be fixed after the wheel is cast.

It must be observed, in the first place, that these projections must be shaped so that the end, *k l*, is inclined to the surface, b^1 a^1, instead of being perpendicular to it. This inclination must be sufficient to allow of the easy disengagement of the piece from the mould. This disposition is necessary, because the lower half of the mould takes the impression of the outside of the crown, with the tooth pieces and the upper portions of the arms, whilst the top part of the mould takes the inside of the crown, the feathers of the arms, and the boss, the position of the whole being the reverse of that in which they are represented in the drawing.

The core pieces for forming the mortises to receive the teeth are made by the moulder in core boxes, similar to those described in reference to Figs. 10 and 11, Plate 21, which we have reproduced in Plate 23, Figs. 12, 13, and 14, as modified to suit the different form of tooth. Fig. 12 is a face view, and Figs. 13 and 14 are sections made through the lines 9, 10, and 11, 12, of Fig. 12. It will be observed, that, at the larger end of the tooth, the part to project is formed with an inclination corresponding to *k l*, in Figs. 7 and 8, already referred to as required in this case.

The operations called for in delineating these patterns are all fully indicated, and are analogous to those in the preceding plates. The observations, also (Pars. 151 and 202), already made, with reference to calculating the allowance to be made for shrinking, and for the turning and finishing processes, are equally applicable to the case before us.

INVOLUTE AND HELICAL TEETH.

PLATE 24.

DELINEATION OF A COUPLE OF SPUR-WHEELS WITH INVOLUTE TEETH.

FIGURES 1 AND 2.

220. In the various systems of gearing just discussed, wherein epicycloidal teeth have been employed, it will have been observed—

1st. That the outline of the teeth of one wheel depends on the diameter of the other wheel with which it is in gear.

2d. That the distance between the centres of any couple of wheels cannot be altered in the slightest degree without deteriorating the movement.

3d. That the distance from the respective centres of the point of contact varies throughout the duration of the contact; from which must obviously result irregularity in the action and inequality in the amount of friction.

The practical defects arising from these causes have induced a search after other forms, and amongst these a modification of the involute has been tried. The form in question possesses the following advantages:—

1st. The form of the teeth of such a wheel is quite independent of the diameter of the wheel with which it is in gear.

2d. The distance between the centres of the wheels may be varied without disadvantage.

Some authors also attribute to this form the property of transmitting the pressure uniformly throughout the duration of the contact. This, however, cannot be the case altogether, for the distance of the point of contact from the centres of the wheels is constantly varying—the variation not being accurately proportional in the two wheels. This system of gearing is constructed on the following principles:—

Let O and O^1 be the centres of a wheel and pinion, given in face view, Fig. 1, the radii, O A and A O^1, of the respective pitch circles being given; also, let O B be the radius of any circle described with the centre, O; to the circumference of this last draw a tangent, A B, passing through the point, A, and prolong it indefinitely in either direction. From the centre, O^1, let fall on this line a perpendicular, O^1 C, on which will accordingly be the radius of a second circle tangent to the same line. These circles of the radii, O B and O^1 C, are those from which are derived the involute curves, *a b* and *c d*, forming the outline of the teeth. For the rest, the wheels are drawn just as in Plate 18 (Par. 185).

It must be observed that the curve, *a b*, which is the involute of the circle of the radius, O B, is that for the tooth of the spur-wheel, the centre of which is the same and the radius, O A; and, in like manner, the curve, *c d*, the involute of the smaller circle, is that for the teeth of the pinion of the radius, O^1 A. It thus follows that the form of the teeth of the spur-wheel is quite independent of the diameter of the pinion, whilst that of the pinion teeth is independent of the diameter of the spur-wheel. From which it follows, that wheels constructed in this manner may be set to gear with any wheels whose teeth are formed on the same principle, and whose pitch is the same, whatever may be their respective diameters. The epicycloidal system does not admit of this, although, when the wheels are large, and there is not much difference between their diameters, a slight deviation from strict mathematical proportions is not found practically inconvenient.

The involute curves, *a b* and *c d*, are repeated symmetrically on either side of the division lines representing the centre lines of the teeth. If we now suppose the two involutes, A b^1 and A c^1 to be in contact at the point, A, on the line of centres, O O^1, and we measure off on the common tangent, A B, a distance, A *e*, equal to the pitch, *f g*, as measured at the pitch circle, and then, with the centre, O^1, describe a circle passing through the point, *e*, this circle will be the external limit of the pinion teeth.

In like manner, if, on the other portion, A C, of the tangent, we measure a distance, A e^1, also equal to the pitch, *f g*, and with the centre, O, describe a circle passing through the point, e^1, it will be the limit of the spur-wheel teeth. It is further obvious, that circles passing a little within the point, e^1, on the one hand, and *e*, on the other, will determine the depth of the intervals, or the line of the web of the pinion and spur-wheel respectively.

Fig. 3 is a diagram to show—first, how that the point of contact of the two involute curves is always in the line of the common tangent, B C. Thus, referring again to Fig. 1, and supposing the pinion to turn in the direction of the arrows, the point of contact, at A of the involute, $a\,b^1$, is gradually removed away from the centre, O^1, of the pinion, whilst it approaches nearer and nearer to the centre, O, of the spur-wheel. Returning to Fig. 3, it is shown, in the second place, that the distance between the two centres, O, O^1, may be varied without its being necessary to alter the curves; but, in such case, the inclination of the tangent will be different, becoming, for example, as B C^1, when the two centres are brought nearer together.

In practice, instead of determining the radius, O B, arbitrarily, and then deriving the other radius, O C, from it, or *vice versa*, the circles which serve for generating the involutes may be found, as well as the inclination of the tangent, by the following method:—On one of the pitch circles, that of the pinion, for example, take an arc, A *i*, equal to the pitch of the teeth; draw the radius, O^1 *i*, and on it let fall a perpendicular, A *m*, from the point, A; O *m* will then be the radius of the generating circle for the involute curve of the teeth of the pinion, and by prolonging *m* A to *n*, which is, in fact, the common tangent, and drawing the radius, O *n*, perpendicular to it, or, what, is the same thing, parallel to O^1 *m*, O *n* will be the radius of the generating circle for the involute of the teeth of the spur-wheel.

If this rule is applied to wheels of large diameters, it will give curves differing very slightly from epicycloids.

By taking for the generating circles, as in the first case, radii, O B and O^1 C, sensibly less than the radii of the pitch circles, the inclination of the common tangent to the line joining the centres is greater, and the resulting form of tooth possesses greater proportionate width and strength at the roots, which is desirable for gearing intended to transmit great or irregular strains.

It will be observed further, that, according to this syetem, the rectilinear portion of the flank of the tooth is almost reduced to nothing, indeed the curve may be continued down to the line of the web with advantage, as the tooth will, in consequence, be much stronger near the web, which is not the case with the epicycloidal teeth, for in these the flanks all converge towards the centre of the wheel, and the tooth is, in consequence, narrower at the neck, close to the web, than at the pitch circle.

Fig. 2 is a fully shaded elevation, or edge view, at right angles to Fig. 1, of the spur-wheel separated from the pinion. The portions of these wheels not particularly referred to, are constructed on the same general principles as those previously discussed.

HELICAL GEARING.

FIGURES 4 AND 5.

221. If to a worm-wheel we apply, instead of a worm, a pinion with teeth helically inclined to correspond to the similarly inclined teeth of the worm-wheel, we shall have a spur-wheel and pinion constructed on the helical principle.

This system, invented in the seventeenth century by Hooke, but reproduced since by White and others, claims to possess two properties which have been often thought to be incompatible with each other—namely, uniformity of angular velocity, and freedom from other than rolling friction between the teeth. In other words, the arcs described by driver and follower will be equal in equal times, and the contact between the teeth will resemble that of circles rolling on planes.

Added to these properties, and consequent to them, are the advantages of a constant contact, and of an insusceptibility to the play between the teeth, which invariably exists more or less palpably in gearing constructed according to the systems before described.

The form of the helical teeth, as taken in a sectional plane at right angles to the axis of the wheel, may be derived either from a couple of epicycloids, or a couple of involutes; it is only the sides which, in common spur-gearing, are parallel to the axis that here follow the inclination of a succession of helices coming in contact one after the other. The arrangement is such that the contact of each tooth commences at one side of the wheel and crosses over to the other, and does not cease until the following tooth shall have commenced a fresh contact.

The helicoidal system may be applied either to wheels having their axes parallel, as spur-wheels, or intersecting, as bevel-wheels, or again inclined, but not intersecting, as skew bevels.

In Figs. 4 and 5 are represented, in face and edge view, a spur-wheel and pinion, constructed according to this system of Hooke's, this being its simplest and most common application:—Let A O and A^1 O be the radii of the respective pitch circles of the two wheels, these radii being, of course, in the same ratio as the numbers of the teeth, as in common gearing. The radii are supposed to lie in a vertical plane, B^1 C^1, and it is on this plane, as turned at right angles, that the operations represented in Fig. 4 are supposed to be performed.

These operations have for their object the obtainment of the outline of the teeth, and are precisely the same as for any other epicycloidal system of gearing. Thus, the curves, A *b* and A *c*, are derived from the generating circles, O D A and A D^1 O^1, as also the flanks, A *d* and A *e*; but it is unnecessary to repeat a detailed explanation of the operation.

Supposing, then, the outline of the teeth to be drawn as on the plane, B^1 C^1, representing say the anterior face or base of the wheels, next draw the line, E F, Fig. 5, representing the opposite face, and parallel to the first, limiting also, the breadth of the wheels.

To proceed methodically, the teeth should also be drawn as seen on this plane, E F being behind the outlines of the anterior ends of the teeth, a distance equal to A A^1, or rather more than the pitch. These last outlines need only be represented in faint dotted or pencil lines in Fig. 4, as the parts they represent are not actually seen in that view when complete. Thus, starting from the point, A^1, on the pitch circle of the spur-wheel, and from the point, G^1, on the pitch circle of the pinion, we repeat the contours of the teeth, as obtained at *e* A *i* and *d* A *n*, respectively.

As the result of this disposition, it will be observed, that if the curve, A *i*, of the tooth, A, of the spur-wheel is in contact, at the pitch circle, with the flank, G *d*, of the tooth, G, of the pinion at the anterior face, B^1 C^1, and if the wheels be made to turn to a certain extent in the direction of the arrows, the curve, A^1 i^1, on the opposite face, E F, will in time be found to be in contact with the corresponding flank, G^1 d^1, of the pinion. In other words, if the space between the curves, A *i* and A^1 i^1, be filled up by a helicoidal surface, as also the space between the flanks, G *d* and G^1 d^1, all the points of one such surface will be in contact successively with the corresponding points on the other; so that when, for example, the curve, A *i*, shall have reached the position, A^2 i^2; that is, when it shall have passed through a distance equal to A A^1, the posterior curve, A^1 i^1, will have assumed the position held originally by A *i*; or rather, a position directly behind this in the plane passing through the axis, and the point of contact between A^1 i^1 and G^1 d^1 will then obviously be in the line of centres, O O^1, on the plane, F E. It thus follows, that any two teeth which act on each other will be constantly in contact on the line of centres throughout a space equal to $A\,A^1$. This space, $A\,A^1$, is, as before stated, somewhat greater than the pitch of the teeth, so as to allow a following couple of teeth to act on each other, and be in contact on the line of centres before the couple in advance shall be quite free, and thus a constant contact on the line of centres is preserved throughout the entire revolution.

In order to delineate the lateral projection, Fig. 5, it will be necessary to find the curves which form the outline of the helicoidal surfaces of the teeth. The principle, according to which this is to be done, is precisely what has already been explained (196). In the present case, however, as we have but fragments of helices to draw, in place of finding the pitch of the helix, and then dividing it and the circumference proportionately, it will be sufficient to divide the width, $B^1\,E$, of the wheels, into a certain number of equal parts; and through the points of division, to draw lines parallel to $B^1\,C^1$. Further, the arcs, $A\,A^1$, $e\,e^1$, $i\,i^1$, must be divided into a like number of equal parts.

To render the diagram clearer, these divisions are transferred to 1, 2, 3, 4, &c., and $1^1, 2^1, 3^1, 4^1$, &c., Fig. 4. Each point, 1, 2, 3, 4, being squared over, in succession, to the corresponding lines in Fig. 5—namely, the lines of division first obtained, and lying parallel to the faces of the wheels, the operation will give the curve, 1, 3, 5, 6, in Fig. 5, corresponding to the outline of the external edge, extending from i to i^1. The similar curve, $1^1, 3^1, 5^1, 6^1$, gives the other edge, $j\,j^1$. It is also obvious that the line of junction of the tooth with the web will be represented by the helical curve, $a\,a^1$, Fig. 5, having the same pitch as the last, but lying on the cylindrical surface of the web, which is smaller in diameter by twice the depth of the teeth.

The lateral projections of all the teeth are determined in the same manner, but they will, of course, assume various aspects, from the different positions in which they lie with respect to the vertical plane.

222. In construction, in order to determine the exact inclination of the teeth, the following proportional formula is employed. The four terms of the formula being, the radius of the wheel, its width, the given circumferential distance, corresponding to $A\,A^1$, and the pitch of the helix; that is, $A\,A^1 : A\,O :: B^1\,E : x$; x being the helical pitch for the spur-wheel, or the quantity sought. It may be obtained geometrically, simply thus:—Make the straight line, $M\,N$, Fig. 6, equal to the arc, $A\,A^1$, as developed; at the extremity, N, of this line, erect a perpendicular, $N\,L$, equal to the width, $B^1\,E$, of the wheels; join $L\,M$, which will give the mean inclination of the tooth, corresponding to the pitch circle. Then make $N\,I$ equal to the arc, $i\,i^1$, rectified, and $N\,J$ equal to the arc, $e\,e^1$, rectified, which will give the inclinations, $L\,I$ and $L\,J$, of the helices, passing through the extremity, i, of the tooth, and the line, e, of junction of the tooth with the web.

It will be understood that the helices of the pinion-teeth will have the same inclination as those of the spur-wheel teeth, with which it is in gear, and the helical pitch is, in consequence, different; for, the radius is smaller, and the corresponding proportional formula becomes $A\,A^1$ or $A\,G^1 : A\,O^1 :: B^1\,E : x$.

The motion of wheel-work, constructed according to the helical system, is remarkably smooth, and free from vibratory action, but it has the defect of producing a longitudinal pressure upon the axes, from the obliquity of the surfaces of contact to the plane of rotation. This, however, may be obviated, and the longitudinal action balanced, by making the wheels duplex; that is, as if two wheels, on each axis, were joined together—the inclination of the helices being in contrary directions, or right and left handed. Such wheels, though duplex, need not be wider or thicker, in proportion, than simple ones; for the arrangement would permit of a much greater obliquity of the teeth, the only limit, indeed, to the degree, being the tendency to jam, which would arise were the inclination very great.

When the wheels are placed on axes which are inclined to each other, as in common bevel-wheels, the helices become such as are described upon conical surfaces, and require to be drawn in the manner already shown (171), the form of the tooth being previously determined, for each end, by means of the developed planes of the opposite faces of the wheels.

223. Besides the epicycloid and involute and their various combinations, other and more complex curves have at different times been proposed for the forms of wheel teeth. The most worthy of notice amongst these is that derived from the "hour-glass" curve, the properties of which have lately been investigated in a very scientific manner by Professor Sang of the Imperial School at Constantinople.

If a couple of discs, with their pitch circles touching, be made to revolve at a rate proportionate to the required number of teeth in each, a point may be imagined as travelling along a curve, returning upon itself in such a manner that it will describe the forms of the respective teeth on each disc. In the system of teeth alluded to, this point is made to travel along the "hour-glass" curve, a curve similar to that described by the piston-rod attachment in Watt's parallel motion, and also exhibited by the vibration of a straight wire, whose breadth is double its thickness. The form of tooth obtained in this manner is demonstrated by its inventor to be theoretically superior to all others yet known. The chief advantage appears to be, that whilst according to the epicycloidal and involute systems, the form of the entire tooth is made up of two curves of different natures, whose junction cannot, in consequence, be perfectly smooth or fluent, the point of inflexion or passage from one curve to the other, occurring, moreover, precisely where the best action would otherwise be. The "hour-glass" curve produces one continuous analytic curve for the entire outline of the wheel, thereby avoiding all sudden transitions, such outline, at the same time, allowing of the interchange, in any way, of wheels of the same pitch.

The great exactness and nicety obtainable by and called for in the construction of teeth on this system, is, however, far beyond the requirements of ordinary machinery. Indeed the practical engineer and machinist will not be at the trouble of employing even epicycloidal or involute curves, but contents himself with arcs of circles approximating pretty nearly to these curves. The method generally pursued in determining the best proportions for the radii of these substitutive arcs is as follows: A pair of templets or thin boards are cut to the curvature of the pitch circle and generating circle, respectively, of the wheel, the shape of whose teeth is sought. The generating templet carries a point which is made to describe the outline of the tooth on an additional board, by rolling its edge on that of the pitch templet. The operator then finds by trial with a pair of compasses, a centre and radius which will give an arc agreeing as nearly as possible with the curve traced by the templet. Through the centre thus found he describes a circle concentric with the pitch circle, and in which the centres for the arcs of all the teeth will obviously lie, and retaining the radius, he steps from tooth to tooth in both directions, until all the teeth are marked out.

224. A very ingenious and useful scale was invented some years ago by Professor Willis, which renders unnecessary this preliminary operation for obtaining the radii and centres. This scale, termed the "Odontograph," is now largely employed, and is found to give very excellent forms of teeth. Its application is very convenient. A graduated side of the instrument has a certain inclination to another, which is first made to coincide with a radius of the wheel, whilst its point of intersection with the first is placed in the pitch circle. The graduated side gives the direction in which the centres lie, whilst the lengths of the radii are obtained from tables calculated for the purpose, indicating the respective distances on the graduated scale, and corresponding to the given pitch and number of teeth.

Wheels with teeth formed according to this scale are capable of being interchanged, which is not the case with those in which the arcs are determined according to other rules.

After going through the explanations given, and rules laid down in the last few sections, the student should be quite competent to design practical arrangements and combinations of toothed gear according to whichever of the systems may be preferred.

CONTRIVANCES FOR OBTAINING DIFFERENTIAL MOVEMENTS.

THE DELINEATION OF ECCENTRICS AND CAMS.

PLATE 25.

225. Eccentrics and cams are employed to convert motion, whilst toothed-wheel work is for the simple transmission of it.

Endued themselves with a continuous circular movement, they are so constructed as to give to what they act upon, an alternate rectilinear movement, or an alternate circular movement, as the case may be, the motion so produced being obtainable in any desired direction.

CIRCULAR ECCENTRIC.

226. There are several descriptions of eccentrics. The simplest and most generally employed, consists of a circular disc, completely filled up, or open, with arms, according to its size, and made to turn in a uniform manner, being fixed on a shaft which does not pass through its centre. Such eccentrics are represented as working the valves and pumps of a steam-engine in Plate 39.

The stroke of such a piece of mechanism is always equal to twice the distance of its centre from that of the shaft on which it turns; that is to say, to the diameter of the circle described by its centre during a revolution of the shaft. The motion of the piece acted upon is uninterrupted during either back or forward stroke, but it is not uniform throughout the stroke, although that of the actuating shaft is so; the velocity, in fact, increasing during the first half of the stroke, and decreasing during the second half.

HEART-SHAPED CAM.

FIGURE 1.

227. When it is required to produce an alternate rectilinear motion which shall be uniform throughout the stroke, the shape of the eccentric or cam is no longer circular; it is differentially curved, and its outline may always be determined geometrically when the length of the stroke is known, together with the radius of the cam, or the distance of its centre from the nearest point of contact.

An example of this form of cam is represented in side elevation or face view, Fig. 1.

Let $a\ a^1$ be the rectilinear distance to be traversed, and *o*, the centre of the shaft on which the cam is fixed, it is required to make the point, *a*, advance to the point, a^1, in a uniform manner during a semi-revolution of the shaft, and to return it to its original position in the same manner during a second semi-revolution.

With the centre, *o*, and radii, *o a*, and $o\ a^1$, describe a couple of circles, and divide them into a certain number of equal parts by radii passing through the points, 1, 2, 3, 4, &c. Also divide the length, $a\ a^1$, into half as many equal parts as the circles, as in the points, 1^1, 2^1, 3^1, &c. Describe circles passing through these points, and concentric with the first. These circles will successively intersect the radii, *o* 1, *o* 2, *o* 3, &c., in the points, *b*, *c*, *d*, *e*, &c., and the continuous curve passing through these points will be the theoretical outline of the cam, which will cause the point, *a*, to traverse to a^1, in a uniform manner, for the equal distances, $a\ 1^1$, $1^1\ 2^1$, $2^1\ 3^1$, &c., passed through by the point, *a*, correspond in succession to the equal angular spaces, a^1 1, 1—2, 2—3, &c., passed through by the cam during its rotation.

As it is not possible to employ a mathematical point in practice, it is usually replaced by an anti-friction roller of the radius, *a i*, which has its centre constantly where the point should be; and it will be seen, that in order that this centre may be made to travel along the path already determined, it will be necessary to modify the cam, and this is done in the following manner:—With each of the points, *b*, *c*, *d*, &c., on the primitive curve as a centre, describe a series of arcs of the radius, *a i*, of the roller, and draw a curve tangent to these, and such curve will be the actual outline to be given to the cam, B.

It will be seen from the drawing that the curve is symmetrical with reference to the line, *a e*, which passes through its centre; in other words, the first half which pushes the roller, and consequently the rod, A, to the end of which the roller is fitted, from *a* to a^1, is precisely the same as the second half, with which the roller keeps in contact during the descent of the rod from a^1 to *a*. Thus the regular and continuous rotation of the cam, B, produces a uniform alternate movement of the roller, and its rod, A, which is maintained in a vertical position by suitable guides.

In actual construction, such a cam is made open, and with one or more arms, like a common wheel, or filled up, and consisting of a simple disc, according to its dimensions; and it has a boss, by means of which it is fixed on the shaft. When it is made open, it is cast with a crown, or rim, of equal thickness all round, and strengthened by an internal feather, curved into the boss at one side, and into the arm or arms at the other.

Examples of the heart-shaped cam are found in an endless variety of machines, and particularly in spinning machinery.

CAM FOR PRODUCING A UNIFORM AND INTERMITTENT RECTILINEAR MOVEMENT.

FIGURES 2 AND 3.

228. In certain machines, as, for example, in looms for the "picking motion," cases occur where it is necessary to produce a uniform rectilinear and alternate motion, but with a pause at each extremity of the stroke. The duration of the pause may be equal to, or greater, or less, than that of the action. Figs. 2 and 3 represent a side elevation and sectional plan of a cam designed to produce a movement of this description; and in this case the angular space passed through by the cam, in making the point, *a*,

traverse to the position, a^1, is supposed to be equal to half the angular space described by it, whilst the point, a, is stationary, whether in its position nearest to the centre, or its furthest, a^1, from it. For this reason, the circles of the radii, $o\ a$, and $o\ a^1$ are each divided into six equal parts in the points, a^1, 1, f, g, h, and j. Of these portions, the two opposite, 1 f and $j\ h$, correspond to the eccentric curves, $b\ f$ and $l\ h$, which produce the movement, whilst the other portions correspond to the pauses.

After having drawn the diameters, 1 h, and $f\ j$, the eccentric curves, $b\ f$, and $l\ h$, are determined in precisely the same manner as the continuous curve already discussed, and represented in Fig. 1. That is to say, the arcs 1 f, and $j\ h$, are to be divided into a certain number of equal parts by radial lines; and the line, $a\ a^1$, being divided into a like number of equal parts in the points, 2^1, 3^1, 4^1, &c., concentric circles are to be drawn through these points, which will be intersected in the points, c, d, e, by the radial division lines. Lines passing through these points of intersection will be the curves sought, $b\ f$, and $l\ h$.

The arcs, $b\ a\ l$, and $f\ g\ h$, which unite the extremities of the curves, are concentric with the shaft, and consequently, as long as the point remains in contact with these arcs, it will continue without motion, although the cam itself continues its rotation.

The observation made with reference to the preceding example of a cam, applies equally to the one, C, under consideration—that is, with regard to the actual shape to be given to it, which is derived from the substitution of an anti-friction roller of the radius, $a\ i$, for the mathematical point, a. The operation is fully indicated on the diagram.

This eccentric not being intended to overcome any great resistance, is made very light, a considerable portion of the metal being cut away, and merely a couple of arms left for stiffness. The crown, arms, and a great part of the boss, are, in fact, all of one thickness, as will be more plainly seen in Fig. 3, which is a sectional plan taken through the line, 1—2, in Fig. 2. Fig. 3 also shows the proportions of the roller, and its spindle.

When the moving point, or the roller, is constrained to move through an arc, instead of a straight line, being, for example, at the end of a vibratory lever, the curves of the cam are no longer symmetrical, but the operations by which they are determined are still the same, the difference arising from the divisions of the arc, which takes the place of the straight line, $a\ a^1$.

INVOLUTE CAM.

FIGS. 4, 5, AND 6.

229.—In certain industrial arts, an instrument is employed for pounding, crushing, and reducing substances, such as plaster or tanbark, for example, in which the direct-acting force is the weight of the instrument itself brought into play by its descent through a determined height. The mechanical forge hammer is a well-known application of this expedient.

In these cases, the stamp, or hammer, has to be raised or tilted up preparatory to each succeeding stroke, and it is obvious that this may be most economically done in a gradual manner. It is generally effected by a cam, the outline of which is the involute curve already described; this form being preferable on account of the uniformity of its action.

The office, then, which the cam under consideration has to fulfil, is the raising of the stamp, or load, to a certain height, and then the letting it fall, without impediment, upon the object submitted to its action.

The diameter of the cam-shaft being predetermined, as well as that of the generating circle, which last is usually the same as that of the boss of the cam, the design is proceeded with as follows:—

Letting A be the cam-shaft, and taking A o as the radius of the generating circle of the involute cam, B, whilst $a\ a^1$ is the height to which the projection piece, C, Figs. 4 and 5, formed on the stamp, D, is to be raised, develop the circumference (185) of the circle of the radius, A o, by means of a series of tangents which give the points, b, c, d, e, &c., the curve passing through which forms the involute, $b\ f\ i$. The inner portion, $b\ o$, is not a continuation of the involute, but simply joins the boss by an arc tangentially, because the stamp projection, C, does not approach the cam-shaft, A, nearer than the point, a, to which b corresponds. Through the point, a, draw the vertical, $a\ a^1$, and make it equal to the height to which the stamp has to be raised; then with the centre, A, and a radius equal to A a^1, describe the arc, $a^1\ m\ i$, which will cut the involute in the point, i, and this point is consequently the outer limit of the cam, which is completed by drawing the line, $i\ o$, tangent to the eye or generating circle, A o. A little consideration will show that if the cam-shaft, A, be turned in the direction of the arrows, supposing that it is originally placed, so that the point, b, coincides with a, it must necessarily raise the lifting piece, C, the lower side of which is indicated by the dotted line, $m\ a$, and will carry it by equal increments up to the position, $m^1\ a^1$. The point, i, will then have attained the position, a^1, and the rotation continuing, the next moment it will pass it, when the cam will be entirely clear of the lifting-piece, C, and this last being unsupported, must fall by its own weight.

The involute might have been derived from a generating circle of the radius, A a, and had this radius been adopted, the resulting curve would have been shorter, notwithstanding that it would give the same extent of lift. The angular space passed over would also be less, and this would admit of a higher velocity of the cam-shaft, and the strokes might be given in more rapid succession, whilst on the other hand, a greater power would be required to raise the same weight.

The cam we have represented in Fig. 4 is such as is employed to actuate the chopping stamp of mills for reducing oak, or other bark, for the preparation of tan. The bark is placed in a kind of wooden trough, E, solidly fixed into the floor. The stamps are armed with a series of cutters, F, in the form of crosses. The side of the trough next to the stamp is vertical, whilst the opposite side is elliptical in shape, and the matter under operation has, consequently, always a tendency to fall under the stamp. The stamps are kept vertical by guides in which they work. They are generally from 450 to 700 pounds weight, and fall through a height of from 16 to 20 inches.

Fig. 5 is a side elevation of the cam showing the shaft, A, in transverse section; and Fig. 6 is a plan of the cam, B, and a portion of its shaft corresponding to Fig. 5, indicating the widths of the rim and boss, while the thickness of the feather or disc uniting the two is shown in dotted lines.

A series of such cams are frequently employed in different planes on the same shaft, actuating a corresponding series of stamps, and in such case they are arranged in steps so as to come into action one after the other. Two or more are also sometimes employed in the same plane, and working a single stamp. In this latter case, the generating circle requires to be of much larger diameter in proportion, but the principle of construction is, however, the same.

TRIANGULAR CAM.

FIGURES 4, 5, 6, 7, AND 8.

230. A species of cam, in the form of a curvilinear equilateral triangle, is sometimes employed in the steam-engine to give motion to the slide valve. This valve is generally of cast-iron or brass, of a rectangular form, as shown at A, Figs. 7 and 8. It is hollowed out at its inner or face side to form a passage. This face, when planed (as well as the edges all round), is applied to the face, *a b*, of the cylinder, B, which is also planed true; both these surfaces are ground together, steam-tight, and are termed the valve faces. Its function is to allow the steam to pass alternately to the upper part of the cylinder, by the port, *c*, or to the lower part, by the port, *d*, whilst the hollow part of the valve forms a communication alternately between either of these ports, *c*, *d*, with the escape or exhaust pipe, E. To obtain the desired effect, it is necessary that the slide valve, A, be actuated with an alternate rectilinear or reciprocatory movement; for this purpose it is attached to a vertical rod, C, passing through a stuffing-box in the valve casing, and connected to the rod, D, represented in Fig. 9, and forming one piece, with the rectangular frame, F, inside, which works the triangular cam, G.

It is the last piece which has to effect the raising and lowering of the valve a certain distance and intermittently, in such a manner that the port, *c*, for example, may be open to the entering steam for a certain time, whilst the other, *d*, is in communication with the exhaust port and pipe, E, and so on reciprocally.

Let *o e*, Fig. 10, be the whole stroke of the valve, or the distance through which the valve traverses; with the centre, *o*, and with this distance, *o e*, for a radius, describe a circle, and divide it into six equal parts, in the points, *e*, 1, 2, 3, 4, and 5. With any two adjacent points, as 1 and 2, and with the same radius, *o e*, describe two arcs, *o* 2, and *o* 1, so as to form the curvilinear triangle, *o*—1—2, which is exactly the outline of the eccentric, G, each side of which is equal to a sixth of the circumference.

Draw the parallels, 5—1 and 4—2, tangential to the two sides of the triangle, G, and we shall thus obtain the upper and lower internal surface of the frame, F.

The bearing surface of the cam is made of steel, as well as the two sides of the frame, F, which bear upon it. It is adjusted and secured by the screw-bolt, *h*, to the disc, H, keyed on the shaft, J, as shown in the horizontal section, Fig. 11, taken through the line, 3—4, in Fig. 10.

It will be easily conceived, that if the shaft turns in the direction of the arrow, the curved side, *o* 1, of the cam, acting against the upper side of the frame, will cause it to rise, carrying with it the rod, D, in such a manner, that when the point, 1, shall have reached the position, *e*, that is, when the cam shall have made a sixth of a revolution, this side of the frame will occupy the position, *m n*, thereby indicating that the slide-valve has been elevated to a distance equal to half *o e*, and that, in consequence, the port, *d*, is uncovered, so as to allow the steam to enter the lower part of the cylinder, Fig. 7; whilst, on the other hand, a communication is established between the upper port, *c*, and the escape orifice, E, so that the steam can pass out from the upper end of the cylinder. If the movement of the cam be continued during a second sixth of a revolution, the slide-valve will remain in the same position, because the arc, 1—2, which is concentric with the axis, does not change the position of the frame, as long as it is in contact with its side, *m n*. As soon, however, as the point, 2, of the cam reaches the position, *e*, the side, *o* 1, will be in the position, *o* 5, and it will, in consequence, be in contact with the lower side of the frame, which is in the position of the horizontal centre-line, 3—4. The further revolution of the cam, therefore, makes the frame descend from its pressure on the lower side, until the side, *o* 1, of the cam, occupies the position, *o* 3, when the lower side of the frame will occupy the position, m^1 n^1, corresponding to the position of the valve, represented in Fig. 7.

It follows from this arrangement, that the valve will remain stationary when it arrives at each extremity of its stroke, and the pause each time will be of a duration corresponding to one-sixth of a revolution of the cam shaft. The upward and downward movements each take place during a third of a revolution, and the velocity of the valve will not be uniform, although the rotation of the cam shaft is so. In actual construction, the angles of the cam are slightly rounded off, to avoid a too sudden change of motion, and to prevent the too rapid wear of the sides of the frame.

CAM TO PRODUCE INTERMITTENT AND DISSIMILAR RECTILINEAR MOVEMENTS.

FIGURES 12, 13, 14, 15, AND 16.

231. In certain examples of steam engines, the valve movement is obtained from a species of duplex cam, which being formed of two distinct thicknesses, affords a means of adjustment whereby the valve may be made to move intermittently and at different rates, the proportions of which are variable at pleasure. The object of this is to form and shut off the communication between the cylinder with the steam-pipe, at any required point of the stroke. In other words, the arrangement permits of the working of the machine on the *expansive* principle, and of varying the "cut-off" point at pleasure within certain limits. We shall see, at a more advanced period, what is to be understood by the foregoing expressions. In designing cams of this class, we primarily determine the radius, *o a*, of the cam boss, and the entire length, *b c*, of the stroke to be given to the valve-rod. This distance, which in the present instance we shall take as equal to three times the height of the port, must not be traversed at one movement. On the contrary, a third only of this is at first passed through, with some rapidity, and the remaining two-thirds are traversed at a later period, in a continuous manner: in other words, after a third of the stroke has been traversed, a slight pause takes place before the remainder is traversed, and a second pause also occurs before the commencement of the return stroke.

After describing a couple of concentric circles with the respective radii, *o a*, and *o c*, and having determined the angular spaces, *a d*, and *f g*, corresponding to the times during which the valve is to remain stationary, and the spaces, *g h*, and *c f*, corresponding to the duration of the movements; divide the whole stroke, *b c*, into three equal parts in the points, *i*, *j*, through which describe circles concentric with the preceding. Through the points, *f*, *g*, *h*, draw radii, and produce them to f^1, g^1, and h^1.

As the cam will act on two friction rollers, G and G^1, diametrically opposite to each other, their radius is determined, as *a e*; one is drawn with its centre, *e*, on the radius, *o a*, produced, and tangential to the circle described with that radius; the other, with its centre, e^1, on the radius, *o c*, produced, is, in like manner, tangential to the circle described with this radius.

Between the two points, *d* and *k*, and comprised within the given angle, *g o h*, a curve, *k l d*, is drawn and united by tangential arcs at either extremity with the circles of the radii, *a o* and *o i*, respectively, in such a manner as to avoid any sudden change of direction. Next divide the arc, *g h*, into a certain number of equal parts in the points, 1, 2, &c., and carry the radii across to 1^1, 2^1, &c., then, on each of these radii, as a centre line, describe an arc corresponding to the radius of the roller, G, and tangential to the curve, *k l d*. By this means will be obtained the points, *r*, *s*, *t*, indicating the successive positions of the centre of the roller, G, on the line, $e\,e^1$, when impelled by the curve, *d l k*. If, then, starting from these several points, we measure on the corresponding cross lines, 1—1^1, 2—2^1, &c., distances equal to $e\,e^1$, which is obviously constant, we shall obtain the positions, r^1, s^1, t^1, of the centre of the opposite roller, G^1, corresponding to those of the first. Then, with these points, r^1, s^1, t^1, as centres, describe arcs of the radius of the roller, and draw the curve, $d^1\,l^1\,g^1$, tangential to them, and unite them to the circles of the radii, *c o* and *j o*, in a similar manner to the opposite curve. The curve, $d^1\,l^1\,g^1$, will obviously, from its construction, be in contact with the roller, G^1, whilst the first, *d l k*, is in contact with the other roller, G.

In order that the rollers, G, G^1, may maintain their relative position, and move in the same rectilinear direction, they are carried in bearings, H, forming, with four tie-rods, I, a frame which embraces the cam and cam-shaft, the middle of the rods being planed to rest and slide upon the latter.

To one end of the frame is bolted the cast-iron connecting-rod, J, Fig. 14, jointed to the bell-crank lever, K. This last vibrates on the centre, L, and by its second arm actuates the link rod, D, connected to the rod, C, of the valve, A, Fig. 12, above. In the position given to the cam and roller frame, in Fig. 15, the valve is not covering the upper port, c^1, and this remains open whilst the cam rotates through the angle, *a o d*, because the arc, *a d*, and its opposite, $c\,d^1$, are both concentric with the axis of the cam shaft, O.

When, however, the point, *d*, shall have arrived at the position, *a*, supposing the cam shaft to continue to turn in the direction of the arrows in the two views, 14 and 15, (which are turned as looking on the reverse sides,) the cam will shortly pass through the angle, *d o g*, and the projecting curve, *d l k*, will push the roller, G, to the right, in Fig. 15, and to the left in Fig. 14, and the opposite roller, G^1, being drawn in the same direction, will roll along the corresponding curve, $d^1\,l^1\,g^1$. This movement will cause the valve to be raised to the extent of a third of its stroke, corresponding exactly to the width of the port, c^1. This port will, in fact, be completely closed when the radius, *o k*, of the cam shall have reached the position, *o e*. At this point, the valve is required to remain stationary for a short time, during which the cam, in continuing to revolve, describes the angle, *g o f*. As soon, however, as the radius, *o f*, reaches the position, *o e*, the valve, and its actuating gear, will again move, and continue to do so, until the lower port, c^2, is completely open. This movement will take place whilst the cam, G, describes the angle, *f o c*, and is caused by the curve, $a^1\,b^1\,c$, which pushes the roller, G, and the frame still further to the right. The curve, $a^1\,b^1\,c$, is united by a gradual turn to the circles of the radii, *o k* and *o c*, in the same manner as the curves previously drawn. The opposite and corresponding curve, *a m n*, is obtained in the same manner as $d^1\,l^1\,g^1$, opposite to, and derived from, the first curve, *d l k*. The operations in both cases are fully indicated on the diagram, and it has merely to be borne in mind that the object is to keep the two rollers, G and G^1, in contact with the cam in every position of the latter.

After the cam has passed through the angle, *f o c*, the valve, with its gear, remains stationary during another interval, in which the angle, $c\,o\,d^1$, is traversed, and then the first curve will commence to act upon the roller, G^1, and cause it, with the frame, to return from right to left in Fig. 15, and from left to right in Fig. 14, and the movements and intervals will take place in the same order as to time as in the upstroke of the valve already described in detail; but the direction will be reversed—that is, the valve will perform its return stroke—until it reaches its original position, as represented in Fig. 12.

To proceed:—(as seen in Fig. 16,) the cam is constructed in two pieces, M and M^1, precisely alike in all respects, and placed close to each other on the shaft, O, that marked M being fast to the shaft, whilst the other, M^1, is capable of being adjusted to the first in any relative position. Since the rollers, G, G^1, are long enough to be in contact with both, it will follow they will, in any given position, be acted upon by that half of the cam which projects most at that particular point; so that, if the curved portion, *d l k*, of one is turned slightly in advance, it will come into action sooner, and, by consequence, will cause the valve to shut off the communication between the steam pipe and the cylinder at an earlier period of the stroke. In this manner is obtained a means of varying the rate of expansion at which the engine is worked.

Fig. 16 is a horizontal section, showing the two halves, M M^1, of the eccentric, and the arrangement of the details of the anti-friction rollers, G G^1, and frame, H I.

Fig. 13 is a front view of the valve face of a steam-engine cylinder, showing the disposition of the ports.

An innumerable variety of movements may be produced by the agency of cams; but the principles of their construction are mostly the same as those just discussed, and the examples given will be a sufficient guide to the student in designing others.

RULES AND PRACTICAL DATA.

MECHANICAL WORK, OR EFFECT.

232. To work, considered in the abstract, is to overcome, during any certain period of time, a continuously replaced resistance, or series of resistances. Thus, to file, to saw, to plane, to draw burdens, is to work, or produce mechanical effect.

Mechanical work is the effect of the simple action of a force upon a resistance which is directly opposed to it, and which it continuously destroys, giving motion in that direction to the point of application of the resistance. It follows from this definition, that the mechanical work or effect of any motor is the product of two indispensable quantities, or terms:—

First, The effort, or pressure exerted.

Second, The space passed through in a given time, or the velocity.

The amount of mechanical work increases directly as the increase of either of these terms, and in the proportion compounded of the two when both increase. If, for example, the pressure exerted be equal to 4 lbs., and the velocity 1 foot per second, the amount of work will be expressed by $4 \times 1 = 4$. If the velocity be double, the work becomes $4 \times 2 = 8$, or double also; and if, with the velocity double, or 2 feet per second, the pressure be doubled as well—that is, raised to 8 lbs.—the work will be, $8 \times 2 = 16$, or the quadruplicate of its original amount.

In the term "velocity," "time" is understood; so that, in fact, just as space or solidity is represented in terms of three dimensions, length, breadth, and depth, so also is mechanical effect defined by the three terms representing pressure, distance, and time. This analogy gives rise to the possibility of treating many questions and problems relating to mechanical effects by means of geometrical diagrams and theorems.

The unit of mechanical effect (corresponding to the geometrical cubical unit) adopted in England, is the horse-power, which is equal to 33,000 lbs. weight or pressure, raised or moved through a space of 1 foot in a minute of time. The corresponding unit employed in France is the kilogrammètre, which is equal to a kilogramme, raised 1 mètre high in a second. The kilogrammètre is equal to 2·2060 lbs. English, and the mètre is equal to 3·2809 feet English. Thus, supposing the pressure exerted be 20 kilog., and the distance traversed by the point of application be 2 mètres in a second, the mechanical effect is represented by 40 k. m.; that is, 40 kilog., raised 1 mètre high. When small amounts of mechanical effect are spoken of in English terms, it is generally said that they are equal to so many pounds raised so many feet high; that is to say, this takes place in some given time, as, for example, a minute, or it is expressed in the fractional part of a horse power. *Time* must always be expressed or understood. It is impossible to express or state intelligibly an amount of mechanical effect, without indicating all the three terms—pressure, distance, and time. It is to the losing sight of this indispensable definition, that we may attribute the vagueness and unintelligibility of many treatises on this subject. The French engineers make the horse-power equal to 75 kilogrammètres; that is, to 75 kilog., raised 1 mètre high per second.

The motors generally employed in manufactures and industrial arts are of two kinds—living, as men and animals; and inanimate, as air, water, gas, and steam.

The latter class, being subject only to mechanical laws, can continue their action without limit. This is not the case with the first, which are susceptible of fatigue, after acting for a certain length of time, or duration of exertion, and require refreshment and repose.

What may be termed the amount of a day's work, producible by men and animals, is the product of the force exerted, multiplied into the distance or space passed over, and the time during which the action is sustained. There will, however, in all cases, be a certain proportion of effort, in relation to the velocity and duration which will yield the largest possible product, or day's work, for any one individual, and this product may be termed the maximum effect. In other words, a man will produce a greater mechanical effect by exerting a certain effort, at a certain velocity, than he will by exerting a greater effort at a less velocity, or a less effort at a greater velocity, and the proportion of effort and velocity which will yield the maximum effect is different in different individuals.

TABLE OF THE AVERAGE AMOUNT OF MECHANICAL EFFECT PRODUCIBLE BY MEN AND ANIMALS.

NATURE OF THE WORK.	Mean weight elevated or effort exerted.	Velocity or distance per second.	Mechanical effect per second.	Duration per diem.	Mechanical effect per diem.
	Lbs.	Feet.	Lbs. raised 1 foot high.	Hours.	Lbs. raised 1 foot high.
A man ascending a slight incline, or a stair, without a burden, his work consisting simply in the elevation of his own weight,	143	·50	71·5	8	2,059,200
A labourer elevating a weight by means of a chord or pulley, the chord being pulled downwards,	40	·65	26·0	6	561,600
A labourer elevating a weight directly, with a chord, or by the hand,	44	·56	24·6	6	531,360
A labourer lifting or carrying a weight on his back, up a slight incline, or stair, and returning unladen,	143	·13	18·6	6	401,760
A labourer carrying materials in a wheel-barrow, up an incline of 1 in 12, and returning unladen,	132	·06	8·5	10	306,000
A labourer raising earth with a spade to a mean height of five feet,	60	·13	7·8	10	280,000
ACTION ON MACHINES.					
A labourer acting on a spoke-wheel, or inside a large drum.					
At the level of the axis,	132	·50	66·0	8	1,900,800
Near the bottom of the wheel,	26	2·30	59·8	8	1,722,240
A labourer pushing or pulling horizontally,	26	1·97	51·2	8	1,474,560
A labourer working at a winch-handle,	17½	2·46	43·0	8	1,238,400
A labourer pushing and pulling alternately in a vertical direction,	11	3·61	39·7	8	1,143,360
A horse drawing a carriage at an ordinary pace,	154	2·95	454·3	10	16,354,800
A horse turning a mill at an ordinary pace,	99	2·95	292·0	8	8,409,600
A horse turning a mill at a trot,	66	6·56	433·0	4½	7,014,600
An ox doing the same at an ordinary pace,	143	1·97	281·7	8	8,112,960
A mule do. do.	66	2·95	194·7	8	5,607,360
An ass do. do.	31	2·62	81·2	8	2,338,560

It may be gathered from this table that a labourer turning a winch handle can make its extremity pass through a distance of 2·46 feet per second, or 60 × 2·46 = 147·6 feet per minute. Then, supposing the handle has 13¾ inches, = 1·147 feet radius, which corresponds to a circumference of 6·28 × 1·147 = 7·2 feet at the point of application, the labourer is capable of an average velocity of

$$\frac{147·6}{7·2} = 20 \text{ turns (nearly) per minute.}$$

Also, the same labourer exerting a force equal to 17½ lbs. with the velocity of 2·46 feet per second, will produce a mechanical effect equal to

$17\tfrac{1}{2} \times 2·46 = 43$ lbs. raised 1 foot high per second, or of
$43 \times 60 = 2580$ per minute, and
$2580 \times 60 = 154,800$ lbs. raised 1 foot high per hour.

And as he can work at this 8 hours per diem, the total mechanical effect during this time will be, as indicated in the table, equal to 1,238,400 lbs. raised 1 foot high.

We may then calculate that, as a day's work, a labourer turning a winch-handle can elevate in a continuous manner 17½ lbs. 2·46 feet high per second; when, however, the labourer has only to apply his strength at intervals to a crane, a windlass, or a capstan, he can develop a much greater force for a few minutes. According to experiments tried in England with a discharging crane, a man can in 90″ raise a load of 1048·6 lbs. to a height of 16½ feet. Now, to compare this with the tabulated quantities, we must multiply the weight raised, 1048·6 lbs., by the height, 16½ feet, and divide the product by the duration of the action, or 90″; the quotient, 192, indicates the number of pounds raised 1 foot high in a second, to which the mechanical effect is equal.

It has been proved by experiments, that under the most favourable circumstances, an Irish labourer of extra strength can, by great exertion, raise to the same height, 16½ feet, a load of 1474 lbs. in 132″, which is equal to a mechanical effect of

$$\frac{1474 \times 16\cdot5}{132} = 184\cdot25 \text{ lbs. raised 1 foot high per second.}$$

A man can evidently only exert such a force during a very limited period; we cannot, therfore, compare this kind of labour with that which continues through several consecutive hours. Although the load and velocity as given in the table are those most conveniently proportioned to each other, still, when the case requires it, they might be altered to some extent; thus, if it is necessary to apply a force of 25 lbs. to the extremity of the winch-handle instead of 17½, then the velocity would be reduced, and would become

$$\frac{43}{25} = 1\cdot72 \text{ feet per second, instead of } 2\cdot46.$$

It has been ascertained from actual observations, that a horse, going at the respective rates of 1, 3, 5, and 10 miles per hour, cannot exert a greater tractive force than the corresponding weights, 194, 143, 100, and 24 lbs., and cannot draw anything appreciable when going at the rate of 15 miles per hour.

Thus, when it is wished to increase the force exerted, a decrease takes place in the velocity; and reciprocally, when it is wished to gain time and speed, it can only be done at the expense of the load. Thus, in the case of the winch-handle, the two factors must always produce an effect equal to 43 lbs. raised 1 foot high per second, whatever ratio they may have to each other.

In all cases of the direct action of forces, a certain *velocity* is impressed, for without *movement* there could not be the action of a force.

There are two kinds of motion—uniform and varied motion.

233. UNIFORM MOTION.—A body is said to have a uniform motion when it passes through equal distances in equal times. Thus, for example, if a body traverses 5 feet in the first second, 5 feet in the second, and so on throughout, its motion is uniform.

Putting D to represent the distance, V the velocity, and T the time, the formula, $D = V \times T$, indicates that the distance is equal to the velocity multiplied by the time.

First Example.—The velocity of a body subject to a uniform motion is 3 feet per second, through what distance will it have passed in 15 seconds?

$$D = 3 \times 15 = 45 \text{ feet.}$$

From the preceding formula, $D = V \times T$, is obtained $V = \frac{D}{T}$; that is to say, the velocity per second is equal to the distance divided by the time.

Second Example.—The distance passed through in 15 seconds is 45 feet, what is the velocity?

$$V = \frac{45}{15} = 3 \text{ feet.}$$

The wheel-gear and machinery, as well as many other instruments of transmission, is, for the most part, actuated in a uniform manner.

234. VARIED MOTION.—When a body passes in equal times through distances which augment or decrease by equal quantities, the motion is called uniformly varied.

The distance in motion uniformly varied is equal to half the sum of the extreme velocities multiplied by the time in seconds.

First Example.—What is the distance passed through in 4 seconds by a body in motion, the velocity of which is 2 feet per second at starting, and 6 feet per second at the termination?

$$D = \frac{2 + 6}{2} \times 4 = 16 \text{ feet.}$$

Second Example.—What is the distance passed through in 4 seconds by a body in motion, which at starting has a velocity of 6 feet per second, but which is gradually reduced to 2 feet?

$$D = \frac{6 + 2}{2} \times 4 = 16 \text{ feet.}$$

It will be seen from these two examples, that, with like conditions, the total distance is the same for motions uniformly accelerated or retarded.

The velocity at the end of a given time, in uniformly accelerated motion, is equal to the velocity at starting, plus the product of the increase per second into the time in seconds.

First Example.—What velocity will a body have at the end of 8 seconds, supposing the initial velocity = 1, and that it increases at the rate of 3 feet per second?

$$V = 1 + (8 \times 3) = 25 \text{ feet.}$$

The velocity which, at the end of a given time, a body uniformly retarded should have, is equal to the initial velocity minus the product of the diminution per second, multiplied into the time in seconds.

Second Example.—A body in motion starts with a velocity of 22 feet per second, and its velocity decreases at the rate of 2 feet per second, what will be the velocity at the end of 10 seconds?

$$V = 22 - (2 \times 10) = 2 \text{ feet.}$$

235. The motions of which the various parts of machines are capable are of two principal kinds—continuous and alternate, or back and forward motion.

These two kinds of motion may take place either in straight or curved lines, the latter generally being circular.

In the actual construction of machinery, we find that, from these principal descriptions of motions, the following combinations are derived:—

Continuous rectilinear motion is converted into	Continuous rectilinear. Continuous circular. Alternate circular.
Alternate rectilinear motion is converted into	Continuous rectilinear. Continuous circular. Alternate circular.
Continuous circular motion is converted into	Continuous rectilinear. Alternate rectilinear. Continuous circular. Alternate circular.
Alternate circular motion is converted into	Alternate rectilinear. Continuous circular. Alternate circular.

THE SIMPLE MACHINES.

236. This term is applied to those mechanical agents which enter as elements into the composition of all machinery, whether their function be to elevate loads, or to overcome resistances.

The simple machines (or "mechanical powers,") are generally considered to be six—the lever, the wheel and axle, the pulley, the inclined plane, the screw, and the wedge.

A more scientific and comprehensive arrangement of the elementary machines, is that lately suggested by Mr. G. P. Renshaw, C.E., of Nottingham. According to his system, the elementary machines, or mechanical powers, are five—namely, the lever, the incline, the toggle or knee-joint, the pulley, and the ram.

The wheel and axle, of the first system, is evidently but a modification of the lever, and the screw and wedge are modifications of the inclined plane; whilst no mention is made of the toggle-joint and ram—the last so well represented by the hydrostatic press.

All these machines act on the fundamental principle, known as that of *virtual velocities*. According to this principle, the pressure or resistance is inversely as the velocity or space passed through, or that would be passed through, if the piece were put in motion.

The *momentum* of the power and resistance is equal when the machine is in *equilibrio*. By *momentum* is understood the product of the power by the space passed through by the point of application.

Time is occupied in the transmission of all mechanical force. In any mechanical action we do not see the effect and the cause at the same instant. Thus, in continuous motion, in which the time expended is not apparent at first sight, each succeeding portion of the motion is due to a portion of the impelling action exerted a certain time previously. This will be more obvious on observing the commencement and termination of any motion. The motion does not commence at the instant that the power is applied, nor does it cease at the exact moment of the power's cessation. The fiction of the *vis inertiæ* has been invented to account for these latter observed facts, but it explains them very awkwardly. Thus, bodies are said to possess a certain force which is opposed to a *change* of state, whether from rest to motion or motion to rest. If such a resistive *force* existed, it would require an effort to overcome it, in addition to what is actually accounted for by the motion. If it is said that this is again given back at the termination of the motion, another fiction is required to account for it in the meantime, that is, during the continuation of the motion. Moreover, there is nothing analogous to it throughout the entire range of physical science.

The facts are described in a much more simple and philosophical manner, when they are said to arise from the *time* taken in the transmission of motive force. Why there should be this expenditure of time is a more abstruse question. It probably arises from the elasticity of the component particles of bodies and resisting media, and is regulated by the laws which govern the relation to time of the vibrations of the pendulum.

In all machines, a portion of the actuating power is lost or misapplied in overcoming the friction of the parts.

237. THE LEVER.—The lever, in its simplest form, is an inflexible bar, capable of oscillation about a fixed centre, termed the *fulcrum*. A lever transmits the action of a power and a resistance, or load; the distance of the power, or load, from the centre of oscillation, is called an arm of the lever.

There are two kinds of *power* levers, distinguished by the position of the fulcrum as regards the power and the resistance. These become *speed* levers, by transposing the power and resistance. By a *power* machine, is meant one which gives an increase of power at the expense of speed, and by a *speed* machine, one that gives an increase of speed at the expense of power, and all the simple machines are one or the other, according to the relative position of the power and resistance.

In all cases of the lever, *the power and the resistance are in the inverse ratio to each other of their distances from the centre of oscillation.* That is to say, that when, in *equilibrio*, the momentum of the power, $P \times A$, or the product of this power into the space described by the lever arm, A, is equal to the product, $R \times B$, of the resistance, into the space described by the lever arm, B: whence the following inverse proportion:—

$$P : R :: B : A;$$

Any three of which terms being known, the fourth can be found at once.

238. The wheel and axle is a perpetual lever. As a power, the advantage gained is in the proportion of the radius of the circumference of the wheel to that of the axle. That is to say, the power, P, is to the resistance, R, as the radius, b, of the axle, is to the radius, a, of the wheel, or the length of the winch handle—in the simpler form of this machine, consisting of an axle and a winch handle. The same rules and formulæ obviously apply to this, as to the first described form of lever.

Thus, multiply the resistance by the radius of the axle, and divide by that of the handle, and the quotient will be the power.

In windlasses and cranes, consisting of a system of wheel-work, the power is applied to a handle fixed on the spindle of a pinion, which transmits the power to a spur-wheel, fixed in the spindle of the barrel, about which the cord, or rope, carrying the load to be raised, is wound.

Where there are several pairs of such wheels, it is necessary to include in the calculations the ratios of the pinions to the spur-wheels.

The proportional formula will, in this case, be the same as for a system of levers:—

$$P : R :: b \times b' \times b'' : a \times a' \times a'';$$

Or, the power is to the resistance, as the product of the radii of the pinions and barrel is to the product of the radii of the spur-wheels and handle.

From this we derive the following rules:—

I. *Multiply the load to be raised by the product of the radius of the barrel into the radii of the pinions, and divide the sum obtained by the product of the radius of the handle into the radii of the spur-wheels, and the quotient will be the power, which, when applied to the handle, will balance the load.*

II. *Multiply the power applied by the radius of the handle, and by the radii of the spur-wheels, and divide the product by the radius of the barrel, and by the radii of the pinions, and the quotient will be the resistance which will balance the power.*

III. *Multiply the radii of the pinions and barrel, and divide the product by the radii of the handle and spur-wheels, and the quotient will be the ratio of the power to the resistance.*

239. THE INCLINE.—When a body is raised up a vertical plane, its whole weight is supported by the elevating power, and this power is consequently equal to the weight elevated.

When a body is drawn along a horizontal plane, the tractive power has none of the weight of the body to sustain, but merely to overcome the friction of the surface.

If, however, a body is drawn up an inclined plane, the power required to elevate it is proportionate to the inclination of the plane, in such a manner, that

If the power acts parallel to the plane, the length of the plane will be to the load as the height is to the power.

The advantage gained by the use of the inclined plane, as a power, is the greater, the more its length outmeasures its height; it is then the ratio of the length to the height which determines that of the power to the resistance, whence we obtain the following rules:—

I. *The resistance, multiplied by the height and divided by the length of the plane, is equal to the power required to balance a body on the inclined plane.*

II. *The power, multiplied by the length of the plane and divided by the height, is equal to the resistance.*

III. *The resistance, multiplied by the height of the plane and divided by its length, is equal to the load on the plane.*

The wedge and the screw are noticeable modifications of the incline. An incline wrapped round a cylinder generates a screw. When used as a power-machine, it is generally combined with a lever, as in presses. The advantage gained depends upon the length of the lever and the pitch of the screw. Multiply the actuating force by the circumference described by the end of the arm or lever, and divide the product by the length of the pitch of the screw; the quotient, minus the friction, which is very considerable in these machines, will be the pressure exerted by the screw, and the velocity will, of course, be in the inverse ratio of the theoretical pressure to the actuating force.

240. **The Toggle.**—This is met with chiefly in punching presses. Deflected springs and rods are also examples of it, and also the twisted cords used by carpenters to stretch their saws in frames. As in the other machines, the resistance is to the power, as the space passed through by the latter is to the space passed through by the former.

241. **The Pulley.**—There are two kinds of pulleys, the one turning on fixed centres, the other on traversing centres.

The pulley, which turns on a fixed centre, serves simply to change the direction of the motive force, without altering the relations of power and velocity. It is, in fact, only the moveable pulleys which can be classed amongst the elementary machines.

A single moveable pulley, acting as a power, doubles it at the expense of the speed; thus, if a weight of 10 lbs. be suspended to one extremity of the cord, it will balance 20 lbs. hung to the axis of the pulley. This arises from the fact that, from the arrangement of the cord, the pulley only rises through half the height passed through by the motive force; thus, if the latter pass through 6 feet, the pulley will only rise 3 feet, and the resulting momentum of the power, 10×6, will be equal to that of the resistance, or 20×3, so that the two will be in equilibrium.

Though the stationary pulley cannot be considered as a mechanical power, yet, in changing the direction of the motion, it affords great facilities in the application of force; thus, it is easier to pull downwards than upwards, as the labourer brings his weight to bear in the former case.

When several pulleys or sheaves are placed on one axis in a suitable frame, it is called a block. Where two or more blocks are employed, it is only the moveable ones which increase the power, and this increase is equal to double the number of sheaves, or pulleys, in the block or blocks.

The mechanical advantage of the block, as a power, arises from the fact, that the space traversed by the motive power is equal to the sum of doublings of the cord round the pulleys, whilst the load is only elevated to a distance corresponding to this space divided by the number of these doublings.

A clock line and weight, in which the line goes round a pulley fixed in the weight, is an example of a speed-pulley, that is, one in which the power, or resistance, is transposed, for the weight, or motive power, causes the moveable end of the cord to pass through twice the space it passes through itself.

242. **The Ram.**—This is the most economical augmentor of power that we have. It is freer from friction and other disadvantages than the other simple machines, and it is, in its action, very closely allied to the pulley. Each derives its advantage from the division of the points of support, for the proportionate area of the piston in the Bramah press represents the number of points over which the pressure, or resistance, is diffused.

243. **Remarks.**—It is essential that, to avoid illusive mistakes, the student should perfectly understand, that when, in employing mechanical forces, the effect of the power applied is augmented, the distance passed through by the resistance, or load, is diminished, with reference to that passed through by the power, *in exactly the same ratio that this is increased.* This is true in all cases, and may be stated thus: What we gain in force, by means of machinery, we lose in speed, and reciprocally.

It follows from this, that the true object of machinery cannot be to augment the work performed by the motive agent, but to convert any primary action in a manner appropriate to the circumstances in which the power is to be used. Thus we can make a very small force, as that of a man, elevate an enormous weight, but with a speed proportionately slow.

Finally, *The mechanical effect developed in a given time by a given force, through the instrumentality of machinery, must always equal the useful effect obtained, plus the amount lost in overcoming frictional and other resistances; and the useful effect of machinery will be the greater, according as the causes of these resistances are diminished.*

CENTRE OF GRAVITY.

244. All bodies are equally subjected to the action of weight. Gravity, or weight, is the action of that universal attraction which draws all bodies towards each other, and by which, in the case of bodies on the surface of the earth, these are drawn towards its centre. The power, of whatever nature it may be, which balances this action, is equal to the weight of the body.

The curvature of the surface of the earth being quite inappreciable for small distances, gravity is considered as acting in parallel lines, and its direction is given by the plumbline.

The centre of gravity is that point in any body in which the action of its entire weight may be said to be concentrated. If the body be suspended by this point it will be in *equilibrio*, in whatever position it is put.

The position of the centre of gravity depends upon the nature and form of any body; it may generally be found in the following manner:—

Suspend the body by a thread attached to any point whatever in it; when the body is motionless, the line of the suspension thread will pass directly through the centre of gravity. Suspend the body by any other point, and the centre of gravity will also be in the continuation of the line of the thread, so that the actual centre must be at the point of intersection of the two lines thus obtained. This simple expedient reminds us of the application of the square to the finding of a shaft, for example, where the intersection of any two lines, drawn along the blade of the square, when the head is laid against the periphery of the shaft in two different positions, gives the required point of centre.

The centre of gravity of regular bodies, as spheres, cylinders, prisms, is in the centre of their configuration.

The centre of gravity of an isosceles triangle is one-third up the centre line which bisects the base.

The centre of gravity of a pyramid, with a triangular or polygonal base, is one-fourth up the line which joins the summit with the centre of gravity of the base. It is the same with a cone.

The centre of gravity of a hemisphere is situated three-eighths up the radius at right angles to the base.

The centre of gravity of an ellipse is in the point of intersection of the axes.

When a body is placed in a vertical or inclined position on a plane, it is necessary, in order that it may rest upon it in that position without falling, that the vertical line passing through the centre of gravity shall fall within the external outline of the side in contact with the plane. This limit, however, allows of considerable deviation from the vertical in the general contour of bodies, as is instanced in the case of leaning or inclined edifices. The stability of bodies increases as the extent of their bases is greater in comparison with their height, and also, as the vertical line, passing through the centre of gravity, meets the plane on which the body rests nearer to the centre of the base. A body is said to be more stable when it requires a greater force to overturn it. A cone is more stable than a cylinder of the same height and base. The stability of walls depends greatly on the kind of foundations given to them, and on the proportionate extension of their bases.

ON ESTIMATING THE POWER OF PRIME MOVERS.

245. As we shall see further on, the power of prime movers may be calculated from the dimensions of the various parts of the engine. Still, the many different modes of construction tend to modify considerably the actual useful effect, and engineers have endeavoured to construct an apparatus, by means of which the actual power, or useful effect of engines, may be measured with exactitude.

Prony's brake, which is the instrument most generally used for this purpose, acts on the principle of the lever, and consists of a cast-iron pulley in two halves, united by screws. This is fixed on the main shaft of the prime mover, the force of which it is wished to measure. It is embraced by two jaws, which may be tightened down upon the pulley by screws. To the lower jaw is attached a long lever, from the end of which is suspended a scale for weights. If it is known what power the engine was designed to possess, it is simply necessary to put into the scale the weight corresponding to this power, that is, the weight which, by the action of the lever, will give a pressure equal to the supposed power of the machine.

Having fixed the apparatus on the engine, and provided means of efficiently lubricating the frictional surface of the pulley with soap and water, and having balanced the apparatus in such a manner that it will not be necessary to take into the calculation anything but the weight placed in the scale, the steam may be gradually let on. The engine will perhaps shortly acquire a greater velocity than that for which it was designed; if this is the case, the jaws are gradually screwed closer and closer upon the pulley. As the friction thereby increases, the velocity will diminish, and full steam may be let on. After a short time, and when the friction is so great that the lever is raised slightly above the horizontal line, and the engine is going at its proper velocity, and the pressure of the steam at its correct point, so that the power of the engine balances the load on the lever, it may be concluded that the engine develops the power for which it was intended. If the lever rises considerably, it will be necessary to increase the weight in the scale, so as to obtain the actual maximum power of the engine; and, on the contrary, if the engine does not appear to have the desired power, the weight must be reduced, by which means its actual power will be ascertainable.

CALCULATION FOR THE BRAKE.

246. The weight which will balance the force of a machine may be calculated when the length of the lever arm is known, or, more correctly, the radius from the centre of the shaft to the point of suspension of the weight, and the nominal horse-power, by the following rule:—

Multiply the nominal horse-power by 33,000, *and divide the product by the circumference described by the end of the lever, and by the number of revolutions per minute, and the quotient will be the weight sought.*

Let us take, for example, the main shaft of a steam-engine of 16 horse-power, which runs at the rate of 30 revolutions per minute, the radius of the brake being nine feet—

$$\text{Here we have } \mathrm{w} = \frac{16 \times 33{,}000}{6{\cdot}28 \times 9 \times 30} = 311{\cdot}4 \text{ lbs.}$$

Such is the net weight to be suspended from the end of the lever, the brake being previously balanced by being suspended on its centre of gravity.

247. The actual power, or maximum effect of an engine, may likewise be calculated by means of the following rule:—

Multiply the circumference described by the lever, by the number of revolutions of the shaft per minute, and by the weight in the scale, and divide the product by 33,000, *and the quotient will be the actual force of the engine in horses-power.*

For example, let us suppose that the main shaft of a steam-engine makes 30 revolutions per minute, that the radius of the lever is 9 feet, and that the net weight in the scale is 311·4 lbs., what is the maximum force of the engine?

$$\mathrm{F} = \frac{6{\cdot}28 \times 9 \times 30 \times 311{\cdot}4}{33{,}000} = 16 \text{ H. P.}$$

THE FALL OF BODIES.

248. When bodies fall freely of their own weight, the velocities which they acquire are proportionate to the time during which they have fallen, whilst the spaces passed through are as the squares of the times.

It has been ascertained by experiment that a body falling freely from a state of rest, passes through a distance of 16 feet and a small fraction, in the first second of time. At the end of this time it has a velocity equal to twice this distance per second.

From this it follows that if the times of observation are—

	1″	2″	3″	4″
The corresponding velocities will be	32 ft. ...	64 ft. ...	96 ft. ...	128 ft.
The spaces passed, through from the commencement,	16 “ ...	64 “ ...	144 “ ...	256 “
The spaces passed through during each second,	16 “ ...	48 “ ...	80 “ ...	112 “

That is to say, that the times are as the numbers,	1,	2,	3,	4
The velocities also as,	1,	2,	3,	4
The spaces passed through as the squares,	1,	4,	9,	16
And the space for each interval as the odd numbers,	1,	3,	5,	7

These principles apply equally to all bodies, whatever may be their specific gravity, for gravity acts equally on all bodies; the effect, however, being modified by the resistance of the media through which the bodies pass, which is greater in proportion, as the specific gravity is less.

249. The velocity which a body will acquire in a given time when falling freely will be found by multiplying the time expressed in seconds by 32 feet.

Example.—Let it be required to ascertain the velocity acquired by a body falling during 12 seconds.

$$V = 12 \times 32 = 384 \text{ feet per second.}$$

When a body falls from a given height, H, the ultimate velocity, or that acquired by the time the base is reached, will be given by the formula (g being the velocity gravity causes a body to acquire in the first second)

$$V = \sqrt{2\,g\,H}, \text{ or } V = \sqrt{64 \times H},$$

which leads to the following rule:—Multiply the given height in feet by 64, and extract the square root, which will be the velocity in feet per second by the time the body shall have fallen through the height, H, not taking resistance into consideration.

Example.—What will be the ultimate velocity of a body falling a distance of 215 feet?

$$V = \sqrt{64 \times 215} = 117{\cdot}3 \text{ feet per second.}$$

From the above formula,

$$V = \sqrt{2\,g\,H},$$

we obtain $V^2 = 2\,g\,H$, then

$$H = \frac{V^2}{2\,g} = \frac{V^2}{64};$$

whence we have this rule:—Divide the square of the velocity in feet per second by 64, and the quotient will express the height through which a body must fall unimpeded, from a state of rest, in order to obtain that velocity.

TABLE OF HEIGHTS CORRESPONDING TO VARIOUS VELOCITIES OF FALLING BODIES.

Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
·1	·0001	5·7	·165	16·5	1·388	44·5	10·094	72 5	26·794
·2	·0002	5·8	·171	17·0	1·473	45·0	10·322	73·0	27·164
·3	·0005	5·9	·177	17·5	1·561	45·5	10·553	73·5	27·538
·4	·0009	6·0	·184	18·0	1·651	46·0	10·786	74·0	27·914
·5	·0013	6·1	·190	18·5	1·745	46·5	11·022	74·5	28·292
·6	·0019	6·2	·196	19·0	1·840	47·0	11·260	75·0	28·673
·7	·0026	6·3	·202	19·5	1·938	47·5	11·501	75·5	29·057
·8	·0034	6·4	·209	20·0	2·039	48·0	11·744	76·0	29·443
·9	·0043	6·5	·215	20·5	2·142	48·5	11·990	76·5	29·832
1·0	·0051	6·6	·222	21·0	2·248	49·0	12 239	77·0	30·223
1·1	·0062	6·7	·229	21·5	2·356	49·5	12·490	77·5	30·617
1·2	·0074	6·8	·236	22·0	2·467	50·0	12·741	78·0	31·013
1·3	·0087	6·9	·243	22·5	2·580	50·5	13·000	78·5	31·412
1·4	·0101	7·0	·250	23·0	2·696	51·0	13·258	79·0	31·813
1·5	·0115	7·1	·257	23·5	2·815	51·5	13·520	79·5	32·217
1·6	·0131	7·2	·264	24·0	2·936	52·0	13·784	80·0	32·624
1·7	·0148	7·3	·272	24·5	3·060	52·5	14·050	80·5	33·033
1·8	·0166	7·4	·279	25·0	3·186	53·0	14·319	81·0	33·145
1·9	·0185	7·5	·287	25·5	3·315	53·5	14·590	81·5	33·859
2·0	·0204	7·6	·295	26·0	3·446	54·0	14·864	82·0	34·275
2·1	·0225	7·7	·302	26·5	3·580	54·5	15·141	82·5	34·695
2·2	·0247	7·8	·310	27·0	3·716	55·0	15·420	88·0	35·116
2·3	·0270	7·9	·318	27·5	3·855	55·5	15·701	83·5	35·541
2·4	·0294	8·0	·326	28·0	3·996	56·0	15·986	84·0	35·968
2·5	·0319	8·1	·334	28·5	4·140	56·5	16·272	84·5	36·397
2·6	·0345	8·2	·343	29·0	4·287	57·0	16·562	85·0	36·829
2·7	·0372	8·3	·351	29·5	4·436	57·5	16·854	85·5	37·264
2·8	·0400	8·4	·360	30·0	4·588	58·0	17·148	86·0	37·701
2·9	·0429	8·5	·368	30·5	4·742	58·5	17·445	86·5	38·141
3·0	·0459	8·6	·377	31·0	4·899	59·0	17·744	87·0	38·583
3·1	·0490	8·7	·386	31·5	5·058	59·5	18·046	87·5	39·028
3·2	·0522	8·8	·395	32·0	5·220	60·0	18·351	88·0	39·475
3·3	·0555	8·9	·404	32·5	5·384	60·5	18·658	88·5	39·925
3·4	·0589	9·0	·413	33·0	5·551	61·0	18·968	89·0	40·377
8·5	·0624	9·1	·422	33·5	5·721	61·5	19·280	89·5	40·832
3·6	·0660	9·2	·431	34·0	5·893	62·0	19·595	90·0	41·290
3·7	·0697	9·3	·441	34·5	6·067	62·5	19·912	90·5	41·750
3·8	·0735	9·4	·450	35·0	6·244	63·0	20·232	91·0	42·212
3·9	·0775	9·5	·460	35·5	6·424	63·5	20·554	91·5	42·677
4·0	·0816	9·6	·470	36·0	6·606	64·0	27·879	92·0	43·145
4·1	·0856	9·7	·480	36·5	6·791	64·5	21·207	92·5	43·615
4·2	·0899	9·8	·490	37·0	6·978	65·0	21·537	93·0	44·088
4·3	·0942	9·9	·500	37·5	7·168	65·5	21·869	93·5	44·563
4·4	·0986	10·0	·510	38·0	7·361	66·0	22·205	91·0	45·041
4·5	·1032	10·5	·562	38·5	7 556	66 5	22·542	94·5	45·522
4·6	·1078	11·0	·617	39·0	7·753	67·0	22·883	95·0	46·005
4·7	·1125	11·5	·674	39·5	7·953	67·5	23·225	95 5	46·490
4·8	·1174	12·0	·734	40·0	8·156	68·0	23·571	96·0	46·978
4·9	·1228	12·5	·797	40·5	8·361	68·5	23·919	96·5	47·469
5·0	·1274	13·0	·861	41·0	8·569	69·0	24·969	97·0	47·962
5·1	·1325	13·5	·929	41·5	8·779	69·5	24·622	97·5	48·458
5·2	·1378	14·0	·999	42·0	8·992	70·0	24·978	98·0	48·956
5·3	·1431	14·5	1·072	42·5	9·207	70·5	25·336	98·5	49·457
5·4	·1486	15·0	1·147	43·0	9·425	71·0	25·696	99·0	49·960
5·5	·1541	15·5	1·225	43·5	9·646	71·5	26·060	99·5	50 466
5·6	·1598	16·0	1·305	44·0	9·869	72·0	26·425	100·0	50·975

Example.—A body has acquired a velocity of 117·3 feet per second, through what height must it have fallen?

$$H = \frac{117{\cdot}3^2}{64} = 215 \text{ feet, the height of the fall.}$$

To obviate the necessity of calculating the corresponding heights and velocities, we give a very extensive table, calculated for tenths of inches. The numbers, however, being equally correct as representing feet or yards, those of both columns being of the same denomination.

MOMENTUM.

250. The force with which a body in motion strikes upon one in a state of rest, is equal to the product of the mass of the moving body multiplied into the velocity; this product is termed its momentum. If a body with a mass, m, is animated with a velocity, v, its momentum is equal to $m\,v$. The term, m, however, may be taken as signifying the mechanical effect of a weight falling during a second of time, or through 32 feet, therefore, $m = \frac{w}{g}$ that is, the weight in pounds divided by 32 feet, whence $m\,v = \frac{w \times v}{g}$.

What distinguishes the simple momentum or force of impact of a body from the mechanical effect of a prime mover is, that whilst the former is due to a single impulse, we have in the latter to consider the continuous action of the impelling force.

251. When a motive force imparts continuously a certain velocity to a body, the result of its action is what may be termed *vis viva*, or continuous momentum; it is numerically the product of the (moving) mass multiplied into the square of the velocity imparted to it.

Putting M to represent the mass of a body, and V the velocity impressed upon it,

$$M\,V^2 \text{ or } \frac{W\,V^2}{g}$$

is the expression of the *vis viva* of the body. This force is double that developed by gravity. For, in fact, when a body of the weight W, falls from a height, H, it acquires from its fall an ultimate velocity, V, which we have already shown to be equal to

$$\sqrt{2\,g\,H} = \frac{V^2}{2\,g},$$

and the mechanical effect, W H, is consequently expressed by

$$\frac{W\,V^2}{2\,g};$$

now, putting for P its value, M g, the formula becomes

$$\frac{M\,V^2}{2}.$$

Thus, the mechanical effect developed by gravity is equal to half the momentum imparted to a body.

CENTRAL FORCES.

252. When a body revolves freely about an axis, it is said to be subjected to two central forces; the one, termed "centripetal," tends to draw the body to the axis; the other, termed "centrifugal," or tangential, and due to the tendency of bodies in motion to proceed in straight lines, strives to carry the body away from the centre. These forces are equal and act transversely to each other.

The centrifugal effort exerted by a body in rotative motion, and which tends to separate the component particles, is expressed by the following formula:—

$$F = \frac{W\,V^2}{g \times R},$$

in which W represents the weight of the body; V the velocity in feet per second; and R, the radius or distance of the centre of motion from the centre of the revolving body.

Example.—Let a ball of the weight W = 23 lbs., attached to a radius, R, measuring 5 feet, rotate with a velocity, V = 40 feet per second, what is the centrifugal effort, or the pull of the ball on the radius?

$$F = \frac{23 \times 40 \times 40}{32 \times 5} = 230 \text{ lbs. raised 1 foot high per second.}$$

CHAPTER IX.

ELEMENTARY PRINCIPLES OF SHADOWS.

253. We have already, when treating of shadow lines, laid it down as a rule to be observed generally, in mechanical or geometrical drawing, that the objects represented shall be supposed to receive the light in parallel rays, in the direction of the diagonal of a cube running from the upper left hand corner of the anterior face of the cube, down to the lower right hand corner of the posterior face, in the vertical elevation, and in the horizontal plane of projection they are taken in the direction of the cube running from the upper left hand posterior corner down to the right hand lower anterior corner of the cube.

We have also shown that the horizontal and vertical projections of these cubic diagonals make angles of 45° with the horizontal or base line, that is in the direction of the arrow, R^1, Fig. 22, Plate 7.

The advantages of this assumption of the direction of the rays of light will, no doubt, have been appreciated. Amongst these, it has the merit of at first sight plainly pointing out the relative degrees of prominence of the various parts of an object, even with the aid of a single projection or view.

254. This point, then, being determined, on considering an object of any form whatever, as receiving in this way the parallel rays of light, it may be conceived that these rays will form a cylindrical or prismatical column, the base of which will be the illuminated outline of the object. The part met by these rays of light will be fully illuminated, whilst the portion opposite to this will be as entirely void of light. The absence of light on this latter part may be termed the *shadow proper* of the object—that is, its own shadow upon itself, and the line of separation of these two parts on the object itself, is termed the line of separation of light and shade of the shadow proper.

255. If further, we suppose the luminous rays surrounding the object to be prolonged until intercepted by the surface upon, or adjacent to which it lies, a portion of such surface will be in shadow, because of the interception of some of the rays by the object; the outline of this unilluminated portion will be limited by, and depend upon the contour of the object, and the adjacent surface; it is termed the shadow *cast* or *thrown*, by an object on such a surface.

EXAMPLES IN THE PR

Fig. 1.

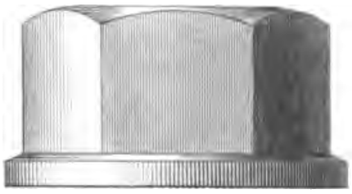


Fig. 2.

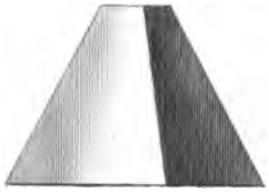
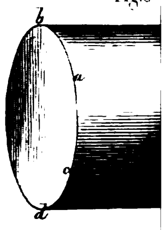


Fig. 3



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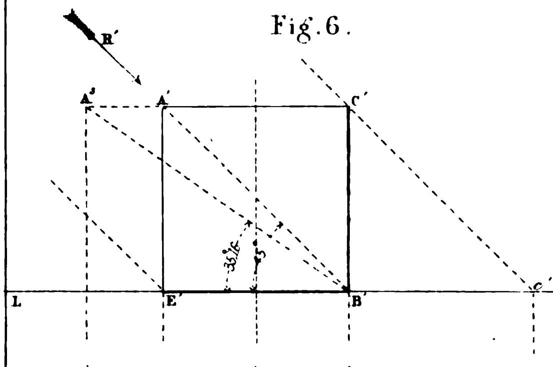
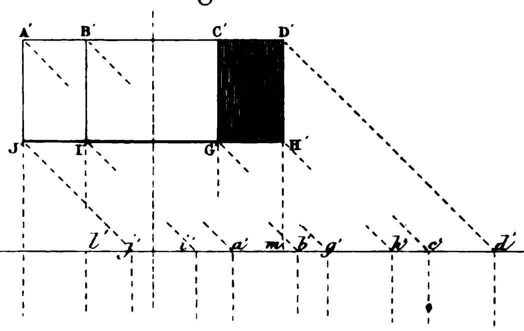


Fig. 8.



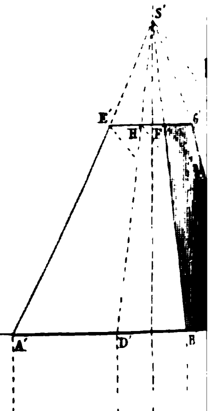


Fig. 7.

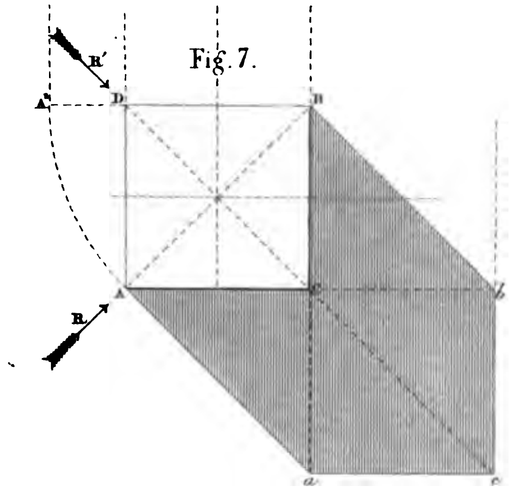
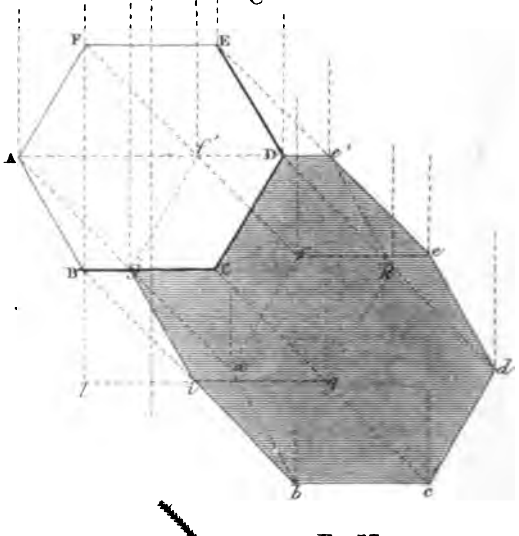


Fig. 9.



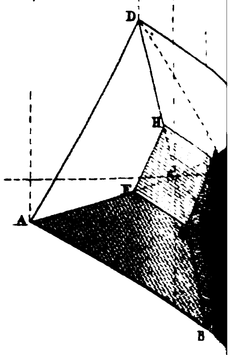


Fig. 15.

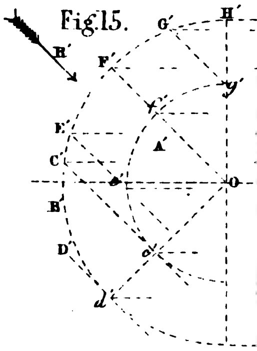


Fig. 14.

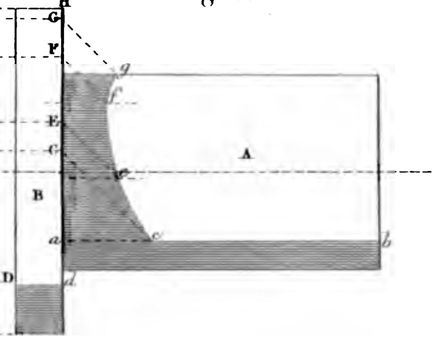


Fig. 17.

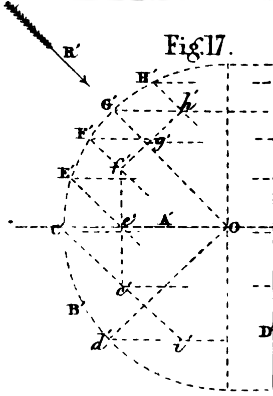
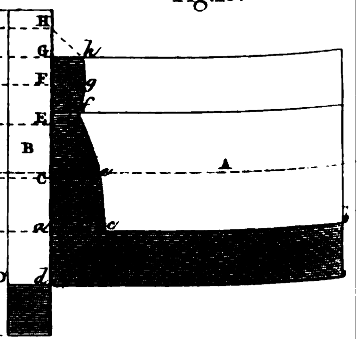


Fig. 16.



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ECTION OF SHADOWS.

Fig 4

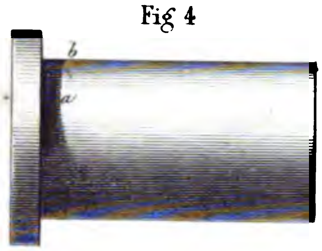

Fig. 5.

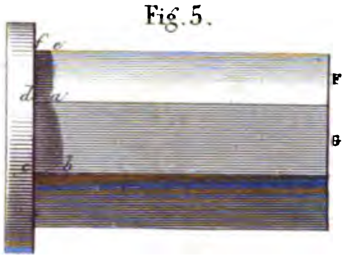

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Fig. 12.

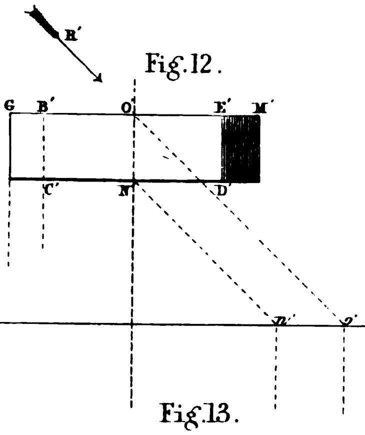
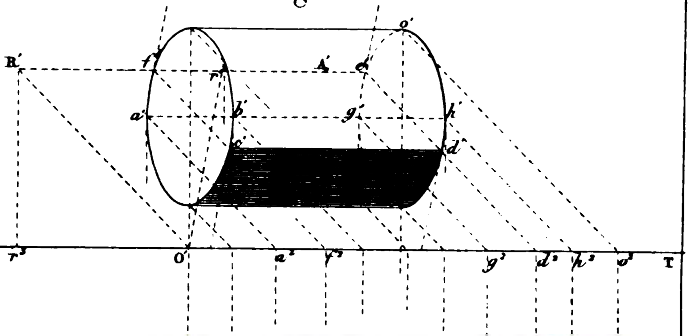


Fig. 22.



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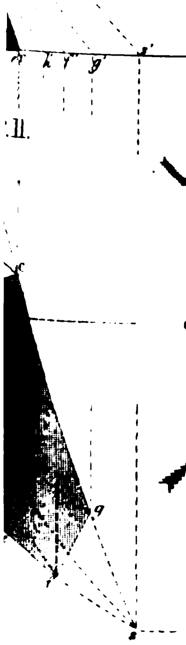


Fig. 13.

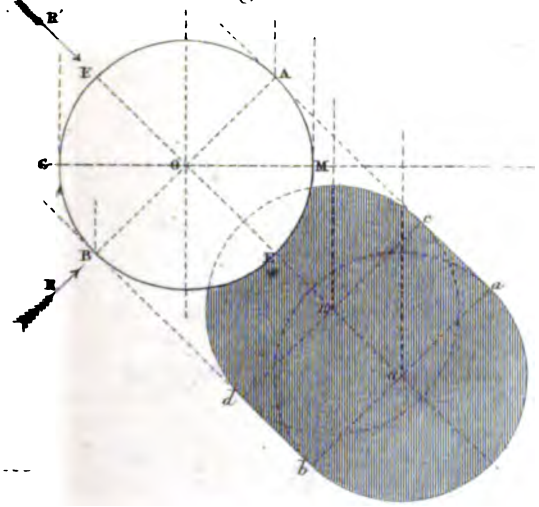


Fig. 23.

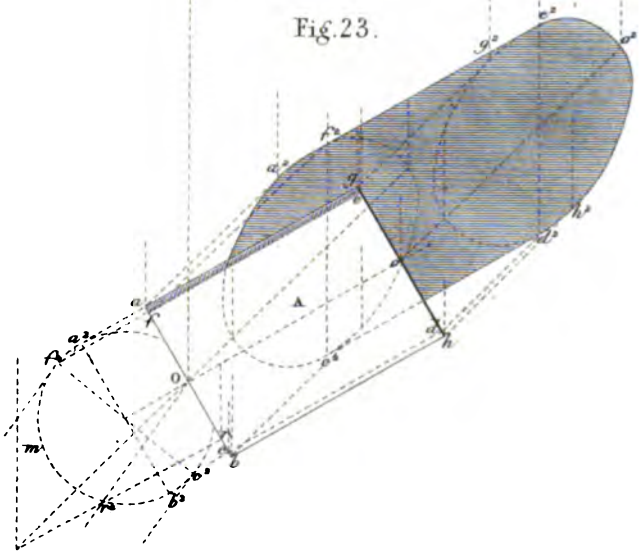


Fig. 19.

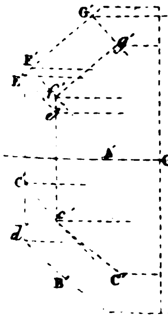


Fig. 18.

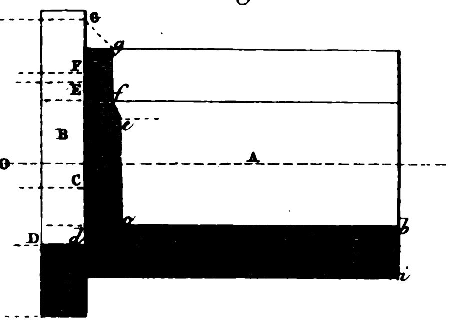


Fig. 21.

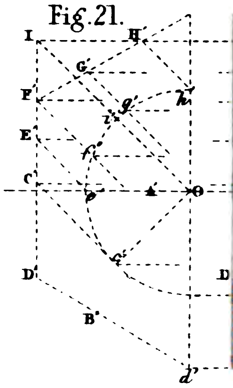
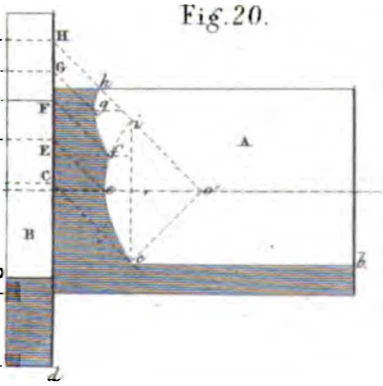


Fig. 20.



The line which separates the illuminated from the unilluminated portion is termed the line of separation of light and shade, or the outline of the shadow. This is modified by the form of the recipient surface, as well as by that of the object which gives rise to it. It is always bounded by straight lines when the generating surfaces are planes; and by curves, when either or both are cylindrical, conical, spherical or otherwise curved.

256. As a general rule, the determination of the outlines of shadows proper, and cast, reduces itself to the problem of finding the point of contact of a straight line representing a luminous ray, with a plane or other surface. The application, however, of this general principle, though apparently so simple, gives rise to many difficulties in practice, from the variety of cases presented by the different forms of objects, and it is necessary to give several special examples, to explain the most simple and expeditious modes which may be employed in such cases, always with a due regard to geometrical accuracy.

We shall primarily choose for these applications objects of simple form, and bounded by plane surfaces; next such as are wholly or partially cylindrical; and we shall proceed, in succession, to objects of more complex forms. The objects which we have taken in preference, as examples, are such as are most frequently met with in machinery and architecture; they will, notwithstanding, afford quite sufficient illustration in connection with what has to be said respecting the study of shadows. Several of the most simple objects we propose taking for this purpose, are represented in vertical elevation in Figs. 1, 2, 3, 4, and 5, at the top of this plate, completely shaded, and having the shadows well defined according to the principles now to be discussed under various heads.

SHADOWS OF PRISMS, PYRAMIDS, AND CYLINDERS.

PLATE 26.

PRISMS.

257. Let the Figures 6 and 7 be given, the vertical and horizontal projections of a cube, it is required to determine the form of the shadow cast by this cube on the horizontal plane.

In the position given to this cube it is easy to see that the sides which are in the light are the top face, A D B C, of the cube, and those represented by the lines A D, and D^1 B, in the horizontal projection, and projected vertically in A^1 C^1, A^1 E^1, and A^1 C^1 B^1 E^1, the side D B being exactly posterior to A C, or A^1 C^1 B^1 E^1, the front face of the cube. The opposite faces, C B and C A, Fig. 7, and B^1 C^1, B^1 E^1, Fig. 6, are consequently in the shade; as, however, these latter faces are reduced to mere lines in the representations, the shadow proper can only be shown by a thick shadow line, produced by China ink in line drawings, and by a narrow stroke of the brush in water-colour drawings.

These lines, which distinguish the illuminated sides of the cube from those which are not so, are termed, as we have said, the lines of separation of light and shade. It now only remains to find the shadow cast by the cube on the plane, L T.

258. When the cube rests on the horizontal plane, as shown in this case, and has a side parallel, also to the vertical plane of projection, the width of the shadow cast by it in the horizontal plane, is exactly equal to its height; thus to determine its outline, we draw straight lines from each corner of the cube, representing the rays of light, as A *a*, C *c*, B *b*, parallel to the arrow, B^1, Fig. 7, which is the direction of the rays of light, proper to it. The arrow, B, being only the horizontal projection of the arrow, B^1, in Fig. 6, and the direction of the rays of light, proper to the elevation. Having drawn these lines, it is now merely requisite in such cases as this to limit or intersect them by the two lines, *c a*, *c b*, drawn parallel to C A, C B, and at a perpendicular distance or width, C *a*, C *b*, from the latter equal to the height of the cube, A^1 E^1, which gives the lines, A *a*, *a c*, *c b*, and *b* B, forming the outline of the shadow cast by the cube, A D B C, in the horizontal projection or plan, Fig. 7, the shadow being indicated by a light ruled flat tint. Another mode of finding the points, *a*, *c*, *b*, in which these lines, A *a*, C *c*, B *b* meet in the horizontal plane, is to draw through the points, C^1 and A^1, Fig. 6, the projections of the two first, A, C, the rays, C^1, *c*^{1} and A^1 B^1, parallel to B^1, and meeting the base line, L T, in *c*^{1} and B^1, If now, through these points, we draw perpendiculars to the base line, as *c*^{1} *c*, B^1 *a*, these will cut the first rays in *a*, *c*, and *b*. The contour of the shadow cast is thus limited by the lines C *c*, *c b*, *b d*, and *d* D, as in the first case.

The face, E^1 B^1, being that on which the cube rests, has no prominence, and cannot therefore cast any shadow. It follows, then, that the shadow, as above determined, is all that is apparent. It is generally represented by a flat, uniform shade, laid on with the brush, and produced by a greyish wash of China ink.

259. It will be observed that the lines, *a c* and *c b*, are parallel to the straight lines, A C and C B. This is because these are themselves parallel to the horizontal plane; for when a line is parallel to a plane (85), its projection on this plane is a line parallel to itself; and hence we have this first consequence, that—

When a straight line is parallel to the plane of projection, it casts a shadow on the plane, in the form of an equal and parallel straight line.

260. It will also be observed, that the straight lines, A *a*, C *c*, B *b*, which are the shadows cast by the verticals, projected in the points, A, C, and B, in the horizontal plane of projection, are inclined at an angle of 45° to the base line; whence we derive the second consequence, that—

When a straight line is perpendicular to the plane of projection, it casts a shadow on the plane in the form of a straight line, parallel to the rays of light, and consequently inclined at an angle of 45° to the base line.

261. These observations suggest a means of considerably simplifying the operations. Thus, in place of searching separately for each of the points, *a*, *c*, *b*, where the rays of light pierce the horizontal plane, it is sufficient to determine one of these points, such as *c*, for example, and through it to draw the straight lines, *c a*, *c b*, parallel and equal to the sides, A C and C B, of the cube and intersecting lines, inclined at an angle of 45° drawn from the points, A, B.

In the actual case before us, we may even entirely dispense with the vertical projection, Fig. 6, since it would have been sufficient to prolong the diagonal, D C, to *c*, making C *c* equal to C D, or to make the inclined lines, A *a*, or C *c*, equal to the diagonal, D C; because the vertical projection, C^1 *c*^{1}, and horizontal projection, C *c*, of the same ray of light, are always of the same length, which follows from our having taken the diagonal of the cube for the direction of this ray, the two projections, A^1 B^1, A B, and D C, of this diagonal being obviously equal. Whence follows the third consequence, that—

If, through any point of which the two projections are given, we draw a straight line, representing the ray of light, and if we ascertain the point in which this ray meets either plane, the length of the ray in the other plane of projection will be the same.

262. Finally, it is to be observed that the distance, C *a* or C *b*, taken on the prolongation of the lines, C B, or C A, are equal to the entire height, C^1 B^1, namely, that of the cube; and consequently, in place of employing the diagonal to obtain the various points, *a*, *c*, *b*, we may make the distance, C *a*, equal to the height of the cube, and draw, through *a*, a straight line, *a c*, parallel and equal to A C, and through *c*, a second, *c b*, parallel and equal to C B, and then join A *a*, B *b*, as already shown (258), whence follows the fourth consequence.

Thus the shadow cast on a plane by a point, is at a distance from the projection of the point, equal to the distance of the point itself from the plane.

263. Figs. 8 and 9 represent a prism of hexagonal base, in vertical and horizontal projection, but instead of resting on the base line it is supposed to be elevated some distance above and parallel to the base line.

In the elevation, Fig. 8, it will be seen that the vertical faces which are illuminated are those projected in the horizontal view, Fig. 9, by the lines, F A, A B, and B C, the two latter only of which are visible, whilst the opposite ones, represented by the lines, F E, E D, and D C, are in the shade.

Of these lâtter the face, D C, only is visible in the vertical projection, Fig. 8, and represented by the rectangle, C^1 D^1 H G, which should be shaded to a deeper tint than the cast shadows, to distinguish it.

264. The operations by which we determine the shadow cast upon the horizontal plane, is evidently the same as in the preceding case; still, since the lower base, J^1 H^1, does not rest upon the horizontal plane, it will not be sufficient merely to draw the rays of light through the points, B, C, D, E, of the upper side; it is, in addition, necessary to draw corresponding rays through the points, J^1, I^1, G^1, H^1, of the base.

It is to be observed, as in the preceding case, that as these two faces are parallel to the horizontal plane, the shadow cast by each upon this plane will be a figure equal and parallel to itself; so that, in place of seeking all the points of the shadow, it would have been quite sufficient to obtain one of these points, as *d*, for example, of the upper side, and *h* or *i*, of the lower side, and then, starting from them, to draw a couple of hexagons, parallel and equal to A B C D E F.

It will also be understood, that as it is only the outside lines, those of the separation of the light and shade, which make up the contour of the shadow, it is not necessary to determine the points which fall within this contour, and correspond to those points in the objects itself which do not lie in the lines of separation of the illuminated and shaded parts.

265. Thus it is unnecessary to find the points, *a*, *f*, *g*, *h*; and generally, in making drawings, we do not seek the shadows cast by points fully illuminated, or within the borders of the shaded portion; and the contour of the shadow is derived simply from points lying in the line of separation of the light and shade on the object.

266. From what we have already explained, it will be gathered, that the projection in one plane of the shadow, cast by a point, can be obtained by drawing the diagonal of the square, a side of which is equal to the distance of the point from the plane, as shown in the other projection.

For example, the shadow, *i*, on the horizontal plane of the point, I^1, the two projections of which are I^1 and B, Figs. 8 and 9, may be got by forming the square, B *l* *i*, a side of which, B *l*, is equal to the distance, I^1 l^1, of the point from the horizontal plane.

In the same way, we have the points, *h*, *g*, *j*, corresponding to H, G, J, which are the same height as the first above the plane.

For the points, *b*, *c*, *d*, *e*, which correspond to the upper side, A^1 D^1, of the prism, we draw the diagonal, D^1 d^1, of the square, having for a side the height, D^1 *m*, of the point, D^1, above the horizontal plane, and set out this diagonal from B to *b*, C to *c*, D to *d*, and E to *e*.

PYRAMID.

267. When several straight lines converge to a point, the shadows they cast on either plane of projection must necessarily, also, converge to a point. Thus, in the pyramid, Figs. 10 and 11, the apex of which is projected in the points, S and S^1, the edges of all the sides being directed to this point, cast shadows on the horizontal plane, bounded by lines converging to the point, *s*, the shadow cast by the apex on the same plane. In order, then, to find the shadow cast by a pyramid, on either of the planes of projection, it is sufficient to draw the ray of light through the apex, and ascertain the point at which this ray meets the plane; then to draw lines to this point from all the angles of the base of the pyramid, if this rests upon the plane. If, however, the pyramid is raised above the plane, it will be necessary to find the shadows cast by the various angles of the base, and then draw straight lines from these to the shadow of the apex.

TRUNCATED PYRAMID.

268. When we have only the frustum of a pyramid to deal with, and the apex is not given, it is necessary to find the shadows cast both by the angles of the base, and by those of the surface of truncation. Thus, the points, E, F, G, H, of the upper side, cast their shadows on the horizontal plane, in the points, *e*, *f*, *g*, *h*, which are obtained by drawing through each point, in the vertical projection, E^1, F^1, G^1, H^1, the rays inclined at an angle of 45°, meeting the base line in the points, e^1, f^1, g^1, h^1, which are squared over to the horizontal projection, so as to meet the corresponding rays, drawn through the points E, F, G, H. Then, if we draw lines from the points, *e*, *f*, *g*, *h*, to the angles, A, B, C, D, situated in the horizontal plane, we shall obtain the shadows cast by each of the lateral edges of the pyramid.

For the same reason that these edges are diversely inclined to the horizontal plane, the shadows cast by them on this plane have also different inclinations to the base line; but the edges of the upper side or surface of truncation being parallel to this plane, cast a shadow, which in figure is equal and parallel to this side; this would not have been the case had it been inclined to the plane. It is evident that, in the position in which the pyramid is represented with regard to the rays of light in the horizontal projection, Fig 11, the two faces A E H D and D H G C, are in the light, whilst their opposites, A E F B and C G F B, are in the shade. But in the direction, the light falls upon the vertical projection, Fig. 10, it is the sides D H G C and C G F B, which are in the shade, the last one being visible in this figure; all these shaded sides being distinguished by a moderate tint of colour.

CYLINDER.

269. A cylinder with a circular base being a regular solid, all that is wanted, in determining the lines of separation of light and shade, is, when the cylinder is vertical, to draw a couple of planes tangential to it, and parallel to the rays of light, as in Figs. 12 and

13. These tangential planes are projected in the horizontal plane, in the lines, A *a*, B *b*, tangents to the circle; and inclined at the angle of 45°. Then by the points of contact of the diameter E F, at right angles to the arrow, R, with the circle, give the lines of separation of light and shade, proper to Fig. 12, which are projected vertically in B^1 C^4 and E^1 D^1. The last of these lines is apparent on this view, but the other is not. We have thus, in Fig. 13, the portion, A E B, of the cylinder, in the light, and its opposite, A F B, in the shade, and in Fig. 12, the portion, E B F, in the light, and E A F, in the shade. A small portion of this last only is seen there, and it is slightly shaded to distinguish it from the light part.

270. With reference to the cast shadow, it is to be remarked, that for the very same reason that the lines of separation of light and shade are vertical, the shadows they will cast on the horizontal plane will be in two lines, *c a* and *d b*, with an inclination of 45°, as already explained, these lines being identical with the prolongation of the tangential rays. The two bases of the cylinder being parallel to the horizontal plane, their shadows will be circles equal to themselves; and all that is required is to find the shadows, *n*, *o*, cast by their respective centres, N^1, and O^1, and with the points, *n*, *o*, as centres, to describe circles, with a radius equal to O A. The entire shadow cast by the cylinder is comprised between the two semicircles and the two tangents, *c a*, *d b*.

SHADOW CAST BY ONE CYLINDER UPON ANOTHER.

271. Hitherto we have only considered the shadow cast by an object upon one of the planes of projection. It frequently happens, however, that one body casts a shadow on another, or that the configuration of the body itself is such, that one part of it casts a shadow on another.

Let Fig. 14 be the vertical projection of a short cylinder, A, with a concentric cylindrical head, B. We have, in the first place, to find the line of separation of light and shade upon these two cylinders; and for this purpose we require to draw a second vertical projection, Fig. 15, at right angles to the first, and in the line of its axis. In these figures the projection of the ray of light is in the direction of the arrow, R^1, at an angle of 45° with the base line. We must, consequently, draw the two straight lines, C^1 c^1 and D^1 d^1, tangential to the circles, A^1 and B^1, and project, or square over, the points of contact, c^1, and d^1, to Fig. 14, drawing the lines, *a b* and D *d*, which separate the light from the shaded part of the objects. Instead of drawing these tangents, we can directly obtain both points of contact, by drawing the radius, O c^1 d^1, at right angles to the ray of light, R^1.

272. The shadow cast by the projecting head, B, upon the cylinder, A, is limited to that due to the portion, d^1 C^1 H^1, of the circumference. Different points in the outline of this shadow are determined, by first taking any points, C^1, E, F^1, G^1, upon the arc, d^1 C^1 H^1, and drawing through each of them lines representing the parallel rays of light, and meeting the circumference of the cylinder, A^1, in the points, c^1, e^1, f^1, g^1. Having projected the first-mentioned points on the base, *d* H, Fig. 14, draw through the points, C, E, F, G, a series of lines parallel to the first, and likewise representing the rays of light, and square over the points of contact, c^1, e^1, f, g^1, which will give the points, *c*, *e*, *f*, *g*, of the curve, which is the outline of the shadow upon the cylinder, A.

As shown in a former example, instead of squaring over the points, c^1, e^1, f^1, g^1, we can obtain the same result by making the corresponding rays, C *c*, E *e*, F *f*, G *g*, equal to the lines, C^1 c^1, E^1 e^1, F^1 f^1, G^1 g^1.

SHADOW CAST BY A CYLINDER UPON A PRISM.

273. Figs. 16 and 17, represent two vertical projections of a prism, A, of an octagonal base, having a cylindrical projecting head, B.

As in the preceding case, draw the radius, O d^1, perpendicular to the ray of light, thereby obtaining the point of contact, d^1, and in consequence, the line of separation, D *d*, of light and shade on the cylindrical head, B.

The inclined facet, c^1 i^1, of the prism, being in the direction of the ray of light, and, consequently, inclined at an angle of 45° with the vertical plane, is considered to be completely in the shade. The edge line, *a b*, Fig. 16, is therefore the line of separation of light and shade on the prism-shaped portion of the object, and the surface, *a b i d*, is consequently tinted. The shadow cast upon the prism by the overhanging head, B, reduces itself to that due to the portion, C^1 F^1 H^1, merely, of the circumference of the latter, and it falls upon the two faces, c^1 f^1 and f^1 h^1, of the latter.

The lines indicated on the diagram, with their corresponding letters, when compared with those of the preceding example, will show that the operations are precisely the same in both cases, and, in the latter, the curves, *c e f* and *f g h*, are the resulting outlines of the shadow. In general, it is unnecessary to obtain more than the extreme points of the curve, and another near the middle. Through the three points thus obtained, arcs of circles can then be drawn. The curves are, however, in reality elliptical.

SHADOW CAST BY ONE PRISM UPON ANOTHER.

274. Figs. 18 and 19 represent a couple of vertical projections, at right angles to each other, of a prism of an octagonal base, surmounted by a similar and concentric, but larger prism. Although the operations called for in this case are precisely the same as in the two preceding, still it is an exemplification which cannot be omitted; and its chief use is to show, that

The shadow cast by a straight line upon a plane surface is invariably a straight line; and, consequently, it is sufficient to determine its extreme points, in order to obtain the entire shadow in any one plane.

Thus, the straight line, E^1 C^1, casts a shadow upon the plane facet, f^1 c^1, which is represented by the straight line, *e c*, in Fig. 18.

It is further obvious, that

The shadow cast upon a plane surface, by any line parallel to it, must be parallel to that line.

Thus, the straight line, E^1 G^1, of the larger prism, B, being parallel to the plane facet, f^1 g^1, of the prism, A, casts a shadow upon the latter, which is represented by the straight line, *f g*, parallel to the line, F G, Fig. 18, the vertical projection of the edge, F^1 G^1. It is not, however, the same with the portion, *e f*, because the corresponding portion, E^1 F^1, of the edge of the larger prism, is not parallel to the facet, f^1 e^1, or f^1 c^1, but, having found the two points, *f*, *e*, they are simply joined, and *f e* is, as above, the outline of the shadow cast by the edge part, F E, of the prism, B, on the face, f^1 c^1, of the prism, A.

SHADOW CAST BY A PRISM UPON A CYLINDER.

275. Figs. 20 and 21 represent vertical projections, at right angles to each other, of a portion of an iron rod, A, surmounted by a concentric head, B, of a hexagonal base. The main object of this diagram is to show, that

When a right cylinder is parallel, or perpendicular, to a plane of projection, any straight line, which is perpendicular to the axis of *the cylinder, and parallel to the plane of projection, casts a shadow upon the cylindrical surface, which is represented by a curve, similar to the cross section of such surface.*

If, therefore, the cylinder is of circular base or cross section, as we have supposed in the present case, the shadow cast upon it will be a portion of a circle, of the same radius as the cylinder. Thus, the straight line, D^1 F^1, situated in a plane, at right angles to the axis of the cylinder, A, and being, at the same time, parallel to the vertical plane, casts a shadow upon the cylinder, which is represented by the portion, *c e f*, of a circle, the centre, o^1, of which is obtained by drawing through the point, O, a line, O I, representing the ray of light, and extending to the prolongation of the edge, D^1 F^1. The line, O I, cuts the circumference of the cylinder in the point, i^1, which is squared over to *i*, upon the other projection, H *i*, Fig. 20, of the ray, O I. The lower point, *c*, is obtained from the upper one, *i*, being symmetrical with reference to the axis of the cylinder. The ray, H *i*, being continued to the axis, cuts it in the point, o^1, which is, consequently, the centre of the arc, *c e i*, the radius, $i\ o^1$ or $c\ o^1$, of which is equal to that, O i^1, of the cylinder.

276. The edge, F^1 H^1, although situated in a plane perpendicular to the axis of the cylinder, is not parallel to the vertical plane, and does not, therefore, cast a shadow of a circular outline upon the cylinder, but one of an elliptical outline, as *f g h*, which is obtained by means of points, the operations being fully indicated on the diagrams. If the head, B, which casts a shadow upon the cylinder, were square, instead of hexagonal, as is often the case, one of the sides of the square, as I H^1, Fig. 21, being perpendicular to the vertical plane, would cast a shadow on the cylinder, having for outline the straight line, H *i*, making the angle of 45° with the axis.

Thus, whenever a straight line is perpendicular to the plane of projection, not only is its shadow, as cast upon this plane, a straight line, inclined at the angle of 45° *but it is also the same on an object projected in this plane, no matter of what form.*

OBSERVATION.—In the four examples last discussed, we have only represented half views of the objects in the auxiliary vertical projections, Figs. 15, 17, 19, and 21, this being quite sufficient for determining the shadow, as it is only that produced by this half which is seen. It is obvious, that the same operations will answer the purpose, whether the axis of the object be horizontal or vertical, the only difference being the different direction of the light, which must always be attended to.

SHADOW CAST BY A CYLINDER IN AN OBLIQUE POSITION.

277. In Figs. 22 and 23 we have given the vertical and horizontal projections of a right cylinder, having its axis horizontal, but inclined to the vertical plane. As in this oblique projection we cannot obtain the points of contact of the luminous rays with the base in a direct manner, it becomes necessary to make an especial diagram, in order to determine the lines of separation of light and shade, which are always straight lines, parallel to the axis of the cylinder. In this instance we have supposed the light to come in the direction of the line, R, O, in the horizontal projection, Fig. 23, and R O is the projection of R^1 O^1, in the horizontal plane, and R^1 O^1, the projection of R O, in the vertical projection.

To this effect, we shall make use of a general construction, susceptible of application to a variety of such cases. This construction consists in determining the projection of the luminous ray, in any given plane, perpendicular to either of the geometrical planes, whence may be derived its form and aspect in either of the latter planes. It follows, that if we have any curve in the given plane, we can easily find the point of separation of the light and shade situated upon this curve, by drawing a couple of tangents to it, parallel to the ray of light projected in this plane, and transferred to the other plane of projection.

Thus, let R O and R^1 O^1 be the projections of the luminous ray: it is proposed to find the projection of this ray upon the plane, *a b*, of the base of the cylinder. To obtain this, project the point, R, to *r*, by means of a perpendicular, to *a b*, and *r* O represents the horizontal projection of the ray of light upon the plane, *a b;* and the vertical projection, r^1 O^1, is obtained by squaring over the point, O to O^1, on the base line, and the point, *r* to r^1, on the horizontal, R^1 e^1, and then joining O^1 r^1. Next, draw tangents to the ellipses, which represent the vertical projections of the ends of the cylinder, Fig. 22, making these tangents parallel to the ray of light, r^1 O^1. Their points of contact give, on the one hand, the first line, $c^1\ d^1$, of separation of light and shade, which is visible in the vertical projection, and, on the other hand, the second line, $e^1\ f^1$, which is not visible in that projection.

By squaring over these points of contact, respectively, to the two ends, *a b* and *g h*, of the cylinder, in the horizontal projection, we obtain the corresponding lines of separation of light and shade, *c d* and *f e*, as in the horizontal projection; the former of which is invisible, whilst the latter is visible.

These same lines, *c d* and *f e*, Fig. 23, can be obtained independently of the vertical projection, Fig. 22, in the following manner: —Draw an end view of the cylinder, as at $a^2\ b^2$, having its centre in the continuation of the cylinder's axis. Upon this end view, also, draw the ray of light, as projected upon the base, after describing the circle, $a^2\ m\ b^2$, with the radius, O *a;* make $r\ r^2$ equal to the height of the point, R^1, above the bottom of the cylinder, thereby obtaining the line, $o\ r^2$, representing the ray of light upon the end view of the cylinder. Next, draw a couple of tangents to the circle, $a^2\ m\ b^2$, parallel to O r^2, and their points of contact, c^2, f^2, will represent the end view of the lines of separation of light and shade, which are transferred to the horizontal projection, Fig. 23, by perpendiculars drawn from them to the straight line, *a b*.

278. When the shadow proper, of the cylinder, has been thus determined, it will not be difficult to find the outline of its shadow cast upon the horizontal plane. In the first place, the shadows of the two bases are found, being in the form of ellipses; and next, those, $c^2\ d^2$ and $f^2\ e^2$, cast by the lines, *c d* and *f e;* namely, those of the separation of light and shade upon the object itself. These lines will necessarily be tangents to the ellipses, representing the shadows of the bases. It may be observed that the transverse axis of the ellipses are parallel to the line, r^2 O.

If the cylinder were inclined at an angle of 45° to the vertical plane, still remaining parallel, however, to the horizontal plane, the lines of separation of light and shade would, in the horizontal projection, be confounded with the extreme generatrices, or outlines, of the cylinder, the visible semi-cylinder being wholly in the light, and the opposite semi-cylinder wholly in the shade. In the vertical projection, the line of separation would be in the line of the axis, and would divide the figure horizontally into two equal parts.

OBSERVATION.—The shadows proper and cast, shown in the several Figures, 1 to 23, are not to be taken as examples of the strength or gradation of light and shade proper to the object in the circumstances described, as they are merely represented by a light flat tint, formed of fine lines, to indicate the form of the shadow, and yet not obscure the dotted lines necessary to the construction and projection of the shadow in the several planes of projection.

EXAMPLES

Fig. 1.

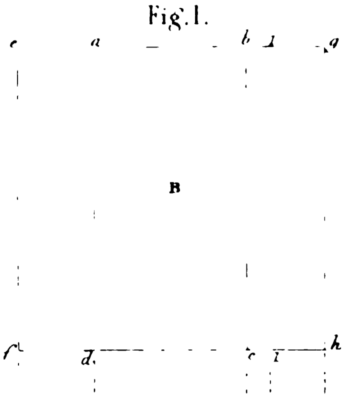


Fig. 2.

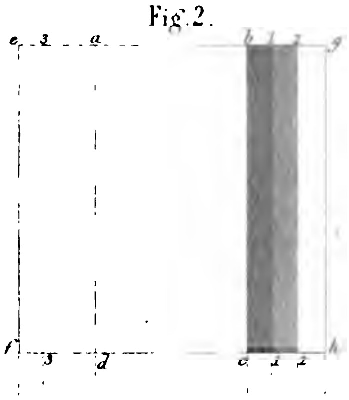


Fig. 3.

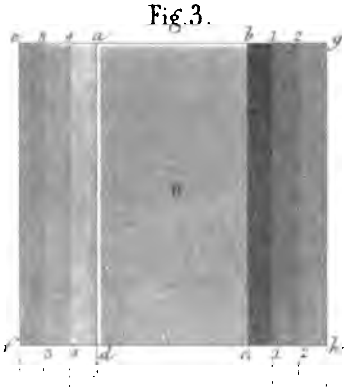


Fig. 4.

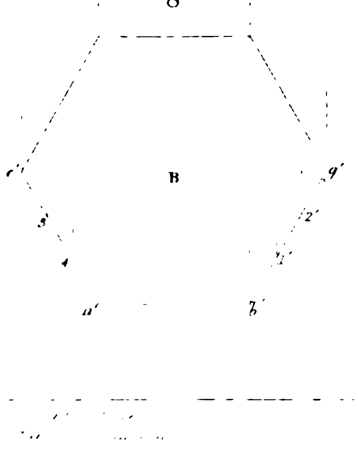


Fig. 5.

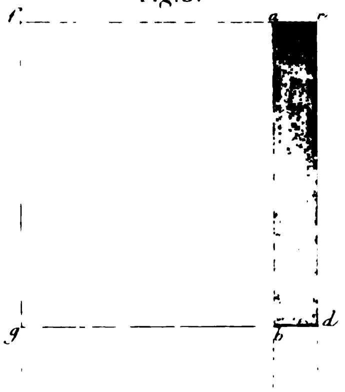


Fig. 6.

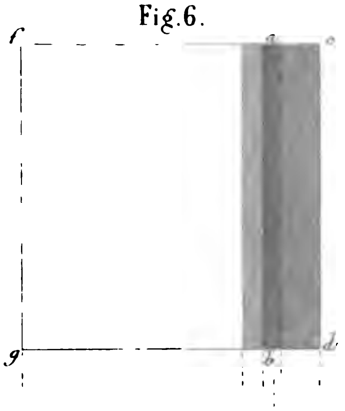


Fig. 7.

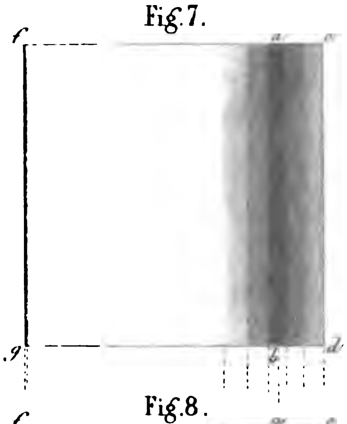
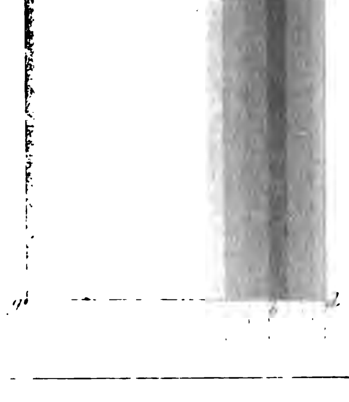
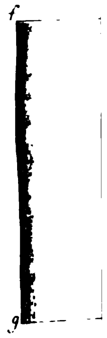
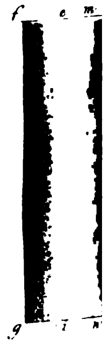


Fig. 8.







SHADING.

Fig. 17.

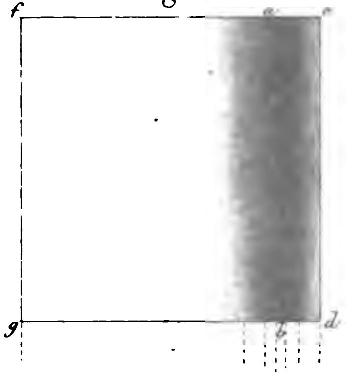


Fig. 18.

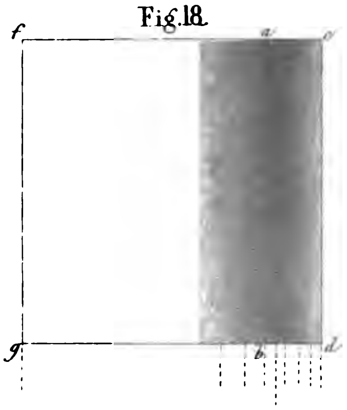


Fig. 19.

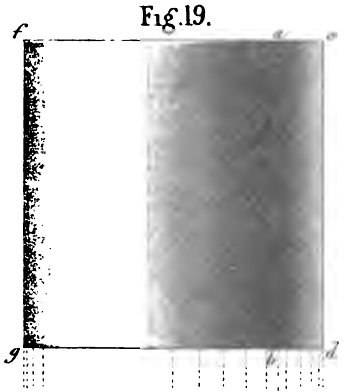


Fig. 20.

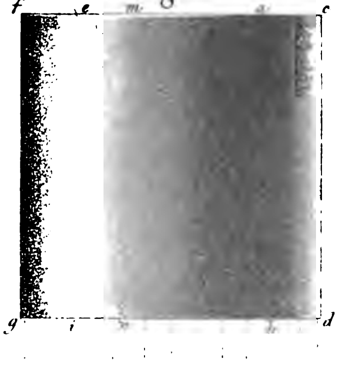


Fig. 13.

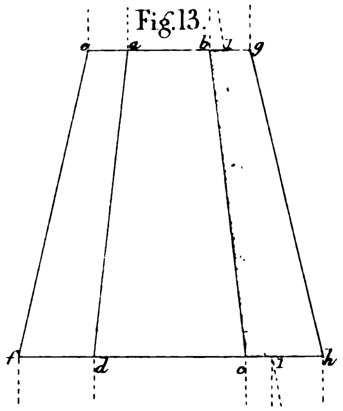


Fig. 14.

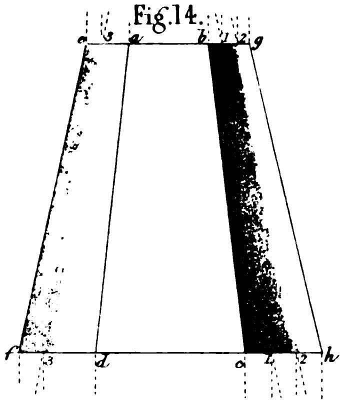


Fig. 15.

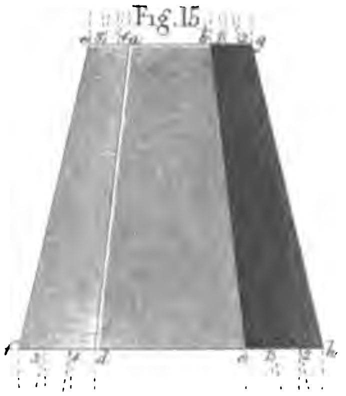
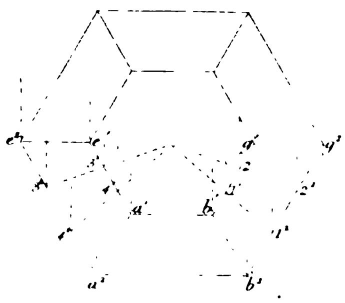


Fig. 16.



279. Figs. 1 to 5, at the top of this plate, represent several simple objects completely shaded, in accordance with the principles just discussed, in reference to Figs. 6 to 23, and besides being given as examples of the depth of shade and shadow due to the several parts may be taken as good examples in line shading. When shading a cylinder, Fig. 3, according to this system, the student ought to commence at *b*, with moderately thick lines, drawn very closely together, but gradually getting wider and finer until they cease to be visible at the light part, *a*, where a small portion of the surface is left pure white, and then a reverse gradation of fine lines are recommenced, gradually becoming stronger and closer until they attain their maximum strength at *c*, where they again take a reverse gradation and become rather wider and lighter, and finally end at the lower circumference, *d*, with a line rather thicker, and producing a darker shade than that of the darkest part, *b*, of the light side of the cylinder. This shading, when properly executed, gives the shading due to the rotundity of the object, as well as that of the shadow proper cast on the shade side of the cylinder itself, and due to the strength and direction of the light. In regard to the shadow cast on the cylinder, Fig. 4, the lines in it ought to get stronger and closer at the part, *a*, producing a reverse shade to that just described, in reference to the shading of the object itself, the lines of this shadow die away into the proper shading of the cylinder, at *b* and *c*. The principles for such line shadows, are in a great measure the same as that for colour shading, which will be treated of in reference to Plate 27.

PRINCIPLES OF SHADING.

PLATE 27.

280. Before proceeding to the further study of shadows, we must observe that the shadows, proper and cast—which have been simply represented by flat tints, so as not to render the diagrams confused—should be modified in intensity according to the form of the objects, and likewise the position of their surfaces in reference to the light, and to the eye of the spectator. The study of shading carries us somewhat into the province of the non-mechanical painter, who is guided by his taste rather than by mathematical rules; still, whilst we acknowledge the difficulty of laying down an exact theory on this subject, we would recommend the following systematic methods, which will render the first difficulties of the study more easily surmountable.

In painting, and in every description of drawing, the effects of light and shade depend upon the following principles:—

ILLUMINED SURFACES.

281. *When an illumined surface has all its points at an equal distance from the eye, it must receive a clear shade of uniform intensity throughout.*

In geometrical drawing, where all the visual rays are supposed to be parallel and perpendicular to the plane of projection, all surfaces parallel to this plane have all their points equally distant from the eye: such is the plane and vertical surface, *a b c d*, of the prism, Fig. 3.

282. *Of two such surfaces, disposed parallel to each other, and illumined in the same manner, that which is nearer to the eye should receive a shade of less intensity.*

283. *Any illumined surface, inclined to the plane of the picture, having its points at varying distances from the eye, should receive a shade of varying intensity.*

Now, according to the foregoing principle, it is the most advanced portion of an object which ought to be the lightest in colour; this effect is produced on the face, *a d f e*, which, as shown in the plan, Fig. 4, is inclined to the vertical plane of projection.

284. *Of two illumined surfaces, that which is more directly presented to the rays of light should receive a shade of less intensity.*

Thus the face, e^1 a^1, Fig. 4, being presented more directly to the light than the face, a^1 b^1, is covered with a shade which, being graduated because of the inclination to the plane of the picture, is still, at the more advanced portion, of less intensity than that of the latter face. It is near the edge, *a d*, that the difference is more sensible.

SURFACES IN THE SHADE.

285. *When a surface in the shade is parallel to the plane of projection, or of the picture, it must receive a deep tint of uniform intensity throughout.*

An exemplification of this will be seen on the fillet, B, Fig. 7, Plate 28, which is parallel to the vertical plane: the difference of shade upon this fillet, in comparison with that upon the more projecting portion, A, which is parallel to it, but in the light, distinctly points out the difference between an illumined surface and one in the shade, in conformity with the two principles, 281, and 285, just enunciated.

286. *Of two parallel surfaces in the shade, that nearer the eye should receive the deeper tint.*

Thus, the shadow cast upon the fillet, B, Fig. 7, Plate 28, is sensibly deeper than that cast by it upon the vertical plane, which is more distant.

287. *When a surface in the shade is inclined to the plane of the picture, the part nearest to the eye should receive the deepest tint.*

The face, *b g h c*, Fig 3, Plate 27, projected horizontally in b^1 g^1, Fig. 4, is thus situated. The shade is made considerably deeper near the edge, *b c*, than near the more distant one, *g h*.

288. *When two surfaces in the shade are unequally inclined, with reference to the direction of the rays of light, the shadow cast by any object should be deeper upon that which receives it more directly.*

Thus, the shadow, *a d f e*, cast upon the face, F, of the prism, Fig. 5, Plate 26, should be slightly stronger than that cast upon the face, G, because the first is more directly presented to the light than the second, as shown by the lines, f^1 h^1 and f^1 c^1, Fig. 17.

These first principles are exemplified in the finished figures, 1, 2, 3, 4, and 5, Plate 26, also in those of 3, 11, 16, and 19, of plate 27, and in subsequent ones.

As in order to produce the gradations of shades, it is important to have some knowledge of actual colouring or shading by means of the brush, we shall proceed to give a few short explanations of this matter.

Two methods of producing the graduated shades are in use—one consisting in laying on a succession of flat tints; the other, in softening off the shade by the manipulation of the brush.

We have already said two or three words about the laying on of flat tints, when treating of representing sections by distinguishing colours (139). These first precepts may serve as a basis for the first method of shading, which is the less difficult of the two for beginners. In fact, according to it, the graduated shade is produced by the simple superposition of a number of flat tints.

FLAT-TINTED SHADING.

289. Let it be required to shade a prism, Fig. 3, Plate 27, with flat tints :—

According to the position of this prism, with reference to the plane of projection, as seen in Fig. 4, it appears that the face, a^1 b^1, is parallel to the vertical plane, and is fully illumined; it should, consequently, receive a clear uniform tint, spread over it by the brush, and made either from China ink or sepia, as shown upon the rectangle, *a*, *b*, *c*, *d*, Fig. 3. When the surface to be washed is of considerable extent, the paper should first be prepared by a very light wash, the full intensity required being arrived at by a second or third (139); a very narrow light part being left round the edges nearest the light, as shown at *a b*, *a d*, Fig. 3.

The face, b^1 g^1, being inclined to the vertical plane, and completely in the shade should receive a tint (285) deepest at the edge, *b c*, and gradually less intense towards *g h*; this is obtained by laying on several flat shades, each of different extent. For this purpose, and to proceed in a regular manner, we recommend the student to divide the face, b^1 g^1, Fig. 4, into several equal parts, as in the points, 1^1, 2^1, and through these points to draw lines parallel to the sides, *b c*, *g h*, Fig. 3. These lines should be drawn very lightly indeed, in pencil, as they are merely for guides. A first greyish tint is then spread over the surface comprised between the first line, 1—1, and the side, *b c*, as in Fig. 1; when this is quite dry, a second like it is laid on, covering the first, and extending from the side, *b c*, to the line, 2—2, as in Fig. 2. Finally, these are covered with a third wash, as in Fig. 3, extending to the outer edge, *g h*, and completing the graduated shade of the rectangle, *b c h g*.

The number of washes by which the gradation is expressed, evidently depends upon the width of the surface to be shaded; and it will be seen that the greater the number of washes used, the lighter they should be, and the lines produced by the edges of each will be less hard, and a more beautiful effect will result.

The student must remember to efface the pencilled guide-lines, as soon as the first washes coming up to them are sufficiently dry.

290. This method of overlaying the washes, and covering a greater extent of surface at each succeeding time, is preferable to the one sometimes adopted, according to which the whole surface, *b g h c*, is first covered by a uniform wash; a second being then laid over *b* 2—2 *c*; and, finally, a third over the narrow strip, *b* 1—1 *c*, Fig. 3. When the shade is produced in this manner, the edges of the washes are always harder than when the washes are laid on as we recommend—the narrowest first—for the subsequent washes, coming over the edge of each preceding one, soften it to a considerable extent.

The face, e^1 a^1, Fig. 4, being likewise inclined to the vertical plane, but being wholly illumined, should receive a very light shade (283), being, however, a little bolder towards the outer edge, *e f*, Fig. 3. The shade is produced in the same way as that of the face, b^1 g^1, but with much fainter washes as shown in the same, Figs. 1, 2 and 3.

291. Let it be proposed to shade a cylinder, as shown in vertical and horizontal projections in Figs. 11 and 12, with a series of flat tints :—

In a cylinder, it is necessary to give the gradations of shade, both of the illumined and of the non-illumined portion. In reference to this, it will be recollected that the line of separation, *a b*, of light and shade, is determined by the radius inclined at an angle of 45° to the vertical plane, or as O *a*, Fig. 12, perpendicular to the ray of light, R O, which is the projection of the ray of light proper to the vertical projection, Fig. 11; consequently, all the shadow proper, which is apparent in this projection, is comprised between the line, *a b*, and the extreme generatrix, *c d*. Consequently, according to the principle already laid down (286), the shade of this portion of the surface should be graduated from *a b* to *c d*, as was the case with the inclined plane surface, b^1 g^1, Fig. 4, the greater intensity being towards *a b*.

On the other hand, all that part of the cylinder comprised between the line, *a b*, and the extreme generatrix, *f g*, is in the light; at the same time, from its rounded form, each generatrix is at a different distance from the vertical plane of projection, and makes different angles with the ray of light. Consequently, this portion of the surface should receive graduated shades (283). To express the effect in a proper manner, it is necessary to know what part of the surface is the clearest and most brilliant; and this is evidently the part about the generatrix, *e i*, Fig. 11, situated in the vertical plane of the ray of light, R O, Fig. 12. In consequence, however, of the visual rays being perpendicular to the vertical plane and parallel to the line, V O, the portion which appears to the eye to be clearest will be nearer to this line, V O, and may be limited on the one hand, by the line, T O, bisecting the angle made by the lines, R O and O V, and on the other, by the line, R O; squaring over, then, the points, e^1 and m^1, Fig. 12, and drawing the lines, *e i* and *m n*, Fig. 11, we obtain the surface, *e i m n*, which is the most illumined.

292. This surface is bright, and remains white, when the cylinder is polished, as a turned iron shaft, for example, or a marble column; it is covered with a light shade, being always clearer, however, than the rest of the surface, when the cylinder is unpolished, as a cast-iron pipe.

293. After these preliminary observations, we may proceed to shade the cylinder, f^1 m^1 a^1 c^1, Fig. 12, dividing it into a certain number of equal parts, the more numerous according as the cylinder is greater. These divisions are squared over to the vertical projection and straight lines, drawn lightly with the pencil, as limiting guides for the colour. We then lay a light gray shade on the surface, *a c d b*, Fig. 5, to distinguish at once the part in the shade; when this is dry we lay on a second covering, the line, *a b*, of separation of light and shade, as shown in Fig. 6, and extending over a division on either side of it, or as near it as possible, as the line of separation of light and shade will sometimes fall in the centre of a division, as in this instance; we afterwards lay on a third shade, covering two divisions to the right and to the left, as in Fig. 7; and proceed in the same manner, covering more and more each time, always keeping to the pencil lines. The different stages are represented in Figs. 8, 9, and 10.

294. We next shade the part, *f e i g*, laying on successive shades, but lighter than the preceding, as indicated in Figs. 7, 8, 9, and 10.

The operation is finally terminated by laying a light wash over the whole, leaving untouched only a very small portion of the bright surface, *e m n i*, Fig. 11. This last wash gives a beautiful and softening effect to the whole, and leaves the cylinder completely shaded as shown in this figure.

SHADING BY SOFTENED WASHES.

295. This system of shading differs from the former in producing the effects of light and shade by imperceptible gradations, obtained by manipulation with the brush in the laying on of the colour: this system possesses the advantage over the first, of not leaving any lines, dividing the different degrees of shade, which sometimes

PROJECTION

Fig. 1.

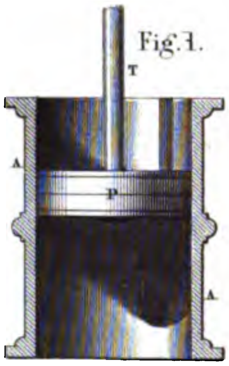


Fig. 4.

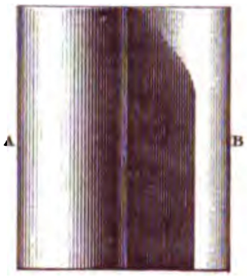


Fig 7

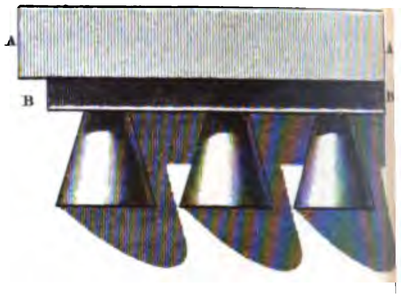


Fig. 2.

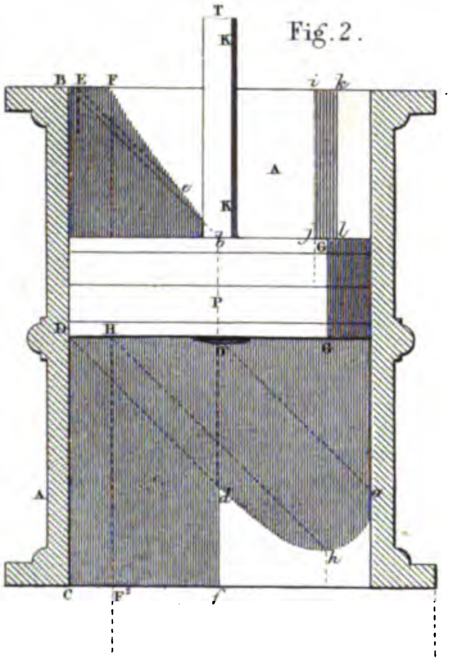


Fig. 5.

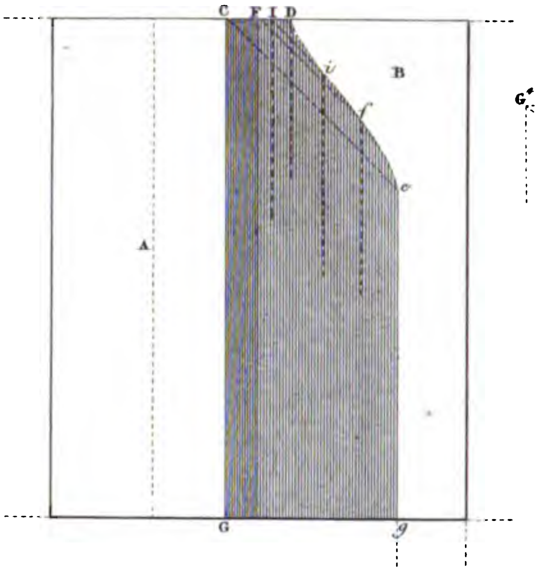


Fig. 8.

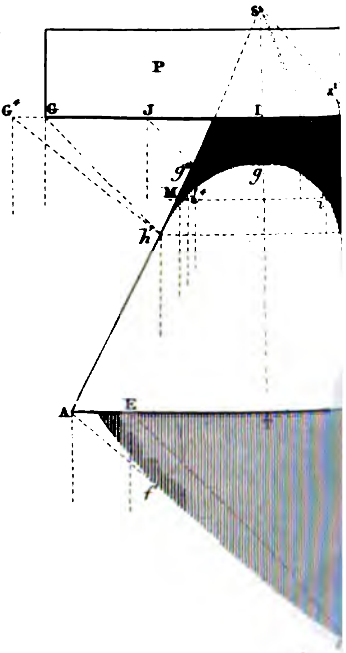


Fig. 3.

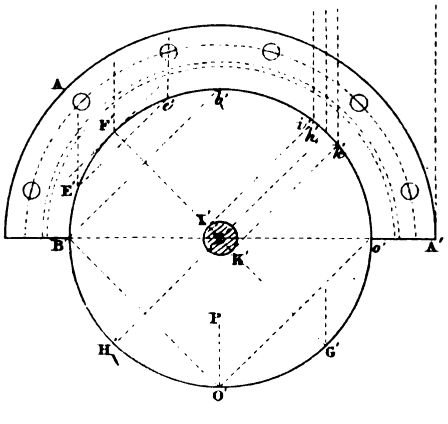


Fig. 6.

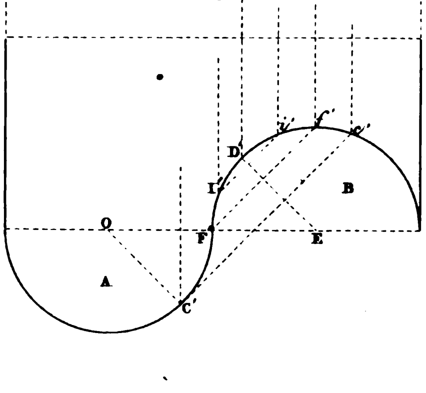
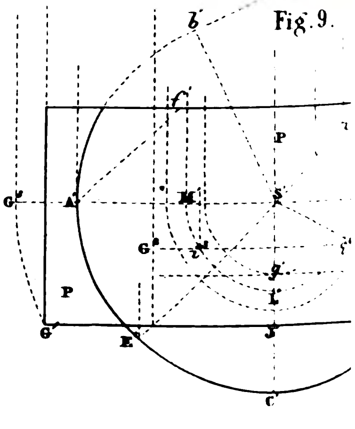


Fig. 9.



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F SHADOWS.

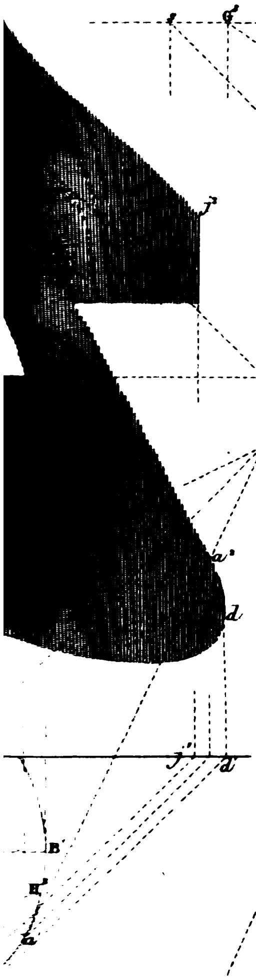


Fig.10.

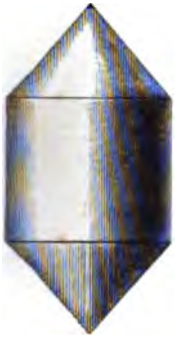


Fig.11.

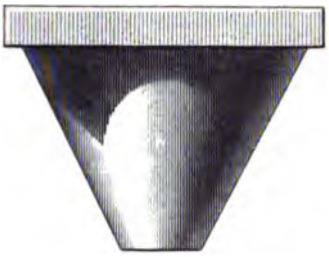


Fig.14.

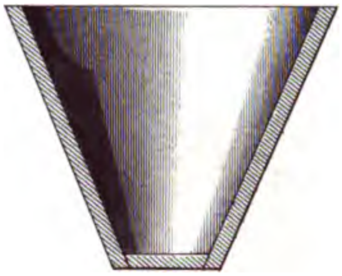


Fig.12.

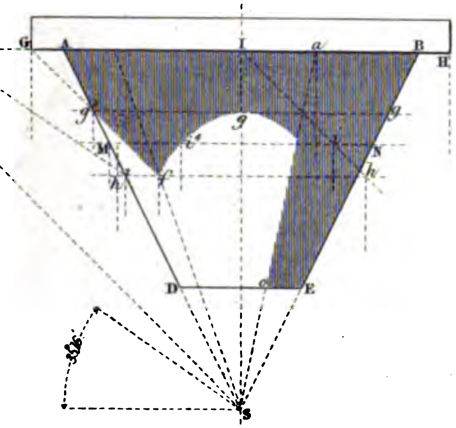


Fig.15.

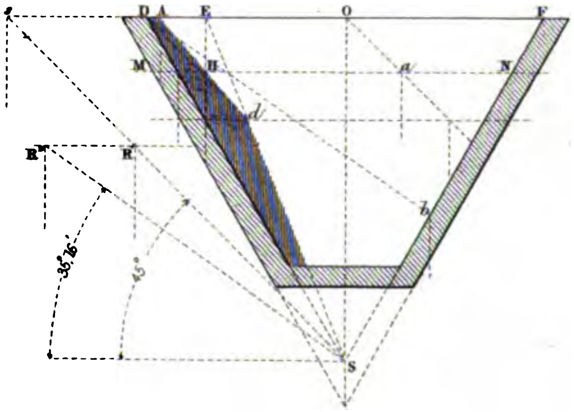


Fig.13.

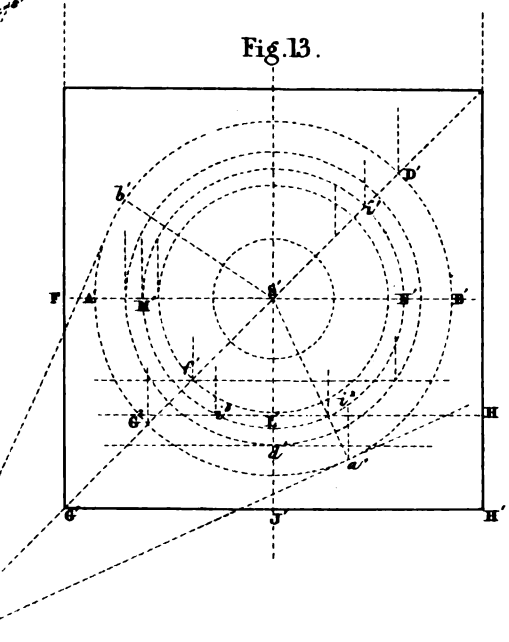
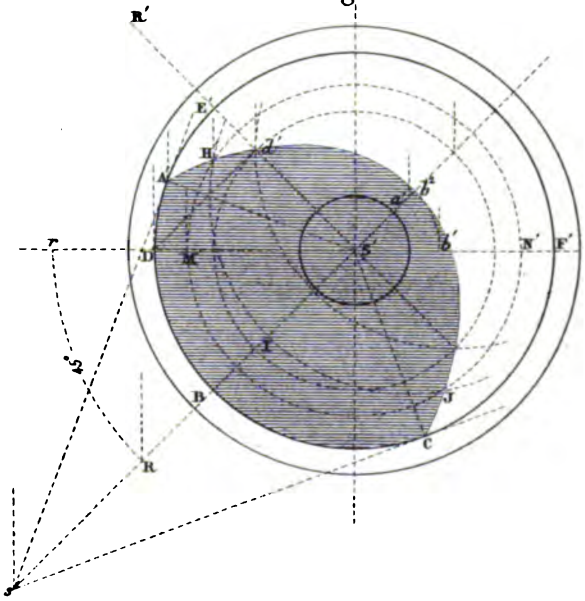


Fig.16.



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appear harsh to the eye, and seem to represent facets or flutings, which do not exist.

For machinery, generally, the former system is very effective, bringing out the objects so shaded in a remarkable manner. However, where very soft effects are desired, as in the finer class of drawings of machinery and in architectural subjects, we recommend them to be treated according to the second system.

In this, the laying on of the shade is much more difficult, requiring considerable practice, which will be aided by proceeding in the following systematic course.

296. Let it be proposed to shade a truncated hexagonal pyramid, as shown in vertical and horizontal projections, in Figs. 15 and 16.

The position of this solid, with reference to the vertical plane of projection, is the same as that of the prism, shown in Figs. 3 and 4 of this plate. Thus the face, *a b c d*, should receive a uniform flat shade of little intensity; rigorously keeping to the rules already given, this should be slightly darkest at the top, and graduated regularly to the bottom, as the face is not quite parallel to the vertical plane.

The face, *b g h c*, being inclined, and in the shade, should receive a deep shade, graduated from *b c* to *g h*; to produce this, apply a first light shade to the side, *b c*, Fig. 13, softening it off to the right, taking the line, 1—1, as a limiting guide in that direction: this softening is produced by clearing the brush, so that the colour may be all expended before the lighter side is reached; and when the shade is wide, a little water should be taken up in the brush once or twice, to attenuate the colour remaining in it. By these means an effect will be produced like that indicated in Fig. 13, care being taken not to extend the wash beyond the outline of the object.

When this first wash is well dry, a second is laid over it, produced exactly in the same manner, and extending further to the right, covering the space, *b c* 2—2, as shown in Fig. 14. Proceeding in the same manner, according to the number of divisions of the face, we at length cover the whole, producing the graduated shade, *b g h c*, Fig. 15.

The operations are the same for the face, *e a d f*, which is nearly perpendicular to the rays of light, but is considerably inclined to the plane of projection.

In rigorously following out the established principles, the shade on this face should be graduated, not only from *e f* to *a d*, but also from *e a* to *f d*. Also, on the face, *b g h c*, in the shade, the tint should be a trifle darker at the base, *c h*, being graduated off towards *b g*. But for objects so simple in form as the one under consideration, this nicety may be neglected—at any rate, by the beginner—as only increasing his difficulties: the proficient on the other hand, is well aware how attention to these refinements assists in producing effective and truthful representations.

297. Let it be proposed to shade a cylinder with softened washes, as shown in vertical projection in Fig. 20.

By following the indications given in Fig. 12, for the regular imposition of the shades, as explained with reference to the flat-wash shading, the desired effect may be similarly produced, by substituting the softened washes. It is scarcely necessary to divide the circumference into so many parts as for the former method; a first shade must be laid on at the line, *a b*, of separation of light and shade, and this must be softened off in both directions, as in Fig. 17; a second and a third wash must then be applied and similarly softened off, and in this manner we attain the effects rendered in Figs. 18 and 19, and as completely shaded in that of Fig. 20.

OBSERVATION.—We have not deemed it necessary to give diagrams of all the stages, as the method of procedure will be easily understood from the preceding examples. The student should practise these methods, both of line and wash shading, upon different objects of simple form, and we would also recommend him to adopt a much larger scale in his practice, as it is desirable to be able to produce large washes with regularity and smoothness of effect; a desideration which he will soon acquire, as well as the necessary facility of execution, by adhering systematically to the rules just laid down.

298. When spots or inequalities arise in laying on a wash, from defects in the paper or other accidents, they should be corrected with great care. If they are on the dark side, they should, if possible be washed out; the best means of doing this, in very bad cases, is to let the drawing become perfectly dry, and then slightly moisten the spots, and gently rub off the colour with a clean rag. Lights may be taken out in this way, where, from their minuteness or intricate shape, it would be difficult to leave them whilst laying on a flat shade, in the midst of which they may happen to be. A defect on the light side is more easily corrected, by applying more colour to the spots in question—being careful to soften off the edges, and to equalize the whole wash.

CONTINUATION OF THE STUDY OF SHADOWS.

PLATE 28.

SHADOW CAST UPON THE INTERIOR OF A CYLINDER.

299. When a hollow cylinder, as a steam-engine cylinder, a cast-iron column, or a pipe, is cut by a plane passing through its axis, we have, on the one hand, a straight projecting edge, and, on the other, a portion of one of the ends, which cast shadows upon the internal surface of the cylinder.

We propose, then, to determine the form, of the shadow cast upon its interior by a steam-engine cylinder, A, sectioned by a plane passing through its axis, Figs. 1, 2, and 3. In the first place, we seek the projection of the shadow cast by the rectilinear projecting edge, B C, of the vertical section, Fig. 2, which is, in fact, produced by the intersecting plane, B^1 A^1, Fig. 3. This straight line, B C, being vertical, is projected horizontally in the point, B^1, and casts a shadow upon the inner surface of the cylinder, as represented by the straight line, *b f*, which is also vertical, and is determined by the point, b^1, of intersection of the ray of light, B^1 b^1, with the surface of the cylinder, B^1 b^1 o^1. *Thus, when a straight line is parallel to a generatrix of the cylinder, the shadow cast by it will be a straight line parallel to the axis.* It is, therefore, evidently quite sufficient to find a single point, whence the entire shadow may be derived.

300. We next proceed to determine the shadow cast upon the interior of the cylinder by the circular portion, B^1 E^1 F^1, of the upper end. If we take any point, E^1, on this circle, and square it over to E in the vertical projection, and draw through these points rays of light, E^1 e^1, E *e*, the form will be found to meet the cylindrical surface in the point, e^1, which is squared over to intersect the second in *e*, the length of the ray being equal in both projections, according to the rule before given. This applies to any point in the arc, B^1 F^1, so that any number of points, as *e*, can be obtained, and the line drawn through them. The extreme point on one side is obtained by a tangent to the circle in the point, F^1, giving the point, F, in the vertical projection; the opposite extreme point, *b*, being already given as the top point of the vertical edge, B C; we have,

therefore, the curve, F *e b*, for the upper outline of the shadow due to the circular portion, B^1, E^1, F^1.

301. If, as shown in these figures, we suppose the piston, P, with its rod, T, to be retained unsectioned in the cylinder, we shall have to determine the form of the shadow cast by the projecting part of the piston upon the interior of the cylinder, and represented by the curve, *d h o*, Fig. 2. For this purpose we take any points, B^1, H^1, O^1, on the circumference of the piston, and draw through them, in both projections, the rays of light which meet the surface of the cylinder, B^1 b^1 o^1, in the points, b^1, h^1, o^1, which are projected in *d*, *h*, *o*, in the vertical plane; the curve passing through these points is the outline of the shadow sought. The curved portions of these shadows are elliptical.

The piston-rod, T, being cylindrical and vertical, casts a shadow, of a rectangular form, upon the interior of the cylinder, the vertical sides, *i j*, *k l*, being determined by tangents to the piston rod, I^1 i^1, K^1 k^1, parallel to the luminar ray, H^1 h^1, passing through the axis, and squared up to the vertical plane. The line of separation of light and shade on the piston, P, and rod, T, in Fig. 2, is found by squaring up the points, G^1, and K^1, Fig. 3, found by the line, G^1, K^1, intersecting their respective surfaces at right angles to the ray of light through their axis, and squaring them over to G G, and K K, in the vertical section, Fig. 2.

SHADOW CAST BY ONE CYLINDER UPON ANOTHER.

302. Let Figs. 5 and 6 be the vertical and horizontal projections of a convex semi-cylinder, A, tangential to a concave semi-cylinder, B, forming a pattern often met with in mouldings, and shown as finished in Fig. 4.

This problem, which consists in determining the shadow proper of a convex cylinder, together with that cast by it upon the surface of a concave cylinder, in addition to that cast by the latter upon itself, is a combination of the cases discussed in reference to Figs. 12 and 13, Plate 26; and to Figs. 1, 2, and 3, in the present plate. The operations called for here are fully indicated on the diagram; and we have merely to remark, that it is always well to start by determining the extreme points, as C^1, D^1, which limit the shadow proper, C G, and cast shadow, D *c g*; these points may be obtained more exactly, as already pointed out, by drawing the radii, O C^1, and D^1 E, perpendicular to the projection of the luminous rays in Fig. 6.

SHADOWS OF CONES.

303. In this branch of the study, we propose to determine, first, the shadow proper, or the line of separation of light and shade upon the surface of the cone; second, the shadow cast by the cone upon the vertical plane of projection; and third, the shadow cast upon the cone, and upon the vertical plane of projection, by a prism of a square base, placed horizontally over the cone, as shown in a finished state in Fig. 7.

304. First: We have laid it down as a general principle, that, in order to determine the shadow proper of any surface, it is necessary to draw a series of parallel luminous rays tangential to this surface. When, however, the body is a solid of revolution generated by a straight line, as a cylinder or a cone, it is sufficient to draw tangential planes parallel to the luminous rays, to obtain the lines of separation of light and shade.

In the case of the cone represented in Figs. 8 and 9, and of which the axis, S T, is vertical, the operation consists in drawing from the apex, S and S^1, two lines, making angles of 45°, as S *s* and S^1 s^1, giving, in the point, s^1, the shadow cast by the apex as projected in the horizontal plane. From this point we draw a straight line, a^1 s^1, tangential to the base, A^1 C^1 B^1, of the cone. This straight line represents the plane, tangential to the cone, as intersecting the horizontal plane of the base; and the contact generatrix is then obtained by letting fall from the centre, S^1, a radius, S^1 a^1, perpendicular to the line, a^1 s^1; and this line, S^1 a^1, is the horizontal projection of one of the lines of separation of light and shade. The vertical projection of this straight line is obtained by squaring over the point of contact, a^1, to *a*, and then drawing the straight line, S *a*. The other line, S^1 b^1, of the separation of light and shade, is similarly obtained by means of the tangent, s^1 b^1. Its vertical projection is, however, not apparent in Fig. 8.

305. Second: The shadow cast by the cone upon the vertical plane is limited, on the one hand, by the line of separation of light and shade, and, on the other, by the portion of the illumined base comprised between the two separation lines. Now, the straight line, S *a*, casts a shadow, represented by the straight line, s^2 a^2, as indicated in Fig. 8; and the base, A^1 E^1 C^1 B^1, casts a shadow, represented by the elliptic curve, *f e d* a^2, which is determined by points, as in the case considered in reference to Fig. 23, Plate 26.

306. Third: The shadow cast by the lower side, G H, of the rectangular prism, P, upon the convex surface of the cone, is found in accordance with the principle already enunciated—that when a straight line is parallel to a plane of projection, it casts a shadow upon this plane, which is represented by a straight line, equal and parallel to itself. It follows, then, that if we cut the cone by a plane, M N, parallel to its base, the shadow cast by the straight line, G H, upon this plane, will be found by drawing from the point, I, of the base, situated upon the axis of the cone, and projected horizontally in the point, S^1, a luminous ray, which meets this plane, M N, in the point, *i*, projected horizontally in the point, i^1, upon the projection of the same ray. If, next, we make i^1 i^2 equal to S^1 J^1, and through i^2 draw the straight line, G^2 H^2, this last will be the shadow cast by the edge, G H, of the prism, upon the plane, M N. This plane, however, cuts the cone in a circle, the diameter, M N, of which is comprised between the extreme generatrices, whilst the circle is projected horizontally in M^1 L^1 N^1. The intersection of the straight line, G^2 H^2, with this circle, gives the two points, i^2 and i^3, which being projected vertically in *i*, i^4, upon the straight line, M N, constitute two points in the outline of the shadow cast upon the cone.

Continuing the operations in this manner, and taking any other intersecting plane parallel to M N, any number of points may be obtained. It will be observed that these planes are taken at a convenient height, where the projections of the straight line, G H, cut the corresponding circles; and with regard to this, much useless labour may be avoided, by at first determining the limiting points of the curve. Thus, in the example before us, we get the summit, *g*, of the curve, by making I J, Fig. 8, equal to J^1 S^1, Fig. 9. Through the point, J, we then draw a luminous ray, and the point, g^2, at which it meets the extreme generatrix, A S, of the cone, is squared over to the generatrix, S T, by means of the horizontal, g^2 *g*, which gives the point, *g*, as the summit of the curve. We next obtain the extreme points, *h*, h^1, of the same shadow, by making S^1 G^3, Fig. 9, equal to S^1 G^1, and squaring over G^3 to G^4, in Fig. 8. Through G^4 draw the straight line, G^4 h^1, parallel to the luminous ray, as situate in a vertical plane, passing through it; the ray, as we have already seen, making in this plane, an angle of 35° 16′ with the base line. The point, h^1, at which this ray meets the extreme generatrix, S A, determines the plane, h^1 *h*, which is intersected by the luminous rays making angles of 45°, and drawn through the points, I and G, in the points, *h*, h^1, the limits of the curve sought.

The projection of the shadow cast by the prism, P, upon the vertical plane is fully indicated: but as this presents no peculiarity apart from the principles already fully explained in reference to the shadows cast by a prism, Figs. 6 and 7, Plate 26, it is unnecessary to say much upon this part—the luminous rays through the points, *j* and H, being limited by the projection of the same rays through the point, h^1, till it meets the vertical plane in j^1, Fig. 9, and squared up to j^2, gives the outline of the shadow corresponding to the edge, *j* H, cast on the vertical plane.

SHADOW OF AN INVERTED CONE.

307. When the cone, instead of resting upon its base, has its apex downwards, as is the case with the one represented in Fig. 11, as fully shaded, and also in vertical and horizontal projection, in Figs. 12 and 13; in this case the rays of light illumine a less portion of its surface; and the lines of separation of light and shade are determined by drawing from the apex, S S^1, lines at an angle of 45°, which are prolonged towards the light, until they intersect the prolongation of the plane of the base, A B.

It will be observed, that the points of intersection, *s* s^1, lie to the left, instead of to the right of the cone. Through the point, s^1, the horizontal projection of the point, *s*, draw a couple of lines, $s^1 a^1$, $s^1 b^1$, tangential to the circumference, $A^1 G^2 B^1 D^1$, of the base. The radii, $S^1 a^1$ and $S^1 b^1$, drawn to the points of contact, represent the horizontal projection of the two lines of separation of light and shade, and show that the illumined portion of the cone, consisting of the surface, $b^1 G^2 a^1 S^1$, is smaller than the portion, $b^1 D^1 a^1 S^1$, in the shade. In the case of the cone with its apex uppermost, the contrary would be observed, the portion in the shade being there less than that in the light; and the method given of determining the proportion of shadow of the inverted cone is suggested by the consideration, that this proportion must be exactly the reverse of that for the cone with its apex uppermost.

The first-mentioned line, $S^1 a^1$, is the only one apparent in the vertical projection, Fig. 12. It is found by squaring over the point, a^1, to *a*, and joining this last to the apex, S. As the cone is truncated by the plane, D E, the line of separation obviously terminates at the point, *c*, of its intersection with this plane.

308. The cone, thus inverted, is surmounted, moreover, by a square plinth, the sides of which, F G^1 and $G^1 H^1$, cast shadows upon its convex surface. The side, F G^1, as projected vertically in G, Fig. 12, is perpendicular to the vertical plane, and consequently its cast shadow is a straight line, making an angle of 45°, as G *f*. The extreme limit, *f*, is determined by proceeding as in previous examples; that is to say, by making I G^3, in Fig. 12, equal to $S^1 G^1$, in Fig. 13, and then drawing through the point, G^3, the straight line, $G^3 h^2$, parallel to the ray of light, as in the diagonal plane, that is, at an angle of 35° 16′ to the horizon; next draw the horizontal line, h^2 *h*, and it will be intersected by the straight line, G *f*, in the point, *f*, which is consequently the shadow cast by the corner, G.

The following method, although more complicated, is of more universal application:—Draw the vertical projection of the outline of the body which receives the shadow, as sectioned by the vertical plane, in which the ray of light lies, which passes through the point whose shadow is sought; draw the same ray of light as projected in the vertical plane, and its intersection with the projection of the sectional outline will be the projection of the shadow of the point.

Thus, in the present instance, as the plane of the ray, $G^1 S^1$, passes through the apex of the cone, the latter will present a triangular section, the vertical projection of which may be obtained by squaring over the point, G^2, Fig. 13, to the base line, A B, Fig. 12; then, if a straight line is drawn from the vertical projection of this point to the apex, S, it will represent the projection of the section of the cone, and it will be intersected by the luminous ray, G *f*, in the point, *f*, which is the point sought.

If the plane passing through the point does not likewise pass through the axis of the cone, the section will be a parabola, which may be drawn according to methods already discussed. If the object is a sphere instead of a cone, the section will be a circle, whether the plane passes through the centre or not, and the vertical projection will, in all cases, be an ellipse. As a good idea of the whereabouts of the point sought may always be formed on inspection, it will generally be sufficient to find one or two points in the parabola or ellipse, near the supposed position, when a sufficient length of the curve may be drawn to give the intersection of the luminous ray, as G *f*.

As the plinth is square, the summit, *g*, of the curved outline, corresponding to the shadow of the front edge, G H, is obtained directly by the intersection in g^2 of the line, G *f*, with the extreme generatrix, A S, the horizontal line, g^2 *g*, being drawn through this point, gives the point, *g*, as the summit. Any other point in the curve, as i^1, is afterwards found by means of the sectional plane, M N, Fig. 12, and G^2 H, Fig. 13, which is the projection of the shadow cast by the edge, G H, in that plane, and it cuts the circle representing the section of the cone in the same plane in the point, i^2, which is obviously a point in the outline of the shadow.

SHADOW CAST UPON THE INTERIOR OF A HOLLOW CONE.

309. Figs. 14 and 15 represent vertical sections through the axis of a hollow truncated cone, and Fig. 16 a plan of the same, corresponding to Fig. 15. It is required to determine the vertical projection of the shadow cast by the sectional edge, D S, and by the small circular portion, $A^1 D^1$, projected vertically in A D, and the horizontal projection of the shadow cast upon the internal surface of the cone by the portion of the edge, A^1 B C.

It is to be observed, in the first place, that the straight line, D S, which is a generatrix of the cone, casts a shadow upon the latter, in the form of a straight line, for the plane parallel to the ray of light, and passing through this line, D S, must cut the cone in a generatrix; we therefore draw through the point, D^1, the ray of light, $D^1 d^1$, making an angle of 45° with the base line, and from the centre, S^1, let fall the perpendicular, $S^1 E^1$, this straight line representing the horizontal projection of the intersection of the cone by the plane passing through the line, $D^1 S^1$, and at the same time parallel to the ray of light. By squaring over the point, E, to E, Fig. 15, and joining E S, we have the vertical projection of this line of intersection, and, consequently, the shadow cast by the line, D S. The diagonal ray of light, D *d*, drawn through the point, D, determines the limit, *d*, of the shadow. The horizontal projection of the extreme points, A and C, of the curved outline of the shadow, is also obtained by means of the tangents, $s^1 A^1$ and s^1 C, drawn from the point, s^1, in which the ray of light passing through the apex intersects the plane of the base of the cone. The determination of the central or symmetrical point, b^2, of the same curve, is derived from the straight line, D *b*, drawn from the point, D, parallel to the ray of light, S B^2, as in the diagonal plane, that is, as at 35° 16′, with the horizontal plane. The point, *b*, in which this straight line meets the generatrix, F S, directly opposite to that passing through the point, D, is projected horizontally in the point, b^1, and carried round the centre, S^1, concentrically to the point, b^2, upon the prolongation of the diagonal ray of light, $s^1\ s^1$.

310. The operation for finding any intermediate point in the curve, is based on principles already explained; namely, that when a line or a surface is parallel to a plane, the shadow cast is also a line or a surface equal and parallel to the first. If, then, we draw a plane, M N, parallel to the base, D F, of the cone, the shadow cast by this base upon the plane, M N, will be a circle; it will, consequently, be sufficient to draw through the centre, O, Fig. 15, a ray, O *a*, which will meet the plane, M N, in *a*, which must be squared down to a^1, on the horizontal projection of the same ray. Next, with the point, a^1, for a centre, and with a radius equal to D O, describe a circle, H^1 I J; this will represent the entire shadow that would be cast by the base, D F, of the cone upon the plane, M N; this plane, however, cuts the cone in the vertical projection, in a circle, of which $M^1\ N^1$ is the diameter, and $H^1\ M^1$ J N^1 the horizontal projection; this circle is cut by the former in the points, H^1 and J, which are, consequently, two points in the outline of the shadow in Fig. 16, and the one of these which is seen in the vertical projection is squared over to H, upon the line, M N.

In the same way any number of points may be obtained, and a line drawn through them will give the outline of the shadow cast in both these projections, supposing the light to be coming in the direction of B^1 S, and B S^1, in the vertical and horizontal planes respectively, each being the projection of the other in the different plane. But when it is required to draw the shadow proper to the plan view, the light must be supposed to come in the direction of $B^1\ S^1$, in Fig. 16, and proceed accordingly as described in reference to the several Figs. on Plate 26, or, after finding the shadow as just projected, to turn it round, as it were, on the centre, S^1, 90°, or making the centre line, R S^1, of the shadow to coincide with $B^1\ S^1$, and outline it accordingly.

APPLICATIONS.

311. In the top row of figures of this plate, as well as in that of Plate 26, we have given shaded and finished representations of several objects, which serve as examples or applications of the several principles we have just pointed out, whether referring to shadows proper, or cast, or to graduated shading. Thus, Fig. 1 represents the interior of a steam-engine cylinder with piston and rod. In this example, regard has been had to the general principle, that shadows are the stronger the brighter the surfaces are on which they fall, if illumined, that is, when such surfaces are perpendicular to the rays of light, any shadow cast upon them will be most intense; the shadow cast by the piston on the inner surface of the cylinder is consequently made deepest or darkest about the generatrix, corresponding to G *h*, in Fig. 2, and situate in the vertical plane of the rays of light passing through the axis of the cylinder: to the right and left of this line, the shadow is softened off into the shadow proper on the one hand, and to the shading due to the shape of the cylinder on the other.

312. In the gradation of the shade, regard has also been had to the effects of the reflected light, which prevents a surface in the shade from being quite black. In a hollow cylinder, for the portion in the shade, it is the generatrix, F F^2, Fig. 2, which should receive the shade of least intensity, as it receives the reflected rays of light more directly. It will be recollected that the point, F^1, is obtained by means of the radius, T F^1, perpendicular to the projection of the ray of light proper to the vertical section, Fig. 2, through the axis, T.

313. Fig. 4 represents a portion of a common moulding, and shows how the distinction made between the shadow proper and the cast shadow tends to bring out and show the form of the object.

314. Fig. 7 is an architectural fragment from the Doric order, given as an application of shadows cast upon cones, as well as those cast by cones upon a vertical plane.

This example also shows how necessary it is, in producing an effective representation, to make a difference in the intensity of shadows cast upon planes parallel to the plane of projection, and at different distances from the eye; and also to give gradations to such shadows when cast upon rounded surfaces.

Fig. 10 is a combination of a cylinder with a couple of cones, with their apices in opposite directions, showing how differently the effects of light and shade have to be rendered upon each according as their surfaces lie more or less directly to the light and the eye.

There is less shadow upon the upper cone than upon the cylinder, whilst there is more upon the lower cone; the reasons of these differences have already been explained in reference to Figs. 8 and 12, and also at (290) in discussing the several figures on Plate 27.

Fig. 11 represents an inverted and truncated cone, showing the manner of shading the same, and the form of the shadow cast by the square tablet above; and Fig. 14 is a view of a hollow cone, sectioned across the axis, presenting further variety and examples of the several principles just discussed; in this case, the shadow cast on the unillumined surface is always shown lighter and less defined than that in Fig. 11, cast on the illumined surface.

TUSCAN ORDER.

PLATE 29.

SHADOW OF THE TORUS.

315. In geometry, the torus is a solid, generated by a circle, revolving about an axis, continuing constantly in the plane of this axis, in such a manner, that all sections made by planes passing through the axis are equal circles, and all sections by planes perpendicular to the axis will also be circles, but of variable diameters.

We have seen, that in architecture, the torus is one of the essential parts of the base, and of the capital of the column, of each order. It will, therefore, be useful to give the methods of determining the shadows upon it, or cast by it, in the quickest and most accurate manner.

Figs. 1 and 2 represent respectively the vertical and horizontal projections of a torus, A, supposed to be generated by the semicircle, *a f c*, revolving about the vertical axis, O P; namely, that of the column.

316. We propose to determine the shadow proper of this torus, or the line of separation of light and shade upon its external surface in Fig. 1. It will be convenient, in the first place, to seek the principal points, which, for the most part, present little difficulty. Thus, by drawing parallel to the ray of light, B^1 O, proper to the vertical projection, Fig. 1, a couple of tangents to the semi-circles, *a f c*, in Fig. 1, which limit the contour of the torus in this projection, we at once obtain the two extreme points, *b*, *d*, of the curved line of separation. These points are more exactly defined by letting fall perpendiculars from the centres, *o*, o^1, of the semi-circles, upon the tangents. Then, by drawing through the point, *b*, the horizontal, *b e*, the middle point, *e*, of the curve will be obtained upon the vertical line, O P.

To obtain the other points in the curve, we first find them in the

EXAMPLES IN THE PROJECT

Fig. 1.

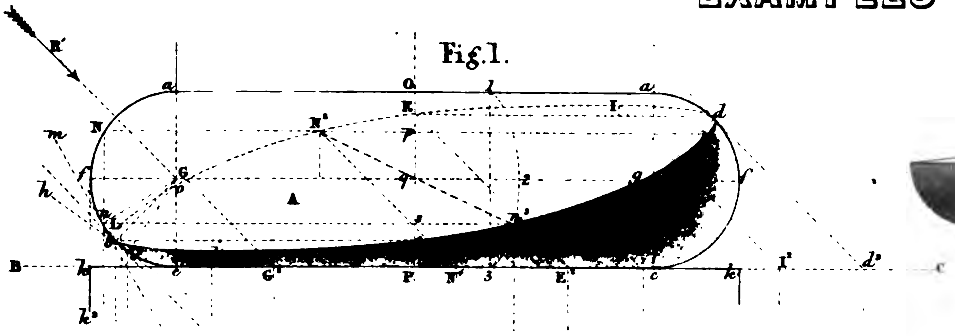


Fig. 2.

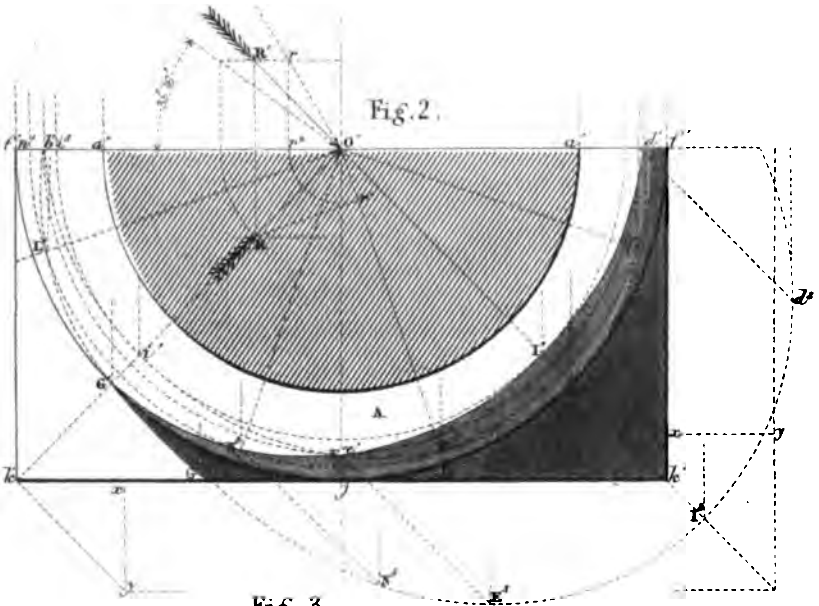


Fig. 3.

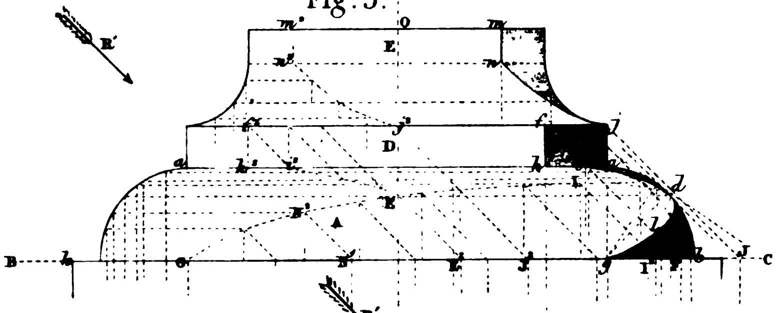


Fig. 4.

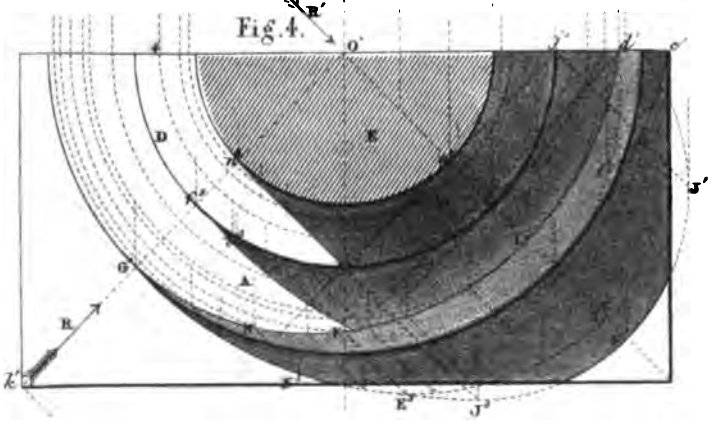
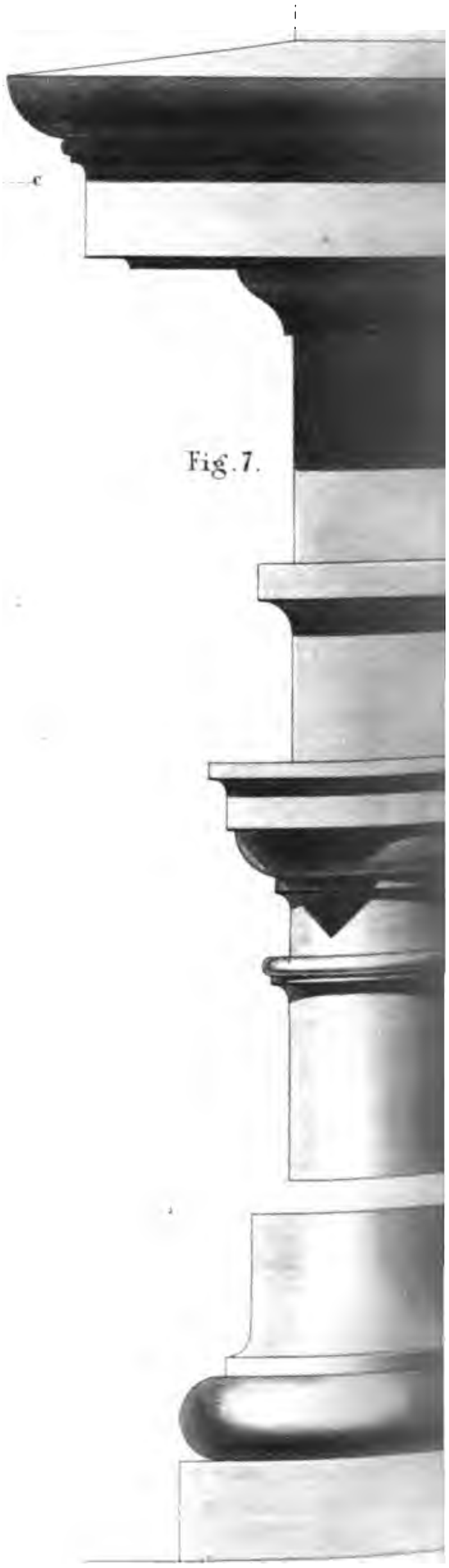


Fig. 7.



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N OF SHADOWS & SHADING.

Fig. 5.

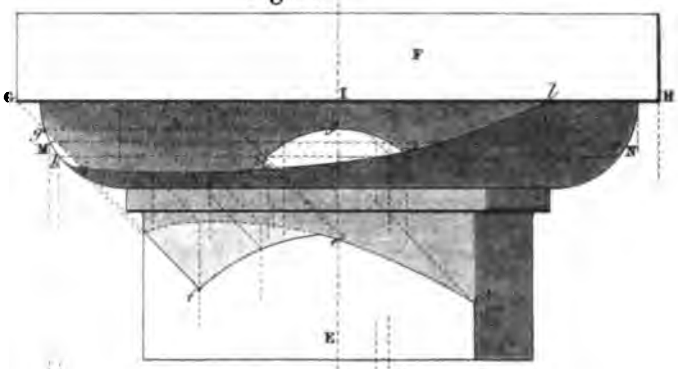


Fig. 6.

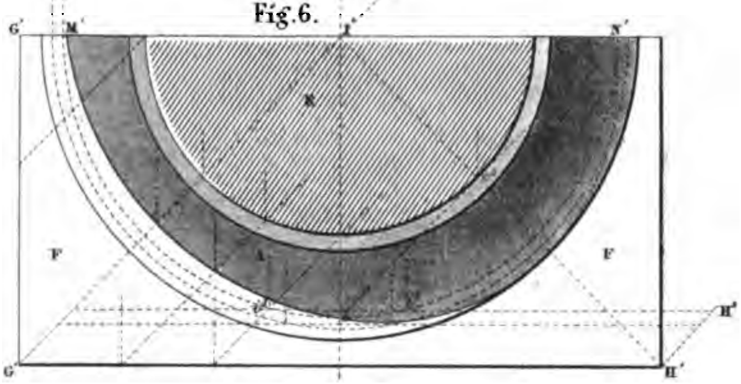


Fig. 8.

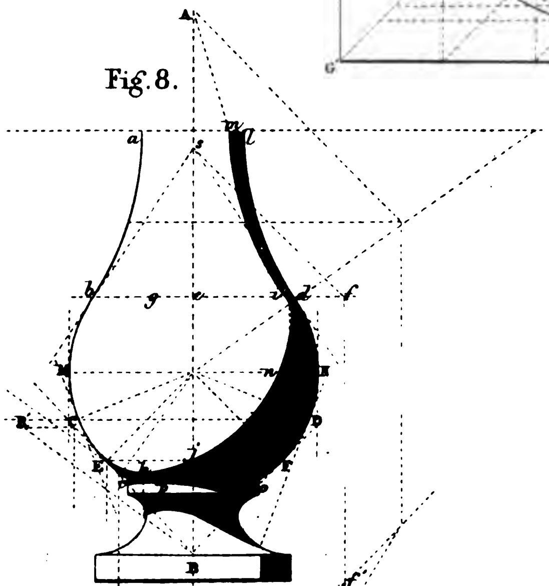


Fig. 9.

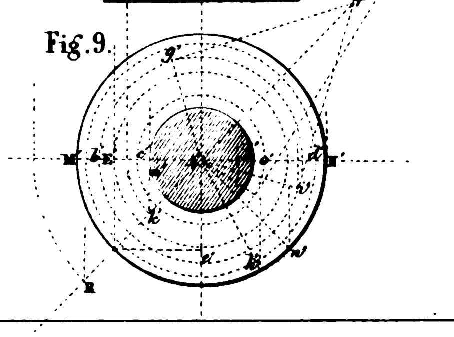


Fig. 10.

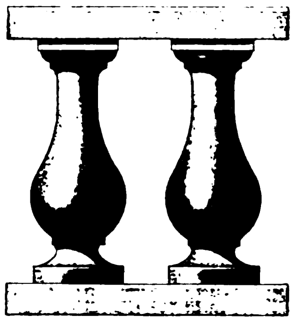
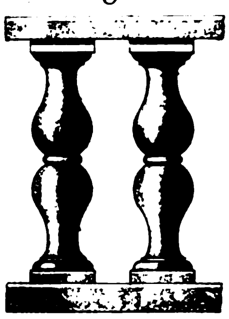


Fig. 11.



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horizontal projection, as follows:—Thus, square down the points, *b*, *e*, *d*, of Fig. 1, to b^1, e^1, d^1, Fig. 2, which will lie in a circle, having *b e*, or E *d*, for radius. An additional point, *g*, is obtained by drawing the diagonal, $\mathrm{o}^1\ g^1$, Fig. 2, perpendicular to the ray, R o^1, which is the projection, in this plane, of the rays represented by the arrow, R^1, and proper to the elevation, Fig. 1; this radius cuts the outer circumference, $f^1\ j\ f^1$, of the torus, in the point, g^1. This circumference, $f^1\ j\ f^1$, is projected vertically in the horizontal line, *f f*, passing through the centres, *o*, o^1, of the semi-circles, and the point, g^1, is squared over to *g*, in Fig. 1.

To find the point, *i*, which seems to be the lowest in the curve, and which is situated in the vertical plane passing through the luminous ray, R o^1, or $i^1\ \mathrm{o}^1$, of the horizontal projection, Fig. 2, we proceed as follows:—Suppose the vertical plane, $i^1\ \mathrm{o}^1$, to be turned about the axis, O P, so as to coincide with the vertical plane of projection, when the section of the torus by the plane, $i^1\ \mathrm{o}^1$, being obviously a semi-circle, will coincide with the semi-circle, *a f c*, draw a tangent, $h\ i^2$, to the last, parallel to the ray of light, as in the vertical plane, $i^1\ \mathrm{o}^1$—that is, at an angle of 35° 16′, as has been already explained (105), the point of contact, i^2, is the one sought. But it has to be transferred to the original position of the vertical plane; and for this purpose it is squared over to i^3, in the horizontal projection. Then $\mathrm{o}^1\ i^1$ is made equal to $\mathrm{o}^1\ i^3$, and the point, i^1, again squared over to *i*, in the horizontal line, $i^2\ i$, drawn through i^2, Fig. 1.

It is generally sufficient to find five principal points, as *b*, *i*, *e*, *g*, and *d*, in the curved line of separation of light and shade; but if, because of the large scale of the drawing, it is wished to obtain intermediate points, this may be done by drawing planes passing through the axis; such, for instance, as $\mathrm{o}^1\ \mathrm{B}^1$, Fig. 2, which cuts the torus in a circle of the same radius as the generating circle. We then proceed to find the point of contact of the ray of light, according to the mode indicated in Figs. 22 and 23, Plate 26 (277); that is to say, we seek the projection of the luminous ray upon this plane, $\mathrm{o}^1\ \mathrm{B}^1$. For this purpose, we let fall upon this plane a perpendicular, R r^1, from any point taken upon the luminous ray, R o^1, and we obtain a face view of this ray, as it would be projected upon the plane, $\mathrm{o}^1\ \mathrm{B}^1$, in the elevation, Fig. 1, by supposing the latter to be turned about the axis, o^1 P, until it coincides with $f^1\ \mathrm{o}^1, f^1$, when the point, r^1, will coincide with r^2, in the horizontal plane. Then, as the height, $r^2\ r$, of the point, r^1, above the horizontal plane, is equal to that of the point, R^1, the line joining $r\ \mathrm{o}^1$, will be the face view of the projection of the ray in the plane, $\mathrm{o}^1\ \mathrm{B}^1$. In turning round the plane, $\mathrm{o}^1\ \mathrm{B}^1$, the section of the torus will become coincident with the semi-circle, *a f c*, as in a previous operation. If, therefore, we draw a straight line, *m n*, tangential to this semi-circle, and parallel to the ray, $r\ \mathrm{o}^1$, the point of contact, *n*, will be the point of separation of light and shade, as in the plane, $\mathrm{o}^1\ \mathrm{B}^1$. Finally, we square *n*, over to n^1, Fig. 2; make $\mathrm{o}^1\ n^3$, equal to $\mathrm{o}^1\ n^1$, by describing an arc with the centre, o^1, and radius, $\mathrm{o}^1\ n^1$, and cutting the line, $\mathrm{o}^1\ \mathrm{B}^1$, in n^3. This point, n^3, we again square over to n^2, upon the horizontal line, $n\ n^2$, in Fig. 1, and n^2 is the point sought. Or we might have drawn the vertical projection of the section of the torus by the plane, $\mathrm{o}^1\ \mathrm{B}^1$, which would have been an ellipse, as 1, 2, n^2 *o*, similar to that in Fig. 22, Plate 26 (277), and we might have proceeded, as shown in reference to that figure, the result being the same in both cases.

If the circular arc be prolonged to beyond the radius, $\mathrm{o}^1\ g^1$, and upon it, $g^1\ l^1$ be made equal to $g^1\ n^3$, another point, l^1, will be obtained, symmetrical with n^3, with reference to the radius, $\mathrm{o}^1\ g^1$, which is at an angle of 45° to the base line, and perpendicular to the luminous ray, R o^1. This point, l^1, is to be squared over to *l*, in the vertical projection, where it intersects the horizontal line, *p l*, drawn at a distance, *q p*, above the centre line, *f f*, equal to the distance, *q s*, of the horizontal passing through the point, n^2, below it. This gives another point, *l*, in the line of separation of light and shade or the shadow proper to Fig. 1, which is obtained by simply drawing a curved line, as shown, through the points, *b*, *i*, *e*, n^2, *g*, *l*, and *d*, thus found.

317. It will be seen that the horizontal projection of this curve lies through the corresponding points, b^1, i^1, e^1, n^3, g^1, l^1, and d^1, in Fig. 2 as dotted in, and that the part, b^1, i^1, e^1, n^3, g^1, of this curve line lies under and within the extreme circumference, f^1, j, f^1, of the torus, A, and therefore would not be seen in the plan, but the part, g^1, l^1, d^1, which lies above the horizontal, *f f*, in Fig. 1, might be seen were it the shadow proper to the horizontal projection; but as it is merged into this shadow, we have indicated it by a slightly darker tint for the purpose of showing its relation to the corresponding part, *g l d*, in the vertical projection, Fig. 1, and of which it is the horizontal projection.

In this instance the torus being symmetrical as regards the horizontal, *f f*, Fig. 1, that is, being generated by a semi-circle revolving round the axis, O P, the posterior portion of the line of separation of light and shade is similar to that of the anterior, that is, the curve and points, *b*, L, G, N^2, E, I, *d*, respectively, correspond to *d*, *l*, *g*, n^2, *e*, *i*, *b*, as dotted in Fig. 1, not being seen in this view. These opposite diagonal points may be found simply in either of these ways, and the corresponding curve drawn through them. Thus suppose *q p*, equal to *q s*, as before, and make $p\ \mathrm{N}^2$, equal to $s\ n^2$, both being parallel to *f f*, then N^2, will be a point in the posterior curve corresponding to n^2, in the anterior one, otherwise, through the centre, *q*, draw $n^2\ q\ \mathrm{N}^2$, making $q\ \mathrm{N}^2$, equal to $q\ n^2$, and N^2 will be a point in the curve as before, and in this way the complete outline of the shadow proper projected; and as many points squared down as desired to complete the horizontal projection.

We have now to determine the line of separation of light and shade, or the shadow proper of the torus in the horizontal projection, Fig. 2.

This may be done in various ways, thus we might proceed as in the last instance and find the principal points, recollecting that $\mathrm{R}^1\ \mathrm{o}^1$, (Par. 104 and 108), is the projection of the ray of light proper in this view, Fig. 2; and that its direction in the vertical elevation, Fig. 1, is projected in the arrow and line, R^1 O. So that the principal points to be found in this case would be b^1, G^1, N^3, E^1, I^1, and d^1, but as the corresponding projection of these points lie in the posterior line of separation of light and shade, already found in Fig. 1, which now becomes the anterior line of separation of light and shade, as projected proper in relation to Fig. 2. It is unnecessary to go through the whole construction of finding these points, since they are easily found from their opposite diagonal points, namely, *d*, *g*, n^2, *e*, *i*, and *b*, as shown in relation to Fig. 1. Thus, then, through any point, N^2, in the curve, Fig. 1, draw the line, N^2 N, parallel to *f f*, till it intersect the arc, *a f c*, in N, a perpendicular let fall from this point will be seen to coincide with *n* and n^1, in the first construction, in both projections, and the point, n^1, as in the first construction, carried round concentric with the centre, o^1, until it intersects perpendiculars squared down from the points, L and N^2, in L^1 and N^3, will give these two points in the line of separation of light and shade on the anterior surface in the horizontal projection, Fig. 2; and if the point, n^1, was carried round to intersect lines squared down from the points, *l*, and n^2, on the posterior face, two other points would be got in the line of separation of light and shade on that surface. As many points may be found in this way as necessary, and the curve drawn through them, as $b^1\ L^1\ G^1\ N^3\ E\ I^1\ d^1$, will give the shadow proper in the horizontal projection, and the part, $b^1\ L^1\ G^1$, will be below the circumference, $f^4\ h\ f^1$, and therefore not seen, but the part, $G^1\ N^3\ E^1\ I^1\ d^1$, being above, it is seen and shown by a sharp line and light tint to represent a wash of China ink.

When the shadow proper of a torus is known, it is very easy to determine the shadow which it will cast upon the horizontal plane—the plinth or pedestal below it, for instance—by drawing through any number of points in the line of separation of light and shade, a number of lines parallel to the luminous ray, and then finding the points at which these lines intersect the horizontal plane.

Thus, a portion of the torus, A, Fig. 2, casts a shadow upon the top of the plinth, or horizontal plane, indicated in vertical projection by the line, B C, very little of which would be seen on the plinth, but for the purpose of showing the curved outline of this shadow, and the mode of finding it, we have indicated a portion of it in dotted lines. Take any two corresponding points in both projections of the curve outline of the shadow proper, as E and E^1, and draw through them lines parallel to the arrows, R^1, and square down the point, E^2, in the first, where it meets the plane, B C, till it intersects the second in the point, E^3, which will be one point in the curve, and when a sufficient number of points have been thus found, a curve line drawn through them, as $G^1\ G^3\ N^5\ E^3\ I^3\ d^3$, will be the outline of light and shade of the shadow cast by the torus, A, on the plane, B C. The edge, $k\ k^2$, and top edge, $k\ k$, of the plinth, would also cast a shadow upon its base in the plan, Fig. 2, which would simply be found by drawing through k^1, k^1, lines parallel to the ray, $R^1\ O^1$, and limiting them by parallels to the edges, $k^1\ k^1$, and $k^1\ f^4$ at a perpendicular distance from them, as $x\ y$, equal to the height or thickness of the plinth from the base.

In Fig. 4, a portion of the torus, A, casts a similar shadow upon the horizontal plane, B C, Fig. 3, the outline of which is a curve, found as above by taking any two points in the projections of the shadow proper to Fig. 3, as N^2, N^3, and determining the point in which the ray of light drawn meets the plane, B C. This point, which is N^5, is a point in the outline of the shadow, $G^1\ N^5\ E^3\ J^3\ F^1\ J^1$, Fig. 4.

318. When the torus is surmounted by a cylindrical fillet, the line of separation of light and shade upon the latter, as well as part of its upper edge, will cast a shadow upon the surface of the torus. Thus, in Figs. 3 and 4, this will be the case with the fillet, D, the line of separation of light and shade of which is $f\ h$ in Fig. 3. This line being vertical, casts a shadow, which is a straight line, as $f^1\ i^1$, parallel to the luminous ray, $R\ O^1$, in Fig. 4, and determined by drawing through the point, f, a luminous ray, $f\ i$, meeting the horizontal plane, $a\ a$, in i, which point, i, being squared over to the horizontal projection, intersects the line, $f^1\ i^1$, in i^1, and gives the line, $f^1\ i$, as the projection in this plane of the shadow cast by the line, $f\ h$, in the vertical projection, Fig. 3. It remains to determine the projection of the shadow cast by the upper edge of the circular portion, $f\ j$, which is in the shade: this may be done according to the general mode explained in reference to the several figures on Plate 26 (268, 269, and 277), and which we shall have occasion to repeat on Figs. 5 and 6 of the present plate. This method is also applicable for the determination of the projection, $n\ j$, $n^1\ j^1$, of the shadow, cast by the line of separation of light and shade of the cylinder or shaft, E, upon the annular gorge, which unites this cylinder with the fillet, D; however, the operations are all fully indicated and lettered in accordance with the principles just discussed and quoted in reference to this and former Plates.

SHADOW CAST BY A STRAIGHT LINE UPON A TORUS OR QUARTER ROUND.

319. Fig. 5 is the vertical projection and Fig. 6, the horizontal projection, as seen from below, of a fragment of a Tuscan capital. In these figures we show the form of the shadow cast by the larmier, F, which is a square prism, upon the quarter-round, A, which is annular.

We again recall the general principle, that when a straight line is parallel to a plane, its shadow upon this plane is a straight line parallel to itself. For the rest, it will be sufficient to compare the operations indicated in these two figures, as well as the two former, Figs. 3 and 4, with those of Figs. 8, 9, and 12, 13, Plate 28, and more particularly with reference to (306, 308, and 310,) to see that they are precisely the same; thus, on the one hand, we have the diagonal, $G\ g^1\ i\ f$, for the shadow cast by the side, $G^1\ G^1$, upon the quarter-round, A, where it is limited at i, by the curve, $b\ e\ l$, the line of separation of light and shade upon this surface; and, on the other hand, we have the curve, $i^1\ g\ i^2$, likewise limited by the same curve at the points, i^1, i^2, for the shadow cast by the edge, G H, of the larmier upon the quarter-round.

In Figs. 5 and 6 are completed what refers to the shadow of the capital of a column; they show the operations necessary to determine the shadow cast by the line of separation of light and shade of the quarter-round upon a cylinder, as well as that cast on the same cylinder by a portion of the larmier. The operation, in fact, simply consists in drawing the luminous rays through various points, $i^1\ e$, in a portion of the line of separation of light and shade upon the quarter-round, finding their intersection with the cylindrical surface of the shaft, E, by means of the horizontal projection. There is no peculiarity or difficulty in this procedure, further than what has been gone over in former figures, and the whole being fully indicated upon the diagrams, we need not pause to detail it further.

To render the diagrams just discussed more generally applicable and intelligible, we have not given to the different parts the precise proportions prescribed in any particular architectural order; such prorortions, however, will be found in Fig. 7, which represents the entablature and column of the Tuscan order fully shaded and finished. The objects intended to be gained by giving this beautiful "aquatinta" engraving as an example of drawing and shading are, namely, to show the application of the principles laid down regarding shadows, and the distinctness and niceties to be observed in the various intensities of the washes, and in the general shading made to represent the effect desired to be produced in shading with China ink. The other figures on this plate are not, however, so shown, but only distinguished by light tints of various intensity, to distinguish the projections of the different shadows, proper and cast, and are therefore not given as examples of the depth of shade due to the particular parts.

SHADOWS OF SURFACES OF REVOLUTION.

320. It will be recollected, that a solid or surface of revolution is that which may be said to be generated by a straight or curved line, caused to turn about a given fixed axis, and maintaining a uniform distance therefrom; thus, the cylinder, the cone, the sphere, the torus, are all surfaces of revolution; so also is the surface generated by the curve, $a\ b\ c$, revolving about the axis,

A B, Figs. 8 and 9. It follows, from the above definition, that every section made perpendicularly to the axis will be a circle, and all such sections will be parallel. Every section made by a plane passing through the axis will give an outline equal to the generating curve, and which may be termed a *meridian.*

321. The shadow proper of a surface of revolution may be determined in two different ways: by drawing sectional planes perpendicular to the axis, and then considering the sections made by these planes as bases of so many right cones; or by imagining a series of planes passing through the axis, and then projecting the ray of light upon these planes, so as to draw lines tangential to the different parts of the outline, and parallel to the projections of the line of separation of light and shade sought. This latter method having been applied in the preceding Figs. 1 and 2 of the plate we are now describing and also in Figs. 8, 9, 12, and 13, Plate 28, (304 and 307,) we deem it more useful, in the present instance, to explain the operations called for in the first mode.

Take, then, any horizontal plane, $b\,d$, Figs. 8 and 9, cutting the surface of revolution in a circle, the radius of which is $b\,e$, and the horizontal projection $b^1\,e^1\,d^1$, through the points, b and d, Fig. 8 draw a couple of tangents to the generating curve which forms the outline of the surface of revolution. These tangents will cut each other in the point, s, upon the axis, this point being the apex of an imaginary cone, $s\,b\,d$; through this apex draw a luminous ray, $s\,f$ and $A^1\,f^1$, meeting the horizontal plane of the section, $b\,d\,f$, in f, f^1; from this latter point, in the horizontal projection, Fig. 9, draw two straight lines, $f^1\,g^1$ and $f^1\,i^1$, tangents to the circle, $d^1\,e^1\,g^1$: then the points of contact, g^1 and i^1, will be the two points of the line of separation of light and shade intersected by the plane, $b\,d$, and they are therefore squared over to g and i, in the vertical projection, Fig. 8, the one point only of which is visible, in the anterior face.

In a similar manner the points, h and j^1, are determined, these points being situated in planes, C D and E F, respectively and parallel to the first $b\,d$. It is to be observed, however, that, in these two last cases, the imaginary cones will be inverted, and the luminous ray must consequently be drawn to the left instead of to the right, as has already been explained in reference to Figs. 8 and 12, Plate 28.

322. When the tangents to the generating curve are vertical, as is the case with the sectional planes, M N and $a\,l$, the points, m and n, of the line of separation of light and shade, are determined by lines, inclined at an angle of 45°, and tangential to the circular sections in the horizontal projection, because these circular sections are the bases of imaginary cylinders and not cones.

When a sufficient number of points have been obtained in this manner, as in Fig. 8, a curved line is drawn through them all, which will give the visible portion, $m\,i\,n\,h\,j\,k$ E, of the line of separation of light and shade upon the surface of revolution. This method is general, and may be applied to surfaces of revolution of any outline whatever.

As it is well to determine directly the lowest point, k, of this and similar curves, it may be done in the same manner as for the torus, Figs 1 and 2, namely, by drawing the ray of light, R B^1, at the same inclination to the base line, as it is in the diagonal and vertical plane, and then drawing parallel to it a tangent to the outline of the surface of revolution, the projection for the moment being supposed to be in a plane parallel to the ray of light, B A^1; the distance of the point of contact, k, from the axis, being then measured upon the horizontal projection, B A, of the luminous ray, gives the point, k^1, which is finally squared over to k, in the horizontal line in the vertical projection passing through the same point of contact.

A portion of this curve, namely, the lower part, E $k\,j$, casts a shadow upon the cylindrical fillet, $c\,o$; to determine this shadow, it will, in the first place, be necessary to delineate the horizontal projection of the curve, E $k\,j$, and then to draw luminous rays through one or two points in the latter, to meet the circle, $c^1\,o^1$, the horizontal projection of the fillet. The points in which these projected luminous rays intersect the circle, are then to be squared over to the vertical projection of the same rays, from the intersection of which is derived the curve, $c\,p\,q$. The various operation lines are not indicated on the figures, to avoid confusion, but the operation will be easily comprehended from what has been before said on cast shadows.

323. Fig. 8 represents the vertical projection of the greater part of a baluster, such as is often seen in balconies of stone or marble, and sometimes also in machinery, serving as an isolated standard, or as a portion of the framing. Below the fillet, $c\,o$, is an annular gorge, upon the surface of which the base of the fillet casts a shadow. It is easy to see that this shadow is obtained in precisely the same manner as those occurring in Figs. 8 and 9, Plate 28 (306, 308, and 310), as well as in subsequent diagrams just described in reference to this plate.

Figs. 10 and 11 represent two descriptions of baluster, consisting of surfaces of revolution. They are drawn to a scale of one-tenth their actual size and shaded complete. We recommend the student to draw them upon a large scale, and to determine the outline of the shadows in rigorous accordance with the principles which we have laid down. Such balusters are generally made of stone, and are susceptible of various sizes and proportions.

Many forms and combinations, of which we have said nothing, will be met with in actual practice; but our labours would be interminable were we to give them all. Our exemplifications involve all the principles that are needed, and each case will suggest the modification of operations applicable to it.

RULES AND PRACTICAL DATA.

PUMPS.

324. There are three kinds of pumps.

I. *Lifting pumps,* in which the piston or bucket lifts the water, first drawing it up by suction. We engrave one of this kind in Plate 37.

II. *Forcing pumps,* in which the piston presses or forces the water to any distance. The feed pumps of steam-engines are of this class, and one is represented in Plate 39.

III. *Lifting and forcing pumps,* in which both the above actions are combined.

HYDROSTATIC PRINCIPLES.

325. Whatever be the height at which a pump delivers its water —whatever be the calibre or inclination of the suction or delivery pipe—the piston has always to support a weight equal to a column of water, the base of which is equal to the area of the piston, and the height is equal to the difference of level of the water below, from which the pump draws its supply, and the point of delivery above.

Thus, putting H to represent the difference in the level, D for the diameter of the piston, and P for the weight or pressure on the piston—

$$P = \frac{\pi D^2 H}{4}.$$

To express this pressure in pounds, it must be multiplied by 62·5, that being the weight in pounds of a cubic foot of water; the formula then becomes—

$$P = 62{\cdot}5 \frac{\pi D^2 H}{4} \text{ lbs.},$$

the measurement being expressed in feet.

326. Independently of this load, which corresponds to the useful effect of the machine, the power employed in elevating the piston has other passive resistances to overcome, namely—

1st. The friction of the piston against the sides of the pump.

2d. The friction of the water itself in the pipes.

3d. The retardation of the water in its passage to the pump by the suction valve.

4th. The weight of this valve.

These resistances can only be determined approximately. Still it follows, from the experiments of M. d'Aubisson, that the load to be overcome in raising the piston is equal to

$$62{\cdot}5 \frac{\pi D^2}{4} \times H \times 1{\cdot}08;$$

or, more simply,

$$52{\cdot}5 \, D^2 H.$$

It is sufficient to add to this the weight of the piston and rod.

The power exerted in depressing the piston, being assisted by the weight of itself and the rod, is always less than that required to raise it.

327. In ordinary pumps, the volume of water delivered for each stroke of the piston, instead of being given by the formula,

$$\frac{\pi D^2}{4} \times l, \text{ or } {\cdot}785 \, D^2 l,$$

where l is the length of stroke, is determined by an expression which varies between

$${\cdot}6 \, D^2 l \text{ and } {\cdot}7 \, D^2 l.$$

The velocity of the piston generally ranges between a minimum of 50 feet and a maximum of 80 feet per minute. The diameter of the suction and discharge pipes is generally equal to $\frac{2}{3}$ or $\frac{3}{4}$ of that of the body of the pump.

It may be remarked, that the height to which the liquids rise in vacuo, by the pressure of the atmosphere, is in the inverse ratio of their specific gravities. Thus the pressure, which is equal to 15 lbs. to the square inch, makes water rise to 33 feet, whilst mercury only rises to 30 inches, its specific gravity being 13·59 times that of water. If the atmosphere presses on a liquid lighter than water, it will cause it to rise higher in vacuo than 33 feet, in proportion to the difference of the specific gravity. In practice, more than 29 or 30 feet cannot be calculated on for the lift of the pump, because of the difficulty of obtaining a perfect vacuum.

FORCING PUMPS.

328. What has been here said of lifting pumps, applies as well to forcing pumps. The resistance, however, to be overcome, is somewhat greater in the latter case—for instance, at the moment of opening the discharge valve; and in general this occurs with all valves having a great body of water above them, and with their upper surface greater than the area of the orifice below.

LIFTING AND FORCING PUMPS.

329. A pump of this description ordinarily consists of a cylinder with a short suction pipe, a discharge pipe, a solid piston, termed a plunger, and suction and discharge valves.

Two such pumps are frequently coupled together, in which case a single suction and discharge pipe serves for both.

330. The power necessary to work one or more pumps is expressed by $52{\cdot}5 \, D^2 H v$; or taking into account the force necessary to work the piston by itself, $55{\cdot}7 \, D^2 H v$; v signifying the velocity in feet per minute.

This velocity is obviously obtained by multiplying the number of strokes per minute by the length of stroke; thus—

$$v = 2n \, l,$$

n being the number of back-and-forward movements per minute: consequently, the power required is equal to

$$55{\cdot}7 \, D^2 H \times 2n \, l = 111{\cdot}4 \, D^2 H n \, l;$$

this product representing pounds raised one foot high per minute, the measurements being in feet.

With these premises, we can solve such problems as the following:—

First: What force, F, is required to work a pump, having a piston 6 inches in diameter, a stroke of 18 inches, and a velocity of 15 double-strokes per minute; the whole height between the well and the point of delivery being 70 feet?

The velocity $v = 2n \, l = 30 \times 1\frac{1}{2} = 45$ feet. Then $F = 55{\cdot}7 \, D^2 \times H \times v = 55{\cdot}7 \times {\cdot}25 \times 75 \times 45 = 46{,}997$ lbs. raised one foot high per minute.

To express this in horses power, we must simply divide it by 33,000; therefore,

$$F = \frac{46{,}997}{33{,}000} = 1\tfrac{1}{2} \text{ horses power, nearly.}$$

Second: What quantity will the same pump raise in ten hours?

Assuming, according to the formula (326), the effective volume $V = {\cdot}6 \, D^2 l$, or $V = {\cdot}6 \times {\cdot}25 \times 1{\cdot}5 = {\cdot}225$ cubic feet per stroke, and the volume per minute,

$${\cdot}225 \times 15 = 3{\cdot}375 \text{ cubic feet};$$

and per hour,

$$3{\cdot}375 \times 60 = 202{\cdot}5 \text{ cubic feet.}$$

The quantity of water raised in ten hours will consequently be

$$202{\cdot}5 \times 10 = 2{,}025 \text{ cubic feet.}$$

Third: What diameter should be given to the piston of a pump which raises 202·5 cubic feet of water per hour, the velocity being 45 feet per minute, the length of stroke 18 inches, and the height to which the water is raised 75 feet?

The formula above, relative to the effective discharge per stroke,

$$V = {\cdot}6 \, D^2 \times l,$$

by transposition, becomes

$$D^2 = \frac{V}{{\cdot}6 \times l}.$$

Now, the volume, 202·5 cubic feet, discharged per hour, is, per minute,

$$\frac{202{\cdot}5}{60} = 3{\cdot}375 \text{ cubic feet.}$$

This last again reduces itself to

$$\frac{2 \times 1{\cdot}5 \times 3{\cdot}375}{45} = {\cdot}225 \text{ cubic feet per stroke;}$$

consequently,

$$D^2 \times \frac{{\cdot}225}{{\cdot}6 \times l.}$$

Whence,

$$D = \sqrt{\frac{·225}{·6 \times 1·5}} = ·5 \text{ foot, or 6 inches.}$$

THE HYDROSTATIC PRESS.

331. This powerful machine is an application of the lifting and forcing pump. It consists of a bulky piston or plunger, termed a ram, working in a cylinder to correspond, and communicating, by a pipe of small bore, with a small but very strong forcing pump. To the top of the large piston is fixed a table or platform, which compresses or crushes, what is submitted to the action of the machine.

The pressure exerted upon the water by the smaller piston, is, by means of the fluid contained in the pipe, transmitted to the base of the ram; and as, according to the well-known hydrostatic law, the pressure is equal on all points, the total force acting on each piston will be in proportion to their area; so that if, for example, the diameters of the pistons are to each other as 1 to 5, the pressure on the larger one, the ram, will be 25 times as great as that exerted by the pump-piston. Suppose a man can apply a force equal to 60 lbs. to the end of a lever 3 feet long, and that the point of connection with the piston-rod is only 1½ inch from the fulcrum, the leverage of the power will be 24 times as great as that of the resistance, and the pressure upon the ram will consequently be 24 × 25 × 60 = 36,000 lbs., an effort equal to that of 600 men acting at once.

In the hydrostatic press, we have, consequently, to consider two mechanical advantages—that of the simple machine, the lever, and that of the ram: these advantages, are, however, necessarily compensated for by the diminution in the velocity of the ram.

On these principles, enormously powerful presses and lifting-machines have been constructed. The one capable of lifting 18,000 tons, at the Menai Tubular Bridge, is an unparalleled example.

HYDROSTATICAL CALCULATIONS AND DATA—DISCHARGE OF WATER THROUGH DIFFERENT ORIFICES.

332. The discharge of a volume of water, in a given time, varies according to the velocity of the water, and depends upon the area and form of discharge orifice.

Surface Velocity.—The velocity of water at the surface of a water-course or river, of which it is wished to ascertain the discharge, is obtained by means of a float, which is thrown into the part where the current is strongest. As the wind, if there is any, affects the result very considerably, the floats must project above the surface as little as possible. A distance of as great a length as convenient is measured on the part of the stream where the current is most regular, and the time occupied by the float in passing that distance is noted by a seconds watch. The space passed through is then divided by the time expressed in seconds, and the quotient will be the surface velocity per second.

It is usual to try several floats in different parts of the current.

Example.—Suppose the space passed through by each float is 150 feet in 35 seconds, what is the surface velocity?

$$V = \frac{150}{35} = 4·28 \text{ feet per second.}$$

If the velocity is not uniform throughout the length of the canal, the velocity at any point may be obtained by means of a small paddle-wheel the floats of which just dip into the water. The number of revolutions per minute of this instrument being multiplied by its mean circumference—that is, the circumference corresponding to the centre of the immerged part of the float—the product expresses the velocity per minute; and by dividing by 60, the surface velocity per second is obtained.

Example.—Suppose that the wheel makes 120 revolutions per minute and that the mean circumference is equal to 1½ foot, what is the surface velocity of the current?

$$\frac{120 \times 1·5}{60} = 3 \text{ feet per second.}$$

333. *Mean Velocity.*—The velocity above obtained is only that at the surface; now, the mean velocity, V′, of the whole body of water, which is what is necessary to know for the gauging of the river or canal, is deduced from the first, by multiplying it by a co-efficient, which varies in the following proportions:—

For a surface velocity equal to	·5 ft.	1·5 ft.	3 ft.	5 ft.	6·5 ft.	8 ft.	10 ft.	11·5 ft.	13 ft.
The ratio of V to V′ is	·77	·78	·81	·83	·85	·86	·87	·88	·89

Example.—What is the mean velocity of a current of which the surface velocity is 5 feet per second?

It is equal to ·83 × 5 = 4·15 feet.

The mean velocity of water in an open water-course or river of uniform cross-section is determined by the following formula:—

$$V' = 56·86 \times \sqrt{\frac{A}{P} \times \frac{H}{L}} - ·236.$$

This formula requires the obtainment of the exact level of the surface of the water throughout a certain length, L, the greater the better; the cross-sectional area, A; the form of the immerged perimeter or profile of the bed; and the height of the fall, H, corresponding to the length, L.

Example.—What is the mean velocity of the water in a water-course of uniform rectangular cross-section, having a width of 35 feet, a depth of 12 feet, and with a fall of ·8 feet in a distance of 1400 feet?

The cross-sectional area, A,

$$= 35 \times 12 = 420 \text{ square feet.}$$

The immerged profile, P,

$$= 35 + (2 \times 12) = 59 \text{ feet.}$$

Then,

$$V = 56·86 \times \sqrt{\frac{420 \text{ sq. ft.}}{59} \times \frac{·8}{1400}} - ·236 = 3·39$$

feet per second.

Thus, according to this formula, it is necessary to extract the square root of the product of the quantities placed under the radical sign $\sqrt{\ }$; next to multiply this root by the co-efficient 56·86; and, finally, to subtract from the product ·236 feet. When the measurements are taken in French mètres this last term must be ·072.

THE GAUGING OF A WATER-COURSE OF UNIFORM SECTION AND FALL.

334. When we know the mean velocity of a water-course of regular section and uniform fall, the discharge per second can be obtained by the following formula:—$D = A \times V'$, in which D signifies the discharge per second; A, the cross-sectional area; and V′, the mean velocity.

Example.—What is the discharge of a water-course, the cross-section of which is 4·2 square feet, and the mean velocity 1·065 feet per second?

$D = 4·2 \times 1·065 = 4·473$ cubic feet, or 27·875 gallons per second.

VELOCITY AT THE BOTTOM OF WATER-COURSES.

335. The velocity of water at the bottom of water-courses is still less than the mean velocity.

Putting V to represent the surface velocity, V' the mean velocity, and V'' the ground velocity, the relation of the three will be expressed by $V'' = 2\,V'' - V$. That is to say, the velocity at the bottom of a canal is equal to twice the mean velocity minus the surface velocity.

Example.—The surface velocity of a water-way is found to be 2 feet, and the mean velocity calculated to be 1·55 feet, what is the ground velocity?

$$V'' = 2 \times 1{\cdot}55 - 2 = 1{\cdot}10 \text{ feet per second.}$$

Too great a velocity at the bottom of a water-course tends to loosen and carry away the bed, undermining the sides and causing a great deal of damage; too small a velocity, on the other hand, by allowing the matter suspended in the water to settle, is a cause of obstruction.

The following table shows the limit of velocity according to the nature of the bed, which cannot be exceeded without danger:—

Nature of the Bed.	Limit of the Velocity per second.
	Feet.
Soft brown earth,	·25
Soft clay,	·49
Sand,	1·00
Gravel,	2·00
Flint stones,	2·02
Shingle,	4·00
Agglomerated stones, soft schist,	5·00
Rock fragments,	6·00
Solid rock,	10·00

CALCULATION OF THE DISCHARGE OF WATER THROUGH RECTANGULAR ORIFICES OF NARROW EDGES.

336. As it is of importance, in a majority of circumstances, to be able to calculate the discharge of water by sluice-gates, or by the vertical discharge-gates of hydraulic motors, so as to know the volume, and, consequently, the value of a stream of water. The following method, which is not only simple and generally applicable, but likewise admitting of considerable accuracy, will therefore be useful. This consists in erecting a *notch* in some convenient part of the watercourse where the velocity is not great. The notch is easily formed in leads of moderate size by a board stretched across the channel, and having a rectangular part cut or notched out, and through which the whole of the water will be made to pass. The notch-board being fixed, a rod must be fixed vertically in the channel a few yards behind, and having a mark upon it at exactly the level of the edge of the notch. The water being then permitted to descend in the lead, let its depth upon the rod be carefully noted in inches, then taking from the second or third column of the annexed table the quantity corresponding to one inch of width at the depth noted, multiply that quantity by the whole width in inches, and the result will be the whole quantity flowing through the notch in cubic feet per minute.

Thus, if the depth on the rod be 16 inches, and the width of the notch be 7 feet = 84 inches, then corresponding to 16 inches is 25·8 cubic feet in the second column, and which multiplied by 84 gives 2167·2 cubic feet as the whole quantity passing through the notch in a minute.

The quantity corresponding to 16 in the third column is 27·143 cubic feet, which multiplied by 84 gives 22·80 cubic feet as the supply per minute, which is 112·8 cubic feet in excess of the result obtained by employing the second column. This difference is the result of the third column being calculated for weirs, which discharge more water in a given time than notches, on account of their offering less impediment to the motion of the fluid. A weir is a wall built generally of solid masonry across the channel, with a parallel plank fixed horizontally on edge along the top of the building. The plank is termed the waste board, and the water flows over it along the whole breadth of the channel, and thus suffers no lateral obstruction as it does in meeting the notch-board, in which the passage is usually contracted. But if the notch be made equal in width to the width of the channel, then this column ought to be employed, since under these circumstances the conditions are strictly analogous, and the notch may be called a weir.

Depth of the upper edge of the waste-board below the surface in inches.	Cubic feet of water discharged in a minute by every inch of the notch, according to Du Buat's formula.	Cubic feet of water discharged in a minute by every inch of the waste-board of a weir, from experiments made by Mr. Smeaton.
1	0·403	0·428
2	1·140	1·211
3	2·095	2·226
4	3·225	3·427
5	4·507	4·789
6	5·925	6·295
7	7·466	7·933
8	9·122	9·692
9	10·884	11·564
10	12·748	13·535
11	14·707	15·632
12	16·758	17·805
13	18·895	20·076
14	21·117	22·437
15	23·419	24·883
16	25·800	27·143
17	28·258	30·024
18	30·786	32·710

When the preceding table cannot be conveniently applied, the value of D, the quantity of water discharged in a minute, will be found very nearly from the expression,

$$D = 200\,H\,L\,\sqrt{H},$$

in which H is the height of the surface level of the water above the sole of the notch, and L, the width, all in feet. Thus taking the example given above, we have $H = \frac{4}{3}$ ft., and $L = 7$ ft., therefore

$$D = 200 \times \frac{4}{3} \times 7\sqrt{\frac{4}{3}} = 2155{\cdot}3 \text{ cubic feet per minute.}$$

EXAMPLES IN THE

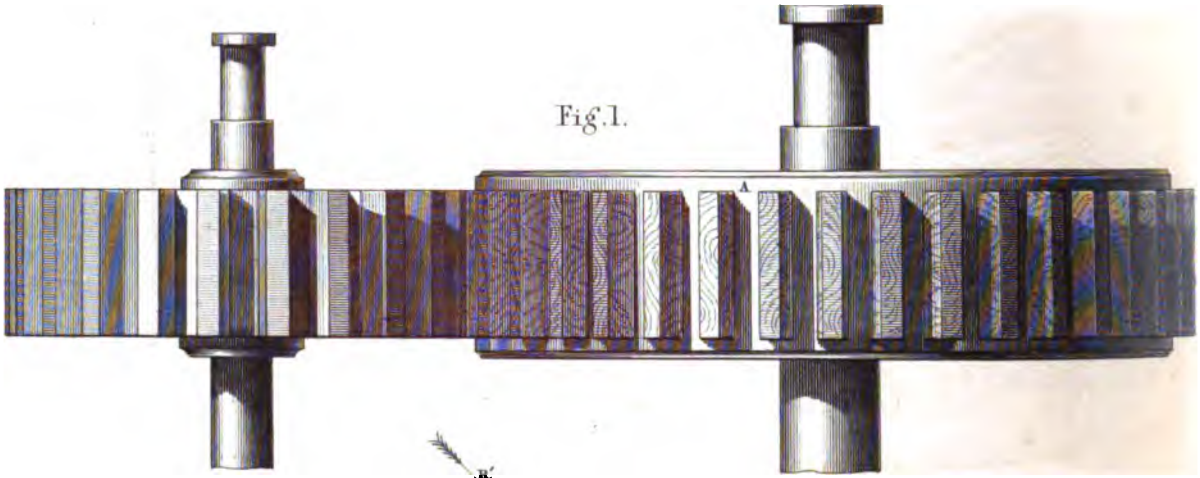


Fig. 1.

Fig. 2.

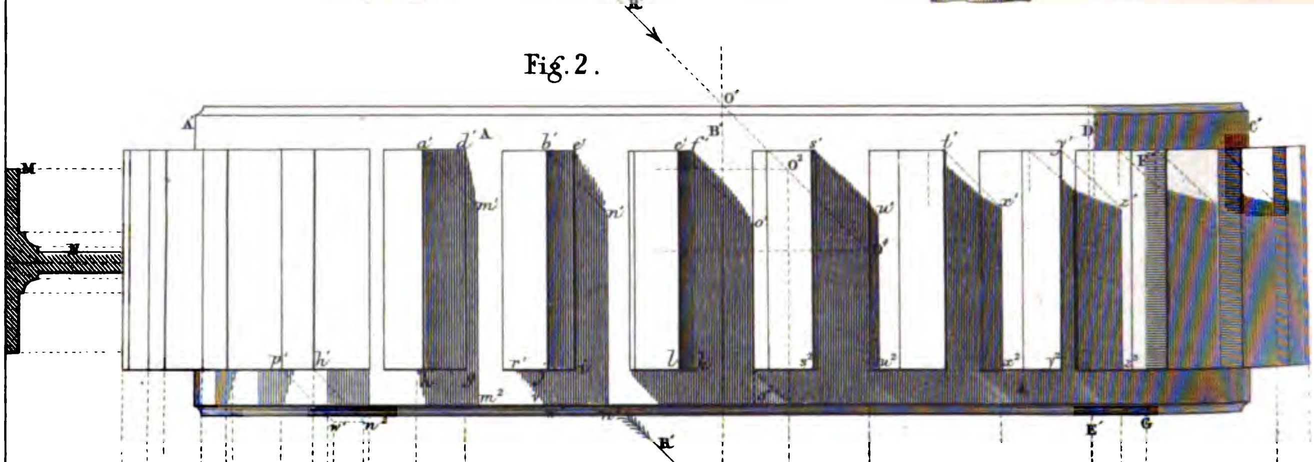
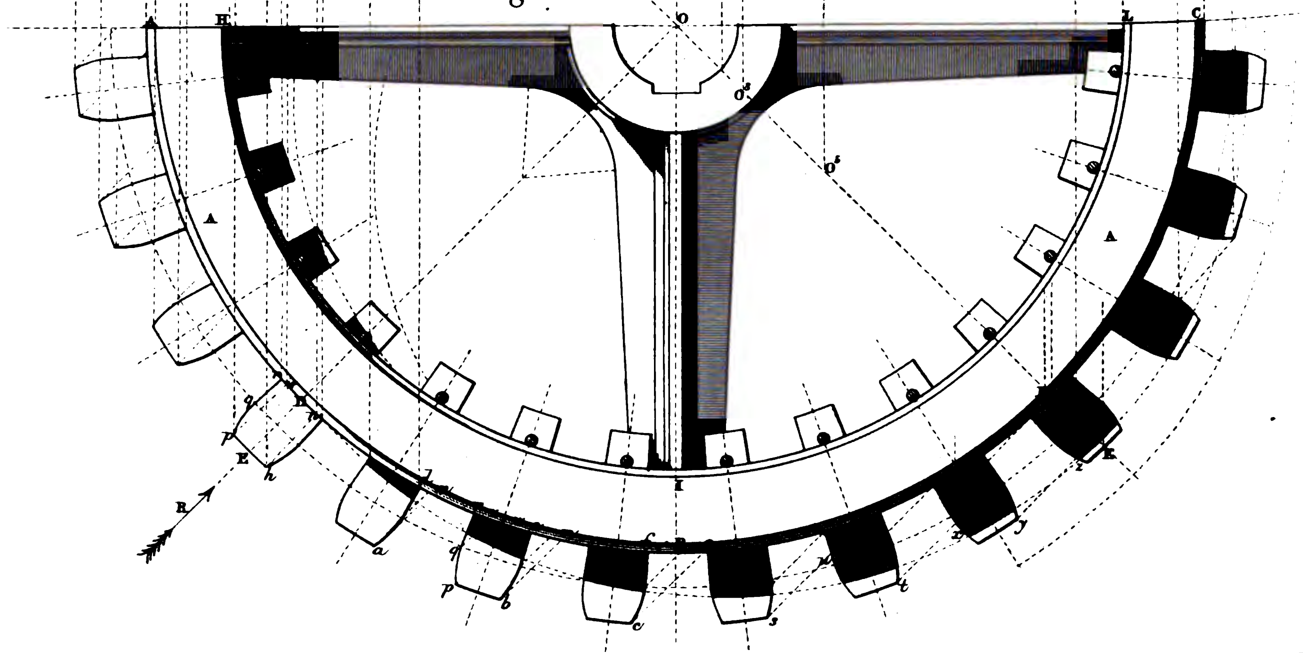


Fig. 3.



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HADING OF WHEELS.

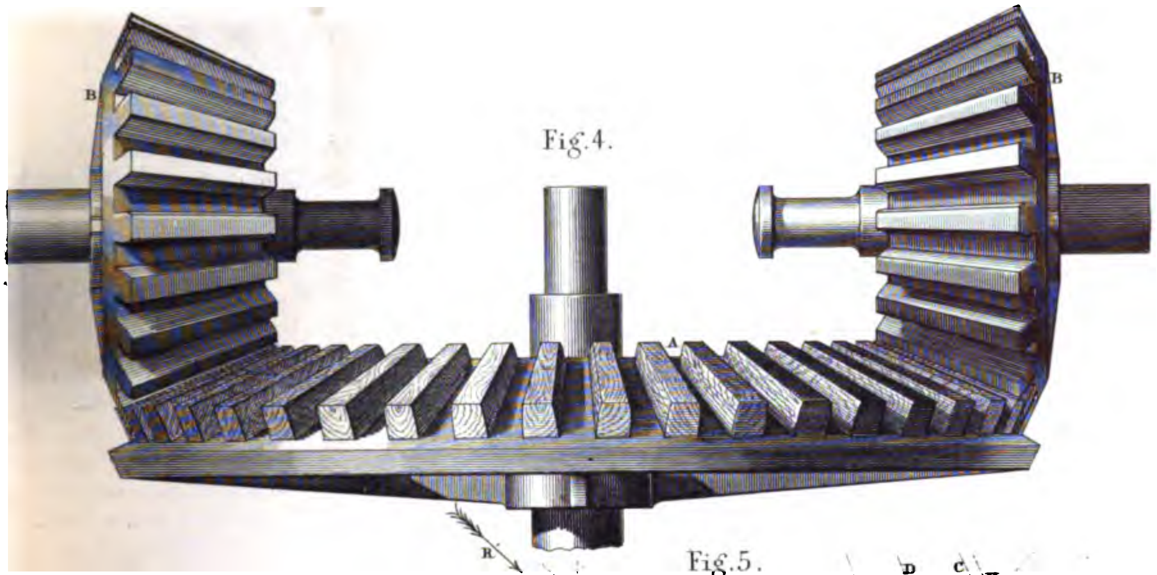

Fig. 4.

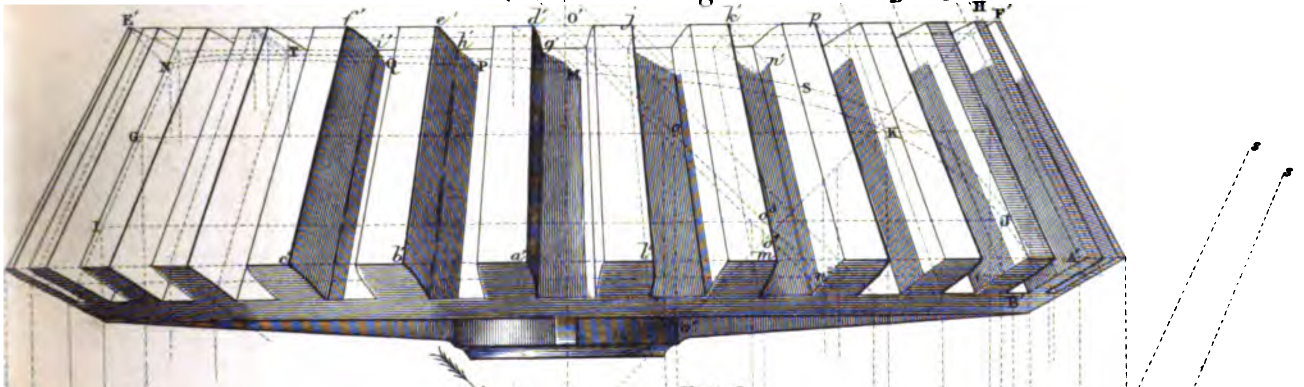

Fig. 5.

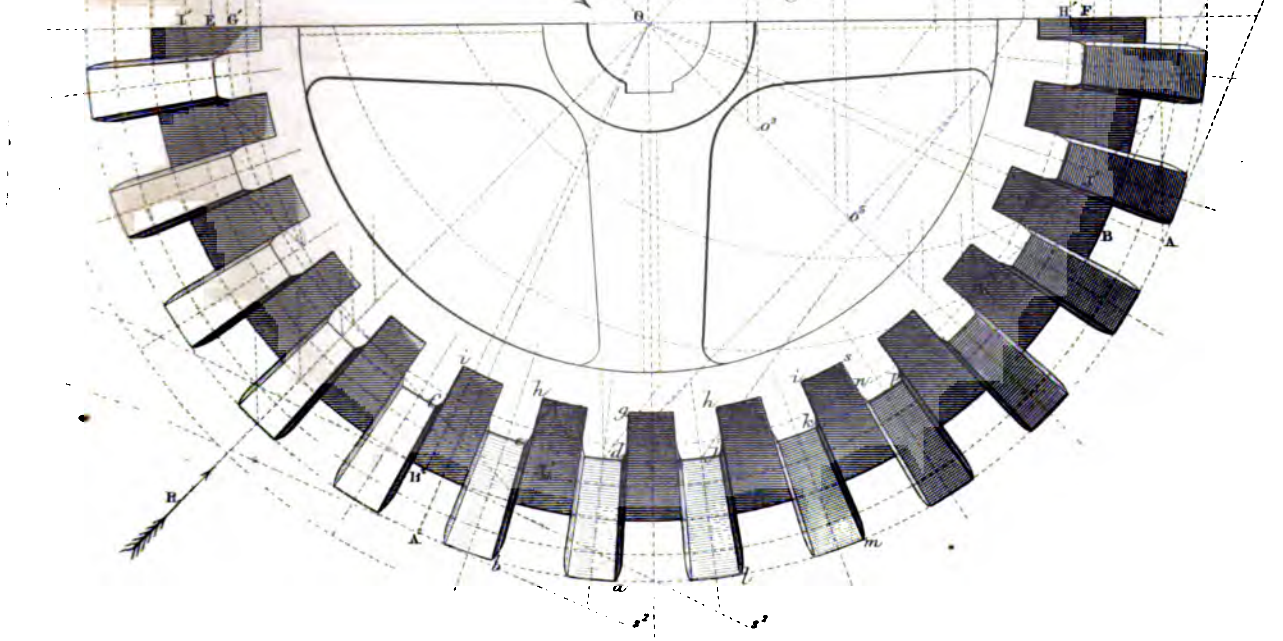

Fig. 6.

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CHAPTER X.

APPLICATIONS OF SHADOWS TO TOOTHED GEAR.

PLATE 30.

SPUR WHEELS.

FIGURES 1, 2, AND 3.

337. We have already pointed out, that before shading an object in a finished manner, it is generally necessary to lay down the outlines of all the shadows, proper and cast, which may happen to be occasioned by the form of each part.

Thus, before proceeding to apply the finishing shades to the spur-wheel and pinion, Fig. 1, we must first determine, separately, on each wheel, both the shadows proper of the external surface of the teeth and web, and also the shadows of the teeth upon the web, and upon themselves. The operations called for to accomplish this with the cog-wheel, to the right of Fig. 1, are indicated in Figs. 2 and 3, which are respectively an elevation and half plan of the same, to a much larger scale.

The external surface, A^1 B^1 C^1, of the web, A, of the spur-wheel, being cylindrical, the line of separation of light and shade will be obviously determined by a tangent parallel to the luminous ray, proper to Fig. 2, and projected in Fig. 3, at R O; or better, by the radius, O D, at right angles to this. By squaring over the point of contact, D, in the horizontal projection, Fig. 3, we obtain the line, D^1 E, in the vertical projection, Fig. 2. Similarly, by squaring over the point, E, we get the straight line, F^1 G, for the line of separation of light and shade on the outer ends of the teeth, which are likewise cylindrical. A portion of the lateral surface of the teeth is also in the shade, as will easily be determined, by drawing lines through the extreme angles, as *a*, *b*, *c*, &c., parallel to the luminous rays as projected in Fig. 3. Thus the surfaces, *a d*, *b e*, and *c f*, do not receive any light, and are, therefore, shaded in the elevation, as within the outlines, a^1 d^1 g h, b^1 e^1 i j, and c^1 f^1 k l.

Each of these teeth, also, casts a shadow upon the vertical cylindrical surface of the web; and as their edges, a^1 h, b^1 j, c^1 l, are vertical or parallel to the surface of the web, their shadows cast on it are also vertical. These last are determined by drawing the luminar lines through the points, a^1, b^1, c^1, Fig. 2, and also those projected in Fig. 3, through the points, *a*, *b*, *c*, and then squaring over the points of contact, *m*, *n*, *o*, to m^1, n^1, o^1, in Fig. 2, which gives the outline of the shadow, m^1 m^2, n^1 n^2 and o^1 o^2, as cast by the edge of the teeth, a^1 h, b^1 j, and c^1 l.

To complete the shadows of the teeth upon the web, it is further necessary to obtain the outline corresponding to the edges, a^1 d^1, b^1 e^1, c^1 f^1, &c. We already have the extreme points, d^1, e^1, f^1, and m^1, n^1, o^1, and in most cases these are sufficient. Where, however, greater exactness is required, it is well to find a few intermediate points. The lower edge of that side of the tooth next the light, also casts a shadow upon the web, which is obtained in the same manner, by drawing luminar lines through the points, *p*, *q*, *r*, meeting the surface of the web in points, *v*, *w*, projected vertically in r^1, v^1, w^1.

Some of the teeth, also, cast shadows upon each other; and as their surfaces are also vertical, these shadows are simply determined in the same way as the lines, m^1 m^2, n^1 n^2, were, by the contact of the luminar lines with them. Thus, the edges, s^1 s^2, t^1 t^2, y^1 y^2, have for shadows the straight lines projected vertically in u^1 u^2 x^1 x^2, z z_r

338. We have now to outline the shadows proper and cast, which occur in the plan, Fig. 3, of the same wheel. First, we have to determine the shadow cast by the web upon the tenons of the teeth, and second, upon the arms, or spokes. All these surfaces being horizontal and parallel, the shadow cast upon each will be a circle equal to the one, H I L, which is the inner edge of the web throwing the shadows. All that is necessary, then is to draw through the centre, O, O^1, a line parallel to the luminous ray; and to find the points of intersection, O^3 and O^5, with the planes, M O^2 and N O^4, in which lie the upper surfaces of the tenons and of the face arms, and to describe arcs on these surfaces respectively with the points, O^3 and O^5, as centres, and with the common radius, O H, (270). In the same manner we obtain the shadows cast by the boss of the wheel, and by the feather arms upon the face arms.

When we have thus gone through the requisite operations for each wheel, we proceed with the shading, according to the principles laid down (280, *et seq.*), covering first the portions which require a more pronounced shade, and leaving the lighter parts to the last.

The specimen, Fig. 1, which we recommend to be copied on a larger scale, indicates the various gradations of shade required to produce the proper effect, according to the different positions of the planes, and to the contour of the surfaces. These wheels are also supposed to be mounted upon their shafts, which are shaded as polished cylinders.

BEVEL WHEELS.

FIGURES 4, 5, AND 6.

339. The procedure here called for will be the same as in the preceding case—that is to say, we must first draw the outlines of the shadows, proper and cast, for each wheel. Fig. 4 is a complete shaded elevation of a bevel cog-wheel, A, and two pinions, B, working into it with iron teeth. Figs. 5 and 6 represent a vertical elevation and half-plan respectively, of a bevel wheel with cast-iron teeth, to about the same proportions as the pinions, B, in Fig. 4, but to a much larger scale, so as to show the outlines and projections of the shadows which are indicated on the different surfaces.

The external surfaces of the teeth and of the web being conical the shadows proper are determined in the same manner as for the cone, by drawing through the apex a plane parallel to the luminous ray, and finding the generatrix at which this plane touches the conical surface (304).

It is in this manner that, for the shadow proper to Fig. 5, on the outer ends of the teeth, we obtain the generatrix projected in O A, and for the outer surface of the web, that projected in O B, Fig. 6. These generatrices, which give the lines of separation of light and shade, when projected vertically in the straight lines, C^1 A^1, and D B^1, Fig. 5, converging to the apex of the cone; since, however, these lines occur between two teeth in the present example, they are not apparent. The lines of separation of light and shade O A^2, and O B^2, found in a similar manner, (304), give the shadows proper to the surface of the teeth and web in the plane, Fig. 6; or having found the lines, $O A$, $O B$, in projecting the shadow proper to Fig. 5, they may be turned round 90°, which would also give $O A^2$, $O B^2$, as the shadow of the surface of the teeth and the web.

Some of the teeth have their lateral faces in the shade, whilst all the lower conical surface corresponding to the wider ends of the teeth is in deep shade, as indicated in Fig. 5 by a darker tint.

We have, also, to determine the shadows cast by the outer edges of the teeth as $a^1 d^1$, $b^1 e^1$, c^1, f^1, and by the curved portions, $d^1 g^1$, $e^1 h^1$, and $f^1 i^1$. Now, the outer edges, $a^1 d^1$, $b^1 e^1$, $c^1 f^1$, cast shadows upon the conical surface of the web, which are represented by straight lines coinciding with generatrices on this surface; and therefore, to determine them, we must draw through the corresponding edges a series of planes parallel to the luminous ray; the whole of these necessarily passing through the common apex, O, it is simply requisite, therefore, to find the shadow cast by any one point in these edges. Let us take, for example, the points, d^1, e^1, f^1, all situate in the same circle, $E d F$, of the horizontal plane, Fig. 6; the operation, then, is to find the shadow of this circle upon the conical surface, and is the same as that which we have already indicated and explained several times (306, 308, 321); it consists, in fact, in drawing any planes, $G H$, and $I J$, perpendicular to the cone's axis, and, consequently, parallel to the plane of the circle, $E^1 d^1 F^1$, in Fig. 5.

340. We have seen that the shadow cast by the circle, $E d F$, upon each of the planes, will be a circle equal to itself; and it is, therefore, simply necessary to find the shadow cast by the centre, O, O^1. This shadow falls in o, o^1, o^2, on the plane, $G H$, and in o^3, o^4, o^5, on the plane $I J$; $o^1 o^4$, are the horizontal projections of the centre, O^1, in these two planes, proper to the direction of the light, $B O$, in Fig. 5; and o^2, o^5, are similar projections of the same points proper to Fig. 6, and due to the direction of the light, $B^1 O$, in it. If then, with the points, o^1 and o^4, as centres, and with the radii, $o^1 K^1$ and $o^4 J^1$, equal to the radius, $O E$, we draw a couple of arcs, these arcs will cut the circles $G^1 K^1 H^1$ and $I^1 L^1 J^1$, the projections of the sectional planes, in the points, K^1 and J^1, which, being squared over to the vertical projection in the points, K and J, gives two points in the curve, $J K M N$, representing the shadow cast by the circle, $E^1 d^1 F^1$, upon the conical surface of the web, in the elevation, Fig. 5. Consequently, if we draw the luminar lines through the points, d^1, e^1, f^1, &c., the respective points of their intersection with the curve, as M, P, Q, will represent their shadows cast upon the surface of the web.

The points, g, h, i, situated upon the upper base of the cone, obviously cast no shadows, the shadows of the teeth, however, springing from them; if it is wished to determine any points between these and those already found, it will be necessary to describe an imaginary circle, such as G^1, K^1, H^1, passing between the points, d and g, the outer and inner angles of the teeth. The curve, $R S T$, as projected in the elevation, will be found to represent the shadow cast by this circle upon the conical surface of the web.

As the edges, $a d$, $b e$, $c f$, cast shadows which coincide with generatrices of the cone, they may be obtained simply by drawing straight lines through the several points, M, P, Q, and M^1, P^1, Q^1, converging in the apex of the cone in both the elevation and plan.

Finally, the shadows cast by some of the outer edges of the teeth, such as $k^1 m^1$, upon the teeth immediately behind, are defined by drawing the luminar line, $k n$, through the point, k, meeting the flank, $n s$, of the other tooth, Fig. 6, which lies in a vertical plane. This point of contact is projected vertically in n^1, on the vertical projection, $k^1 n^1$, of the luminar line. It now remains to draw a line, $n^1 n^2$, through this point, n^1, and through the apex of the cone, and this line will represent the shadow cast by the edge, $k^1 m^1$.

341. In the case where the luminar line passing through the extremity of the tooth—as that, for example, drawn through the point, p—falls upon a curved portion of the tooth behind, it is necessary, if great accuracy is required, to imagine a vertical plane passing through this point and through the luminar line, and then to find the intersection of this plane with the curved surface of the tooth. This would require a separate diagram; but the operation is very simple, and has been explained in reference to previous examples (277).

342. The example, Fig. 4, represents the application of finished shading to a bevel wheel with wooden teeth, in gear with a pinion on each side, each with cast-iron teeth. It is to be remarked, that although the shadows are not the same upon each of these wheels, because of their different positions with regard to the light, these are, nevertheless, determined by means of the same operations as those which we have just explained.

In shading this example, the principles and observations already discussed must be borne in mind, and note taken of the various lights and shades. It is also to be observed, that, from the positions of the two pinions, the inner end of the shaft of one is completely in the shade, whilst the inner end of the other is illuminated; Figs. 2, 3, and 5, 6, are not given as examples of shading, as there is merely a flat tint put on those parts in shade, to define the outline of the shadows proper and cast.

APPLICATION OF SHADOWS TO SCREWS.

PLATE 31.

343. It has already been shown, that a screw may be generated by a triangle, a rectangle, or by a circle, the plane of which passes through the axis of the screw, the generating movement being along a helical path. The screw is, consequently, called triangular, square, or round-threaded. In each of these cases, the outer edges cast shadows upon the core of the screw, or upon the twisted surface of the consecutive convolutions of the thread itself. If the screw is surmounted by a head, there will be, in addition, the shadow cast by this upon the outer surface of the thread, as well as upon the other parts. We shall proceed to explain the methods of determining the various shadows upon these different kinds of screws.

CYLINDRICAL SQUARE-THREADED SCREW.

FIGURES 1, 1^1, 2 AND 3.

344. The limit of the shadow proper upon the screw, is obtained in the same manner as that upon a right cylinder, by drawing the radius, $O A$, at right angles to the projection of the ray of light, $R O$, in Figs. 2 and 3, and then squaring over the point, A, to A^1 and A^2, in Figs. 1 and 1^1, and drawing a line through these parallel to the axis of the cylinder. In the same manner we obtain, by projecting the point, B, the line of separation of light and shade, $B^1 B^2$, upon the surface of the core.

The shadows cast by the outer edge of the threads upon the cylindrical surface of the core, are simply determined by means of the straight lines, $C c$, $D d$, drawn parallel to the luminous ray, $B O$, and meeting the circle, $E d B$, the projection of the core, in c and d;

EXAMPLES IN S

Fig. 4.

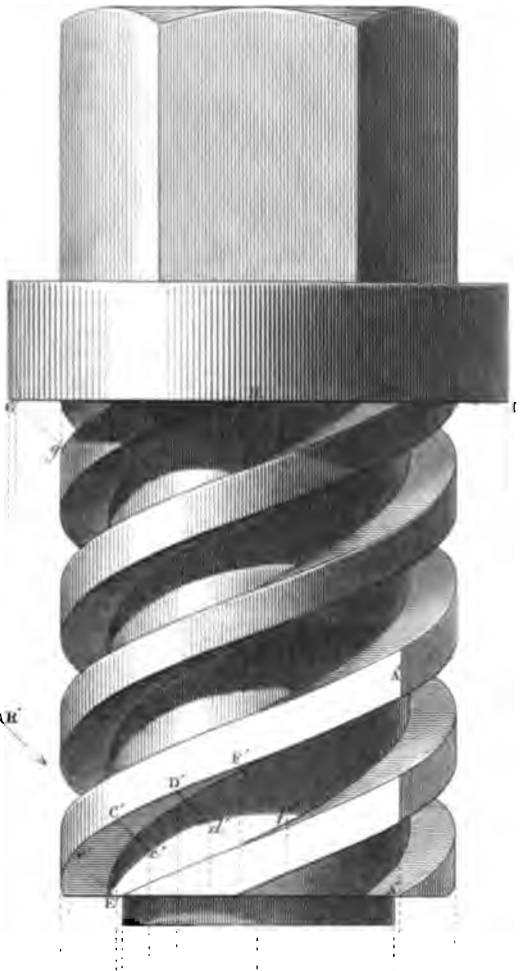


Fig. 2.

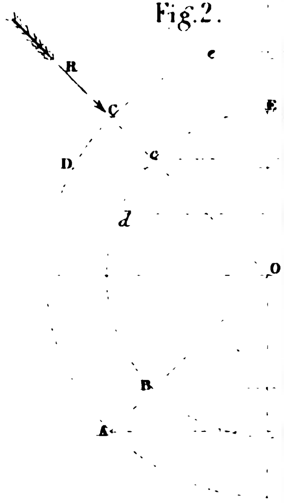


Fig. 1.

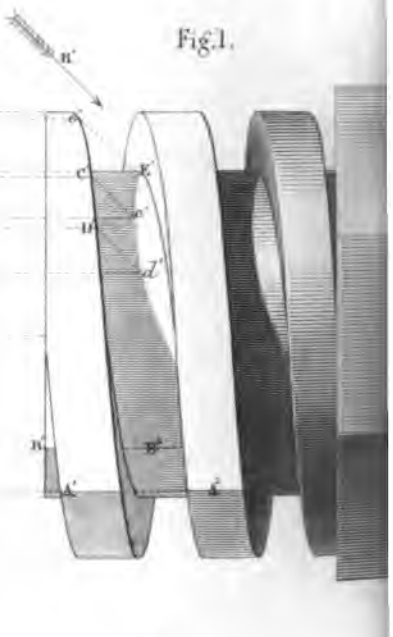


Fig. 5.

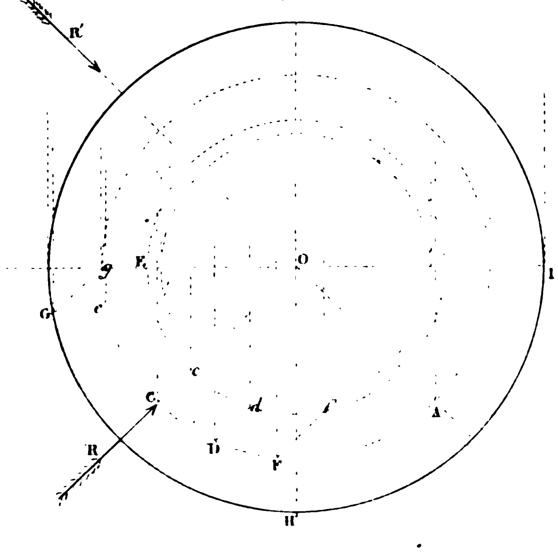


Fig. 7.

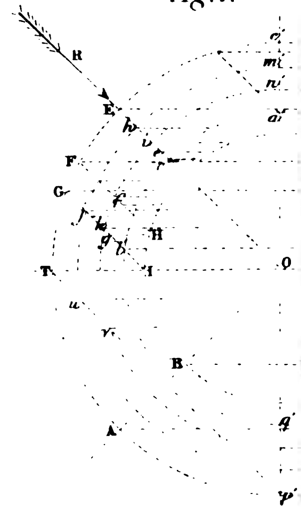
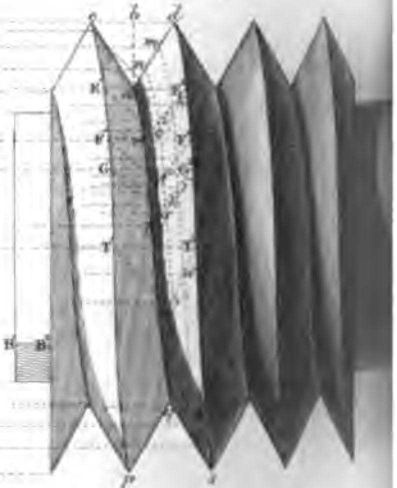


Fig. 6.



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ADING SCREWS.

Fig. 1².

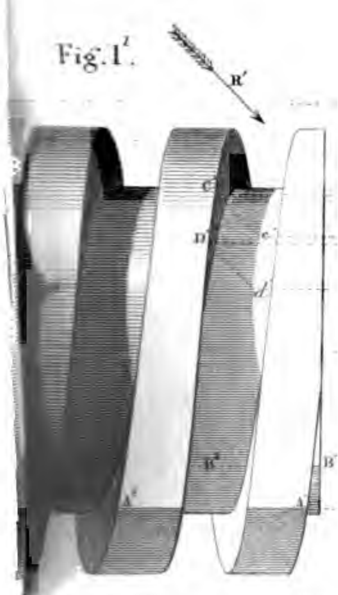


Fig. 3.

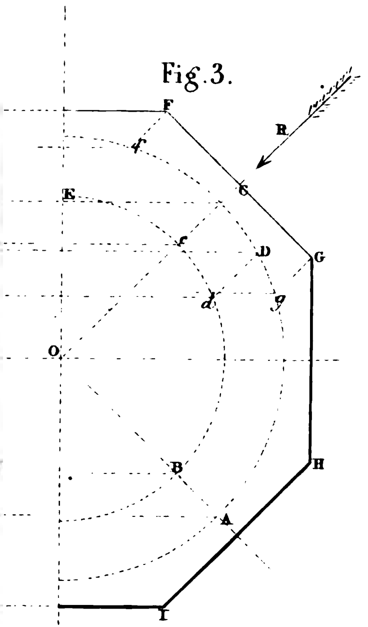


Fig. 9.

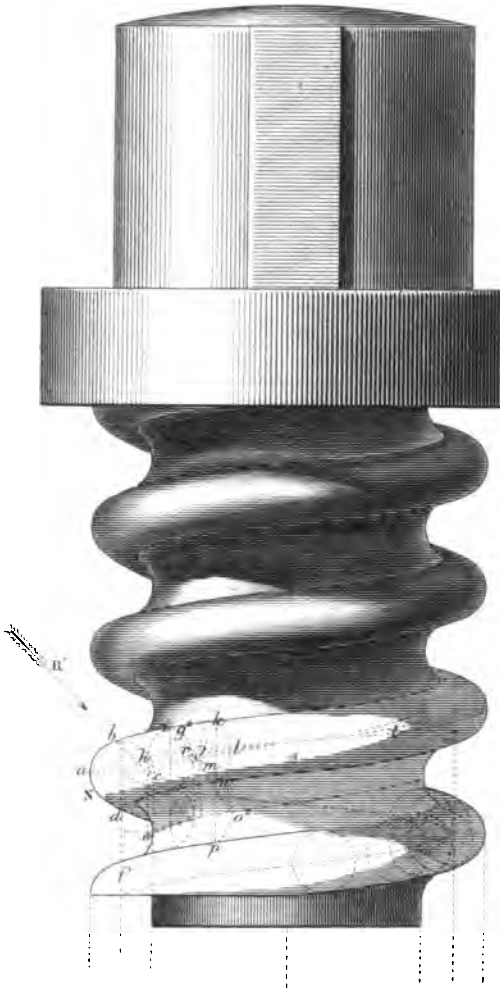


Fig 11

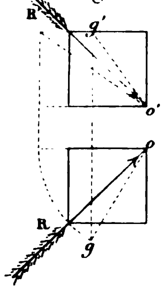


Fig. 6².

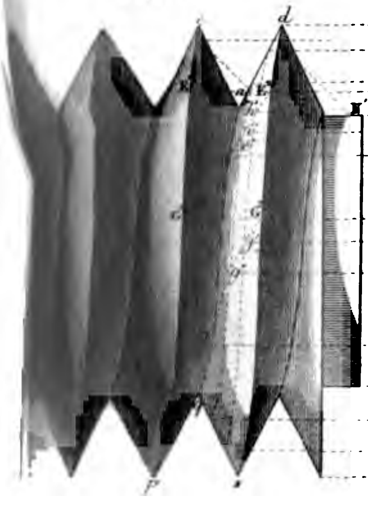


Fig. 8.

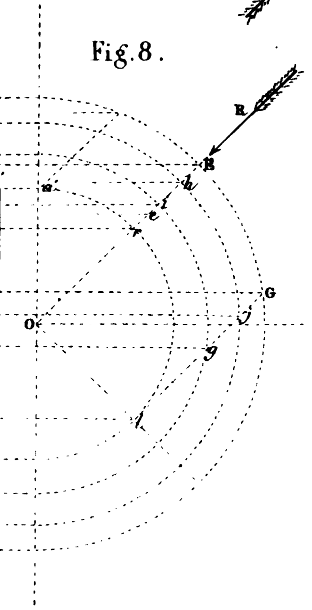
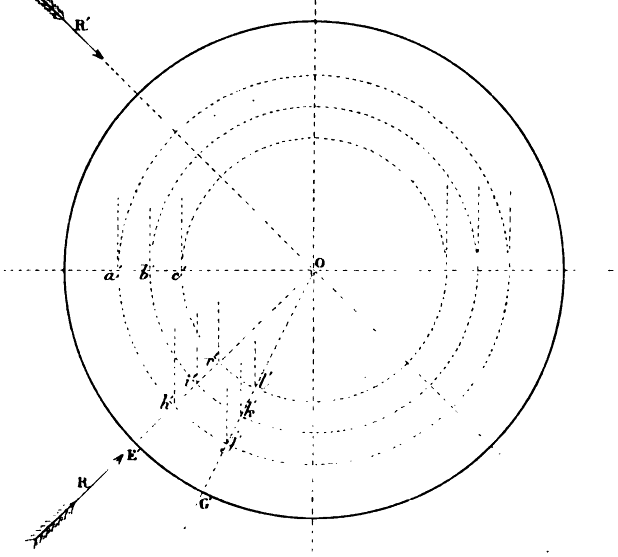


Fig. 10.



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then, by squaring over the points, C D, to C^1 D^1, and drawing through the latter the straight lines, C^1 c^1, D^1 d^1, parallel to the ray of light, R^1, we obtain the points, c^1, d^1, for the shadows sought. We can, in the same manner, obtain as many points as are necessary to complete the curved outline of the shadow.

When the threads of the screw are inclined to the left as in Figs. 1^1 and 3, instead of being inclined to the right, as in Figs. 1 and 2, the operations necessary for determining the curve of the shadows are still the same. This is rendered sufficiently plain by the employment of the same letters to represent similar and symmetrical points; it only requires to be observed, that the end view, Fig. 3, is that of the right half of the screw, whilst that in Fig, 2, is one of the left half, or, one may be supposed to be the turning over of the end of the screw to the right, whilst the other is to the left; the ray of light is similarly respectively represented to the right and left; this, however, does not make any difference, as Figs. 2 and 3 are simple end diagrams to give the length of the line merely, as D *d*, which is required. The luminous ray, in both cases, makes an angle of 45° with the axis of the screw, which is horizontal.

The polygonal head, F G H I, which separates the right-handed from the left-handed portion of the screw, casts a shadow upon a part of the latter, represented by curves, which will be easily determined, in accordance with previous examples (275 and 276), and the principal points in which are *f*, *g*, in Figs. 1^1 and 3.

SCREW WITH SEVERAL RECTANGULAR THREADS.

FIGURES 4 AND 5.

345. The construction of the shadows of a rectangular-threaded screw is the same, whether it be in a horizontal or vertical position, or whether it be right-handed or left-handed. Thus, the screw with several rectangular threads, represented in Figs. 4 and 5, has, in the first place, a shadow proper, limited by the vertical line, A^1 A^2, as squared over from the point, A, and next, the shadow, c^1 d^1 f^1, which is light to show the construction of the shadows cast upon the core by the outer edge of the thread, C^1 D^1 E^1, there is, moreover, a portion of the shadow cast by the circular shoulder, G H I, upon the threads, and also upon the core. The outlines of these shadows are found in precisely the same manner as those in Figs. 1, 2, and 3 (344). In Fig. 5, which is more a horizontal diagram than a proper plan corresponding to Fig. 4, the ray of light, R O, is the horizontal projection of that indicated by the arrow, R^4, of the latter.

TRIANGULAR-THREADED SCREW.

FIGURES 6, 6^1, 7, AND 8.

346. When the screw is generated by an isosceles triangle, such as *c a d*, Fig. 6, of which the height, *a b*, is greater than the half of the base, *c d*, there will be a shadow cast by the outer edge of the thread upon the twisted surface of the succeeding convolution. In proceeding to determine the outline of this shadow in accordance with the general method, which consists in finding the points of contact of the luminous rays with the surface, we are led to seek, in the first place, the curve of intersection of this surface, with a plane passing through the luminous ray, and parallel to the axis of the screw.

For this purpose, let E O, Fig. 7, be the sectional plane; its intersection with the outer edge, *c* G^1 *p*, of the screw-thread will be in the point, E, E^1, Figs. 6 and 7, and similarly its intersection with the inside, *a l q*, will be in the point, *r*, r^1. To obtain intermediate points of the sectional curve, we must describe various circles with the centre, O, and radii, O *m*, O *n*, representing the projections of so many cylinders, on which lie the helices comprised between the inner one *a l q*, and outermost, *d* E^2 *s*, being of the same pitch as the latter. We thereby obtain the points of intersection, *h*, *i*, Fig. 7, which are to be squared over to h^1, i^1, in Fig. 6, and then, by joining the several points, E^2, h^1, i^1, r^1, we get the curve of intersection of the plane with the helical surface of the thread; so that, if we draw a luminar line, E^1 e^1, through the point, E^1, in the same plane, its intersection with the curve, E^1 h^1 i^1 r^1, will give a point, e^1, in the outline of the shadow sought.

In like manner, by drawing other planes, as F H and G I, parallel to the first, E O, we shall obtain the intersectional curves, F^1 f^1 H^1 and G^2 j^1 g^1 *l*, and further upon these the points, f^1 and g^1, of the outline of the shadow. By proceeding thus, we can obtain as many points as may be deemed necessary for the construction of the shadow cast by the outer edge, *c* G^1 *p*, of the thread, and the curve obtained is, of course, repeated on the several convolutions of the thread. We would remark, that there is no shadow cast when the depth of the thread is such, only that *a b*, Fig. 6, is less than the half of the base, *c d*, of the generating triangle.

The diagrams, Figs. 6^1 and 8, which represent a portion of a left-handed screw, will show that the operations required in this modification, to determine the outlines of the shadows, are precisely the same as those last explained.

The core, N, which separates the two portions of the double screw, as well as the end, N^1, receives a shadow cast by the outer edge of the adjacent convolution of the thread.

SHADOWS UPON A ROUND-THREADED SCREW.

FIGURES 9 AND 10.

347. These figures represent a species of screw generated by a circle, *a b c d*, the plane of which passes through the screw's axis, and of which each point describes a helix about the same axis. The intervals or hollows between the convolutions of the thread are also formed with a helical surface generated by a semicircle, *d e f*, tangential to the first. We have, then, to determine the limiting line of the shadow proper upon the screw, and the shadow cast by this line upon the hollows.

The projecting thread being a species of spiral torus or serpentine, the determination of its shadow will be similar to that of the shadow of the ring (316).

Thus, if the screw be sectioned by a vertical plane, G O, passing through its axis, its intersection with the thread will evidently be a circle, as projected in j^1 l^1, Fig. 10. This circle, being inclined to the vertical plane of projection, Fig. 9, is projected therein in the form of an ellipse, the principal points, *j*, *k*, *l*, *n*, of which are obtained by squaring over the points, j^1, k^1, l^1, respectively, upon the helices corresponding to the points, *d*, *a*, *b*, *c*. If, then, upon the plane, G O, which we supposed to be reproduced at *o g*, Fig. 11, we project the luminous ray, R *o*, it will be sufficient to determine the point of contact of this ray with the curve, *j k l*; for this purpose, find the projection of the ray upon the vertical plane in g^1 o^1, Fig. 11; then draw a line, g^2 o^2, tangential to the ellipse, *j k l n*, and parallel to the straight line, g^1 o^1, its point of contact, *m*, with the ellipse will be a point in the line of separation of light and shade upon the outer surface of the screw-thread, as N *m s t*. By proceeding in this manner, any number of points may be obtained, and the line of separation of light and shade found by being drawn through them.

By continuing the sectional plane, G O, across the hollow of the screw, we shall likewise obtain the elliptic curve, *n* o^2, *p*, the principal points in which are equally situated upon the helices which pass through the points, *d*, *e*, *f*; it is sufficient to prolong the luminar line, g^2 o^2, until it cuts the ellipse, *n* o^2 *p*, so as to obtain the point, o^2, which is the shadow cast by the corresponding point, *m*, of the line of separation of light and shade upon the hollows or intervals between the convolutions of the thread.

It is to be remarked, that the prolongation of the line of separation of light and shade, *s t*, casts a shadow upon the outer surface of the convolution immediately below; and, in the same manner, the shoulder above casts a shadow over the projection and hollow of the adjacent thread.

APPLICATION OF SHADOWS TO A BOILER AND ITS FURNACE.

PLATE 32.

SHADOW OF THE SPHERE.

FIGURE 4.

348. It will be recollected, that a sphere is a regular solid generated by the revolution of a semicircle about its diameter. From this definition it follows, that its convex or concave surface, according as it is considered solid or hollow, is a surface of revolution, of which every point is equally distant from the centre of the generating circle. To determine, then, the shadow proper, upon the surface of a sphere, we can proceed according to the general principle (321); but, in this particular case, the following will be the simpler method.

Let us suppose the sphere to be enveloped in a right cylinder, having its axis parallel to the luminous ray; this cylinder will touch the sphere at a great circle, which is, in fact the line of separation of light and shade, and the plane of which is perpendicular to the luminous ray, and, consequently, inclined to the planes of projection; it follows, therefore, that the projection of this line upon these planes will be an ellipse.

Thus let Fig. 4 represent the vertical horizontal projection of a sphere, whose radius is O A, the projections of the extreme generatrices, B C and D E, of the cylinder, parallel to the luminous ray, touch the external contour of the sphere in the points, C, E, which are diametrically opposite to each other, and are the extremities of the transverse axis of the ellipse.

As, in general, this curve can be drawn when its two axis are determined, it merely remains to find the length of its conjugate axis. To this effect let us imagine a vertical plane to pass through the luminous ray, B^1 O, and let us take two lines tangent to the section of the sphere in this plane, and parallel to the luminous ray; if now we turn this plane about the line, B O, considered as an axis, so as to fold it over upon, and make it coincide with, the horizontal plane, the great circle, which is its section with the sphere will obviously coincide with the circle drawn with the radius, A O. The luminous ray will, as already seen (277), be turned over to R O, making an angle of 35° 16′ with the line, B^1 O. It may also be obtained by making the line, B^1 R, perpendicular to B^1 O, and equal to a side of the square, as G R^1, and then joining R O. The two luminous rays tangential to the sphere will then coincide with the straight lines, H L and M N, parallel to R O; their points of contact with the great circle will be the extremities of the diameter, L N, perpendicular to R O. If now we imagine the plane to be returned to its origimal position, the points, L and N, will be projected in L^1 and N^1, thereby giving the length, L N, of the conjugate axis of the ellipse sought.

349. If, in place of constructing this ellipse by the ordinary methods, it is preferred to determine the various points by means of a series of analogous sections, with their subsequent operations, it will be sufficient, for example, to draw the plane *a b*, parallel to B^1 O, and then to turn over, as it were, the section of the sphere formed by this plane, so that it shall coincide with the horizontal plane, making its centre, at the same time, to coincide with the centre, O, of the sphere; the section in question being a circle, described with the radius, *c* O, equal to *a c*. Next draw the tangent, *e d*, parallel to R O, and project the point, *d*, on the diameter, L N, to d^1, upon the original line of section, *a b*, and d^1, is a point in the elliptic curve. In this manner, as many points may be obtained as are wished. The distance, *c* d^2, being made equal to *c* d^1, d^2 will be the symmetrical point in the opposite, and now apparent part of the ellipse.

SHADOW CAST UPON A HOLLOW SPHERE.

FIGURE 5.

350. When a hollow sphere is cut by a plane passing through its centre, and parallel to the plane of projection, the inner edge of the section will cast a shadow upon the concave surface, the outline of which will be an elliptic curve, which may be determined in accordance with the general principle of parallel sections, already explained (277), or by means of the simpler system of sections and auxiliary views adopted in the preceding example, and of which we shall proceed to give another instance, in Fig. 5.

This figure represents the projection of a hollow sphere upon the vertical plane, being, in fact, the section through the line, 1—2, of the boiler, represented in Figs. 7 and 8. If this hemisphere be sectioned by a diametrical plane, A B, parallel to the luminous ray, the section represented in the auxiliary view, Fig. 6, will be a semicircle, A^1 B^1 C^1. The luminous ray, lying in this plane, and passing through the point, A, A^1, will in Fig. 6, be represented by the line, A^1 C^1, parallel to the line, R A, obtained, as indicated in Fig. 5, by the method already explained. This straight line, A^1 C^1, cuts the circle, A^1 C^1 B^1, in the point, C^1, which must be squared over to C, on the line, A B, Fig. 5, when C will be the shadow cast by the point, A.

In the same manner we obtain the points, *b*, *d*, by means of the sectional planes, *a b*, *c d*, parallel to A B, and cutting the sphere in semicircles, represented by a^1 b^1 *e*, in Fig. 6. This semicircle is cut in the point, b^1, by the line, a^1 b^1, parallel to A^1 C^1, and b^1, being squared over gives the two points, *b d*. The extreme points, D, E, are obviously situated at the extremities of the diameter, D E, perpendicular to the luminous ray, R^1 O, and representing the transverse axis of the ellipse. These shadows are frequently met with in architectural and mechanical subjects; as, for example, in niches, domes, and boilers. These two shadows are indicated by a light flat tiut to distinguish their form, which are the same in these two cases, (the objects being spheres and the light in the same direction, B^1 O), whether supposed to be shown in elevation or plan.

APPLICATIONS.

351. Fig. 7 represents a longitudinal section, at the line, 3—4, in Fig. 8, of a cylindrical wrought-iron boiler, with hemispherical ends, and surmounted by a couple of. cylindrical chambers, one of which serves for a man-hole, and has a cover fitted to it.

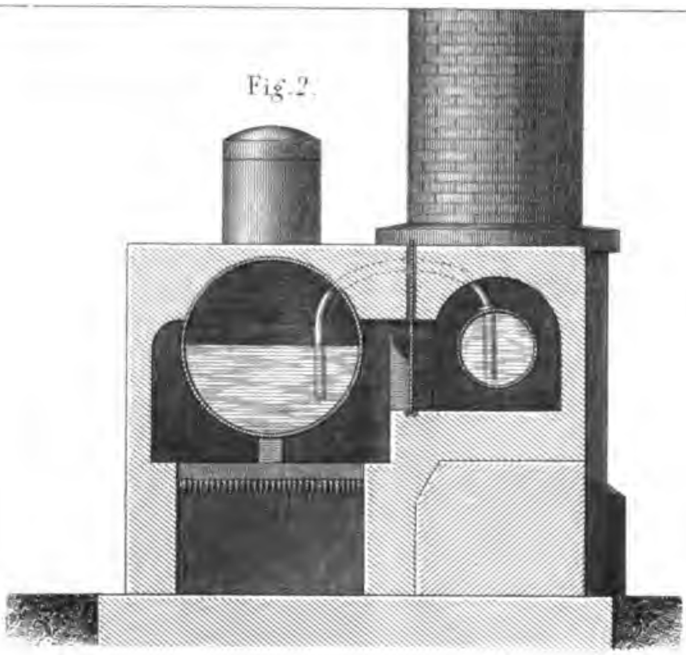
Fig. 8 is a plan of the same boiler, looking down upon it, and showing the cylindricial chambers.

Fig. 9 is a transverse section, made at the line, 5—6, in Figs. 7 and 8.

BOILER

EXAMPLE IN PROJECTING THE

Fig. 2.



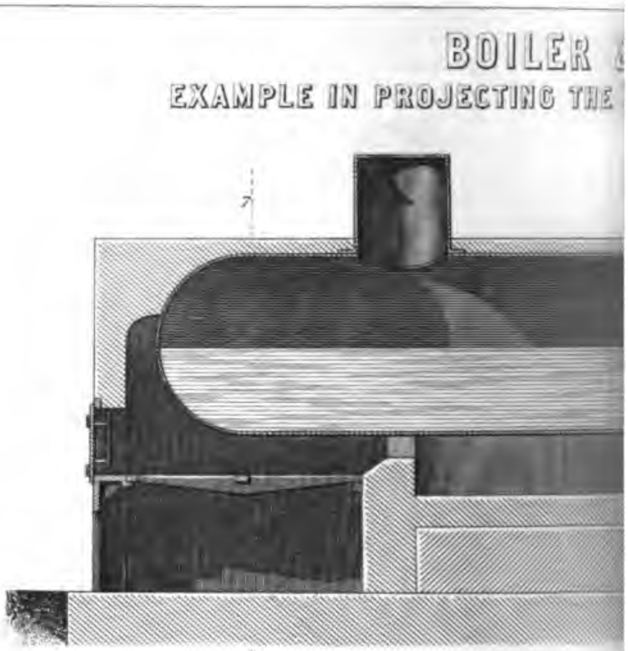


Fig. 6.

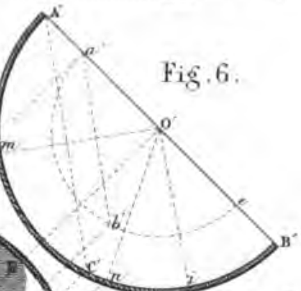
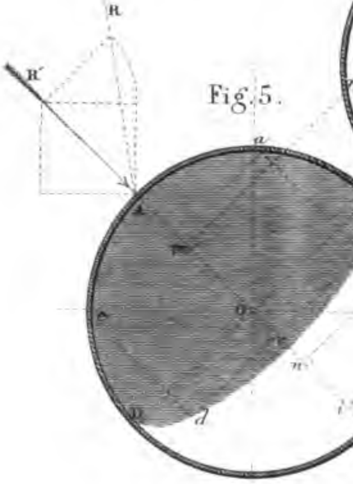


Fig. 5.



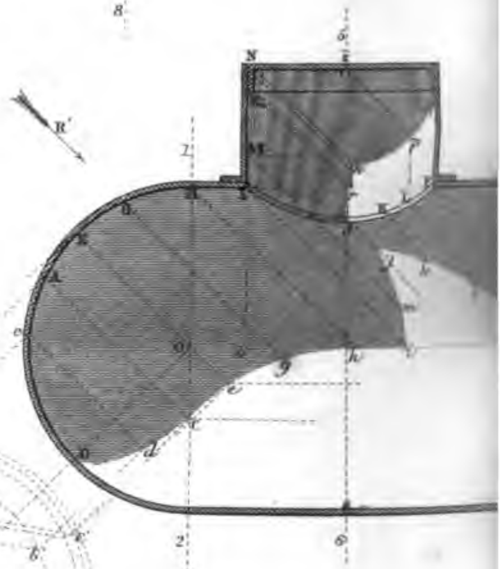


Fig. 10.

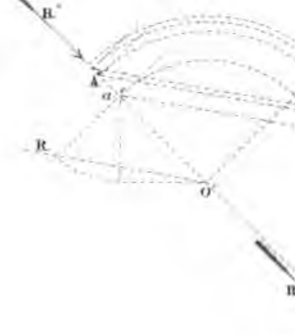
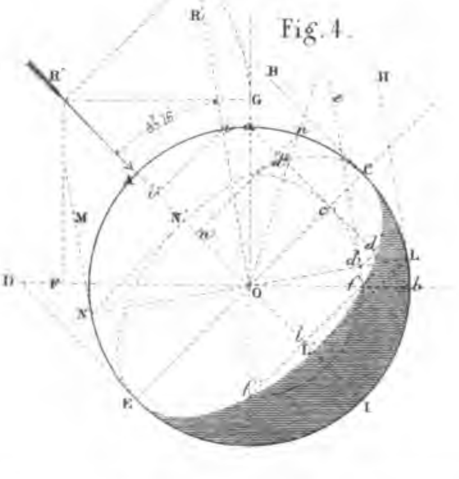
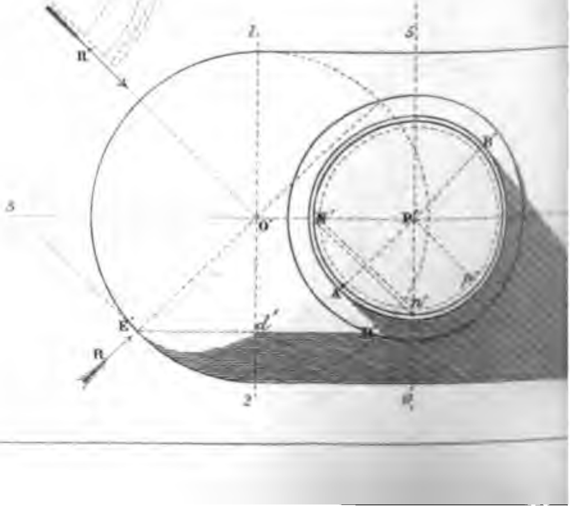


Fig. 4.





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FURNACE,
DOWS & SHADING OF SPHERES.

1.

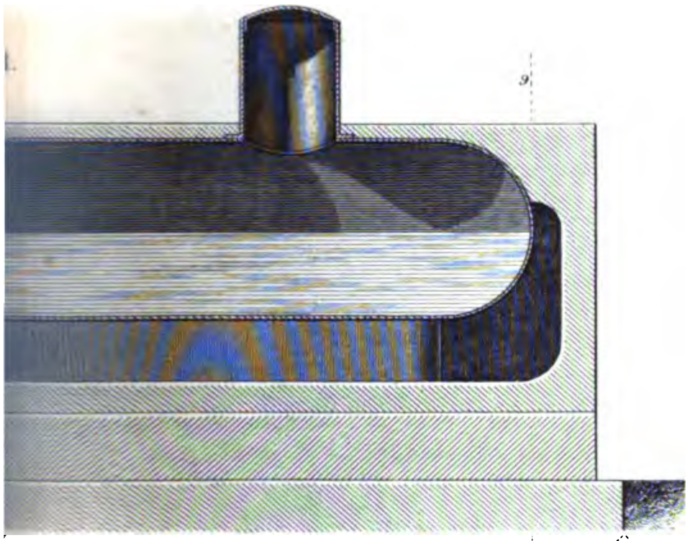


Fig. 3.

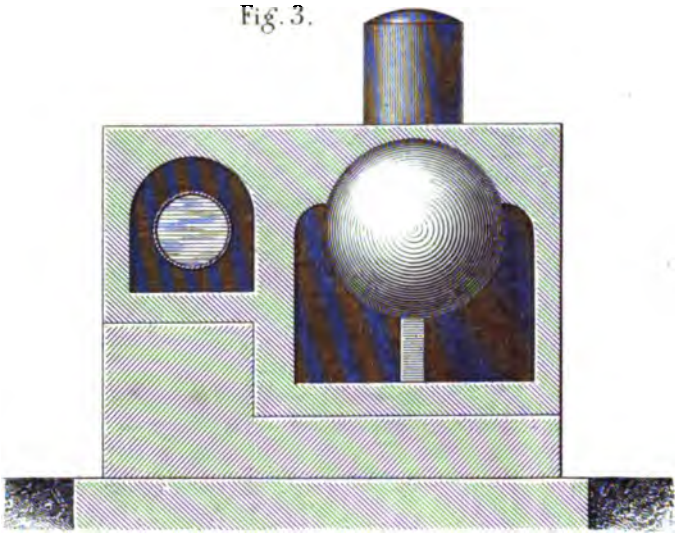


Fig 7

Fig. 11.

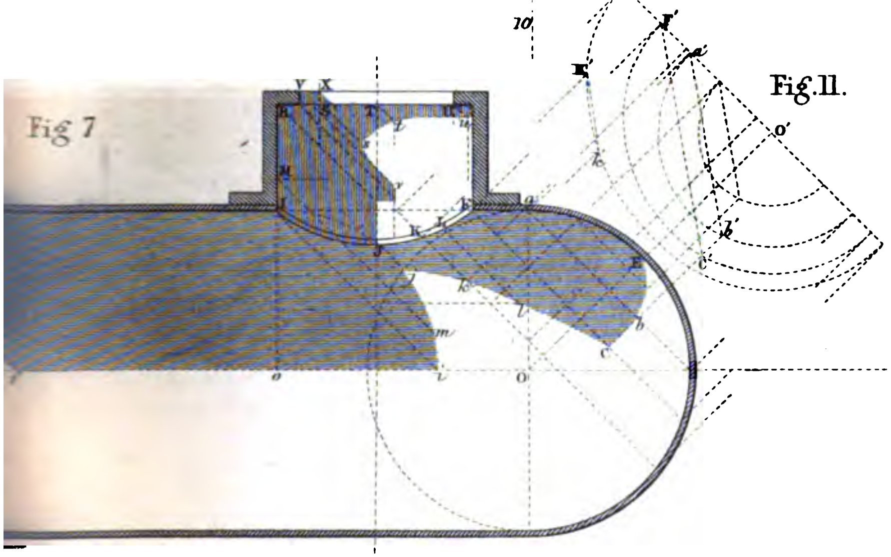


Fig. 9.

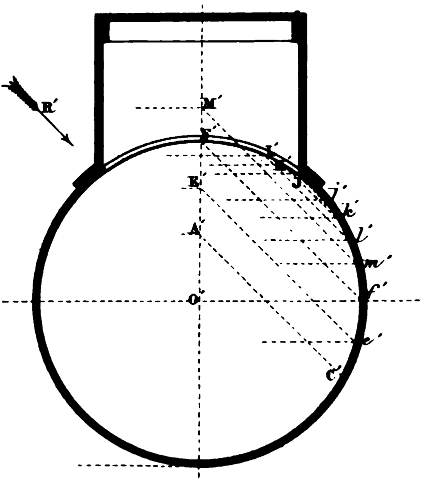


Fig. 12.

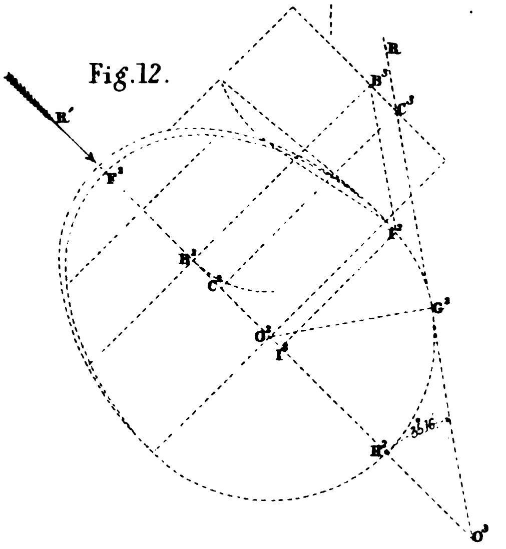
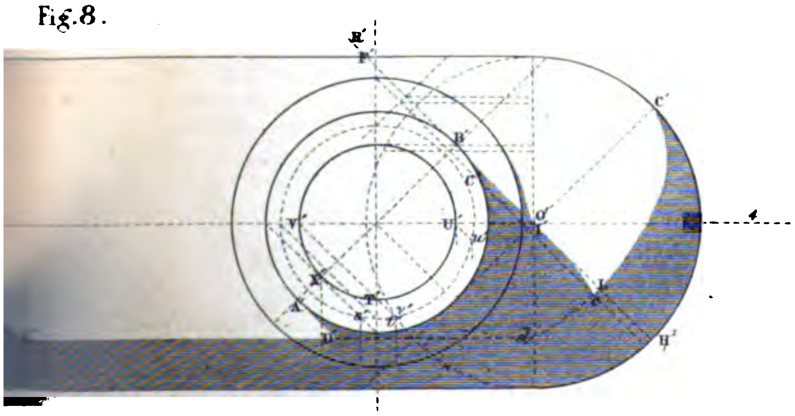


Fig. 8.



For this boiler, we have to determine—

First, in Fig. 7, the shadows, D *d* C and E *b* C, cast upon the spherical surfaces at either end of the boiler, as well as those, C *g i* and *j k l*, upon the cylindrical surface, together with the shadows cast on the interior of the cylindrical chambers.

Second, in Fig. 8, the shadows proper of the external cylindrical and spherical portions of the boiler, and the shadows cast upon these by the cylindrical chambers.

In Fig. 10, we apply the same letters as to the analogous diagram, Fig. 6, this view being drawn for the purpose of obtaining the elliptic curve, D *d* C, of the shadow cast by the circular portion, A C D, upon the internal spherical surface of the end of the boiler.

In the same manner is obtained the portion, E *b* C *l*, by means of the diagram, Fig. 11, observing that the sections made parallel to the ray of light, above the line, *a b*, give semicircles, whilst those made below, such as F C, give the circular portion to the right of the line, *a* O, but an elliptical portion to the left of this line, in consequence of this portion of the plane cutting the cylindrical part of the boiler obliquely. It must be remarked, that the cylindrical chambers, situated on the top of the boiler, give rise to the intersections, I J K F, which cast shadows upon the interior of the boiler, instead of the rectilinear portion, I F, of the extreme generatrix of the cylinder, which would have cast a shadow, had the cylindrical chambers not been there.

The shadow cast by the edge of these intersecting surfaces is limited to the curves, J K F, which may be easily delineated with the aid of the section, Fig. 9, by squaring over the points, J, K, L, F, to the arc, J^1 K^1 L^1 F^1, and then drawing a series of luminar lines through these points; that is, lines parallel to the ray of light indicated by the arrow R^1, and proper to the plane of the picture. These will meet the internal surface of the cylinder in the points, j^1, k^1, l^1, f^1, which are squared over again to the longitudinal section, Fig. 7, by means of horizontals, intersecting the luminar lines, drawn through the corresponding points, J, K, L, in the edges of each chamber, in the points, *j*, *k*, *l*. The rectilinear portion, F I, of the uppermost generatrix of the cylinder, has for its shadow, on the internal surface thereof, the similar and equal straight line, *f i*, which coincides in the projection, with the axis, O O (299).

A part of the extreme left-hand generatrix, I R, of each cylindrical chamber, likewise casts a shadow upon the internal surface of the boiler, the outline of which is a curve, *i m j*, which is simply an arc of a circle, described with the centre, *o*, and with the radius, *o i*, equal to that, *c* O, of the boiler. This shadow is circular, because the straight line, I R, which casts it, is perpendicular to the axis of the cylinder; whilst the axis and itself lie in a plane, parallel to the plane of projection (275).

We can, however, determine the points, *i*. *m*, *j*, of the curve independently, with the assistance of the auxiliary projection, Fig. 9, at right angles to Fig. 7.

It is the same with the curve, *n p q*, which is likewise an arc of a circle, because the straight line, N P, the edge of the cover which closes the top of the chamber, is at right angles to the axis of the latter, and at the same time parallel to the vertical plane of projection. The edges, N R and R M, being vertical, have for shadows upon the internal surface of the chamber, a couple of vertical straight lines, parallel to each other (299). The chamber to the right having a circular opening in the cover, has a shadow upon its internal surface, necessarily different from that in the other chamber. It is, however, easily obtained, and in the same manner as in Figs. 2 and 3, Plate 28 (300). It must be observed, however, that a portion, *s t u*, of this shadow is due to the under edge, S T U, of the cover-piece; whilst the other part, *s v*, takes its contour from the upper edge, V X, of the same piece. A comparison of Figs. 7 and 8, will render these points easy of comprehension.

We may here remark that the horizontal projection of these shadows are taken as projected by the rays in the direction of the arrow, R^1, and thrown on the anterior half of the boiler, which is supposed to be cut away in Fig. 7, and the points thus found squared up to the anterior surface in that figure, give the proper vertical projection of the shadows, just the same as if they had been projected in the direction of the arrow, R, in Fig. 8, which is the horizontal projection of the ray of light, R^1, proper to the elevation, Fig. 7, in which figure we are finding the projection or form of the cast shadows.

There remains, finally, the curve, C *e g h*, and the rectilinear portion, *h i*, together extending from the first, D *d* C, to the straight line, *i i*, and which represents the shadow cast by the arc, A E G H, of the hemispherical end of the boiler, and the straight part, H I, of the upper edge of the cylindrical portion.

The whole curve, D *d*, C *g i*, representing the shadow cast by the edge of the section of the boiler upon the internal surface of the latter, is precisely the same as that distinguished in architecture by the name of the *niche shadow*. It is to be observed, however, that the position in this case is different, as the axis of the niche is vertical.

We have now to draw the shadows, proper and cast, upon the outer surface of the boiler, as seen in horizontal projection, Fig. 8. As for the shadow proper, it consists partly in that limited by the line of separation of light and shade, d^1 d, obtained by the tangential line, making an angle of 45° with the horizon, and touching the circle in the point, E, and partly in that bounded by the elliptic curves, E d^1 and *d c* C^1, upon the spherical ends of the boiler, the manner of determining which has already been thoroughly discussed in reference to Fig. 4.

352. As to the shadows cast by the cylindrical chambers, either on their flanges where they join the boiler, or upon the outside of the boiler itself, they are simply represented by lines inclined at an angle of 45°, as A^1 D^1, B^1 E^1, drawn tangential to the outsides of the cylinders, and which are prolonged in straight lines, as far as the line of separation of light and shade, upon the cylindrical portion of the boiler; that is, in case they stand out far enough from the boiler surface. If, on the contrary, they do not rise very high, as exemplified in the end view, Fig. 12, it will be necessary to determine the outline of the shadow cast by a portion of the upper edge, B^1 C^1, as lying either upon the cylindrical part of the boiler, or upon one of the spherical ends. To find the shadow in this last case, we have supposed an imaginary vertical plane to pass through the luminous ray, R^1 O^1, Fig. 8, producing an elliptical section of the cylinder, and a circular one of the spherical part. This plan being reproduced as turned over into the plane of the picture on R^1 O^2, as an axis in Fig. 12; the curve, F^2 G^2 H^2, representing the section in question. The point of contact, B^1, being transferred to B^2, is also turned down, as it were, upon the horizontal plane, to the point, B^3; so that if we draw a line, B^3 I^2, through this point, B^3, parallel to the luminous ray, R O^3, similarly brought into the horizontal plane, this line, B^3 I^2, will cut the intersectional curve in the point, I^2; the horizontal projection, I^3, of this point, upon the line, B H^2, being obtained by letting fall the perpendicular, I^2 I^3, upon the latter. The corresponding point, I^1, in Fig. 8, is taken at a distance from B^1, equal to B^2 I^2, in Fig. 12. Proceed in the same

manner with another sectional plane, parallel to the first, and passing through the point, c^1, in order to obtain a second point, *c*, of the shadow. The operations necessary for determining the intersectional curves are sufficiently indicated in Figs. 8 and 12.

353. The cylindrical steam-boiler, represented in longitudinal section in Fig. 1, in end elevation in Fig. 2, and in transverse section in Fig. 3, conjoins the various applications of shadows, of which we have been treating, in reference to spheres and cylinders; whilst, at the same time, they serve as examples of shading, by lines or by washes, indicating the effects to be aimed at, and to be attained by the following out of the various principles already laid down.

We must remind the student, that, in order to produce these effects, he must not always confine himself to the representation of the shadows proper and cast merely. He must, further, show the gradations of the light or shadow upon each part, as has already been explained with reference to solid and hollow cylinders. As upon a cylinder or a cone, there is always a line of pre-eminent brilliancy, so likewise, upon the surface of a sphere, will there be a point of greater brilliancy than the rest.

This point is actually situated upon the luminous ray, B^1 O, passing through the centre of the sphere, Fig. 4. Since, however, the visual rays are not coincident with the luminous rays, the apparent position of this point is somewhat changed. Thus, if we bring the vertical plane, B^1 O, Fig. 4, into the horizontal plane, the luminous ray will coincide, as has been seen, with the line, B O, and, consequently, its point of intersection with the sphere will coincide with the point, *i*. On the other hand, the visual rays which are perpendicular to the horizontal plane will coincide with parallels to C O, when brought into the horizontal plane. This latter line intersects the sphere in the point, C; and as the light is reflected from any surface in the direction of the visual rays, so as to make the angle of incidence equal to the angle of reflection, if we divide the angle, *i* O C, into two equal angles, by the line, *n* O, the point, *n*, will be that which will appear to the eye most brilliantly illuminated. The positions, n^1 and i^1, in the vertical plane of the points, *n* and *i*, are obtained by letting fall perpendiculars upon the line, O A, representing this plane.

In shading up a drawing, it is preferable to place the bright or lightest part between the two points, n^1 and i^1, a more pleasing effect being obtained thereby. When the sphere is polished, as a steel, brass, or ivory ball, a circular spot, of pure white, must be left about the point in question. When, however, the body is rough, as is supposed in Fig. 3, this part is always lighter than the rest; but, at the same time, it is covered by a very faint wash.

In the case of a hollow sphere, Figs. 5 and 6, we have to bear in mind, not only to indicate the position of the bright spot, which is projected, in the same manner, upon the luminous ray, A B, and lies between the points, n^1, i^1, but also the point in the cast shadow, which should be the least prominent. This latter will be found to be at m^1, Fig. 5, as determined by the radius, O^1 *m*, Fig. 6, drawn perpendicularly to the ray of light, A^1 C^1, as brought into the same plane as Fig. 6.

The boiler is represented as placed in its furnace, which is built entirely of bricks, with a diaphragm passing down the middle of its length, so that the flame and gases issuing from the grate pass along the flue to the left, then return to the right, and pass through a third flue, before they reach the chimney. In this third flue is placed an auxiliary boiler, full of water, and in communication with the main boiler by a pipe passing to the bottom of each. In this auxiliary boiler, the feedwater becomes heated before entering the main boiler, so as not to reduce the temperature of the latter to a great extent, upon its introduction into it.

The main boiler is represented as half full of water. It should generally be two-thirds full, but is so delineated that the greater portion of the shadow cast upon its interior may be visible. The remainder of the space, as well as the cylindrical chambers, are supposed to be filled with steam. The base of the chimney is of stone, whilst the stalk is of brick. The foundations of the furnace are likewise of stone.

Besides this present example of a boiler, we give a further exercise in finished shading in Plate 28, the objects in which we recommend the student to copy, on a scale two or three times as large, so as to acquire the proper skill and facility of treatment.

SHADING IN BLACK.—SHADING IN COLOURS.

PLATE 33.

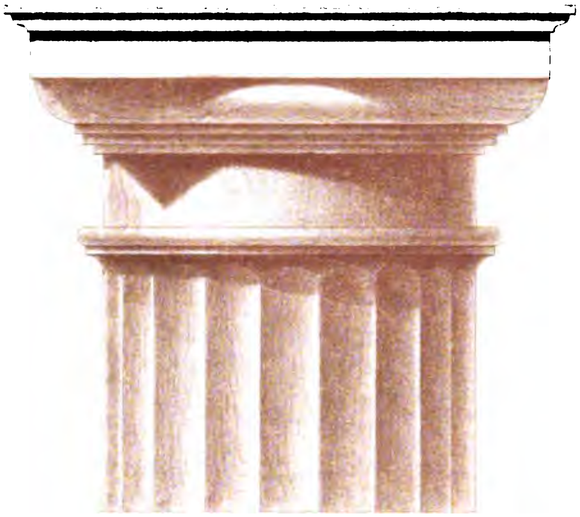
354. In a great number of drawings, and particularly in those termed working drawings, and intended for use in actual construction, the draughtsman contents himself by shading the objects with China ink—sometimes, perhaps, covering this with a faint wash of colour, appropriate to the nature of the material. The shading, on the one hand, brings out the parts in relief, and renders the forms of the object intelligible to the eye; whilst, on the other hand, the colours indicate of what material they are made. This duplex artistic representation makes the drawing much more lifelike, and more easily comprehended. A drawing may be coloured in several ways. The simplest plan is first to shade up the various surfaces with China ink, having due regard to the respective forces and gradations of tone, according to the lights and shades, as has been done in the preceding plates. The entire surface of each object is then covered with the appropriate conventional wash of colour usually employed. It must be laid on in accordance with the instructions given in reference to Plate 10, Chap. V. This first method of operating may suffice in many cases, but it leaves out much to be desired in the effective appearance of the drawing, its aspect being generally without vigour, cold and monotonous. A better result is obtainable by not carrying the China ink shading to so great a depth, and by covering the surfaces by two or three washes of colour, laid on in gradations, as was done with the China ink itself, so as to produce a sufficient strength of colour at the darker parts, whilst the light parts are left very faint; and where the objects are polished, a pure white line or spot is left, which adds considerably to the brilliancy of the whole. A softer and more harmonious effect can be produced by the use of a warm neutral tint, instead of the China ink, for the preliminary shading. This colour, however, is very difficult to mix, and to keep uniform.

When a little practice has given some skill and facility in the preparation and combination of the colours, the draughtsman may proceed, at the outset, in a more direct and vigorous manner, leaving out altogether the preliminary shading with China ink, and laying on at once the successive coloured washes, rendering, at the same time, the effects of light and shade, and indicating the nature of the material. This last mode has the merit of giving to each part of the drawing a richer translucence, more warmth, and a more satisfactory fulfilment of the desirable conditions.

In general, all drawings intended to be shaded should be delineated with faint grey instead of black outlines, as for a simple outline drawing; the faintness of such lines avoids the necessity of

EXAMPLES OF COLOU

Fig. 1.



WOOD.

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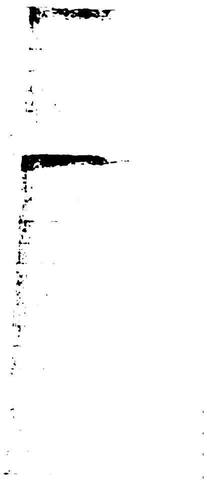

Fig. 2.

STONE.

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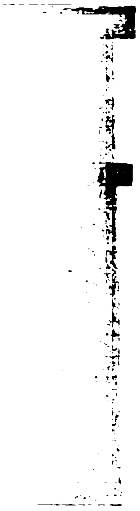

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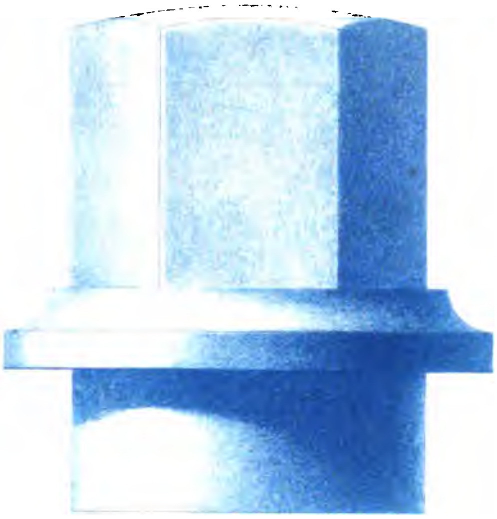
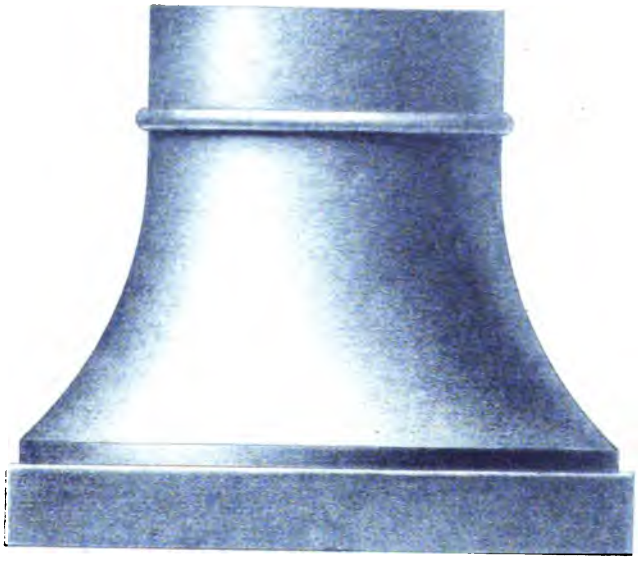
Fig. 5.



MALLEABLE IRON

Fig. 6.



CAST IRON.

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EXAMPLES IN THE P

Fig. 1.

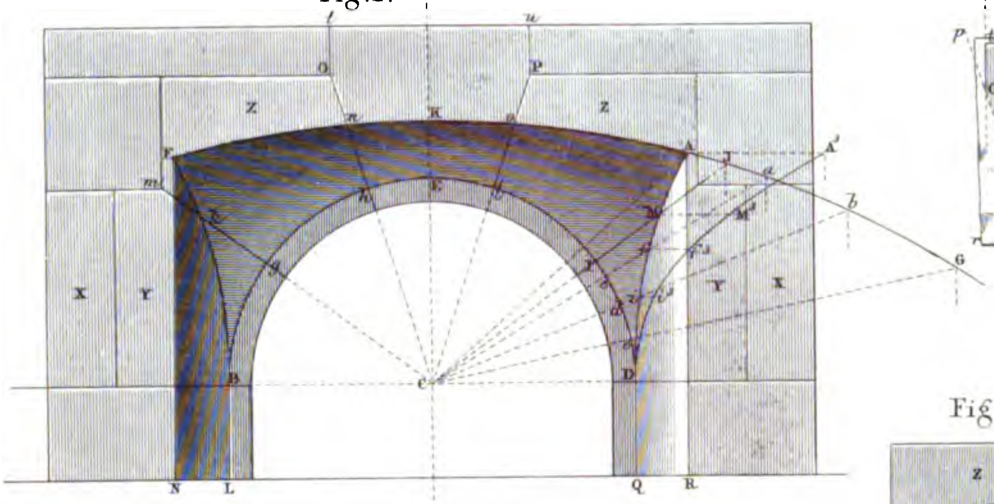

Fig. 3.

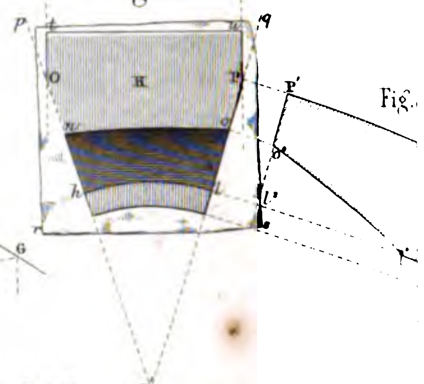

Fig. 6.

Fig. 8.

Fig. 9.

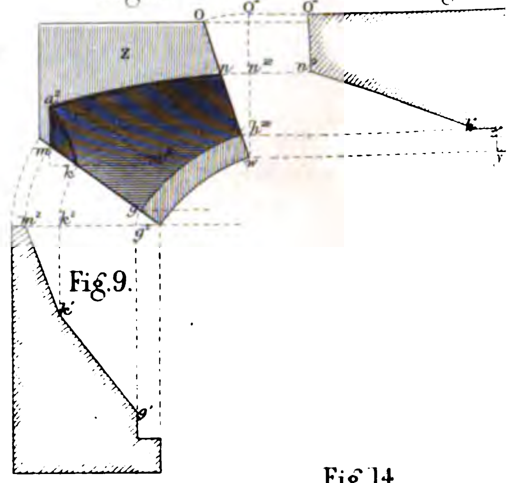

Fig. 2.

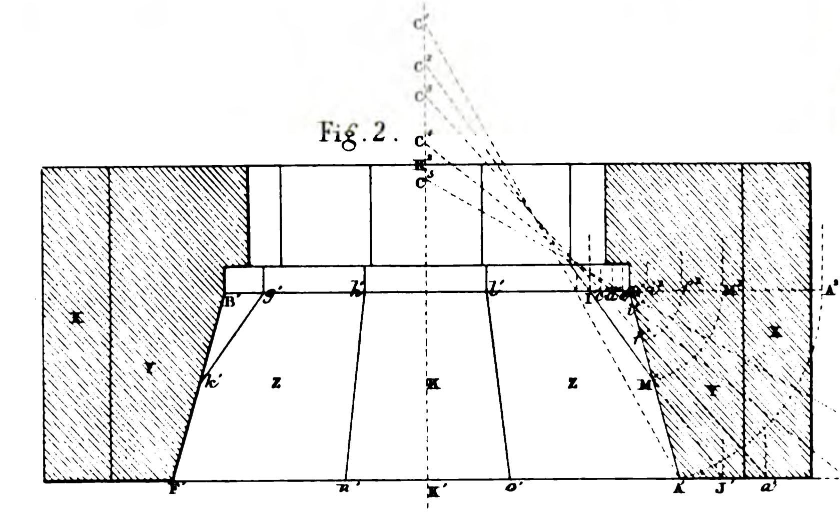

Fig 14

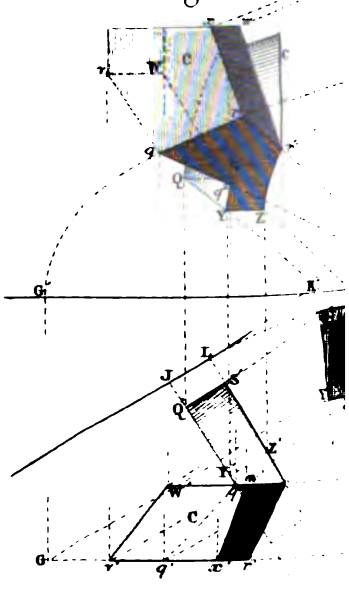

Fig. 15.

Fig. 10.

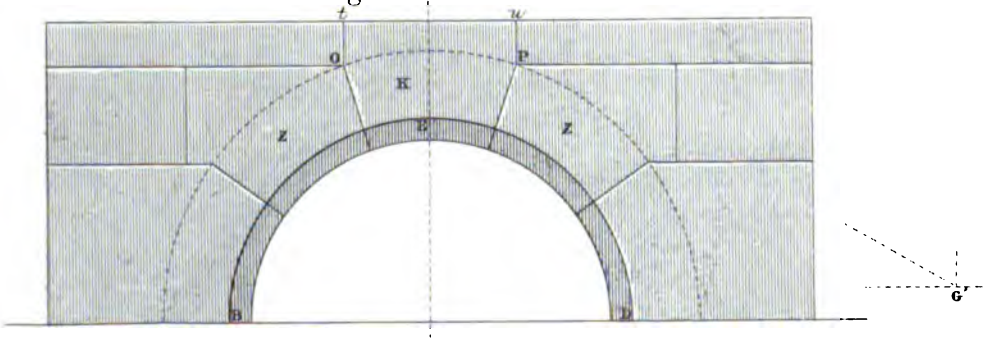

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PLATE 34.

JECTION OF ARCHES.

Fig. 4.

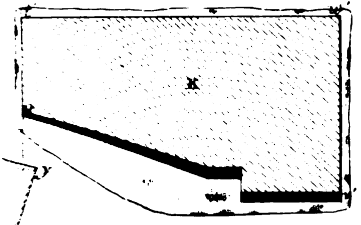

Fig. 7.

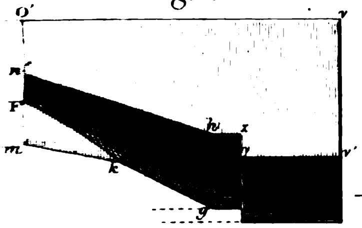

Fig. 11.

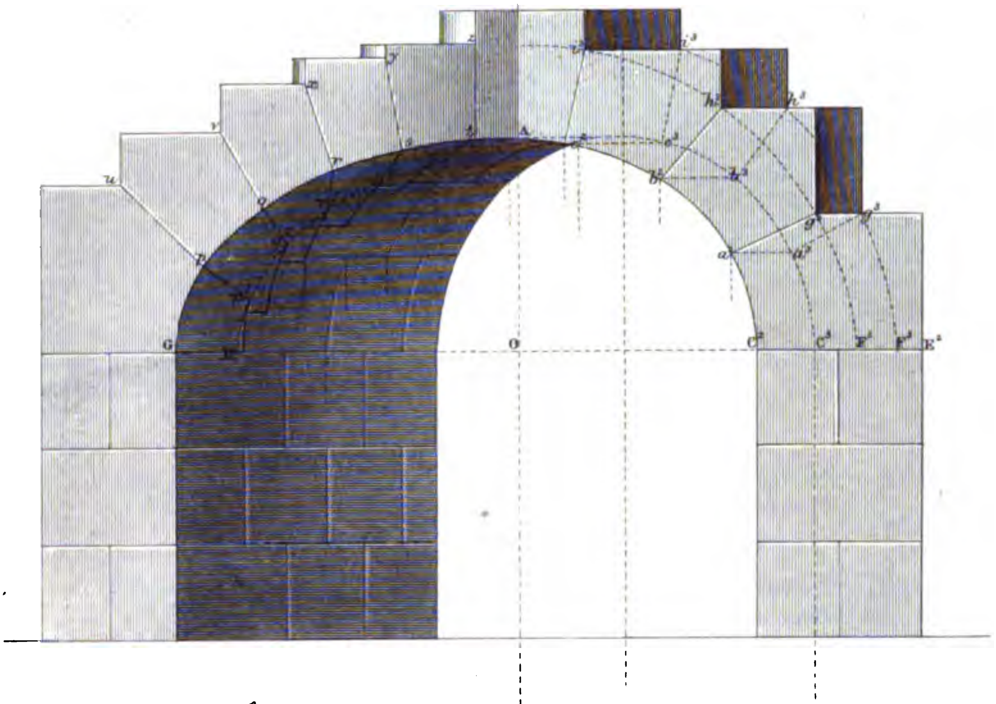

Fig. 13.

Fig. 12.

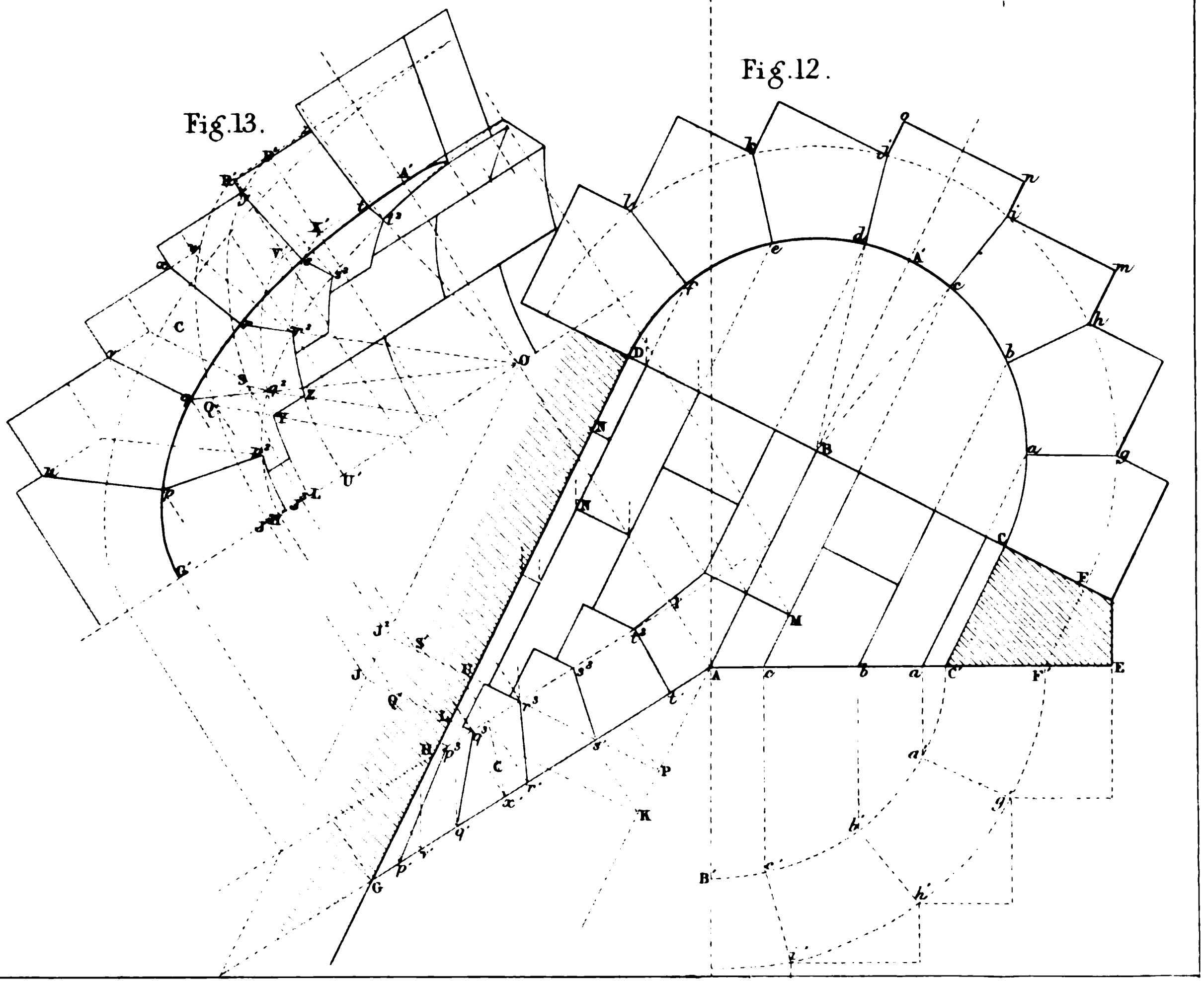

Fig. 16.

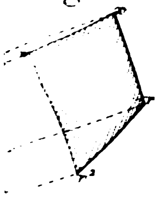

Fig. 17.

making them very fine, and their greater breadth affords a much better guide to the shading-brush. A black outline, however fine it may be, always produces a too sharp and hard appearance, whilst there is much greater risk of overstepping it in laying on the washes.

355. In Plate 33, we give a few good examples of objects shaded in colours, comprising the materials most in use in construction.

Fig. 1 represents the capital of a Doric column in wood. Although the woods are naturally very different in colour, still, in mechanical drawings, a single tint is used indiscriminately; it is, as we have said before entirely conventional.

In fixing upon these colours, the object in view has been to avoid confusion and to employ a distinct and intelligible colour for the representation of each substance, without seeking to copy the natural colour in all its varieties.

In colouring this wooden capital, after the preliminary operations which we have mentioned, for determining the outlines of the shadows, proper and cast, it is first shaded throughout with China ink, and when this shading has reached a convenient depth, which in this case ought to be light and delicate, with bright lights, on the prominent parts, and when thoroughly dry, the whole surface is to be covered with a light wash, which may be a mixture of gamboge, lake, and China ink, or burnt umber alone. The first wash of colour, in fact, should be analogous to that of Fig. 7, Plate 10, (137), which represents the material in section, part being grained both as end and side wood.

The proceeding in the former instance (137) may be easily modified and made to resemble the effect of the present example, Fig. 1, by leaving certain parts of the objects uncoloured, and by softening off the shade in those places where the light is strongest, with a nearly dry brush. If however, the draughtsman has become somewhat familiarized with the use of the brush and the mixture of the colours, he may, as we have said, omit the preliminary shading in black, by modifying each shade as laid on, mixing the China ink directly with the colours, and then gradually bringing up the shades, either according to the system of flat washes, or the more difficult one of softened shades. Care must be taken in laying on these shades to commence at the deeper parts, and then to cover these over again by the subsequent washes which gradually approach the bright part of the object, for in this way a more brilliant and translucent effect will be obtained.

When the objects are of wood, it is customary to represent the graining in faint irregular streaks, care being taken to make these as varied as possible. A general idea of the effect to be produced will be obtained from Fig. 1 of this plate, and Fig. 7, Plate 10.

Side and end wood is often shown in accordance with the general system (354), by shading the object complete, and then graining it all in China ink, until it assumes the appearance of the key, E, in the railway chair, Plate 13, and then laying over the whole surface, a light wash of some of the colours proper for wood as just described.

Following out these principles, the draughtsman may proceed to colour various other objects composed of different materials, merely varying the mixtures of colour according to the instructions given in reference to Plate 10.

Fig. 2 represents the base of a Doric column in stone, showing the flutings. This being an external view, the tint to represent the stone is not made so strong as for the sectional stonework, represented in Plate 10, particularly on the light parts. A yellowish grey may be used for it, made by mixing gamboge, the predominant colour, with a little China ink, adding a little lake to give warmth.

Fig. 3 represents the top of a chimney of brickwork, the form being circular. In this external view, each brick is indicated in outline; and when the scale is large, a line of reflected light is shown on the edges towards the light, near the brighter part of the chimney, to render the bricks more distinct from each other. Indeed, it is generally advisable to leave a narrow, pure white light at the edges of an object which are fully illuminated, as it gives an effective sharp appearance.

These three examples of wood, brick, and stone, represent bodies with rough surfaces, and which, therefore, can never receive such brilliant lights as objects in polished metals; no part, indeed, should be entirely free from some faint colour.

Fig. 4 represents a pair of polished brass bushes in elevation, as detached from their bearing-block or eye of a rod, according to the kind of journal they may be used for.

Fig. 5 represents either a nut or bolt-head of wrought iron; and, as we have supposed it to be turned, planed, and polished upon its entire surface, it has been necessary to leave pure white lights at the brighter parts, to distinguish the surfaces from those which are rough and dull. It is the same in the example, Fig. 6, which represents the base of a polished cast-iron column; both of these being also shown in elevation, and coloured to correspond to the material of which they are made, in accordance with (132, 133).

We hope that the principles of shadows and shading, explained and exemplified in the last two chapters, may serve as sufficient guides for the various applications which may present themselves to the draughtsman—whether his skill be called forth to render the simple effects of light and shadow, or to produce the gradations of shade and colour due to roundness or obliquity of surface—to the various positions of the objects in their polished or unpolished state, and to the various materials of which they may be composed.

Thus, it will be understood, that although two objects are precisely alike in material and form, if they are situated at unequal distances from the spectator, the nearer one of the two must be coloured more strongly and brilliantly than the more distant, greater force and depth being given to the darker shades.

CHAPTER XI.

THE CUTTING AND SHAPING OF MASONRY.

PLATE 34.

356. The operation of stone-cutting has for its object, the preparing and shaping stones in such manner that they may be built up into any desired form in a compact and solid manner; great care and skill, as well as mathematical knowledge, is more particularly required in the preparation of stone for arches, vaults, arcades, and similar structures.

The study of the shaping of stone is based entirely upon descriptive geometry, being indeed but a particular application or branch of it, and in it have to be considered the generating of surfaces, as well as their intersections and development.

In proceeding to adapt the stones to the position they are to occupy, the mason should prepare a preliminary drawing of the actual size of each stone, as well as a general view of the entire erection, indicating the joints of each stone; these according to the various positions to be occupied by them, are called *key stones*, *arch stones*, &c.

It is not our intention to give a complete treatise on the shaping of masonry; but, as this study certainly forms a part of geometrical drawing, we have thought it quite within the design of the present work to give a few applications, sufficient to show the line of procedure to be followed out in operations of this nature.

THE MARSEILLES ARCH, OR ARRIÈRE-VOUSSURE.

FIGURES 1 AND 2.

357. We propose to prepare the designs for the bay and arch of a door or window, to be built of stonework, the upper part being cut away, so as to present a twisted surface, analogous to that known as the *arrière-voussure* of Marseilles.

Fig. 1 represents the vertical projection or front elevation of one form of this kind of arch, and Fig 2 is a sectional plan corresponding to Fig. 1.

The surface of this bay and arch is such as would be generated by a straight line, C A, kept constantly upon the horizontal, $C^1\ K^1$, projected vertically on the point, C, and moved, on the one hand, upon the semi-base, B E D, of a right cylinder, having $C^1\ K^1$ for its axis; and, on the the other, upon the circular arc, F K A, situated in a plane parallel to that of the base, B E D.

The lateral faces, F B L N and A D Q R, of the bay, are vertical, and are projected horizontally in $F^1\ B^1$ and $A^1\ D^1$, Fig. 2. These faces intersect the twisted surface at the curves, F B and A D, which we shall proceed to determine.

For this purpose, the first thing to be done is to seek the projections of the straight generatrix, C A, as occupying different positions, so as to obtain their points of intersection with one of the oblique planes.

We may remark, that if the arc, F K A, be prolonged to the right, for examplo, of Fig. 1, and a number of lines be drawn through the points, C, as C J, C *a*, C *b*, and C G, they will represent so many vertical projections of the generatrix, C A, in different positions. These straight lines meet the semicircle, B E D, in the points, I, *c*, *d*, *e*, which are projected horizontally in the points, I^1, c^1, d^1, e^1. These same lines also cut the circular arc, F A G, in the points, J, *a*, *b*, G, which are projected upon the line, $F^1\ G^1$, the horizontal projection of this line, in the points, J^1, a^1, b^1, G^1. By drawing lines through these last, and through those first obtained, I^1, c^1, d^1, e^1, we obtain the straight lines, $C^2\ J^1$, $C^3\ a^1$, $C^4\ b^1$, $C^5\ G^1$, which are the horizontal projections of the generatrix, C A, and correspond to the vertical projections in Fig. 1.

These straight lines cut the plane, $A^1 D^1$, in the points, $M^1\ f^1\ i^1$, which are then projected vertically to M, *f*, *i*, upon the straight lines, C J, C *a*, C *b*; the curve, A M *f* *i* D, passing through each of these points, is the line of intersection sought, and it is reproduced symmetrically at F B, to the left hand of Fig. 1, so as avoid the necessity of repeating the whole projection over again.

To obtain this line of intersection full size, it is necessary to bring the plane, $D^1\ A^1$, into the plane of the picture, by supposing it to turn about the vertical, D Q, projected horizontally in D^1, as an axis; during this movement, each of the points, A^1, M^1, f^1, will describe an arc of a circle about the centre, D^1, finally coinciding with the points, M^2, A^2, f^2, and i^2. Through the corresponding points, A, M, *f*, *i*, in the vertical projection, we must draw a series of horizontal lines A A^3, M M^3, *f* f^3, upon which, square over the preceding points, M^2, A^2, f^2, i^2, by which means will be obtained the curve, A^3, M^3, f^3, i^3, D^3, representing the exact form or parallel projection of the line of intersection.

358. The preliminary design thus sketched out, gives nothing but the outline of the surface of the erection, and it now remains to divide it into a certain number of parts, to represent the individual stones of which it is to be built up.

The number of divisions necessarily depends upon the sizes and the nature of the stone at the mason's disposal; the number should, however, in all cases, be an odd one, so that a central space may be reserved for the principal piece, known as the key-stone.

The divisions are struck upon the semicircle, B E D, by a series of radii converging in the centre, C; it is these lines which represent the divisions of the stones. Below the arched part, the regular pieces as X, consist of a series of stones of equal dimensions, the joints of which are horizontal.

The horizontal projections of each of the stones forming the arch are straight lines, because the joints lie in planes perpendicular to the vertical plane, whose intersection with the twisted surfaces is always a straight line corresponding to a generatrix; thus, the planes, O C and P C, of the joints on either side of the keystone, K, are perpendicular to the vertical plane, and pass through the axis, $C^1\ K^1$; and portions of them, *n h* and *o l*, comprised between the directing circles, B E D and F K A, are represented in the horizontal projection by the straight lines, $n^1\ h^1$ and $o^1\ l^1$, which are the joints of the stones as seen from below—that is, the lines of their intersection with the twisted surface.

It is the same with the planes, *m* C and J C, in which lie the joints of the corner stones, Y Z; the portions, *k g* and I M, of the joints falling upon the twisted surface, are likewise projected horizontally at the straight lines, $k^1\ g^1$ and $M^1\ I^1$.

359. The design of the erection being thus completed, the stonecutter should delineate each individual stone as detached from the arch, in such a manner as to represent all the faces of the joints, and he then takes for each, a stone of the most convenient dimensions from amongst those at his disposal, which are generally hewn out roughly in the shape of rectangular parallelopipeds; on each of these pieces he marks off the parts to be cut away, to reduce the stone to the required form and dimensions.

Thus, supposing he commences with the key-stone, K, for example, detailed in front view and vertical section through the middle in Figs. 3 and 4, respectively, he takes a parallelopiped, of which the base, *p q r s*, is capable of circumscribing the two parallel faces of the upper part of the key-stone, and of which the height is at least equal to the length, $t^1\ u^1$, or $K^1\ K^2$. After having cut and finished the two vertical faces, t^1 *o* and u^1 *v*, of the prism, as well as the horizontal face, $t^1\ u^4$, he measures off upon the anterior face, Fig. 3, the parallel and vertical sides, *t* O and *u* P, and then the oblique lines, O *h* and P *l*, which, it will be remembered, converge to the same point, C, the axis of the voussure. He next sets off upon a template the arcs, *n o* and *h l*, Fig. 1, and reproduces them thence upon the parallelopiped, Fig. 3, at *n o* and *h l*. After this preliminary marking out, the stonecutter reduces and takes away all the material upon the sides of the parallelopiped which lies outside the lines, O *h* and P *l*; these faces being finished, the shape-designer lays out upon them the lines projected at *n h* and *o l*. In order that the form of this joint may be more easily comprehended, and its actual size seen, we have brought the face, which is projected in the line, $P\ l$, Fig. 2, into the plane of the picture, and representing it in the line Fig. 5, as turned over at right angles to that face, or as parallel to the plane of projection.

This view, it will be seen, is easily obtained; for, on the one hand, we have the line, $P^1\ y$, equal to $K^1\ K^2$, representing the thickness of the wall or of the arch; and, on the other hand, all the other dimensions, as projected horizontally in Fig. 2, so that the inclination of the line, $o^1\ l^1$, can be determined with the most rigorous exactness.

This straight line, as well as the corresponding one on the opposite face, $o\ h$, serves as a guide to the stonecutter in reducing the twisted portion of the surface of the key-stone, comprised between them; and as affording a means of verification, it may be remarked, that this surface should be cut in such a manner, that a rule or straight edge may be applied to all parts of it, being guided by the arcs, $n\ o$ and $h\ l$; the former of which springs from the point, o^1, and the latter from the points, l^1, on the face $P^1\ x$, Fig. 5.

To determine the faces of the joints of either of the two corner pieces, Z, represented in detail, and detached in front and side elevations, in Figs. 6 and 7 respectively, but as these faces cannot be seen in these projections in their full dimensions, it is necessary to proceed in the same manner as before, bringing each face into the plane of projection—that is, delineating auxiliary views of them, as if parallel to this plane.

Thus, to obtain the actual dimensions of the face of the joint projected at $o\ h$, in Fig. 6, with the point, w, as a centre, describe a series of arcs, with the respective radii, $o\ w$, $n\ w$, $h\ w$, so as to reproduce the points o, n, h, at o^2, n^2, h^2, upon the vertical, $o^2\ w$; then, by setting off, $n^2\ h^1$ equal to $n^1\ h^1$, Fig. 2, measured on the horizontal plane, and joining the points, $h^1\ n^2$, Fig. 8, we get the inclination of the generatrix line, $n^2\ h^1$, which is projected vertically at $n\ h$, Fig. 1. The form of the joint face is completed by drawing the horizontal lines, $o^2\ v$, $h^1\ z^1$, $y^1\ v^1$, and the verticals, $o^2\ n$, $v\ v^1$ and $z^1\ y^1$, the lengths between, which last are already given full size in Fig. 7. It will be observed that Fig. 8 is on the plate removed a little to the right of the vertical, $o^2\ w$; but this is a matter of no importance, and is merely done for convenience sake.

The same system of auxiliary projections is applicable to the determination of the dimensions of the other face of this stone—namely, that projected in the line, $m\ g$, Fig. 6, which is brought round to the horizontal, $m^2\ g^2$, and drawn with full dimensions in Fig. 9, measured perpendicular to this line; in this last face, it is necessary to bear in mind the portion of the line of intersection of the sides with the arched part which it contains, and which is obtained in its actual proportions, as at $a^2\ k$, by means of a template formed to the curve, $A^3\ M^3$, in Fig. 1. The stone, Y, beneath, of course, contains the remainder of the intersectional curve.

The methods just explained, in regard to the shaping of the key-stone, and one of the corner-stones, may be extended, without difficulty, to the remaining portions of this Marseilles arch.

In this application, it has been necessary to determine the proportions of the twisted bay of the arch, as well as the faces of the joints; but in the more general case of straight bays, such as that represented in front elevation, Fig. 10, the operations are considerably simplified, and the designer has merely to attend to the form of the joint faces, making use, for this purpose, of the auxiliary projections, as above described. The delineation of the various parts of this figure presenting no new peculiarity, it need not further detain us.

360. Let it be proposed to delineate a circular vault with a full centering, bounded by two plane surfaces oblique to its axis. The example given, which is represented in front elevation and sectional plan in Figs. 11 and 12, respectively, is taken from the entrance to the tunnel on the Strasbourg Railway, near the Paris terminus, and it is a form frequently met with in the construction of railways.

In the representation of this vault, we have supposed one of the oblique planes to be parallel to the vertical plane of projection, and it consequently follows that the axis of the arch is inclined to this plane.

Let $A\ B$ be this axis, and $C\ D$ the horizontal projection of a plane at right angles to it; with the point, B, as a centre, describe the semicircle, $C\ A^1\ D$, representing the arch in its true proportions, as brought into the plane of the picture. Let us suppose this semicircle to be divided into some uneven number of equal parts, as in the points, a, b, c, d, e, f; through each of these points draw straight lines, passing also through the centre, B, and representing the joints, $a\ g$, $b\ h$, $c\ i$, of the arch stones, being, of course, normal to the circular curvature of the arch, and being limited in depth, as we shall suppose, by the second outer semicircle, $g\ i\ l$, concentric with the first. Each of these joint faces intersects the centering of the arch in a straight line parallel with its axis, and the horizontal projections of these intersections, as seen from below, are obtained simply by drawing through the points, a, b, c, d, lines parallel to the axis, $B\ A$; these last extend as far as the vertical plane, $A\ E$, which bounds a portion of the vault. The external faces of the key and arch stones are limited by straight vertical lines, such as $m\ h$, $i\ n$, $o\ j$, and horizontals, as $m\ i$ and $n\ o$.

We have now to obtain the projections on the vertical plane, Fig. 11, of the intersection of each of the arch stones by the plane, $A\ E$.

We may remark, in the first place, that since this plane is oblique to the axis of the cylindrical arch, it produces an elliptical section, having for its semi-transverse axis the length, $C^1\ A$, and for its semi-conjugate axis, the length $A\ B^1$, equal to the radius, $A^1\ B$. As much of this ellipse as is required is drawn according to one or other of the many methods given (*et seq.*)—say as at $C^1\ b^1\ B^1$, Fig. 12, which curve is reproduced at $c^3\ a^3\ b^3$, in the elevation, Fig. 11.

If we, in like manner, obtain the projection of the semicircle, $F\ i\ l$, which limits the radial joints, we shall also obtain the portion of an ellipse, $F^2\ g^2\ i^2$, and we have further merely to project the points, a^1, b^1, c^4, upon the first ellipse, in $a^2\ b^2\ c^2$; as also on the second one, the points, g^2, h^2, i^2, corresponding to $g\ h$ and i. The straight lines, $C^2\ F^2$, $a^2\ g^2$, $b^2\ h^2$, $c^2\ i^2$, represent the intersections of the faces of the arch stone joints, with the plane, $A\ E$.

The vault being supposed to extend no farther back than the plane, $C\ D$, it will be necessary to represent the intersection of this last with the arch stones which extend thus far upon this plane. $C\ D$. We have, therefore, to project the elliptic curves, $C^3\ b^3\ c^3$, and $F^3\ g^3\ i^3$, corresponding to the quarter circles of the radii, $F\ B$ and $C\ B$. As the arch stones cannot extend the entire length of the vault, they are limited by planes, $M\ N$, perpendicular to the axis, and, consequently, parallel to $C\ D$, so that the projections of these joints will be but repetitions of portions of the same elliptic curve; care is taken so to dispose the blocks of stone, that no two joints form a continuous line, the joints in one course being brought between those in the adjacent ones, as is customary in all brick and stone work.

361. We have now to determine the intersection of the oblique plane, $A\ G$, with the remaining half of the same circular vault, and

then to obtain the projection of this intersection upon the vertical plane.

The plane, A G, also produces an elliptical section of the vault; this is reprsented at $G^1\ q\ t$, as brought into the picture in the auxiliary diagram, Fig. 11, which gives its actual proportions; the semi-transverse axis, G^1 O, of this ellipse is equal to G A, and its semi-conjugate axis is equal to the radii, A^1 B, of the vault.

After having divided this curve into a certain uneven number of equal parts, draw normals,* $p\,u$, $q\,v$, $r\,x$, $s\,y$, and $t\,z$, through the points of division representing the joints of the arch stones, the remaining sides of the external faces of which are limited by horizontals and verticals, as before.

If the vault is supposed not to extend beyond the plane, A G, the arch stones will have to be shaped as facing stones, and their joints will require to be set off upon the first ellipse, $G^1\ q\ t$, and to be limited by the second, $H^1\ q^2\ t^2$, obtained from the intersectional plane, H I, drawn parallel to A G; by drawing straight lines from the points of division obtained upon the ellipse, $G^1\ q\ t$, to the centre, O, we obtain the points, p^2, q^2, r^2, s^2, of intersection of these lines upon the second ellipse, and the straight lines, $p\,p^2$, $q\,q^2$, $r\,r^2$, $s\,s^2$, representing the intersections of the arch stones with the inside of the vault; these straight lines are projected horizontally in $p^1\,p^3$, $q^1\,q^3$, $r^4\,r^3$, &c., Fig. 12, where they are visible, because the diagram is supposed to be a projection of the vault, as seen from below; the two diagrams, Figs. 12 and 13, will render the determination of the vertical projection, Fig. 11, very easy, the same lines there being designated by the same letters.

To limit the arch facing stones, and unite them conveniently with the regular courses of the vault, they must be cut by planes, such as J K, and J^1 P, Fig. 12, perpendicular to the axis, A B. The intersections of these planes with the vault, produce portions of circles, which are projected as ellipses in Fig 13, such as $L^1\ q^2$ V and $U^1\ r^2\ X^1$, for the inside of the vault, and $J^1\ Q\ R^1$ and $J^3\ S\ P^1$, for their outer extremities, these various ellipses corresponding to the radii, C B and F B. The joints of these stones are finally completed by planes, such as $K^1\ P^2\ S\ Q$, Fig. 13, passing through the axis, A B, and through horizontal lines, $Y^1\ Z^1$ and Q S, in the vault, the latter and inner one of which only is visible in the auxiliary vertical projection, Fig. 13, and in the elevation, Fig. 11.

362. In constructing this vault, it is necessary to make detailed drawings of each particular stone, showing the dimensions of all the faces. In Figs. 14 and 15, we have represented one of these arch stones, C, in elevation and plan, as detatched from the erection, shown in front elevation, Fig. 11; and showing more particularly the lines and surfaces of this particular stone, which are not apparent in Figs. 12 and 13. Thus, in these views may be distinguished:—

1st. The anterior face, $v\,q\,r\,x$, which is projected horizontally in $v^1\,r^1$, Fig. 15, upon the line of the plane, A G; this face intersects the vault at the elliptic curve projected at $q\,r$, in Figs. 11, 13 and 14.

2d. The face of the joint, $x\,r\,r^2\,w$, Figs. 13 and 14, is projected horizontally in $x^1\,r^1\,r^2\,w^1$, in Fig. 15, and the one edge, $x\,r$, is projected upon the plane, G A, in the line, $x^1\,r^1$, whilst the opposite edge, $w\,r^2$, is projected upon the line, $w^1\,r^2$, parallel to G A; and the lower edge, $r\,r^2$, the line of intersection of this face with the interior of the vault, is projected in the line, $r^1\,r^2$, whilst, finally, the upper and fourth edge, $x\,w$, is projected at $x^1\,w^1$. This face, which is oblique in all the other views, is projected as parallel to the plane of the picture in Fig. 16, so as to show its actual dimensions corresponding to the scale of the other views.

3d. The second joint face, $v\,q\,q^2$ W, is opposite to the first, and projected horizontally in $v^1\,q^1\,q^4\,W^1$, Fig. 15.

4th. The face, Q S Z Y, the horizontal projection of which is $Q^1\ S^1\ Z^1\ Y^1$; this face is situated in a plane passing through the axis of the vault, and is represented by the line, $Q^2\ Y^2$, on the radius, P Q^2, in the additional diagram, Fig. 17, which is the projection of the stone as seen at right angles to that in Fig. 15, and as it were turned over to the right on L P, as an axis.

5th. The species of dovetail joint, $q\,q^2$, Y Z $r^2\,r$, of which the edges, $q\,q^2$ and $r\,r^2$, are projected, as has been seen at $q^1\,q^4$, and $r^1\,r^2$; whilst the sides, Q r and Z r^2, are similarly projected at $q^1\,r^1$ and $Z^1\,r^2$, and finally the side, q^2 Y at $q^4\,Y^1$, in Fig. 15.

6th; and lastly, the posterior face, Q, of which it will be easy to render an account by means of the distinctive letters, which are invariably the same for the same points, although additional special marks are superadded to obviate confusion amongst the various figures. To render the vertical elevation and plan, Figs. 14 and 15, still more intelligible, we have added the subsidiary view, Fig. 17, representing the projection of the block in the plane, L P, as brought into the plane of the picture; thus giving the actual proportions of the faces projected in the planes, $S^1\,r^2$ and P Q^2, in Figs. 15 and 17, respectively. To obtain the various points seen in this view, it is sufficient to set off the vertical distances from the line, G^1 O, of the elevation, Figs. 13 and 14, obtaining in this manner, for instance, the points, r^3, q^3, v^3, x^3, &c., corresponding to r, q, v, and x, in Figs. 13 and 14, and r^1, q^1, v^1, and x^1, in Fig. 15.

The examples chosen for this plate (34) combine the more difficult problems and applications met with in the shaping and arrangement of stonework, and will make the student acquainted with the operations upon which designs for these purposes are based, as well as with the general methods to be adopted in obtaining oblique projections by the employment of auxiliary projections, taken, as it were, in planes parallel to the different surfaces, and then brought into the plane of the picture; this system, at the same time, being of much use in ascertaining the exact proportions of various surfaces, such as the joints of masonry.

RULES AND PRACTICAL DATA.

STRENGTH OF MATERIALS.

363. The various materials employed in mechanical and other constructions differ considerably in their several natures, both with reference to the amount of force they will bear or resist uninjured, and the description of force or mode of applying it, to which they offer the greatest resistance.

Such forces are termed, according to the mode in which they are applied—tension, compression, flexure, and torsion.

A series of practical rules have been deduced from often repeated experiments, which serve as guides for readily calculating the dimensions of any piece of mechanism, with reference to the description and degree of force to which it will be subjected.

RESISTANCE TO COMPRESSION OR CRUSHING FORCE.

Compression is a force which strives to crush, or render more dense, the fibres or molecules of any substance which is submitted to its action.

In general, with oak or cast-iron, flexure begins to take place in a piece submitted to a crushing force, as soon as the length or height reaches ten times the least dimension of the transverse section. Up to this point, the resistance to compression is pretty regular.

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* A line is said to be normal to a curve, when it is perpendicular to a tangent to the curve passing through its point of intersection with the curve.

Rule—To find the greatest compressing weight to which various substances may be submitted with safety:—*Multiply the transverse sectional area of the piece in square inches, by the number of pounds one square inch will sustain when the length is less than twelve times their least dimension, and this co-efficient for the various substances can be found on referring to the annexed and other tables in any Engineers' Pocket-Book and similar works.*

TABLE *showing the weight in pounds per square inch of solids—such as columns and other supports—will sustain, according to their height, without beiny crushed.*

WOODS AND METALS.

Description of Material.	Proportion of length to least dimension.				
	Up to 12.	Above 12.	Above 24.	Above 48.	Above 60.
	lb.	lb.	lb.	lb.	lb.
Sound oak,......	426·750	855·625	213·375	71·125	85 562
Inferior oak,....	270·275	119·490	71·125	"	"
Pitch pine,......	533·437	440·975	266·007	106 687	"
Common pine,..	137·982	116·645	69·702	"	"
Wrought-iron,..	14225·000	11877·875	7112·500	2375·575	1194·900
Cast-iron,........	28450·000	23755·750	14225·000	4741·666	2375·575
Rolled copper,..	11707·175	"	"	"	"

First Example.—What weight can be put with safety upon a pillar constructed of ordinary bricks, the pillar being of a rectangular section, of 50 inches by 60, and the height being below 12 times the length of this cross section?

We have $50 \times 60 = 3000$ square inches of transverse sectional area. A pillar of bricks so formed will safely support 57 lbs. per square inch of sectional area; therefore,

$$3000 \times 57 = 171{,}000 \text{ lbs.}$$

Second Example.—What must be the transverse sectional area of a square post of sound oak, 19 feet 8 inches in height, and which will safely bear a load of 60,000 lbs.?

If we suppose the length to be not more than 12 times the least cross section, the number or co-efficient of compression, in pounds per square inch, is 426·75.

Then, $$\frac{60{,}000}{426{\cdot}75} = 140 \text{ square inches;}$$

and

$$\sqrt{140} = 11{\cdot}8 \text{ inches, the length of the supposed side.}$$

Comparing this 11·8 inches with the given height, we find that

$$\frac{19 \text{ ft. } 8 \text{ in.}}{11{\cdot}8 \text{ in.}} = \frac{236}{11{\cdot}8} = 20.$$

This shows—and we have constructed the example with this view—that in this instance the proportionate length has not been correctly estimated; and therefore, instead of taking the number 426·75, we must take the next number in the second column for a trial division. The calculation will, consequently, have to be rectified thus—

$$\frac{60{,}000}{355{\cdot}625} = 168{\cdot}7 \text{ square inches,}$$

and $\sqrt{168{\cdot}7} = 13$ inches nearly, the proper dimension for the cross section of the post.

In shops and warehouses, builders employ solid cast-iron columns, instead of brick pillars, so as to take up less space. These columns are generally calculated to support loads of above 33,000 lbs. each. They are usually about 3 inches in diameter, and 12 feet high. In which case, supposing a cubic foot of cast-iron weighs 452 lbs., they will weigh (3 inches being equal to ·25 foot)—

$$\cdot 785 \times \cdot 25^2 \times 12 \times 452 = 266 \text{ lbs.}$$

If, instead of these columns being massive or solid, we employ, in place of two of them, a hollow one, to support the proportionate load of 66,000 lbs., and being 6 inches in diameter, this increase in the diameter makes the ratio of the length to it 24, instead of 48; and the co-efficient to be taken will consequently be 14,225, instead of 4,742.

Now, $66{,}000 \div 14{,}225 = 4{\cdot}64$ square inches would be the cross section of a solid pillar, equivalent to that of which the thickness is sought. Since, however, the diameter of the latter is 6 inches, its section of solidity would be—

$$\cdot 785 \times 6^2 = 28{\cdot}26 \text{ square inches.}$$

Then, deducting from this area 4·64 square inches, as above determined, we have $28{\cdot}26 - 4{\cdot}64 = 23{\cdot}62$ for the cross sectional area of the central hollow. From this we deduce the internal diameter, thus—

$$\sqrt{\frac{23{\cdot}62}{\cdot 785}} = 5{\cdot}485 \text{ inches.}$$

And, finally, the thickness of the column will be—

$$\frac{6 - 5{\cdot}485}{2} = \cdot 2575 \text{ inches.}$$

The weight of such a column, if 12 feet in height, will be—

$$\frac{4{\cdot}64}{144} \times 12 \times 452 = 175{\cdot}27 \text{ lbs.}$$

This result shows very markedly how great an economy results from the employment of hollow in place of solid cast-iron columns. The thickness, determined as above, of ·2575 inch is theoretically sufficient, but in practice we seldom find such castings under half an inch thick.

In the above examples the mouldings usually added to the columns are not taken into the account. With these, the weight will be a tenth or so more, according to the description of moulding.

TENSIONAL RESISTANCE.

364. A tensile force is one which acts on a body in the direction of its length, tending to increase the length, and when carried to a sufficient extent to cause rupture.

As with reference to compression, many experiments have been made to determine the sectional area to be given to bodies of various materials submitted to a tensile strain, so that they may safely resist a given force.

First Example.—Required the sectional area for four square tension-rods of wrought-iron, to connect the top and bottom of a hydraulic press, in which the force which tends to separate these two ends, and consequently to rupture the rods, is equal to 500,000 lbs.

Each rod must be capable of resisting

$$\frac{500{,}000}{4} = 125{,}000 \text{ lbs.}$$

The best wrought-iron may be safely subjected to a strain of

14,225 lbs. per square inch of cross section. We have, consequently,

$$\frac{125{,}000}{14{,}225} = 8{\cdot}79 \text{ square inches,}$$

for the area of the cross section; and

$$\sqrt{8{\cdot}79} = 2{\cdot}964, \text{ or nearly three inches,}$$

for a side of the square rod.

If the rod were round, we should have—

$$D = \sqrt{\frac{8{\cdot}79}{{\cdot}785}} = 3{\cdot}345 \text{ inches, for the diameter.}$$

In the same manner, the diameter proper for steam-engine piston-rods may be calculated, when the pressure on the piston is known.

Second Example.—Required the amount of tensile or tractive force which can safely be resisted by a carriage draw-shaft, made of ash, and having a cross-sectional area of 15·5 square inches.

Ash will sustain a tensile strain of 1707 lbs. per square inch of section; therefore,

$$1707 \text{ lbs.} \times 15{\cdot}5 = 26{,}460 \text{ lbs.}$$

RESISTANCE TO FLEXURE.

365. The resistance of a piece of any material to flexure, is the effort which it opposes to all strain acting upon it in a direction perpendicular to its length, as in the case of levers, beams, or shafts.

Bodies may be submitted to the strain of flexure in several ways. Thus, the piece may be firmly fixed in a wall by one end, whilst the straining weight or force is applied at the other; or it may be securely fixed at both ends, and the weight applied in the centre; or it may be supported at the centre, and have the weight applied at both extremities.

We shall first consider the case of a piece fixed by one end, and subjected to a strain at the other.

Let W be the weight in pounds, placed at a distance, L, in inches from the wall, in which the piece under experiment is fixed; C, a co-efficient varying with the material; a, the horizontal dimension in inches of cross section; b, the vertical dimension, similarly expressed;—then the greatest weight that the piece will bear, without undergoing alteration, will be determinable by the following formula, the piece being of rectangular section, and fixed at one end, and weighted at the other.

$$W = \frac{C \times a\,b^2}{6\,L}.$$

Now, C = 8,535, for wrought-iron;
10,668, for cast-iron;
854, for oak and deal.

Substituting these values of C, in the preceding formula, we shall have, for pieces of rectangular section, according to the material—

For Wrought-Iron.

$$W = \frac{8535 \times a\,b^2}{6\,L}, \text{ or more simply, } = \frac{1422{\cdot}5 \times a\,b^2}{L}.$$

For Cast-Iron.

$$W = \frac{10{,}668 \times a\,b^2}{6\,L}, \text{ or more simply, } = \frac{1778 \times a\,b^2}{L}.$$

For Wood.

$$W = \frac{854 \times a\,b^2}{6\,L}, \text{ or more simply, } = \frac{142 \times a\,b^2}{L}.$$

These formulæ lead us to the following rule for pieces of rectangular section:—***Multiply the horizontal dimension in inches of cross section, by the square of the vertical dimension in inches, and by a coefficient depending on the material: then divide the product by the length in inches, and the quotient will be the weight in pounds, which the piece will sustain without alteration.***

This rule is derived from the fact, that the transverse resistance of pieces submitted to a deflective strain is inversely as their length, directly as their width, and as the square or their vertical thickness.

According to this, pieces fixed at one end, and intended to bear a strain at the other, should be placed on edge; in other words, the greatest cross section should be parallel to the direction of the strain.

First Example.—What weight can be suspended, without causing deflection, to the free end of a wrought-iron bar, fixed horizontally into a wall at one end, and projecting 5 feet (= 60 inches) from it; the bar being of a rectangular cross section, having its horizontal dimension, $a = 1{\cdot}2$ inches, and its vertical dimension, $b = 1{\cdot}6$ inches?

We have

$$W = \frac{1422{\cdot}5 \times 1{\cdot}2 \times 1{\cdot}6^2}{60} = 72{\cdot}8 \text{ lbs.}$$

This result is obtained on the supposition that the bar is placed on edge; but what would be the weight other things being equal, supposing the bar to be placed on its side—that is, when 1·6 inches is its horizontal dimension, a, and 1·2 its vertical dimension, b?

We have, in this case,

$$W = \frac{1422{\cdot}5 \times 1{\cdot}6 \times 1{\cdot}2^2}{60} = 54{\cdot}6 \text{ lbs.}$$

This inferior result shows the advantage of placing the bar on edge.

When the piece under experiment is of square instead of oblong section, a necessarily $= b$, and $a\,b^2$ becomes b^3, and this is consequently to be substituted in the formula for the former.

If, however, the piece is cylindrical, the formula will be—D representing the diameter,

$$\text{For wrought-iron, } W = \frac{854 \times D^3}{L}.$$

$$\text{For cast-iron, } W = \frac{1066 \times D^3}{L}.$$

$$\text{For Wood, } W = \frac{85 \times D^3}{L}.$$

In each of the cases just referred to, the transverse sectional dimensions of pieces fixed at one end, and submitted to a strain at the other, are determined by the following formulæ:—

The rule for the determination of the transverse section, whether *rectangular*, *square*, or *circular*, of a bar or beam fixed by one end and loaded at the other is thus stated:—*Multiply the weight in pounds by its distance in inches from the support; divide the product by a coefficient varying with the material and form of section; and extract the cube root, which will give in inches the vertical dimension, the side of the square, or the diameter of the circle, according as the bar or beam is rectangular, square, or circular in cross section.*

First application: What should be the transverse section of a rectangular wrought-iron bar, intended to carry at its free end, and at a distance of 5 feet from its support, a weight of 72·8 lbs., the bar being supposed to be placed on edge?

We have here,

$$a\,b^2 = \frac{72{\cdot}8 \text{ lbs.} \times 60 \text{ in.}}{1{,}422{\cdot}5} = 3{\cdot}071;$$

then, if a be taken $= 1{\cdot}2$ inches,

$$b = \sqrt{\frac{3{\cdot}071}{1{\cdot}2}} = 1{\cdot}6 \text{ inches, the vertical dimension.}$$

Second application: What should the side of the cross section measure, of a *square* bar, under similar circumstances otherwise?

$$b^3 = \frac{72{\cdot}8 \times 60}{1422{\cdot}5} = 3{\cdot}071, \text{ and}$$

$$b = \sqrt[3]{3{\cdot}071} = 1{\cdot}454 \text{ in., the thickness of the bar.}$$

366. OBSERVATIONS.—When the bar or beam, under experiment possesses in itself any weight capable of influencing its resistance; or, besides the weight suspended or acting at one end, has a weight equally distributed throughout its length; the transverse-sectional dimensions are, in the first place, determined without taking the additional weight into consideration. This is, then, calculated approximately, and the half of it added to the suspended load, a fresh calculation being made with this sum as a basis.

A bar, or beam, fixed by one end, and loaded at the other, has always a tendency to break off at the shoulder, or point of junction, with its support, because it is on that point that the weight or strain acts with the greatest leverage. When, therefore, the transverse section of the piece has been determined, in accordance with the formulæ given above, which are calculated for the dimensions of the piece at the shoulder; the section may be beneficially diminished towards the free extremity, thereby economising the material, and lessening its own overhanging weight. The curve proper to give the outline is the parabola, as described in reference to Plates 5 and 12. It may also be obtained in the following manner, for the particular case under consideration:—Calculate the transverse section for different lesser lengths of the piece, the other data remaining as before, and the required curve will be one which passes through the outline of each section, when they are placed at distances from the load equal to the lengths for which they are calculated. This curve is also given to bars, beams, or shafts fixed at both ends and loaded in their middle, or sustaining a uniform weight throughout their length. The cast-iron shaft represented in Plate 12 may be taken as an example of this. Steam-engine beams and side levers are also formed with feathers of this shape, as it gives them a uniform resistance throughout, so that they are not liable to break or give way in any one point rather than another.

A bar, or beam, supported in the centre, and loaded at either end, will support double the weight capable of being carried by one of similar dimensions, supported at one, and loaded at the other end; it is, indeed, evident that each weight will only act with half the leverage, being only half the distance from the point of support.

Similarly, a bar, or beam, freely supported at both extremities, and loaded in the centre, will support a weight double that sustained by a piece of the same dimensions fixed at one, and loaded at the other end. Therefore, in calculating the proportions for these two last-mentioned cases, it is necessary simply to double the coefficient, C, given for the first case.

A bar, or beam, firmly and solidly fixed by both ends, will support a load four times as great as one of the same dimensions fixed at one end, and loaded at the other extremity. It will, conseqently, be necessary to quadruple the above coefficient in this case.

For calculating the diameters of the spindles, or journals, of cast-iron shafts for hydraulic motors, which are intended to sustain great weights, the following particular formula may be employed:—

$$D = \sqrt[3]{W} \times {\cdot}1938,$$

where D expresses the diameter in inches, and W the weight to be sustained in pounds.

According to this formula, the diameter of the cast-iron spindle, or journal, is found by extracting the cube root of the weight, or strain, in pounds, and multiplying it by the constant, ·1938, the product being the diameter in inches.

The diameter for wrought-iron spindles, or journals, may be derived from that for cast-iron, by multiplying the latter by ·863; or, directly, by employing the multiplier, ·1673, in the above formula.

Example.—Of what diameter must the spindle of a water-wheel shaft be, the total strain being equivalent to 70,000 lbs.? Here,

$$D = \sqrt[3]{70{,}000} \times {\cdot}1938 = 7{\cdot}987, \text{ or 8 inches nearly.}$$

A wrought-iron spindle of $(7{\cdot}987 \times {\cdot}863 =)$ 6·9 inches, will answer the same purpose.

RESISTANCE TO TORSION.

367. When two forces act in opposite directions, and tangentially to any solid, tending to turn its opposite ends in different directions or to twist it, it is said to be subjected to *torsion*, and offers more or less resistance to this action according to its form and composition. Taking, for example, the main shaft of a steam-engine, at one end of which the power acts through a crank, set at right angles to it, and at the other the load, by means of wheel gear—the *resistance* which this load presents, on the one hand, and on the other, the *power* applied to the crank, represent two forces which tend to twist the shaft, subjecting it to the action of torsion.

In constructing formulæ for the determination of the diameters of shafts, regard must always be had to the class to which they belong, and also to the description of work they have to perform.

As the journals are the parts of a shaft on which the greatest strain is concentrated, it is obviously to the determination of their dimensions that our investigation should be directed. The practical formula, for ascertaining the diameter proper for the journal of a cast-iron first-mover shaft, is—

$$d = \sqrt[3]{\frac{\text{H P}}{\text{R}} \times 419}.$$

Here, d = diameter of journal in inches.

H P = the horse power transmitted by the shaft.

R = the number of revolutions of the shaft per minute.

This formula is expressed in the following rule.

368. To determine the diameter at the journals of a cast-iron first-mover shaft:—*Divide the horse power of the engine by the number of revolutions of the shaft per minute, multiply the quotient by the constant* 419, *and extract the cubic root, which will be the diameter required in inches.*

For the journals of cast-iron shafts which are second movers, the formula is—

$$d = \sqrt[3]{\frac{\text{H P}}{\text{R}} \times 206};$$

and for third movers—

$$d = \sqrt[3]{\frac{\text{H P}}{\text{R}} \times 106}.$$

These are, in fact, similar to the formula given for first movers, with the exception, that for these the constant multiplier is 419, whilst for the latter it is 206 and 106, respectively.

369. For the journals of wrought-iron shafts the same formulæ are employed, the multipliers only being changed; these are 249 for first movers, 134 for second movers, and 67·6 for third movers.

If, with a view of suppressing the radical sign in the above formulæ, we raise both sides of the equation to their third or cubic power, and further express the multiplier by m, we have

$$d^3 = \frac{\text{H P}}{\text{R}} \times m;$$

from which formula it will be seen, that the cube of the diameter of the journal is proportional to the force transmitted. Similarly, the resistance of a journal is proportional to the cube of its diameter. In other words, one journal, of which the diameter is double that of another, is capable of sustaining a strain eight times greater, since the cube of 2 is 8.

370. To determine the side of a square shaft to resist torsion with a given flexure.

Rule.—Multiply the power in lbs. by the length of the shaft in feet, and by the leverage in feet; divide this product by 92·5 times the degrees of the angle of flexure, and the fourth root of the quotient will be the side of the shaft in inches.

Example.—Find the side of a malleable-iron square shaft 30 feet long, transmitting a power of 4000 lbs. at the circumference of a wheel 2 feet radius, the flexure or twist of the shaft, when the power is applied, not to be more than 1°.

$$\frac{4000 \times 30 \times 2}{92{\cdot}5 \times 1^\circ} = 2600,$$

the fourth root of which is

$$\sqrt[4]{2600} = 7{\cdot}1 = \text{inches in side of shaft.}$$

To determine the diameter of a solid cylinder to resist torsion with a given flexure.

Rule.—Multiply the power in lbs. by the length of the shaft in feet, and by the leverage in feet; divide this product by 55 times the number of degrees in the angle of flexure, the fourth root of the quotient will be the diameter in inches.

Example.—What is the diameter of a shaft 30 feet long, transmitting a power of 4000 lbs. at the circumference of a wheel 4 feet diameter, the twist or flexure of the shaft, when the power is applied, not to be more than 1°.

$$\frac{4000 \times 30 \times 2}{55 \times 1^\circ} = 4364$$

and

$$\sqrt{4364} = 8{\cdot}13 \text{ inches diameter;}$$

if the flexure could be allowed 2°, the diameter would be

$$\sqrt[4]{\frac{4000 \times 30 \times 2}{55 \times 20}} = 7 \text{ inches.}$$

The number of degrees of torsion is fixed at what may be considered best for the length of the shaft and action of the machine; and malleable-iron resists torsion in the proportion of 14 to 9 of cast-iron; cast-iron being the strongest.

The length of the journals and their bearings should always be greater than their diameter. For large sizes, it should be 1·2d to 1·4d, and for smaller sizes, 1·5d to 2d. Thus, the length of a wrought-iron journal, 1·5 inches in diameter, should be from (1·5 × 1·5 =) 2·25 to (1·5 × 2 =) 3 inches.

When shafts have to resist both a torsional and a lateral or transverse strain, the diameter of their journals should be determined with reference to that strain which is the greatest, or which of itself would require the greatest dimensions.

When shafts are not of any great length—3 to 6 feet, for example—their diameter need not be above a tenth greater than that of their journals. Solid cast-iron shafts of above six feet in length should have a diameter one-fifth, or even one-fourth greater than that of their journals.

CHAPTER XII.

THE STUDY OF MACHINERY AND SKETCHING.

VARIOUS APPLICATIONS AND COMBINATIONS.

371. Up to the present we have occupied the student with the study of "industrial drawing," more particularly as regards the geometrical delineation of the principal elements of machinery and architecture, and of what may be termed its artistic delineation or execution, in accordance with the laws of optics, and the conventional usages of the light, colour, and shade given to such mechanical drawings. This study being of the greatest importance, it was necessary to dwell particularly upon it, since it is the basis of all designing in these arts, whether with a view to actual construction or delineation, and comprehending not only the mere outline of objects, but also the proportions between the various parts, as dependent upon the functions which is required of each.

On these elements we have thought it right to dwell at length, since a thorough acquaintance with them is essential to the draughtsman's progress to the higher department of his art, namely, the delineation of complete machines. At this point, however, we have now arrived; and if the student has made himself master of the preceding pages, he will not meet with much difficulty in attempting to delineate an entire piece of machinery, since he will only have to combine the rules and instructions already laid before him and exemplified in detail. The most complicated machine constructed was made up of parts, and was, indeed, but well calculated and thoughtfully arranged combinations of these elements, and afford innumerable applications of the rules and instructions laid down in reference to them.

The objects and examples taken to illustrate these first principles were necessarily of the most simple form, and confined to detached parts or details of mechanical and architectural designs, but if he is capable of drawing these several parts, it will require little more than patience and attention to draw them in combination.

The study of complete structures, in these branches of art, is the next step in our labours; so that with this view we now proceed to the consideration of complete machines.

372. Machines may, in general, be classified under three heads, viz.,—machine tools; productive or manufacturing machinery; and prime movers.

By machine tools are meant those by the instrumentality of which we work upon raw materials, as wood, metal, stone; lathes, wheel-cutting machines; drilling, boring, and shaping machines; mortising, slotting, planing, and grooving machines; riveting ma-

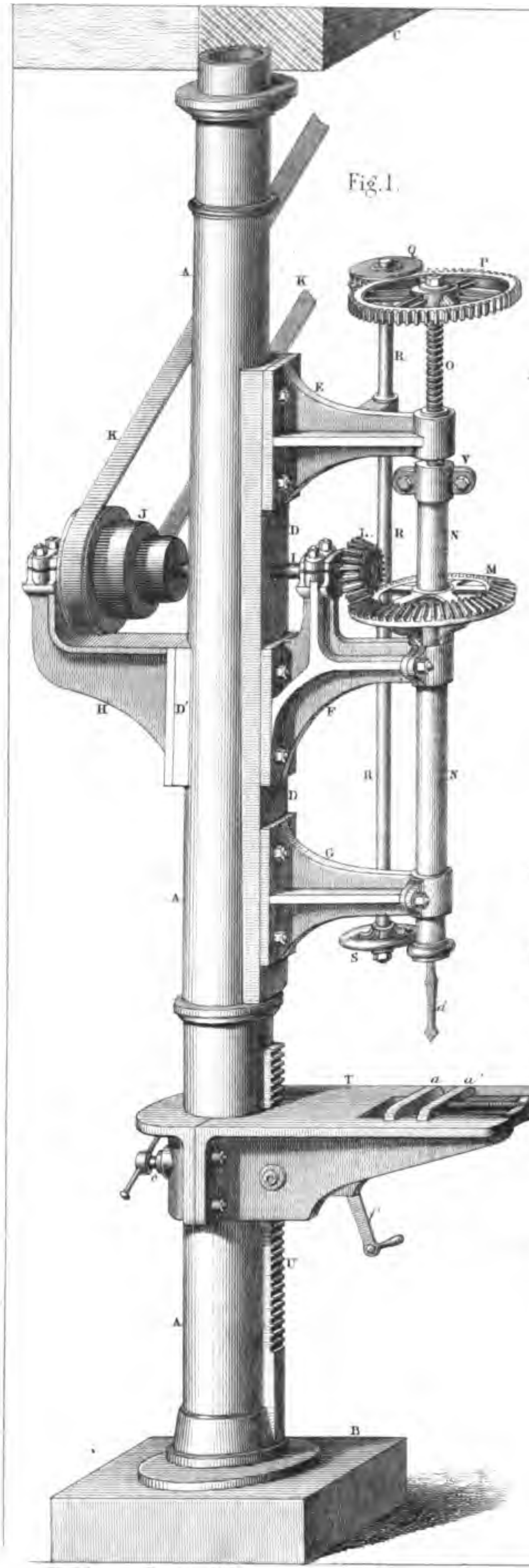


Fig. 1.

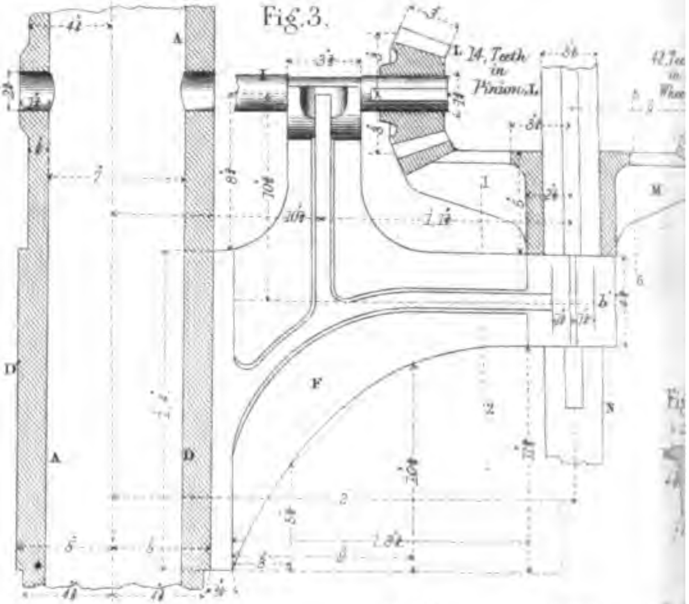


Fig. 3.

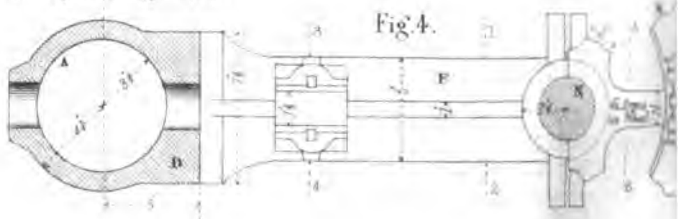


Fig. 4.

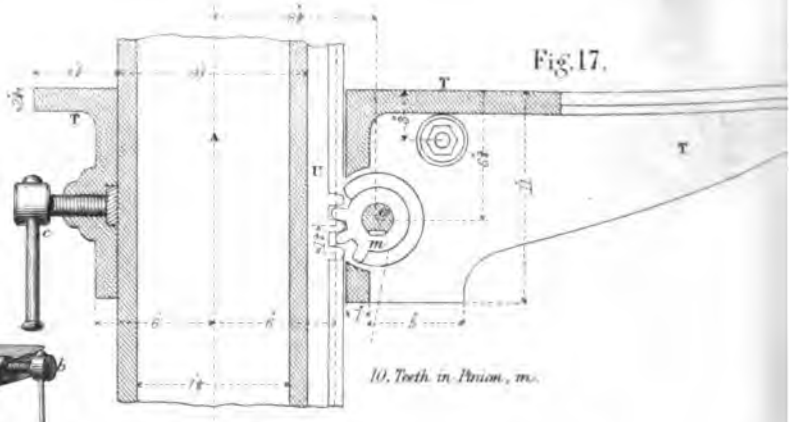


Fig. 17.

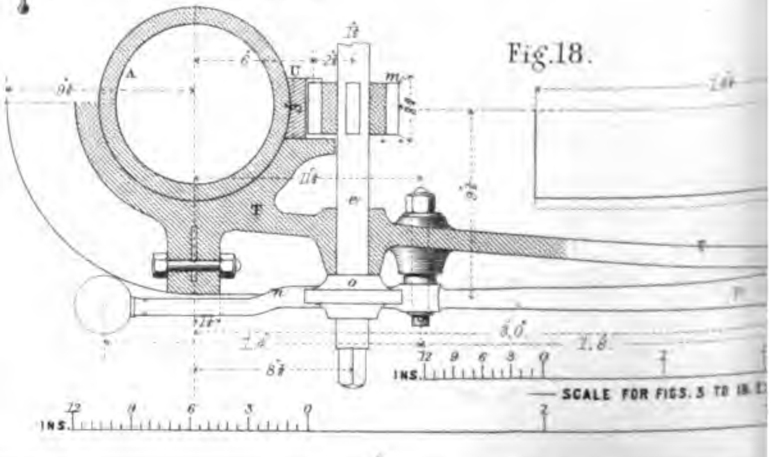


Fig. 18.

W. Johnson, Patent Offices
Lincoln's Inn Fields & Glasgow.

VERTICAL DRILLING MACHINE,

AND

EXAMPLES OF SKETCHING DETAILS.

Fig.5.

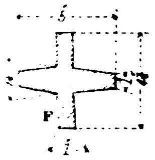


Fig.6.

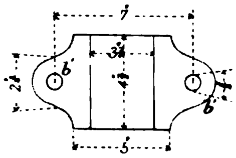


Fig.7.

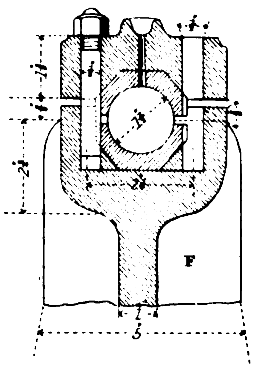


Fig.9.

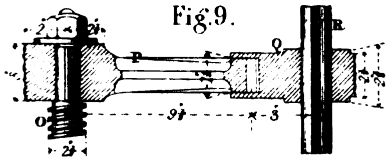


Fig.10.

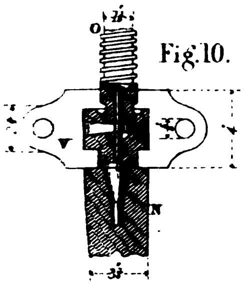


Fig.11.

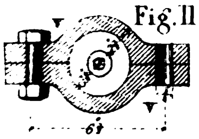


Fig.12.

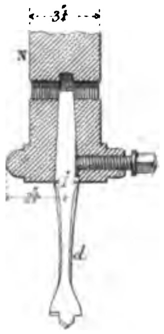


Fig.13.

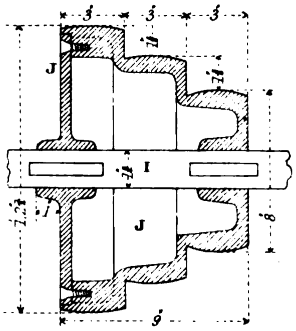


Fig.14

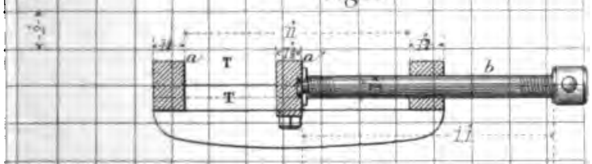


Fig.16.

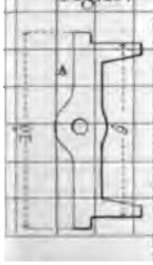


Fig.15.

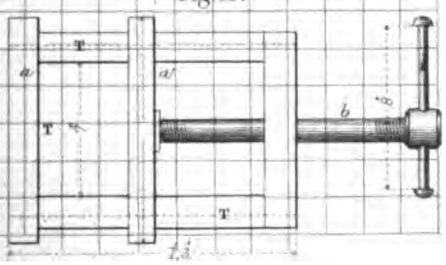
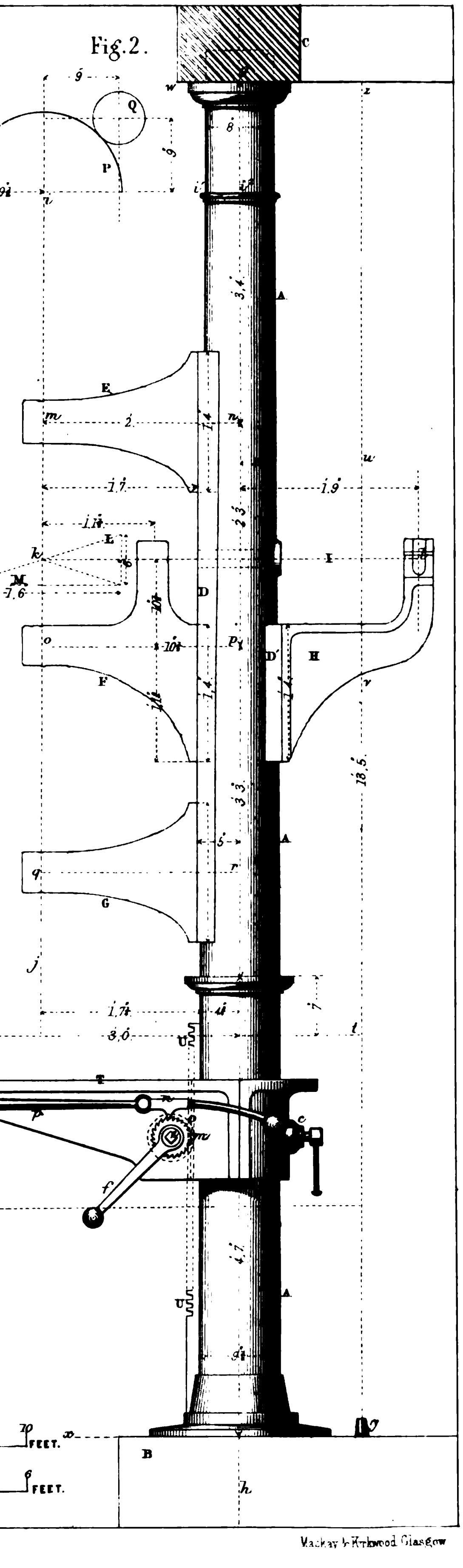


Fig.2.



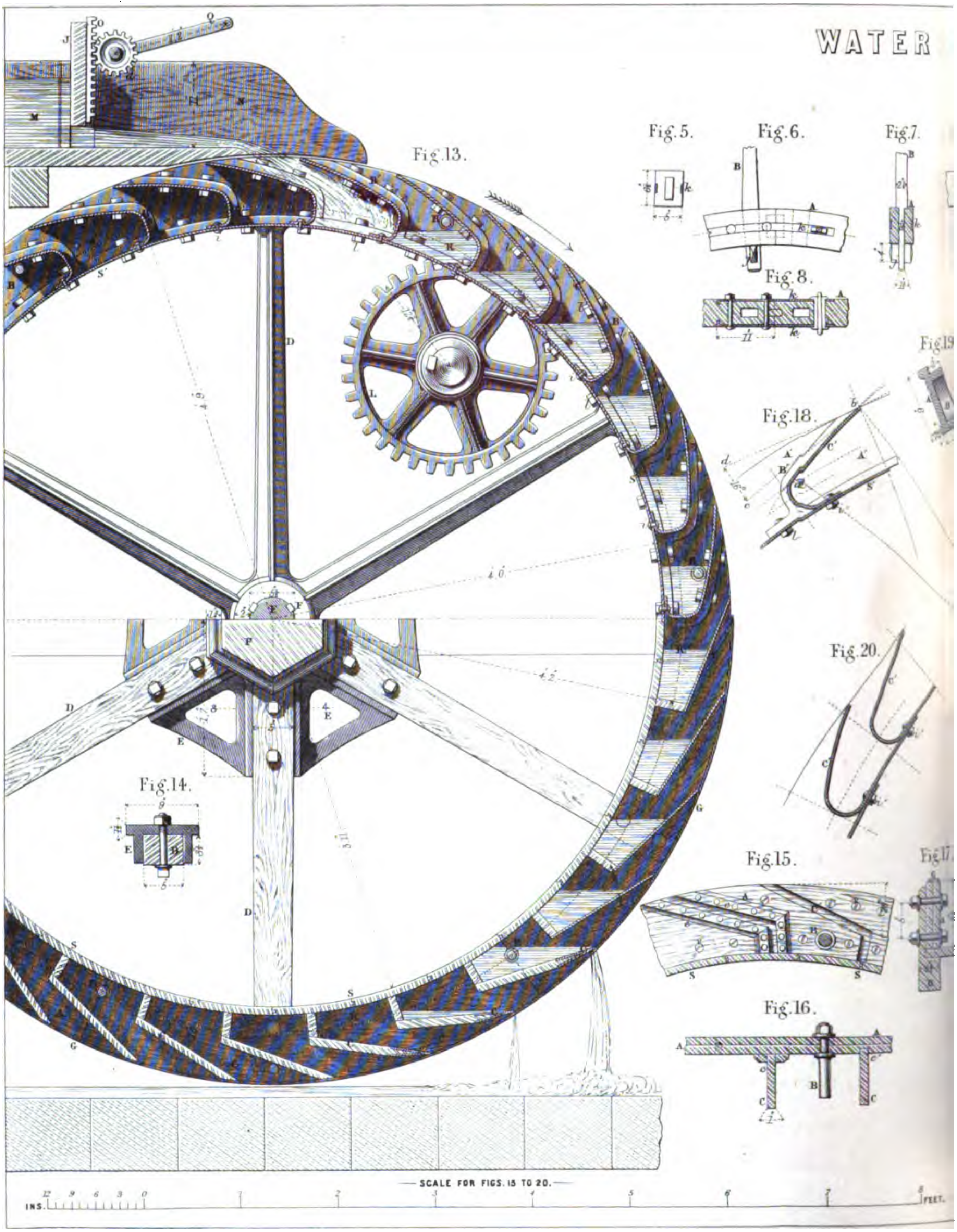
—SCALE FOR FIG.2.—

FEET.

FIG.7. WHICH IS DOUBLE THAT OF THE OTHERS.—

FEET.

Mackay & Kirkwood Glasgow


WATER
Fig. 13.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 18.
Fig. 19
Fig. 20.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17
SCALE FOR FIGS. 15 TO 20.
INS.
12 9 6 3 0
1 2 3 4 5 6 7 8
FEET.
A
B
C
D
E
F
G
J
K
L
M
N
O
Q
S

[illegible]
[illegible]

'HEELS.

Fig. 2.

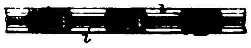


Fig. 3.

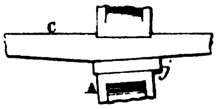
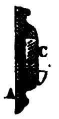


Fig. 4.



;.9.

Fig. 10.

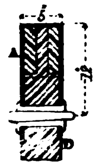


Fig. 11.

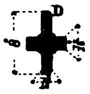
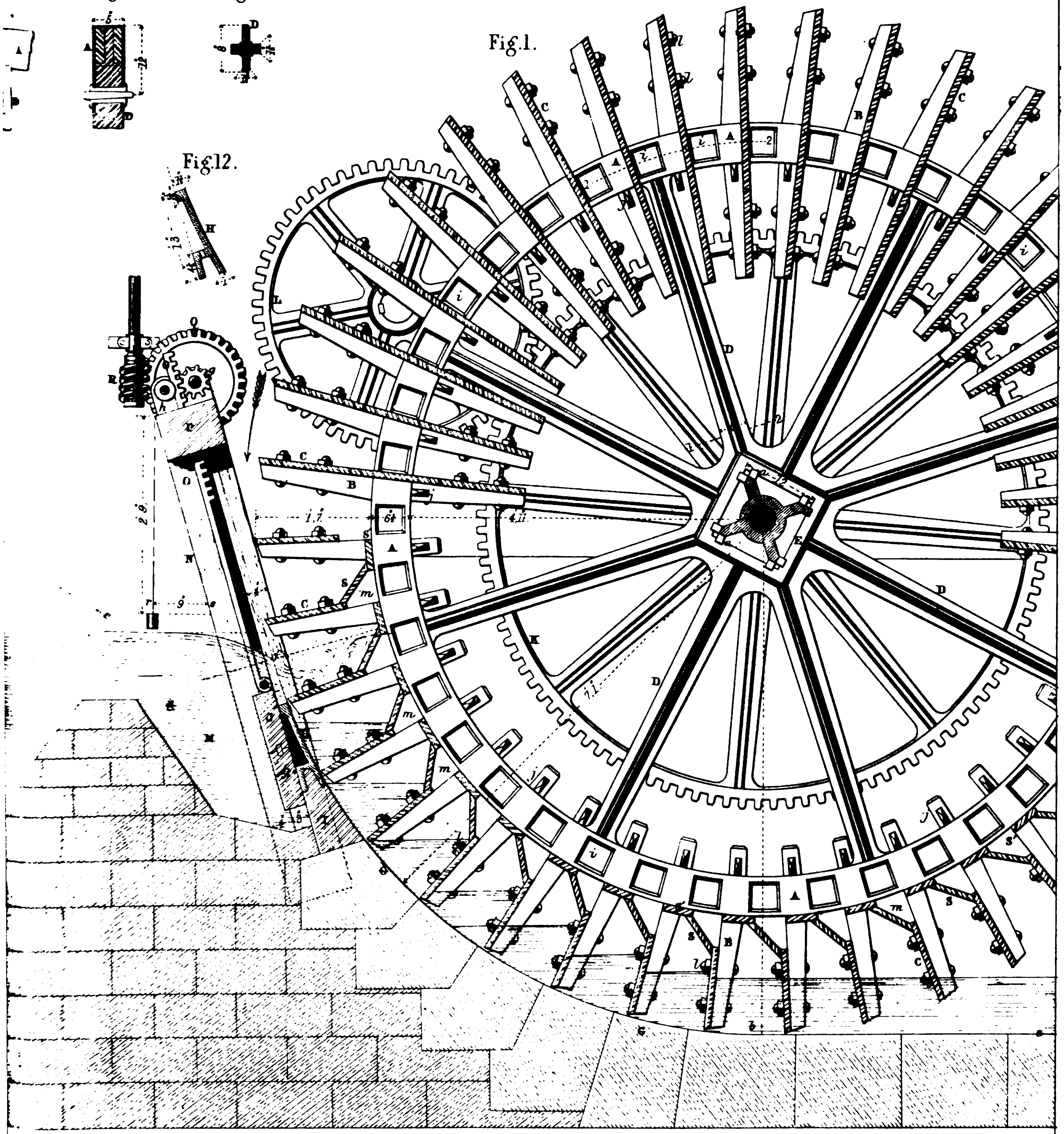


Fig. 1.

Fig. 12.



SCALE FOR FIGS. 1 TO 12.

12 9 6 3 0 1 2 3 4 5 6 7 8 9 10 11 12 FEET.

Maclay & K[illegible] Glasgow

chines; shears, saws, hammers—are of this class. The form, strength, and movements of the several parts, as well as the form of the tool or cutting instrument of these machines, should be so combined, and properly constructed, with a velocity proportioned to the nature of the work and the material acted on.

Amongst productive or manufacturing machinery, are comprised spinning, weaving, and printing machines; pumps, presses, corn and oil mills; and, finally, prime movers, or motors; ore machines, such as those worked by animals; windmills; water-wheels; turbines; steam-engines, and air-engines, &c.

For the study of complete machines, we have selected from each of these classes those possessing most interest and in general use. A drilling machine, as being a very useful tool, and much employed in machine-shops and railway works; a good useful duplex lifting and forcing pump for raising water and other liquids, serving for domestic purposes as well as for large manufacturing establishments; two examples of water-wheels, showing various arrangements and forms of floats or buckets; a high pressure expansive steam engine, with geometrical diagrams, determining the relative positions of the principal pieces in various circumstances; and, finally, other example plates of recent and approved mechanical structures.

To rough draught or sketch from the actual machine, with a view to making a perfect finished drawing of such machine, is the first step to the thorough delineation and understanding of complete machines, as a whole and in all its parts. We therefore now proceed to instruct the student in this kind of sketching, and at the same time to explain the particular machine under consideration, or taken for illustration and practice. The operation in question consists in drawing with the hand, the elevation, plan, sections, and details of a machine, preserving, as much as possible, the forms and proportions of each part; and then taking the actual measurement of each part, and marking it down in figures in its particular position upon the drawing. In this duplex operation of sketching and measuring consists the study and practice of the rough draughting of machinery.

THE SKETCHING OF MACHINERY.

PLATES 35 AND 36.

373. Before commencing the sketch or rough draught of a machine, it is absolutely necessary to look carefully into its construction so as to ascertain the action of the various working parts, the motion of the intermediate mechanical connections, and finally, their object and results, from the smallest part to the whole. This will require on the part of the delineator some general acquaintance at least with mechanics and machinery, but this acquaintance he must obtain from books specially devoted to the exposition of these subjects (such as *Applied Mechanics*, formerly referred to), or in the workshop itself. The object of this preliminary examination is to give the draughtsman a good general idea of the more important parts—those which he will have to render most prominent and detailed when he comes to make a complete drawing of the whole; these rough drawings or sketches will comprise a series of combined views of the machine in elevation, plan, or otherwise, as may be required, together with separate diagrams of such details as may not be apparent in the former, or require to be drawn to a different scale to render them more intelligible. These drawings or sketches should be executed in such a manner, that a perfect representation or drawing of the machine may be made, which, if necessary, may serve in the construction of other similar machines.

DRILLING MACHINE.

PLATE 35.

374. In order to give an idea of the manner of sketching machinery, we take a simple machine as an example; this we have represented in perspective* in Fig. 1, which view is supposed to stand instead of the actual machine itself.

This machine is for drilling metals; it consists of a vertical cast-iron column, A, which forms part of the building or workshop. This column is hollow, and rests by an enlarged base upon a stone plinth, B, imbedded in the ground, and at its upper end it supports the beam, C.

Upon one side of this column is cast the vertical face, D, which is planed to receive three cast-iron brackets, E, F, G, attached to it by bolts. To the opposite side, D^1, of the same column, is in like manner attached the bracket, H, which, with the middle one, F, on the other side, serves to carry the horizontal spindle, I. This spindle carries on one side the cone-pulley, J, over which passes the driving-belt, K, and on the other extremity it has keyed upon it the bevel-pinion, L, which gears with a larger bevel-wheel, M. This last is attached to the vertical shaft, N, which is, in fact, the drill holder, and is moveable in the bracket-bearings, F and G. This shaft receives a duplex movement, that of continuous rotation, which is more or less rapid, according as the belt, K, is on the less or greater diameter of the cone-pulley; and the other vertical and rectilinear, due to the action of the screw, O, which works in an internal screw in the end of the bracket, E. This screw carries at its upper end a spur-wheel, P, gearing with a small pinion, Q, the shaft, R, of which is prolonged downwards, and terminates in a small hand-wheel, S.

The object to be drilled is held between a pair of jaws, *a*, a^1, set in grooves upon the table, T, and capable of adjustment back and forward by means of the screw, *b*, the head of which has a sliding handle. The table, T, is made in two pieces, so as to form a collar about the column, A, and it is fixed at any convenient height upon this column by means of the pressure screw, *c;* the exact distance of the table from the drill, *d*, according to the thickness of the piece of metal to be drilled, is settled by means of the vertical rack, U, which is fitted to the front of the column, and into which gears a pinion on the shaft, *e*, carrying the handle, *f*, at its extremity. The rotation of this handle and the pinion necessarily causes the ascent or descent of the table, T.

The drilling machine, then, fulfils the following conditions: on the one hand, the drill, *d*, is worked at a greater or less speed of rotation, whilst it descends vertically with a very slow motion, which latter varies, of course, with the nature of the material acted upon; and, on the other hand, the table which carries the object to be drilled is capable of being set at the most convenient height, according to the forms and dimensions of the objects, whilst it may also be set eccentrically, when necessary, by turning it to the required extent round the column.

375. After having thus taken note of the construction and action of each individual piece and element of the machine, the draughtsman may proceed to make his sketch. He should commence by

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* In a subsequent chapter, we shall explain the general principles of parallel and exact perspective.

drawing a rough general view, indicating, in mere outline, the relative positions of the various pieces.

For example, in Fig. 2, will be seen the geometrical side elevation of the column, A, with the positions of the brackets and the table, which are merely in outline. It is advisable, even in this rough draught, as well as in the finished drawing, with or without the assistance of a rule, to draw the centre lines for guides; thus, after drawing the first centre line, $g\,h$, of the column, A, draw upon each side of it the portions forming its contour; then draw parallel to it the centre line, $i\,j$, of the drill-stock, N; then the horizontal, $k\,l$, which represents the centre line of the bevel-wheel, L, and the driving-pulleys, J, and likewise the straight lines, $m\,n$, $o\,p$, $q\,r$, which are the centre lines of the brackets, E, F, and G; finally, draw the lines, $s\,t$ and $u\,v$, of the table, T, and of the bracket, H; and likewise the extreme lines, $x\,y$, $w\,z$, of the bottom and top of the column. At this stage it is necessary to lay down the measurements upon the sketch. The column being fitted with the principal parts of the drilling apparatus, so that no clear space can be found upon it for the height to be measured close to it, a plumb-line is suspended from the point, z, on the beam, C, which rests upon the column, and this line is measured either by a foot-rule, or by a measuring tape, and the measurement in feet, inches, and parts of inches, of the whole height, and also of the various parts between the principal cross shafts or brackets, may be written upon the centre line, $g\,h$, of the column as marked and shown in Fig. 2.

The draughtsman must next measure the diameters at the base and summit of the column, as well as those of the various mouldings. These diameters may be measured with callipers, which open to such an extent that they can be applied to the place in question, the amount of opening being then measured upon the rule, and written down upon the corresponding place in the sketch; or the diameters may be obtained by applying a cord, or a very flexible rule, to the circumference. This latter method is always employed for cylinders of large diameter, when it is not possible to obtain the measurement from either base. In this case, to obtain the actual diameter, it is necessary, as has been seen (75) to divide the circumference found, by 3·1416.

To obtain the distance of the line, $i\,j$, from $g\,h$, the centre line of the column, place the extremity of the rule at i^1, against the surface of the column, and let it lie across the centre, i, of the spur-wheel, P, or screw, O; the measurement read off the rule will be that of $i^1\,i$, to which must be added the radius, $i^1\,i^2$, of the column.

If the centre, i, were not approachable with the rule, we should have to take the internal distance between the surface of the column and that of the screw, and then add the respective radii of the screw and column. When these distances are greater than the length of the measuring-rule, a rod or tape must be employed. When, indeed, the draughtsman has attained a reasonable amount of skill, he may take the measurement, $i\,i^2$, directly, by applying the rule upon the surface of the column opposite to its centre; and also opposite to the axis of the screw, in such a manner that the rule shall be tangential to the column, when the space between the two points will be the measurement sought.

It is also necessary to note upon the sketch, the vertical distances between the different horizontal lines, $m\,n$, $o\,p$, $q\,r$, $s\,t$. The measurements indicated upon Fig. 2 will show how all these are obtained.

The preceding operations will allow of the finished drawing being commenced, by laying off the relative position of the main parts which go to compose the machine to be sketched. We have next to sketch and measure all the minor details of each separate piece of the machine. To this end, and to avoid confusion, it is necessary to treat each of these pieces as detached, and to draw different views of them, upon which the dimensions of every part may be properly indicated.

Figs. 3 and 4 represent, in elevation and plan, the detail of the principal bracket, F, and the section of the column, A, which supports one end of the shafts, I, and of the spindle, N, with the bevel-wheels, L and M. Even these views are not sufficient to represent thoroughly all the dimensions of this bracket; thus it is necessary to draw a section such as that made at the line, 1—2, and projected in Fig. 5, so as to show the exact form and strength of the feathers of the bracket; it is likewise necessary to make a front or face view of the cover of the bearing, b^1, Fig. 6, which holds up the spindle, N, to the bracket, and also a vertical section, Fig. 7, taken on the line, 3—4, in Fig. 4, to show the brasses which embrace the journal of that end of the shaft, I, carried by the bracket, F, and which also answers for the bushes and bearing of the other end of the same shaft carried by the part, l, of the bracket, H, Fig. 2, this bracket therefore being plain, and having its feathers and flanges all the same, as the other one, F, requires no detailed sketch, all the sizes necessary being there severally marked. These details should always, if possible, be drawn to a larger scale, so as to indicate the minutiæ and fitting of the parts to each other clearly, and also to give room for marking on the measurements; it may be observed, for a draughtsman who has not much practical knowledge of that, the details in machinery, it will be necessary to take down or separate various parts, such as the cap and upper brass, or the upper bush connection of the spindle, N, with the screw, O, so that he may understand and sketch them in such a manner that he will be able to reproduce them properly on the working or finished drawing. With regard to the wheel-work, it will be sufficient to give the section of the web and boss, as indicated in Fig. 3, and a section, as Fig. 8, of one of the arms taken on the line, 5, when the wheel has any, and then the numbers of teeth and arms must be counted and set down as shown in Fig. 4.

When all the parts of any detail are thus sketched out in elevation, plan, or section, the draughtsman must take the measurements of each, and set them down in their appropriate positions upon the sketches, as indicated in the several figures; being mindful to see that the principal measurements coincide with those laid off in the complete general view, in Fig. 2, already commenced. The measurements of the diameter of the pitch-circle, and of the width across the face of the teeth, along with the number of teeth or pitch, will be sufficient, in addition to what has already been directed to be done in reference to bevel wheel-work, the proper ratios being maintained between those in gear with each other. As many parts of machinery require to be in proportion to each other, a knowledge of such relations will enable the draughtsman to dispense with a great deal of tedious measuring and sketching, as in the case just alluded to, of wheels working together. Fig. 9 shows a sketch of one half of the spur-wheel and pinion, P, Q, at the top of the screw, O, and shaft, R, in vertical section, which would admit of putting on all the sizes necessary for its reproduction to a proper scale, as already explained in reference to the bevel wheels, L and M. Figs. 10 and 11 show, in vertical and horizontal section, the manner of jointing the screw, O, into the upper end of the spindle, N; and Fig. 12 is a vertical section of the lower extremity of the drill-stock, or spindle, N, with the drill, d, in elevation, while, Fig.

13 is a vertical section of the cone pulley, J. The remaining important parts of the machine are to be detailed in a similar manner to Figs. 3 and 4: thus, Figs. 14, 15, and 16, represent a vertical section, a plan, and side view of a portion of the table, T, with its holding-jaws, *a*, a^1, and hand screw, *b*, for working the same To save time, the thread of the screw is only indicated at each end. by a few lines at an angle, this is meant to show that the thread extends from end to end, and the direction of lines shows whether it is a right or left handed thread. Finally, Figs. 17 and 18 give a complete detail of the table, T, in vertical and horizontal section, showing also the rack and pinion, U and *m*, with its shaft, *e*, for elevating it by the crank handle, *f*, as well as the screw, *c*, and ratchet wheel, *o*, with its pawl lever, *n*, *p*, for fixing or adjusting it at any required height.

376. On all the preceding details, we have marked the measurements of the different parts corresponding exactly as they should to those of the actual machine. These measurements are expressed in feet and inches, and parts of inches, as in former examples. We have also slightly shaded various parts, as is generally done where the complication and variety of forms would otherwise lead to confusion and error. Besides, in this manner, a few touches of the pencil show at once whether this or that portion is round, or square, and in many instances, the labour of drawing additional views will thereby be dispensed with.

In order to facilitate the proceedings of beginners in sketching, we would recommend them to delineate the main centre lines with the aid of a rule, and the circles with compasses, though the dimensions of the latter need not be exact. This will give the sketch a much neater appearance, and render the various objects or details more regular. It is with this view that draughtsmen in sketching, frequently employ cross-ruled paper, with horizontal and vertical lines equally spaced. That portion of this plate, upon which are represented, Figs. 14, 15, and 16, is of this description.

It will be seen, that, if the lines ruled upon the paper are at equal distances apart, corresponding to one or more units (as inches) of the scale to which the sketches are being made, these may be drawn in correct proportions at once, in which case it would be unnecessary to write on many of the measurements.

Another good mode of procedure, is for the draughtsman to consider before commencing to sketch (say the general view of the machine, as that shown in Fig. 2) what size he would like the whole machine to occupy on his paper, and then by looking at the machine itself, he finds in a rough way, that it is three or four feet in width over all, by about twelve feet in height, this then enables him to fix on a scale, in his mind, to which he will sketch the machine as nearly as possible. No rule can well be fixed for these scales; since, if the machine is large, and it is desired to take in a whole view of it, the scale must be small in proportion to bring the sketch within a reasonable size. However, we think the scales on or to which Fig. 2 and the other details are executed, on this plate, are good scales for the young draughtsman, for whose benefit the scales are put at the bottom of the plate; although these figures can not be exactly to the scale, since they are only roughly outlined to show the way such sketches ought to be executed by the hand. Beginners have generally a good deal of copying, reducing or enlarging to scales, so that before they come to sketch from the machine they are quite at home in the use of the rule and scales, and can guess the size by looking at the machine, and put it down by hand with a pencil in a rough way, to the scale they have fixed on in the mind, (as if the scale had been laid down in squares on the paper, as before explained in reference to Figs. 14, 15, and 16,) and in this way, though they may not have had much experience in sketching, they will produce a very proportionable view of the machine, which can again be much improved when inking it in with the pen, which, with all the sizes, ought to be done before ever they are used or get rubbed. When the draughtsman gains greater experience in sketching, the operation becomes more mechanical, and he can sketch the machine proportionally by the eye, just as if his hand was guided by a pentagraph, or imaged before him by a camera obscura, requiring little reflection of this kind for the mere execution of the sketch, and thus enabling him to devote his attention entirely to the understanding of the machine as a whole, as well as the bearing of all the parts, one with another throughout, thereby saving much labour in the number of his sketches, and in the measurements required.

The draughtsman ought also to take down in writing notes of such things as cannot be well shown by the pencil sketch, such as the nature of the material of which the different parts are made, the weights and proportions of parts requiring study or calculation, or the relative speeds of the various shafts or moving parts of the machine, and many other things which can alone be taken in the form of a short specification or shown by colouring on the highly finished and detailed drawing.

The example which we have given as an introduction to the study of sketching machines, will have somewhat familiarized the student with his operations even now. The applications contained in the subsequent examples will suffice to complete this study, which is one of great importance to the draughtsman and constructive engineer.

MOTIVE MACHINES.

WATER-WHEELS.

PLATE 36.

377. The water-wheel, represented in Fig. 1, has plane floats, and works through a portion of its circumference, in a concentric circular channel. It receives the water from over a sluice-gate, a little below its centre, and is of the undershot description*.

The wheel is composed of several parallel shroudings, A, in which are fitted the radial wooden bearers, B, carrying the floats, C. When the shroudings are of cast-iron, as is supposed in the present example, they are cast in one piece with the arms, D, and central boss, E, and are firmly secured by keys, *a*, upon the shaft, F, also of cast-iron, see Figs. 7, 8, and 9, Plate 12 (148).

The head of the channel, G, which embraces the lower part of the wheel, is constructed with a piece, H, in cast-iron, called the neck-piece, which is fitted upon the cross timber, I, and let into the two lateral walls. Against this neck-piece works the wooden sluice, J, above which overflows a certain depth of water, falling, in succession, upon each float of the wheel as it comes round, causing it to turn in the direction of the arrow. The rotatory movement of this water-wheel is taken off by the cast-iron spur-wheel, K, mounted upon the end of the shaft, F, and gearing with the cast-iron pinion, L, the shaft of which communicates with the machinery to be set in motion.

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* Undershot water-wheels receive the water below their centres, and their buckets or floats pass through an enclosed circular channel at the place where the water acts on them. Overshot water-wheels receive the water from above.

In giving this example, our object has been to examine this motor, not only with reference to the sketching and accurate delineation of similar wheels, but also with a view to their construction and erection, as well as that of their channel and sluice gear.

THE CONSTRUCTION AND SETTING UP OF THE WATER-WHEEL.

The channel, G, is built up of hewn stones, the lateral joints of which converge towards the centre, *o*, of the wheel, and they are imbedded upon a foundation of ordinary stone-work. All this masonry is put together with mortar, made with hydraulic lime, the joints being finished with Roman cement. In some localities the channel is of bricks or freestone, and sometimes even of wood. The apparent concave surface of the channel, should be perfectly cylindrical, and concentric with the external circumference of the wheel. Also, before placing the latter in its proper position, this surface should be finished, and rendered quite smooth and true, which may be done with the assistance of a temporary shaft, or with the actual shaft of the wheel, in the following manner:—

The shaft, F (which is the same as that described at 148), is adjusted to the exact height at which it is to be afterwards, and it is made capable of rotation in appropriate bearings, adjusted upon iron plates, let into and firmly bolted to the lateral walls. Upon this shaft are fitted the shroudings, A, each connected to its boss by eight arms. To the outside of these arms are then temporarily attached two radial pieces of wood, having a cross piece attached to them, the outer edge of which is made true and parallel to the shaft, and coincident with the external edges of the complete wheel. It will be evident that, if the shaft is now made to revolve, this frame upon it will describe a cylindrical surface, which is precisely that which the channel should possess; it will serve, therefore, as an accurate guide in giving the channel its appropriate contour.

The lower part of the channel is continued in a straight line, commencing at the vertical, *o b*, and in a direction, *b c*, slightly inclined to a short distance away from the wheel, to facilitate the escape of the water.

The cross timber, I, which surmounts the masonry of the channel, and which receives the neck-piece, H, is also rendered concave internally, like the channel, so as to allow the sluice-gate to be brought closer up to the wheel. The neck-piece, H, which forms the crest of the channel, is more frequently constructed of cast-iron than of either wood or stone, as that material does not require to be so thick, for resisting the pressure of the water. The top of the neck-piece is at a distance below the upper water-level: corresponding to the greatest depth of water which it is proposed to admit to the wheel at any time. This depth of water varies very considerably, according to the quantity of water to be discharged, and the width which it is wished to give the wheel. Behind the neck-piece, a cavity, M, is formed in the masonry, which is intended to receive the sluice, J, when lowered, and is of a sufficient size to allow of its being cleaned out, so that it may not become choked up with sediment. The small raised portion of masonry behind this, as well as a grating placed further behind, serves to arrest floating bodies, such as brush wood, &c., and preventing their reacting, and injuring the wheel.

The sluice consists of two strong oaken planks, having grooved and tongued joints, and being made thicker at the middle than at the extremities, where the wheel is of a greater width than four or five feet. The amount of inclination of this sluice, J, is determined by drawing a perpendicular to the extremity of the radius, *o f*, drawn near the middle, or, perhaps, two-thirds of the depth of the overflowing body of water. The sluice is moveable in grooves, in two wooden side-posts, N, entirely imbedded in the lateral walls. At the upper parts of these are iron bearing-pieces, to receive two straight cast-iron racks, O, which rise above the cross timber, P, attached to the two side-posts, N. These racks rest, on one side, upon the anti-friction pulleys, *h*, which also guide them, and, on the other side, they gear with the pinions, *g*, keyed up on one horizontal axis. This latter is at one end prolonged, to receive the worm-wheel, Q, actuated by the worm, R, which may be worked at pleasure from above—a winch-handle, or hand-wheel, being fixed upon the upper extremity of its vertical spindle for this purpose. This arrangement permits of the regulation of the position of the sluice, and, consequently, of the depth of the overflowing water, with the greatest nicety, as well as of the total shutting off of the water from the wheel.

The shrouding, A, of the wheel, being of cast-iron, the weight has been reduced, by making panels in it, as at *i*, shown in the elevation, Fig. 1, and section, Fig. 2, made at the circular line, 1—2. It is also cast with mortises, to receive the tenons, or ends, of the carrier-pieces, B, to which the floats C, are bolted.

When the wheel has counter-floats, as represented at the lower part of Fig. 1, which is only the case when the discharge, and consequently the depth, of water at the sluice-gate is very small, the carrier-pieces are very short, and do not project far upon the inner side of the shrouding. But when the wheel is without counter-floats, which is the case when the discharge, and consequently, the depth, of water at the sluice is considerable, the floats, C, and their carrier-pieces, B, are prolonged to a considerable distance, inside the shroudings, as has been supposed to be the case in the upper part of Fig. 1. In both cases, the tenons, or ends, of the carrier-pieces, always lie in the direction of radii from the centre of the wheel, and they are retained by iron keys, *j*, upon the inside of the shroudings. Sometimes, in order to facilitate the adjustment of the carrier-pieces upon the shroudings, in place of fitting them into closed mortises, they are received into slightly dovetailed recesses, formed upon the side, as shown in Figs. 3 and 4, being retained in position by wedges, *j*. When this last arrangement is adopted, it is unnecessary to cut holes in the carrier-pieces, for the reception of the keys.

When the shroudings are of wood, they must necessarily be composed of several pieces, which are fitted together with mortise and tenon joints, as shown in Figs. 5, 6, 7, and 8, and to consolidate the joints, joint iron straps, *k*, are added, secured at one side by bolts, and at the other by keys, or tightening screws, by means of which the perfect union of the component pieces, can at all times be obtained, should they begin to get loose. In this system, the carrier-pieces are adjusted with tenons, keyed on the inside of the shrouding, and the oak arms, D, are joined to the shrouding with tenons, being further secured by iron straps, as shown in Figs. 9 and 10.

The floats, C, of the wheel, are formed of oak boards, and are attached to the carrier-pieces, *b*, by means of the bolts, *l*.

The counter-floats, S, extend from the inner ends of the floats C, to the bottom pieces, S^1, and are nailed down upon the small triangular pieces, *m*. The open spaces left between the ends of the floats and the bottom pieces serve for the escape of the air.

When the floats are lengthened, they are, of course, formed of several boards, joined edge to edge.

DELINEATION OF THE WATER-WHEEL.

378. The explanations just given will enable the student to comprehend the details and peculiarities of construction of the wheel, channel, and sluice apparatus. He should now proceed to delineate these various objects in the following manner:—Place the centre, *o*, of the wheel, at the intersection of two lines which form a right angle; and with this centre, describe a first circle, with a radius equal to that of the wheel and channel. Divide this circle into as many equal parts as there are to be floats. The number of the floats should always be divisible by that of the arms of the shrouding, so as not to be restricted as to space in fitting in the carrier-pieces. Through each point of division draw lines passing through the centre, and representing the sides of the carrier-pieces upon which each float is placed. Two circles must next be described, expressing the depth of the shrouding. Then the complete outline of one of the carrier-pieces must be drawn, with the dimensions noted on the figure; and the key and bolts may also be indicated upon it. Afterwards, to complete the drawing, it will be sufficient to describe a series of circles, passing through the bolts, the ends of the floats and carrier-pieces of the key, and of the counter-float. With regard to the floats, and to the arms of the shrouding, as well as to the spur-gear for transmitting the motion, the student may refer back to the diagrams and explanations already given concerning similar objects. The same remark applies to the lifting apparatus of the sluice-gate, which is also composed of gearing already treated of in the course of the studies.

DESIGN FOR A WATER-WHEEL.

379. If it is in contemplation to make a design for the construction of a water-wheel, similar to the one above described, it is necessary to ascertain the height of fall, and the amount of discharge per second, of the water at our disposal, and to refer to the calculations and practical rules given at the end of Chaps. VIII. and IX., or to other text books on this subject, to be able to determine, on the one hand, the diameter and width of the wheel, and, on the other, the depth and interstices of the floats, and their number. By referring also, to the data and notes at the end of Chaps. III. and IV., in reference to the measurement of surfaces and solids, and to the rules and examples at the end of Chap. XI., or to other works which treat of the resistance of materials, we shall be able to complete the remaining dimensions for the shaft and its journals, the shrouding and its arms.

The first motion wheels, K and L, will be easily designed to suit the particular work for which they are required, from the data given at the end of Chap. VII., which treats of the strength and dimensions of *toothed gear*.

The study of water-wheels of this description will be much simplified, if we consider that certain dimensions, such as the thickness of the floats, the section of the carrier-pieces and shrouding, and the diameter of the bolts, as well as the details of the sluice apparatus, do not vary so much in ordinary sized water-wheels (unless very large or very small indeed), but what the draughtsman may take many of these minor parts from the drawing given in this Plate, which are themselves examples of actual construction.

SKETCH OF A WATER-WHEEL.

380. The sketch of a water-wheel, already constructed and set up, is a very simple matter; for the apparatus consists of a repetition of various pieces, and it is sufficient to obtain the measurements of one only of each kind. Thus, after having measured the diameter and extreme width of the wheel with the aid of a long rule or tape, and counted the number of buckets or floats, of the shroudings, and of the arms, in accordance with the instructions given in reference to Plate 35; we have merely to take the sketch of a single float, with its carrier-piece and accompaniments, then to make a section of one of the shroudings, another of one of the arms, and, finally, a third of the boss and shaft.

The details given in Figs. 2 to 11 show the various parts of which the sketches have to be made, as detached, together with the corresponding measurements; the latter of which, Fig. 11, is a transverse section of one of the arms, D, of cast-iron, taken on the line, 1—2, near the boss.

The sketching of the sluice apparatus consists in making a section of the side-posts, with their cap-piece, and of the sluice itself; then a detailed view of one of the racks, with its pinion and friction-pulley, and of the worm wheel and worm. As to the amount of inclination of the sluice and side-posts, it has already been seen that it is determined by a perpendicuar to the radius, entering near the middle of the depth of water at the outlet, at the circumference of the wheel. It may, however, be found by means of a plumb-line, let fall from one of the edges of the cap-piece down to the level of the water, by measuring the horizontal distance, *r s*, of the plumb-line, from one of the sides of the side-post, and then the vertical height, *r t*. By applying a rule against the side-post, and down to the neck-piece, H, we can always obtain the actual distance of the top of the latter, either from the prolongation of the horizontal, *r s*, or from the cap-piece, P, of the sluice. To obtain the horizontal distance, *r s*, with exactitude, it should generally be taken at a given distance above the level of the water, and chalked upon one of the side-walls of the channel; it is also advisable to make use of a spirit-level (Fig. 3, Plate 1).

In order to take an accurate sketch of the neck piece and the channel, it is almost always necessary to stop the water behind by means of a dam, so that the parts requiring to be examined may be dry and open. The sluice must also be taken away, as well as a few of the floats of the wheel. We may remark, that this labour may be avoided, when it is known that the height and thickness of the neck-piece are nearly always equal to those indicated in Fig. 12. as a detached sketch; as to the arrangement of the masonry or brickwork, of which the channel may be constructed, it will be recollected that all the lateral joints are pointed towards the centre of the wheel, each stone being dovetailed into each other, and "breaking bond" as shown in Fig. 1.

OVERSHOT WATER-WHEEL.

FIGURE 13.

CONSTRUCTION OF THE WHEEL, AND ITS SLUICE APPARATUS.

381. Overshot water-wheels, with buckets, receive the water from a duct placed immediately above them, and allow it to escape from as low a part only as possible. They are constructed of wood, or of cast-iron. In the first case, which is the simpler and more economical, the shaft, the arms, and the shroudings are of oak. This description of wheel is shown in the lower part of the drawing, Fig. 13, which represents the vertical section of an overshot water-wheel, a little shaded to show the difference between this way of ruling or drawing, and that of plain outline, as shown in Fig. 1. The buckets and the inner rim are likewise of oak, or of iron plates. As this wheel is of small diameter, its shaft, F, has

only six sides; and consequently, each shrouding, A, of the wheel has only six arms, D, which are recessed into, and bolted upon, a central cast-iron frame, E, which is itself keyed upon the shaft. The transverse section, Fig. 14, taken on the line, 3—4, shows the manner in which the arms are attached to this frame or boss, E. The wooden shroudings, A, are generally composed of two rings, formed in segments, placed one on the other, in such a manner that the joints of each are opposite to solid portions of the other, to "break bond," and obviate the tendency to warp. A portion of the shrouding constructed in this way, is represented as detached in Figs. 15 and 16. These rings of segments are held together by screws, *v*, or by nails or pegs; and at their junction with the arms, a couple of bolts are passed through all, as indicated in the transverse section, Fig. 17. The buckets, C, are either let into grooves of small depth, upon the inner face of the shrouding, as seen at c^1, in Figs. 15 and 16, or they are retained by bracket-pieces, *c;* and, added to this, are strong tension-rods, B, to hold the whole together, being secured to the shroudings, A, on either side of the buckets. These tension-rods are fixed, when the inner rim, S, or bottom of the buckets, has been nailed or screwed to the inner edges of the shroudings. The shroudings are further strengthened externally by a circular iron strap or ring, G, similar to the felloe of an ordinary cart wheel, and covers all the joints of the duplex shrouding.

Sometimes the buckets are partly of wood, and partly of iron plate, to give them greater strength. The edges, indeed, should always be defended with metal, as they are apt soon to wear or break away. In the lower half of Fig. 13 is shown three different ways of constructing these buckets.

When the wheel is of cast-iron, if it is not of a very great diameter, but about the size represented in the upper part of Fig. 13, the arms, D^1, and the boss, E^1, may be cast in one piece with the shroudings, A^1. Where the diameter is considerable, however, these consist of several pieces bolted together. The bottom piece, or inside rim, S^1, and the buckets, C, are formed of thin iron plates, of about ⅛th inch in thickness. To secure the latter, a series of feathers, B^1, Figs. 13, 18, and 19, are cast upon the inner faces of the shroudings, A^1, to which they are fixed by screw-bolts, *l*. In the width of the wheel, these buckets, C^1, and the bottom rim, S^1, are riveted together, as at *i*, or are fixed together by small screw-bolts, i^1, Figs. 13 and 19.

The advantage of making the buckets of thin iron plates, consists in the being able to give them a curved form, which enlarges their capacity, and allows of a more favourable introduction of the water; whilst the wooden buckets necessarily consist of two rectilinear portions, one of which is directed towards the centre of the wheel, whilst the other is inclined. When these buckets and inner cleading are well galvanized, they resist the action of the water and last for a great length of time.

The water is generally conducted by a wooden channel, such as that shown at M, to the top of the wheel, and its outflow is regulated by a sluice, J, moving in side grooves, and worked by means of a couple of vertical racks, O, and pinions, *d*, the shaft of which last carries the winch-handle, Q. The two vertical sides, N, of the channel, are prolonged beyond the actual summit of the wheel, and their distance asunder should be a little less than that of the two shroudings of the wheel, with the twofold object of better directing the water into the buckets, and of avoiding the splashing and loss of water by allowing the air to escape laterally. The depth of the outflow of water depends on the distance of the lower edge of the sluice above the bottom of the channel, and should always be less than the smallest distance existing between two consecutive buckets. The pressure of the water upon the buckets, produces the rotation of the wheel in the direction of the arrow, and this motion is given off by an internally-toothed wheel, which is formed in segments and bolted by flanges to the outside of one of the shroudings. This wheel, which in the drawing is simply indicated by dotting in the pitch circle, K, gears with the pinion, L, mounted on the extremity of the shaft, which communicates with the machinery in the interior of the factory or workshop.

DELINEATING, SKETCHING, AND DESIGNING OVERSHOT WATER-WHEELS.

382. The delineation of the principal parts of an overshot bucket water-wheel, is effected in the same way as that of the undershot wheel with floats, the only difference being, in fact, in the receptacles for the water. It has been seen, that when these buckets are of wood, they are composed of two boards, one of which lies in the direction of a radius of the wheel, the other being inclined according to the direction of the water, and making an angle of from 15 to 30 degrees, as the case may be, with the tangent, to the outer circumference of the wheel, drawn through its extremity, as will be seen by the angle, *a b c*, in Fig. 18. When the bucket is made of iron plates, the same angle is adopted near the outer edge, although the whole contour is a continuous curve, which may be made up of two or three arcs of circles, as shown in Figs. 13, 18 and 20.

In sketching this wheel, the directions given in the preceding case may be likewise followed here, by counting the number of buckets, and taking an accurate sketch of one of them, together with the accompanying measurements. We must also measure the internal and external diameters of the shroudings; then the least space existing between two consecutive buckets; also the depth from *b* to *d*, Fig. 18. Finally, if it is required to obtain the exact form or curvature of the bucket; to do this in most instances it is advisable to take one of them down, and to make a pattern of it, by applying a sheet of paper against one edge, and pencilling out the shape, as is done for the forms of wheel-teeth, or other curves, which are difficult to measure, and exact copies are required. As to the sketch of the other parts of the water-wheel, such as the boss, the arms, and also the sluice apparatus, no peculiarity or difficulty will present itself which needs further explanation than what has been given in reference to this and the former plate on sketching. The drawing, moreover, indicates most of the figures and measurements which are necessary for the production of a finished or working drawing, such as that given in the Plate.

In designing an overshot water-wheel, it is necessary to know the height of fall, and the daily discharge of the water. With regard to these particulars, we must refer the student to the rules and practical data given at the end of Chaps. IX. and XI., for measuring and calculating these things, or to some of the numerous works which deal with this subject.

WATER-PUMPS.

PLATE 37.

GEOMETRICAL DELINEATION.

383. Pumps being a class of machines requiring the frequent attention of the practical draughtsman, both as regards the delineation and construction, we have therefore already given, at the end of Chap. IX., some good practical data, relating to various kinds of

WATER–

Fig.7.

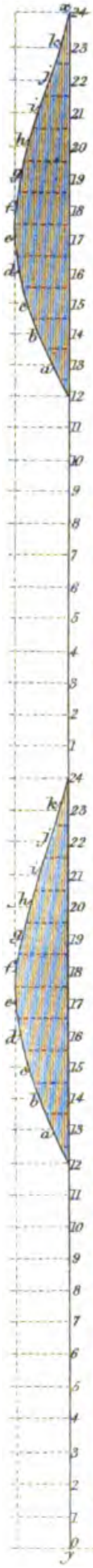


Fig.10.

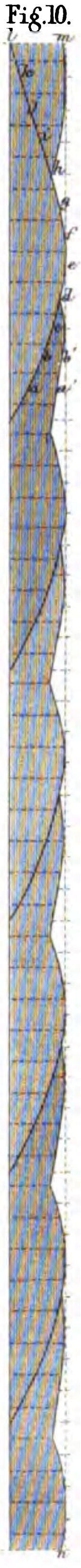


Fig.6.

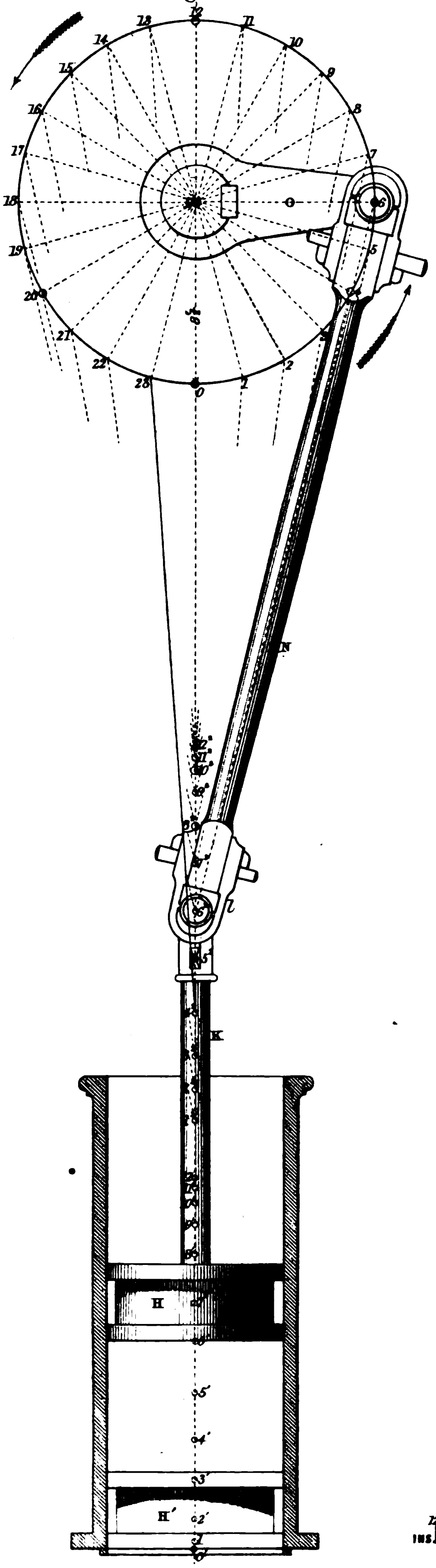


Fig.2.

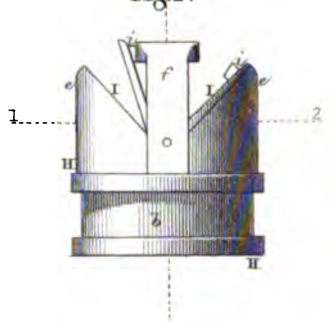


Fig.3.

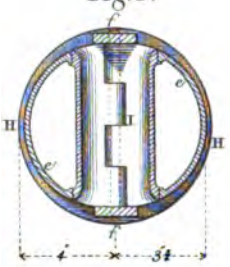
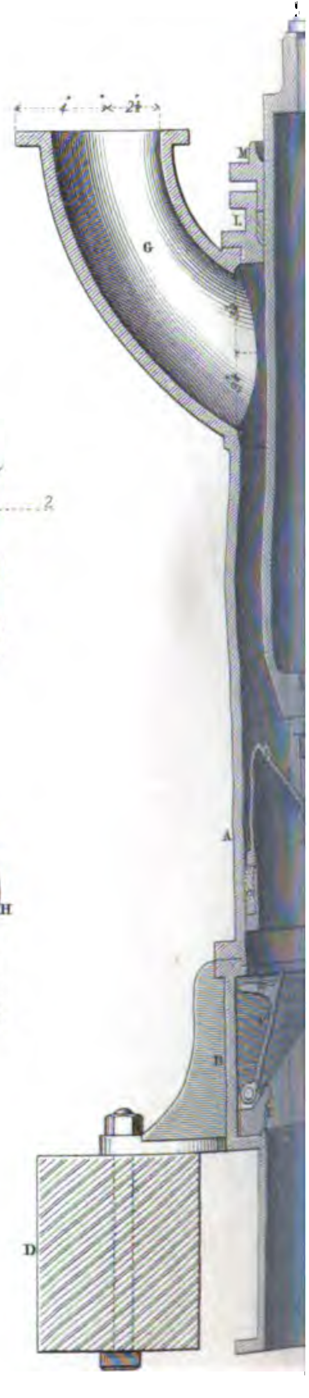
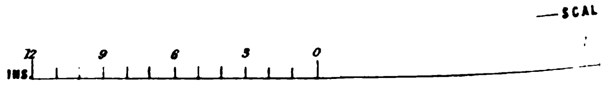


Fig.1.





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47, Lincoln's Inn Fields & Glasgow

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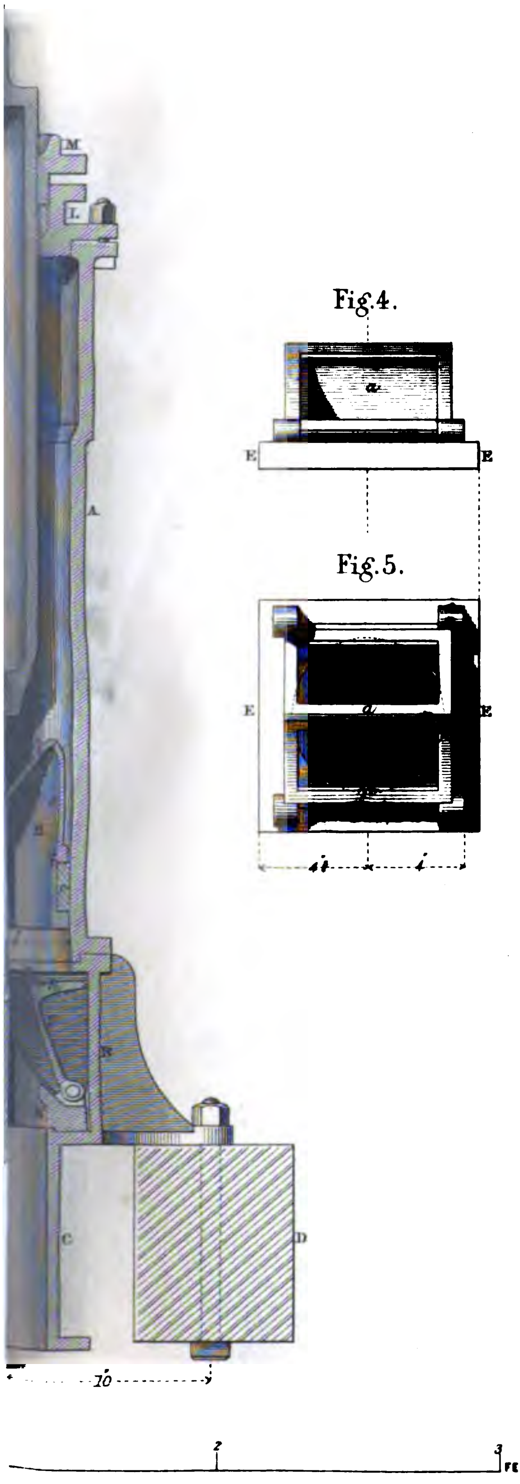


Fig. 4.

Fig. 5.

Fig. 9.

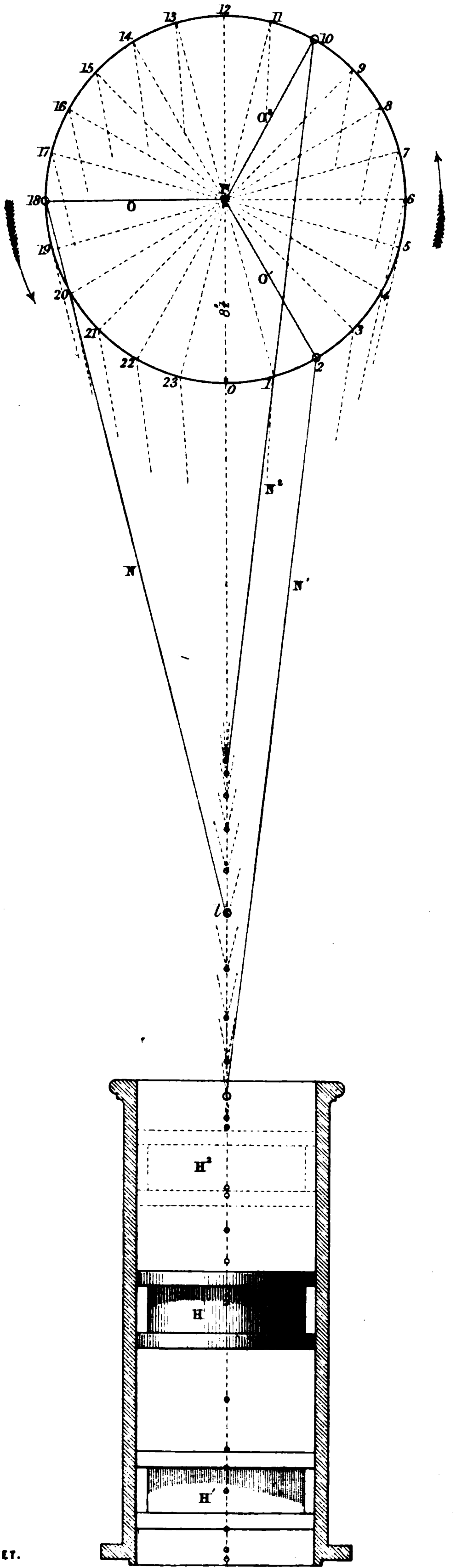


Fig. 8.

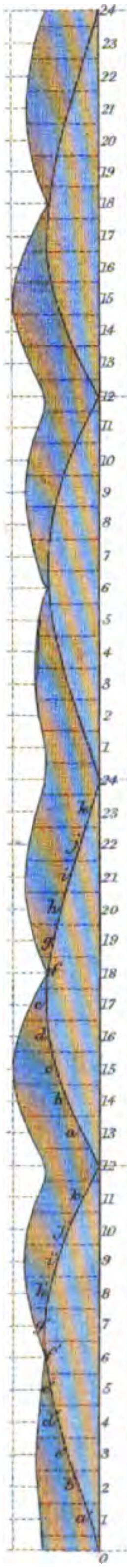
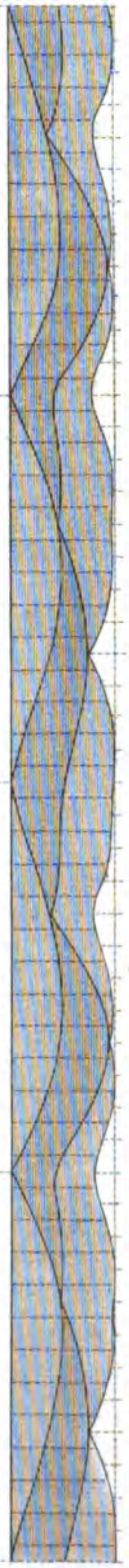


Fig. 11.



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pumps; rules and formulæ for calculating their proper dimensions, in proportion to the quantities of water to be furnished by them. But in addition to this we now propose to enter upon more detailed and complete explanations, with regard to their construction, action, and performance. For this purpose we have selected, by preference, from among the various forms of pumps, what is termed a combined lifting and forcing pump, the discharge of which is almost continuous, although its construction is similar to what is termed a single-acting pump.

Fig. 1, on Plate 37, represents a vertical section, taken through the axis of this pump. It consists of a cast-iron cylinder, A, turned out for the greater portion of its length, and resting upon a feathered base, B, to which it is jointed and screwed by flanges. This base is cast in one piece with the suction or lift-pipe, C, below, and it is bolted down either to stout timbers, D, or to a stonework foundation. It encloses the valve-seat, E, which consists of a rectangular frame, divided by a central partition, *a*, and having the sides formed so as to present two inclined edges, or faces, upon which the brass clacks, F, rest when shut. The pipe, C, terminates below in a flange, by means of which the suction-pipe is attached, extending down to the water to be raised. Towards the upper part of the pump cylinder, A, is cast a curved outlet, G, likewise terminating in flanges, to which the discharge-pipe is secured. The piston, or bucket, of this pump is composed of a brass ring, or short cylinder, H, upon the outer circumference of which is formed a groove, *b*, to receive a packing-ring, *c*, which fits, air-tight, to the inside of the pump cylinder. The bucket, H, has also a central partition, *d*, to the top of which is jointed the two clacks, I, which rest upon inclined seats, formed by the elevated sides, *e*, of the bucket. This is further cast with a bridle, *f*, perforated in the middle, to receive the screw-bolt, *g*, which secures it to the stout hollow piston-rod, or plunger, J. This rod, which in the generality of pumps is made of but small diameter, like the upper part, K, of this figure, or that of K, in Fig. 6, of the same plate, is, in the present instance, of a sectional area, equal to half that of the pump cylinder. It follows from this, as will be more particularly explained further on, that the water is discharged during both the up and down stroke of the piston.

The clacks, F, have projections, *h*, cast upon them, which prevent their opening too far, and falling over against the sides of the casing, B, so as not to shut again when required to do so. The clacks, I, in the bucket, have similar projections, *i*, for a like purpose, these projections striking against the top of the bridle, *f*, when the clacks open. It will have been observed, that the seats of these valves are inclined at an angle of 45°, with the view of facilitating their opening movement, and diminishing the concussive action of their own weight. The edges of these valves are sometimes covered with a surface of thin leather, to effect their tight closing.

Figure 2 is an elevation of the bucket, H, with itsclacks, I; represented as detached from the pump. Fig. 3 is a horizontal section of the bucket, taken at the line, 1—2, and corresponds to Fig. 2. Figs. 4 and 5 give the details of the valve-seat, E, in elevation and plan, the clack valves, F, being removed.

To prevent the entrance of air to the pump cylinder, it is closed at the top by a cast-iron cover, L, which is fitted with a stuffing-box for the passage of the piston-rod, or plunger; the packing is compressed by the gland, M, somewhat similar to that described (142) in reference to Figs. 4, 5, and 6, Plate 11.

ACTION OF THE PUMP.

384. The upper extremity of the pump-rod, K, Fig. 6, carries a cross-head, *l*, and is there jointed to the lower extremity of a connecting-rod, N, which is itself jointed to the pin of a crank, O; this latter is mounted on the end of a horizontal shaft, P, actuated by a continuous rotatory movement. This movement is transformed by means of the crank and connecting-rod into an alternate rectilinear motion—that is, into the up-and-down strokes of the pump bucket, H.—this last being forced to move in a straight line, the cross-head, *l*, sliding in vertical guide-grooves, to maintain the rod, K, in the same line with it.

It follows, from this disposition of parts, that when the crank, O, is in the position, P—*o*, Fig. 6, the piston will be at the bottom of its stroke, that is, at H^1; consequently, during the time the crank turns, the piston must rise, tending to leave a vacuum below it, because the space between the clacks, F, and its under side increases, as well as the volume of air that may be therein enclosed. Consequently, the pressure of this air upon the clacks is diminished, whilst that upon the surface of the water remains the same, and causes the water to rise up the suction pipe, C, and, raising the clacks, F, to enter the pump cylinder, filling it up nearly to the under side of the bucket or piston; or, if the apparatus is in a perfectly air-tight condition, it will rise quite up to it.

When the crank has reached the position, P—12,—that is, when it shall have described a semi-revolution,—the bucket or piston, H, itself will likewise be at the highest point of its stroke, and, in this position, all the space left behind it in the body of the pump will be filled with water; if now the crank, continuing its rotation, makes a second semi-revolution, the piston will descend, and, pressing upon the water below it, will cause the clacks, F, to shut. Now, as the water is incompressible, it must find an exit, or else prevent the descent of the piston; and it therefore raises the bucket-clacks, I, thus opening up for itself a passage through the bucket, or piston, H, above which it then lodges. But as the piston-rod, or plunger, J, is of a large diameter, and therefore occupies a considerable space in the pump cylinder, a part of the water must necessarily escape through the outlet, G, in such a manner that, when the piston shall have reached the bottom of its stroke, there will not remain in the pump cylinder more than half the quantity of water which was contained in it when the piston was at the top of its stroke.

Such is the effect produced by the first turn of the crank, which corresponds to a double stroke of the bucket, or piston—that is, an ascent and a descent.

At the second turn, when the bucket, or piston, again rises, it sucks up, as it were, anew, a volume of water about equal to the length of cylinder, through which it passes, because the suction-clacks, F, which were shut, now open again, and the bucket-clacks, I, which were open during the descent, are now shut by the upward movement of the piston.

During this stroke, all the water which previously remained above the bucket, finds itself forced to pass off through the pipe, G, so that, with this arrangement of bucket and rod, or plunger, of large diameter, it follows that, at each up-stroke of the piston, the quantity of water which rises into the pump is equal to the length of cylinder through which the piston passes, the half of which quantity rises in the discharge-pipe during the descent, and the other half during the subsequent ascent of the piston, and the jet is consequently rendered almost continuous and uniform.

When, on the contrary, the bucket, or piston-rod, is made very small in diameter, as in ordinary pumps, such as that indicated in Fig. 6, the discharge of the water only takes place during the ascent of the piston, and it is consequently intermittent.

In a pump, as in all other machines in which an alternate rectilinear motion is derived from a continuous rotatory motion, by means of a crank and connecting-rod, the spaces passed through in a straight line by the piston do not correspond to the angular spaces described by the crank-pin; in fact, it will be seen from the diagram, Fig. 6, that if the crank-pin is supposed to describe a series of equal arcs, beginning from the point, *o*, the corresponding distances, $o^1 1^1$, $1^1 2^1$, $2^1 3^1$, passed through by the piston will not be uniform; very small at the commencement of the stroke, they will gradually increase towards the middle, after passing which they will similarly decrease whilst the piston approaches the other end of its stroke. The successive positions of the piston may be obtained by describing with each of the points, 1, 2, 2, 3, 4, upon the circumference traced by the crank pin, as centres, and with a radius equal to the length of the connecting-rod, a series of arcs or circles cutting the vertical, passing through the centre, P, in the points, 0^2, 1^2, 2^2, which indicate upon this line the various positions of the point of attachment, *l*, of the connecting-rod to the piston-rod; these points are then repeated at 0^1, 1^1, 2^1, on the same line, at distances from the points, 0^2, 1^2, 2^2, equal to the length of the piston-rod, measured from the point, *l*, to the bottom of the piston.

It will be easily understood, that, in consequence of this irregularity in the motion of the piston, the force and volume of the jet of water will vary throughout the whole stroke. We have endeavoured to show the nature of this variation in the diagram, Fig. 7, which represents the comparative volumes of the jet of water at successive periods for a single-acting pump, such as the one in Fig. 6.

This diagram is constructed by laying off upon any line, *x y*, as many equal parts as we have taken in divisions on the circle described by the crank-pin; then through each of these points, as 1, 2, 3, 4, drawing perpendiculars to *x y*. As during the ascent of the piston from 0 to 12, Figs. 6 and 7, there is no discharge, as the piston only sucks up the water, there is nothing to indicate upon these first divisions; as soon, however, as the crank-pin passes the highest point, and the piston begins to descend, it will produce the jet of water; it is considered then, that when it has passed through the first rectilinear space, 12^1 to 11^1, the quantity of water forced out by the piston may be represented by its base multiplied by the height, 11^1—12^1. It is this distance which is set off from 13 to *a*, upon the perpendicular drawn through the point, 13; in the same manner, when the piston descends from 11^1 to 10^1, it is also taken as represented by its base multiplied by the height, 11^1—10^1, which last is therefore set off from the point, 14 to *b*. It will be seen from this, that, in proceeding with the diagram, it is simply necessary to set off upon each of the perpendiculars drawn through the points of division, 15, 16, 17, the successive distances passed through by the piston during its descent, so as to represent intelligibly the actual volumes of water discharged for each portion of the stroke, since these volumes are proportional to the distances passed through by the piston, the section of the cylinder remaining constant.

If, through the various points, *a*, *b*, *c*, *d*, Fig. 7, obtained in this manner, we trace a curve, we shall obtain the outline of a surface which we have distinguished by a flat shade, and which will give a good idea of the amounts of water discharged in correspondence with any position of the crank. On continuing the rotation of the crank, the piston next ascends and sucks up the water, consequently the jet of water is interrupted during this up stroke, but recommences on the down-stroke due to the subsequent part of the revolution; the quantity of water then discharged is indicated in Fig. 7, by a curve equal to the first, and on which the same points are distinguished by the same letters.

To avoid this irregularity in the discharge, pumping apparatus is sometimes constructed with two, or with three, distinct cylinders, in which the disposition of the pistons is such, that the points of attachment to the several crank-pins divide the circle described by them into two or three equal parts.

Figure 8 represents a geometrical diagram of the performance of a two-cylinder pump; it is evident that the product of each of the pistons is alternately the same, since one descends whilst the other rises; it is thus that one of the pistons, having produced a jet corresponding to, and expressed by, the curve, a^1 b^1 c^1, the other one immediately afterwards produces a jet, expressed by the curve, *a b c d*; so that this diagram only differs from Fig. 7, in that the unoccupied intervals, from 0 to 12, and 24 to 12, in the latter, are in the former filled up by an equal figure, covered by an equal flat shade.

This diagram of the performance of a two-cylinder pump may also be considered as representing that of the pump, Fig. 1, which, because of its trunk piston-rod, acts as a double-acting pump, as already explained.

Figure 9 represents the diagram of the performance of a three-cylinder pump, of which the pistons, H, H^1, H^2, represented, for convenience' sake, as in the same cylinder, occupy the positions corresponding to those of the three crank-pins, o, o^1, o^2, as placed at the angles of an equilateral triangle, inscribed in the circle described by them with the centre, P. In consequence of this disposition, there are at one time two pistons ascending and one descending, and at another time, on the contrary, only one ascending and two descending. It is easy to represent the combined performance of these pumps in a diagram, by using different colours, or different depths of shade, for the performance of each, as dependent upon the successive positions, 1, 2, 3, 4, taken up by their successive crank-pins. By this means all confusion will be avoided, and it will be necessary to find the positions, N, N^1, N^2, of the attachment of the connecting-rod to the piston-rod upon the vertical line passing through the centre, P, only as for one cylinder, as the distances will be the same for all, being merely placed at different parts of the diagram.

In the diagram, Fig. 10, we have laid down the performance of each of the three pumps, supposing them all to be of the same diameter, and taking care, when two pumps are discharging together, to add together their performance; thus, for example, when one of the pistons elevates a quantity of water, corresponding to the perpendicular, 13 *a*, that which is also discharging at the same time furnishes a quantity expressed by the distance, *a* a^1; consequently, the total volume of the discharge at this instance is represented by the total height, 13 a^1; when, on the other hand, only one of the three pumps discharges, whilst the pistons of the other two are ascending, as in Fig. 9, the volume discharged is represented by a single length of perpendicular, such as 18 *f*. Now, it will be observed, that it is precisely at the moment when only one pump is discharging that it gives out its maximum performance; from which it follows, that the jet of water is continuous, and almost uniform throughout its duration, as will be very evident

HIGH-PRESSURE

Fig. 6.

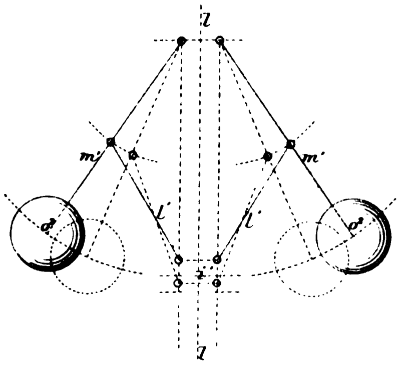


Fig. 3.

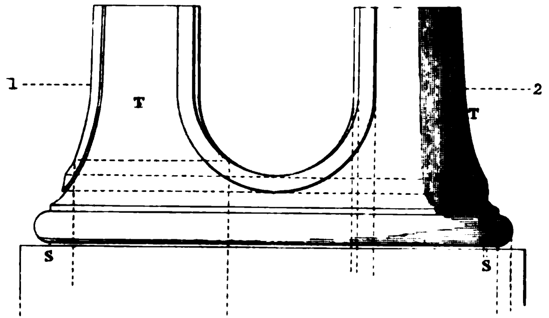
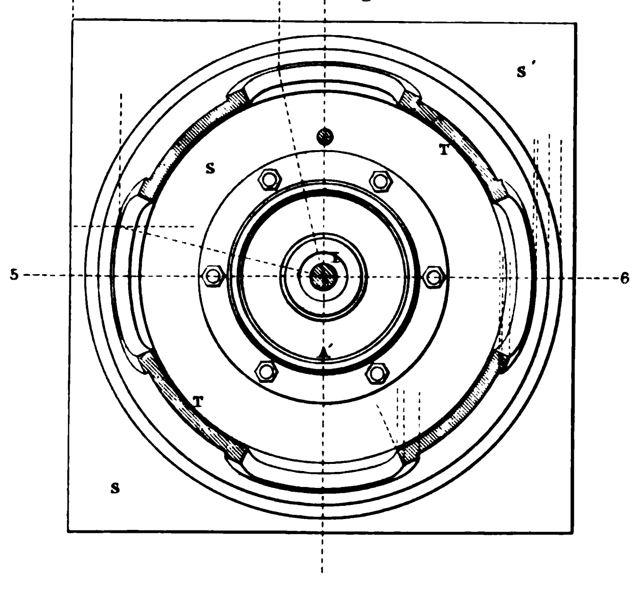
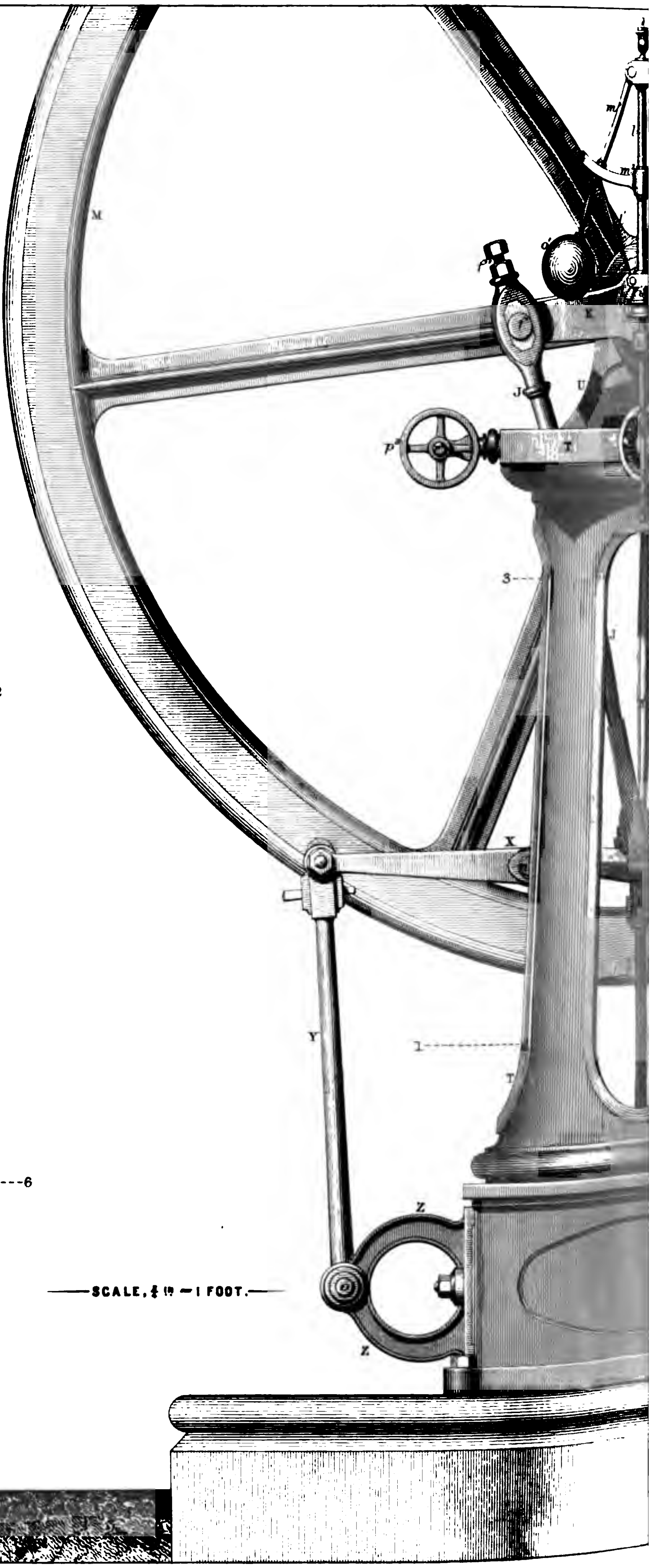


Fig. 2.





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STEAM-ENGINE.

Fig. 1.

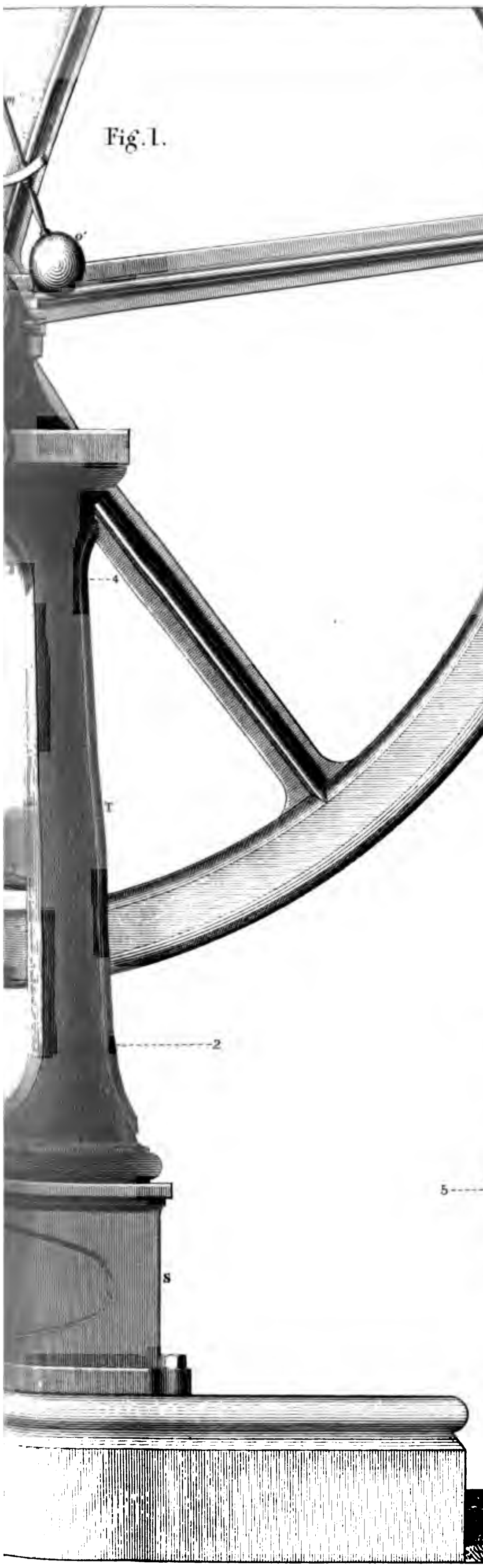


Fig. 7.

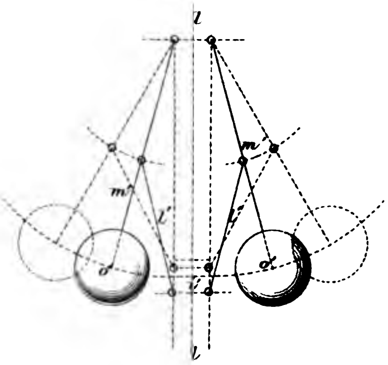


Fig. 5.

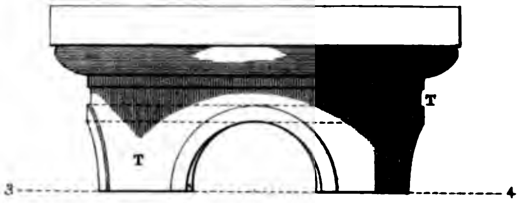
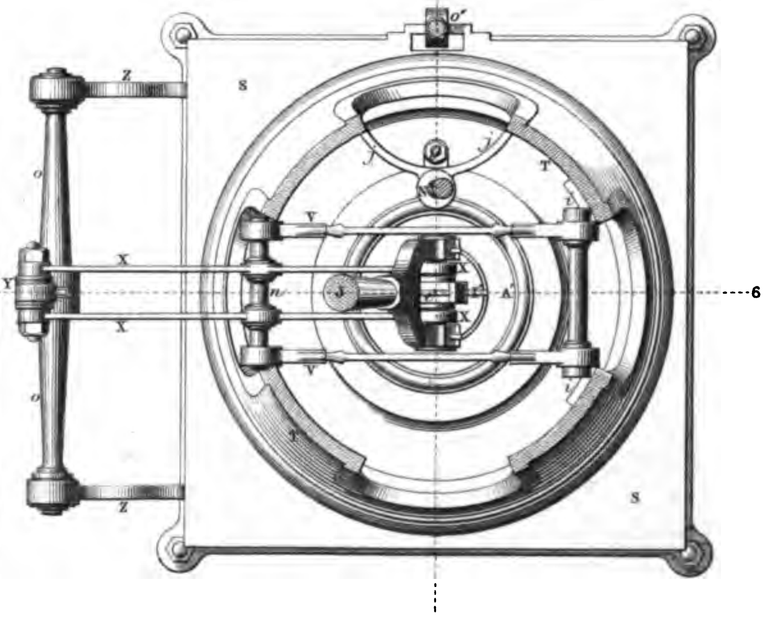


Fig. 4.



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HIGH-PRESSURE

Fig. 6.

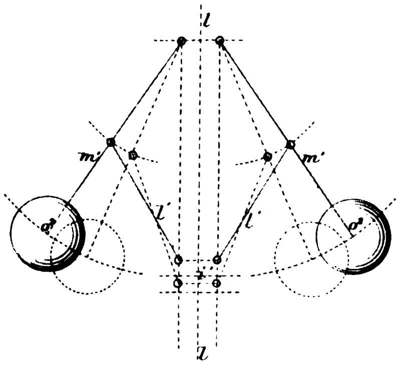


Fig. 3.

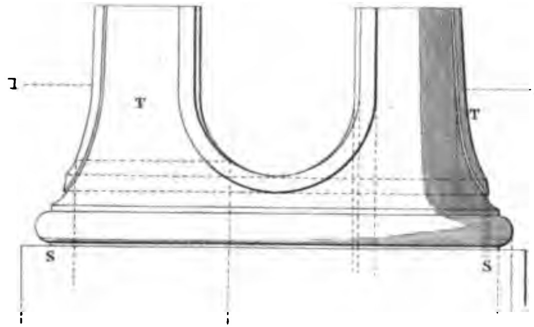
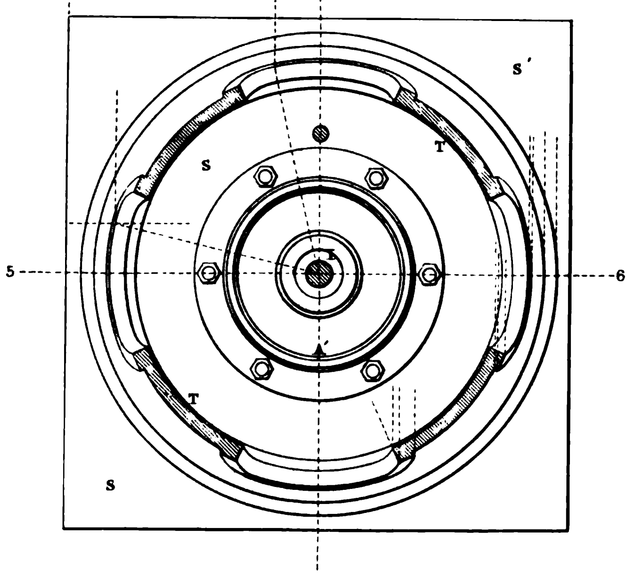
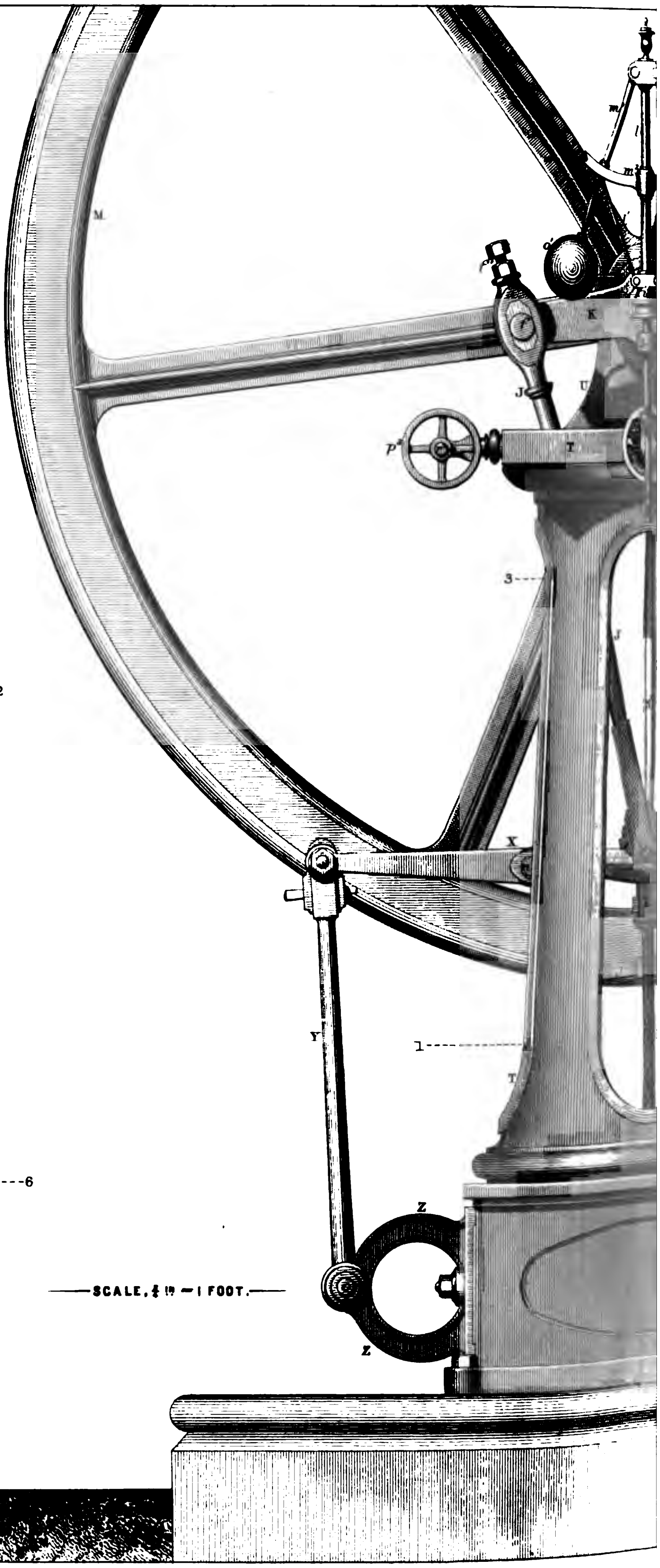


Fig. 2.





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STEAM-ENGINE.

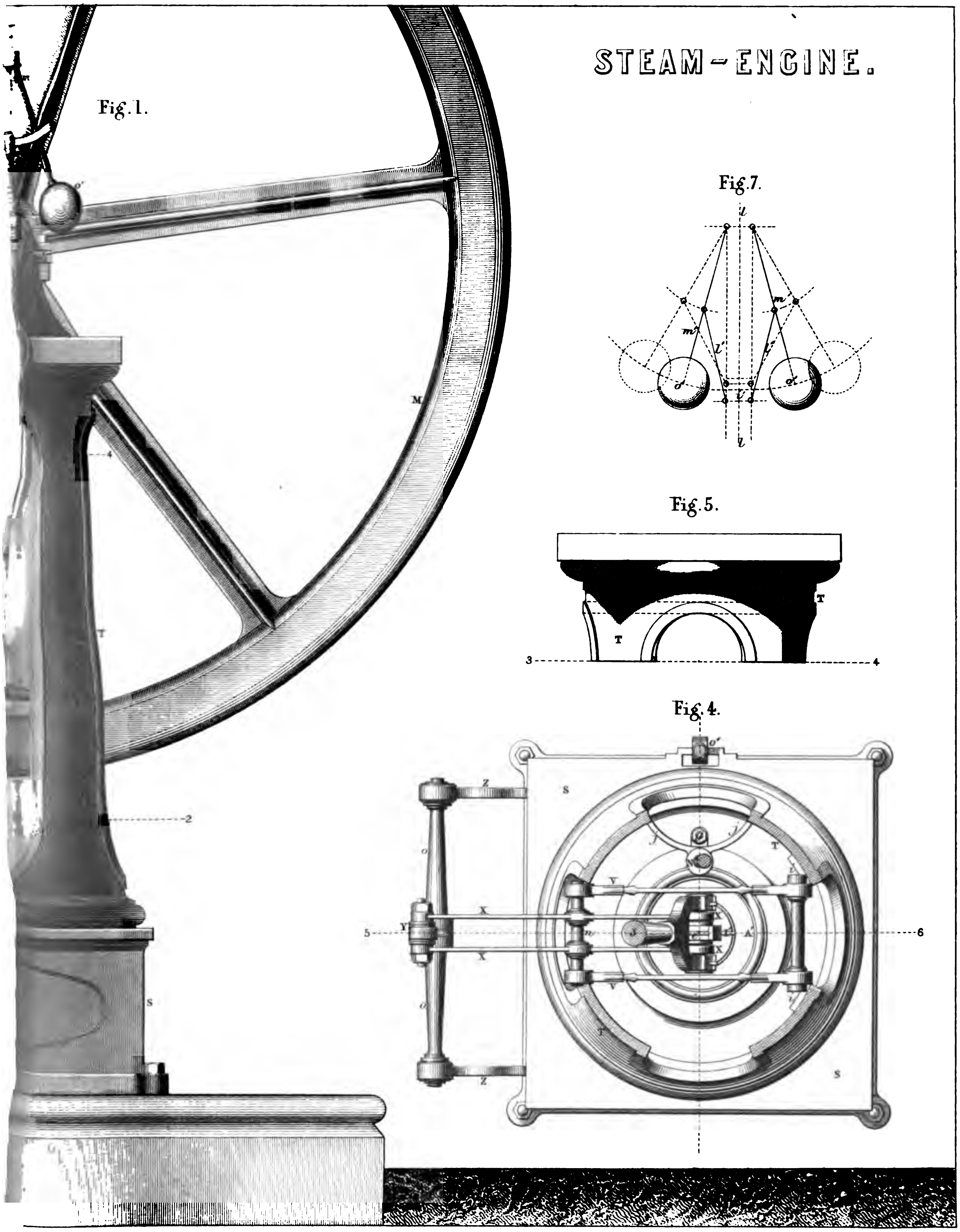


Fig. 1.

Fig. 7.

Fig. 5.

Fig. 4.

Mackay & Kirkwood Glasgow

HIGH-PRESSUR

Fig. 15.

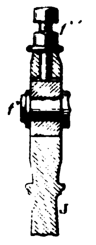


Fig. 16.

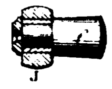


Fig. 8.

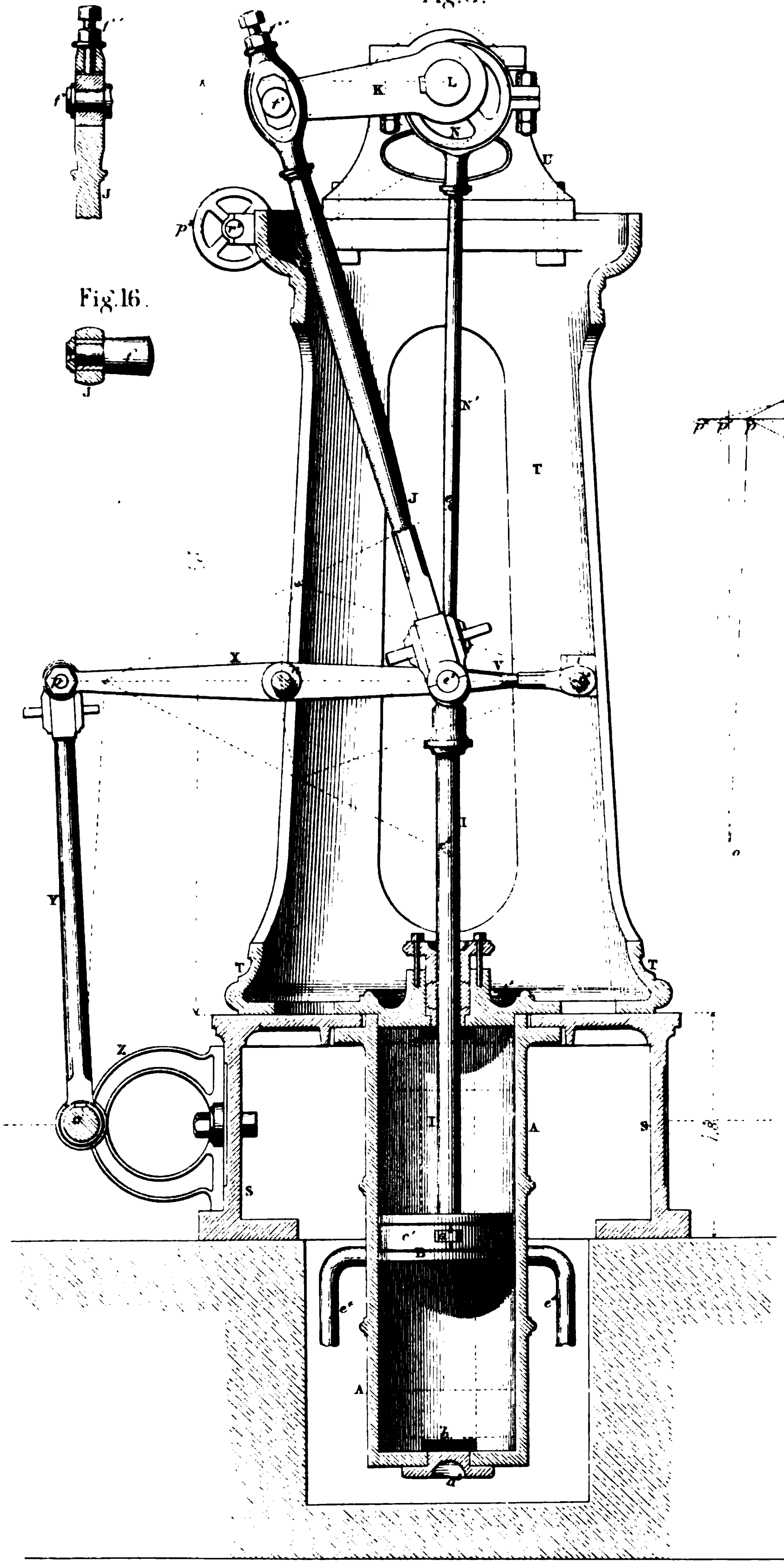


Fig. 11.

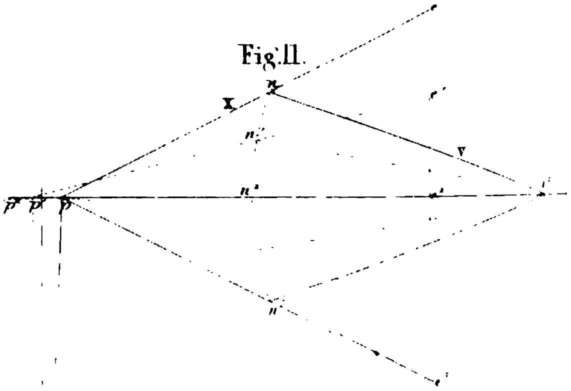
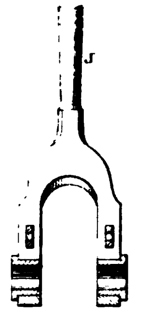
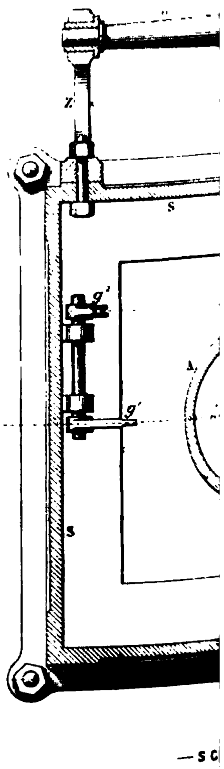


Fig. 14.





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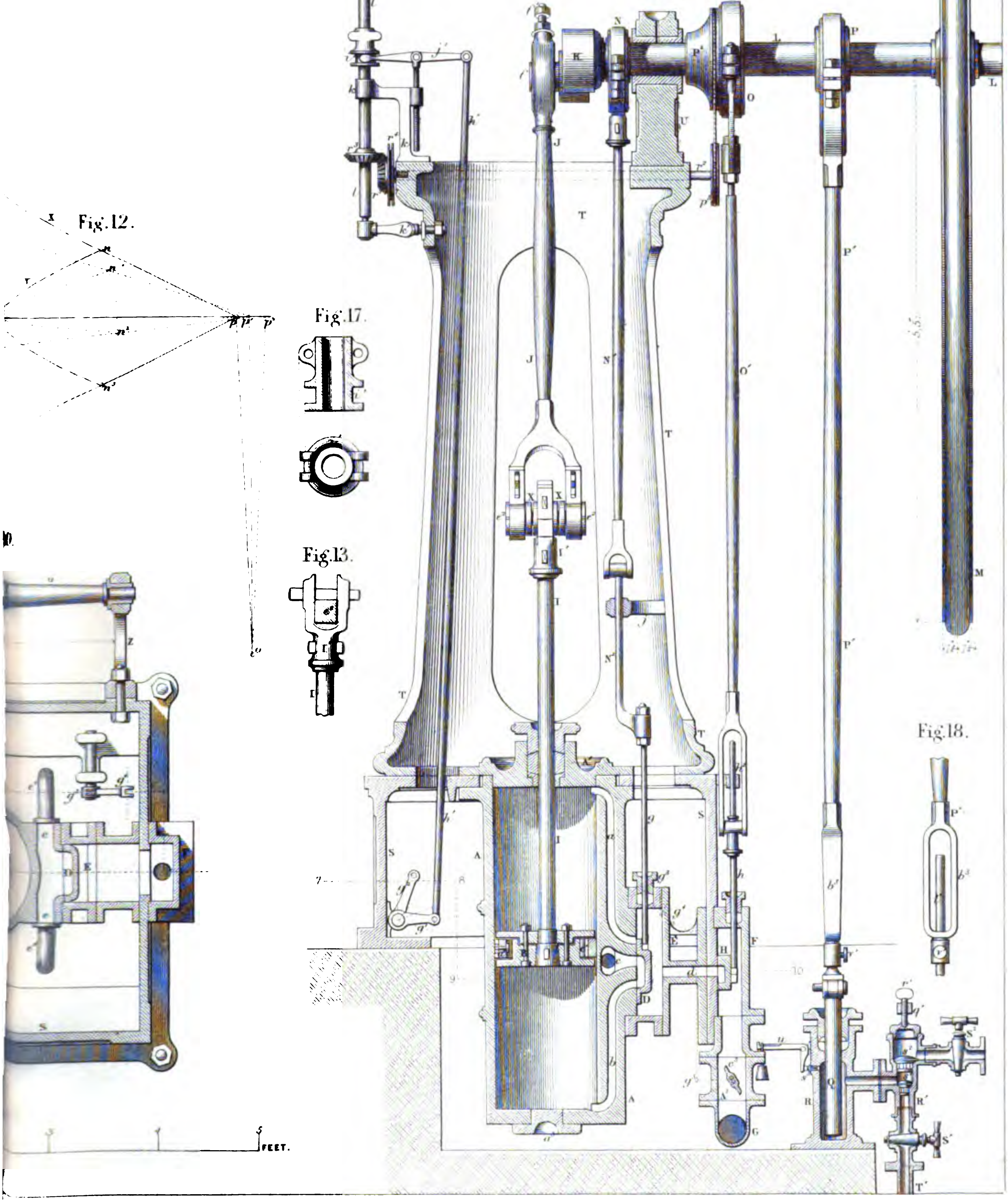
STEAM-ENGINE.



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HIGH PRESSURE S

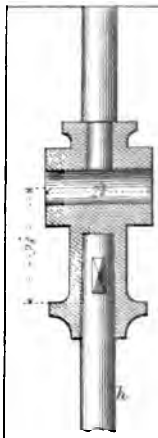
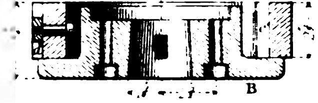


Fig. 19.



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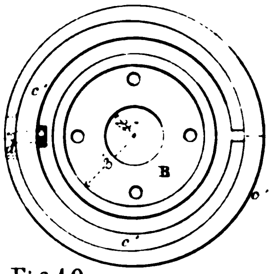

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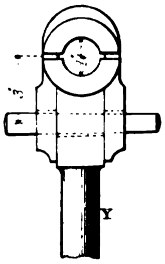

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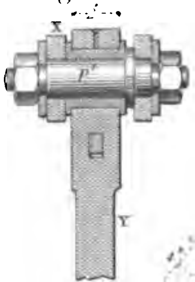

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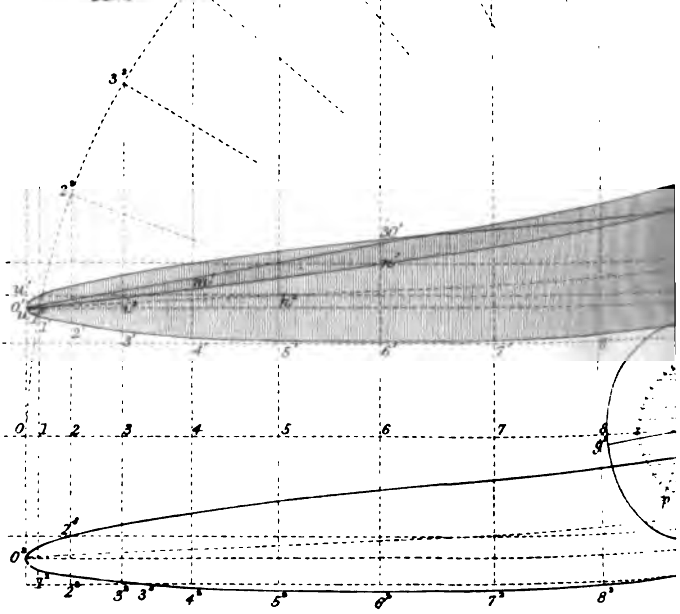
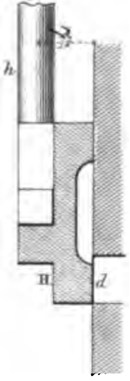


Fig. 41.



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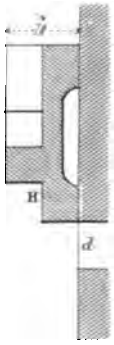

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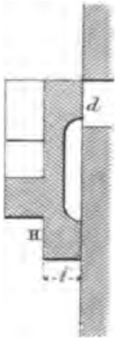

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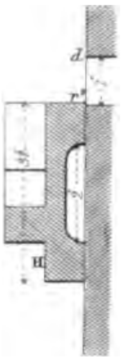

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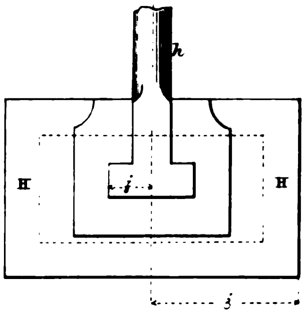

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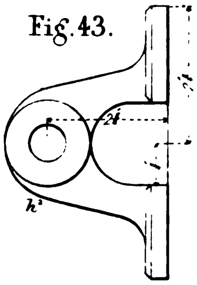

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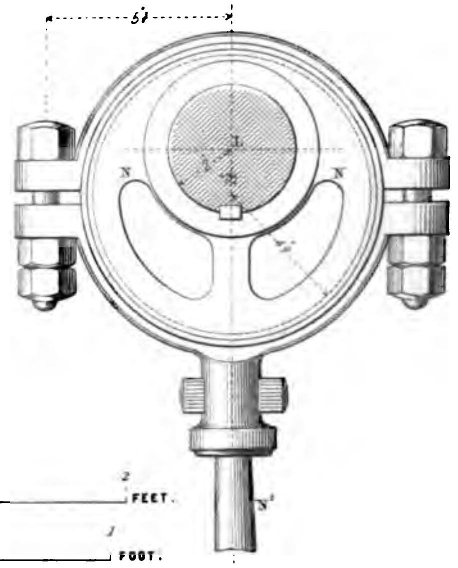

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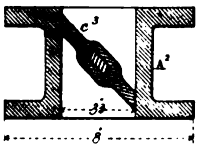
Fig. 26.



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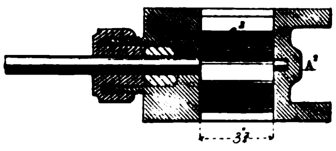

SCALE FOR FIGS 19 TO 23, 26 & 27.

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AM ENGINE DETAILS.

5.

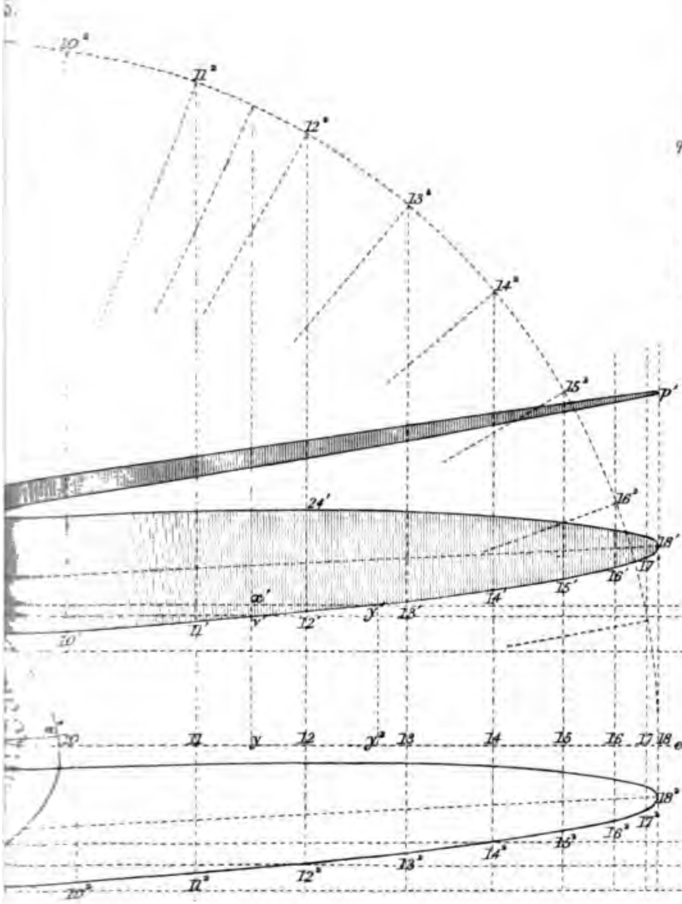


Fig. 23.

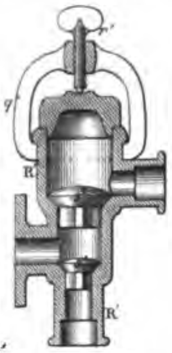


Fig. 24.

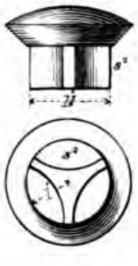


Fig. 25.

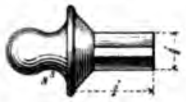


Fig. 31.

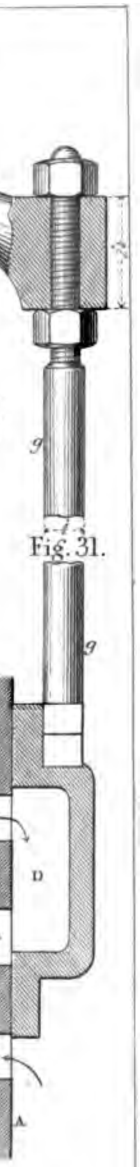


Fig. 28.

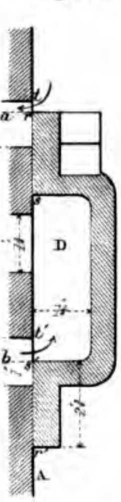


Fig. 29.

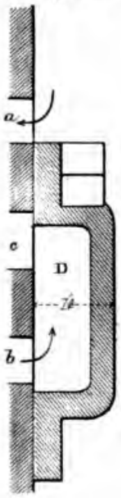


Fig. 30.

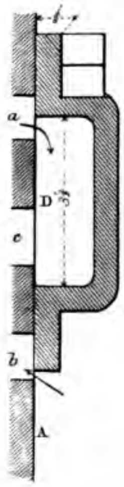
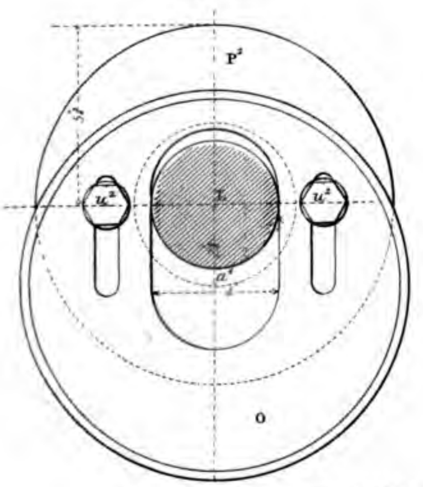


Fig. 36.



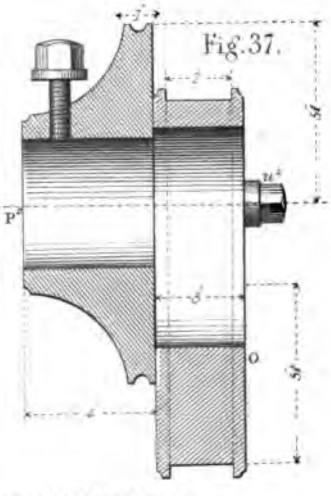
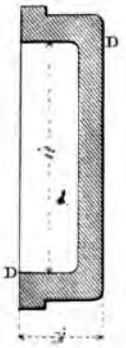
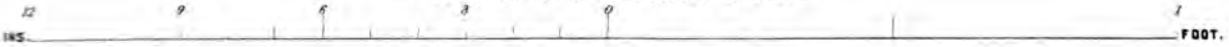


Fig. 37.

Fig. 32.



SCALE FOR FIGS 24, 28 TO 32, 35 & 38 TO 45.



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from a consideration of the diagram, Fig. 10, the outline of which is determined by perpendiculars, or ordinates, reaching nearly to the straight line, *m n*, throughout.

To compare the combined effect of a three-cylinder pump with that of two or of three double-acting pumps, we have, in Figs. 8 and 11, repeated the corresponding diagrams for the two last arrangements; and it will be remarked, that although, with cylinders of an equal area, we necessarily obtain a much larger discharge, yet the regularity of volume is not so great as in the previous example.

STEAM-ENGINES.

HIGH-PRESSURE EXPANSIVE STEAM-ENGINE.

Plates 38, 39, and 40.

385. When the steam generated in a boiler is led into a vessel, or cylinder, which is hermetically closed, it acts with its entire expansive force upon the sides and ends of such vessel, or cylinder, so that if this encloses a diaphragm, or piston, capable of moving through it in an air-tight manner, the force of the steam, in seeking to enlarge its volume, will make the piston move. It is in this way that a mechanical effect is derived from the expansive action of the steam, and it is on the same principle that the generality of steam-engines are constructed.

Thus, in most motors to which the name of steam-engine is given, the action of the steam is caused to exert itself alternately on the upper and under surface of the piston, enclosed in the cylinder, thereby causing it to make a rectilinear back and forward movement or stroke.

Steam-engines are said to be low or high pressure engines, according as the tension of the steam is only of about 1 atmosphere on the one hand, or of 2, 3, and upwards, on the other. Low-pressure engines are generally condensing engines, and high-pressure ones non-condensing; so that the terms, low pressure or condensing, high-pressure or non-condensing, are used indiscriminately, although, in modern engineering practice, what are called high-pressure condensing engines are extensively employed.

When the steam is made to act alternately above and below the piston, the engine is said to be double-acting; and, of this description are most of those employed at the present day; but if the steam acts only on one side of the piston, as is the case in many mine-pumping engines, the engine is called a single-acting one.

Low-pressure engines are generally also condensing engines; that is to say, that after the steam has exerted its expansive action upon the piston, and is on its way out of the cylinder, it passes into a chamber immersed in cold water, and termed a condenser, where it is condensed or reduced to the state of water. This condensation produces a partial vacuum in the cylinder, and consequently considerably diminishes the resistance to the movement of the piston.

In high-pressure engines, the steam which has produced its effect upon the piston escapes directly to the atmosphere, so that the piston has always to overcome a resistance equal to one atmosphere, or about 15 lbs. per square inch, acting in a direction opposite to its motion.

Steam-engines are further distinguished as expansive and non-expansive; of the latter description are those wherein the steam enters the cylinder throughout the entire stroke of the piston; so that the pressure is uniform, since the volume of steam of a given pressure which enters is always equal to the space passed through by the piston.

In expansive engines, on the contrary, the steam is only allowed to enter the cylinder during a portion of the stroke; so that the expansive power of the steam is called into action during the remainder of the movement.

The steam-engine detailed in Plates 38, 39, and 40, is one of the high-pressure class, with a variable expansion valve, and termed an upright high-pressure expansive steam-engine.

Fig. 1, Plate 38, represents an external end elevation or front view of this engine, taken on a plane at right angles to the axis of the crank shaft, the frame of which consists of a hollow column, with lateral openings.

Fig. 2 is a horizontal section, taken at the height of the line, 1—2, in Fig. 1.

Fig. 3 is an elevation of a fragment of the lower part of the column, corresponding to Fig. 2.

Fig. 4 is another horizontal section, taken at the line, 3—4, in Fig. 1; and Fig. 5 is an elevation of the capital of the column, corresponding to the part cut away in Fig. 4; in this figure, and in Fig. 3, the shadows are indicated by a light flat tint.

Figs. 6 and 7 are diagrams, relating to the movement of the governor, with its balls.

Fig. 8, Plate 39, represents a vertical section, taken through the axis of the column and the steam cylinder, on the plane, 5—6, in Figs. 2, 4, and 10, and at right angles to the crank shaft, corresponding to Fig. 1.

Fig. 9 is another vertical section, taken through the axis of the cylinder, column and crank shaft, at right angles to the preceding figure, and shows the most of the working parts of the engine in section.

And, finally, Fig. 10 is a horizontal section, taken at the broken line, 7—8—9—10, in Fig. 9, and corresponding to it.

This machine consists of a cast-iron cylinder, A, truly bored out, and enclosing the piston, B. On one side of the cylinder are cast the passages, *a*, *b*, by which the steam enters alternately above and below the piston. These passages are successively covered over by a cup, or hollow slide valve, D, the details of which are given in Figs. 28 to 31, Plate 40; and the valve is itself contained in the cast-iron chamber, E, called the *valve casing*, and communicating with a second chamber, F, called the *expansion-valve casing*; it is into this latter chamber that the steam is first conducted by the steam-pipe, G, from the boiler. The communication between the two valve casings is intercepted for short periods during the action of the machine, by the expansion valve, H, detailed in Figs. 38 to 41, Plate 40.

The vertical rod, I, of the piston, B, is attached at its upper extremity to a short cross head, or pin, e^2, which connects it to the lower forked end of the wrought-iron connecting-rod, J, the upper end of which is jointed or hung on to the pin, *f*, of the crank, K. This crank is adjusted and keyed upon the extremity of the horizontal shaft, L, termed the crank shaft, having its axis at right angles to that of the cylinder, as seen in Fig 9, and in a line with it, as seen in Fig. 8. This shaft is carried at the one end by the column, and at the other by a wall or a cast-iron frame; and it has keyed on it, at the end near the wall, the fly-wheel, M, and intermediate between the crank and fly-wheel are keyed the eccentrics, N, O, P. The first of these eccentrics is intended to actuate the distributing or ordinary steam-slide valve, D, the valve spindle, *g*, of which is connected to it by the eccentric rod, N^1, and intermediate adjustable rod, N^2. The second, O, works the expansion valve, H, by means of the rods, O^1 and spindle, *h*; and, finally, the third eccentric, P, gives an alternate movement to the piston or plunger, Q, of the feed-pump, R, through the rod, P^1.

The steam cylinder is bolted in a firm and solid manner, by its upper flanges, to the top of the hollow cast-iron plinth or pedestal, S, on which also rests, and is bolted, the column, T. The pedestal is square; and, at the corners of its base, snugs are cast, by means of which it is firmly bolted down to a solid stone foundation.

The column, T, is cast hollow, and with four large lateral openings diametrically opposite to each other, their object being to diminish the weight of the column, and to afford the necessary passages for the introduction of the various pieces when being put together, or when taken down. This column also serves as a frame for the entire machine, and above the capital is placed a cast-iron pillar-block, U, furnished with bearing brasses to receive the principal journal of the first motion, or crank shaft, L, as well as the supporting brackets, *k*, k^1, of the spindle, *l*, of the ball governor. To its inner side are also bolted the two supports, *i*, of the parallel motion, and guide, *j*, of the valve rods, *g*, and N^2.

ACTION OF THE MACHINE.

386. Before proceeding further, we shall give some idea of the general action of the machine. As already mentioned, the steam is generated in a boiler, such, for example, as that represented in Plate 14, and is conducted by the steam-pipe, G, into the first chamber, F; when the valve, H, in this chamber uncovers the orifice, or port, *d*, the steam finds its way into the valve-casing, E, whence its passes either to the upper or to the lower end of the cylinder, accordingly as the valve, D, uncovers one or other of the two ports or passages, *a*, *b*. Now, when the piston is, for example, at the top of its stroke, the passage, *a*, is almost fully open, whilst the channel, *b*, is in communication with the exit or exhaust port, *c*, from which the two pipes, e^4, conduct it to the atmosphere. If, on the introduction of the steam to the cylinder, it has a pressure of, say four atmospheres, it follows that it will act upon the piston with all this force to cause it to descend; since, however, the lower part of the cylinder is at this time in communication with the external atmosphere, there is a resistance equal to one atmosphere opposed to its movement, therefore the actual effective pressure acting on the top of the piston will be equal to three atmospheres.

It is the same when the piston reascends; the valve uncovers the port of the passage, *b*, to allow the steam to enter the lower end of the cylinder, whilst the port, *a*, is put in communication with the exhaust port, *c*, by the cup of the slide-valve, D, to give an outlet for the steam which has just acted on the upper side of the piston during the down-stroke.

It is to be remarked, that if the introduction of the steam takes place during the entire up-and-down-stroke of the piston, which might be the case if the steam-pipe, G, communicated directly with the valve-casing, E, and if the valve, D, kept one of the ports uncovered throughout the entire stroke, the pressure of the steam would remain constant; in such case, it would be said that the machine was a high-pressure non-expansive engine—that is to say, that it worked with a full allowance of steam.

In the engine, however, which at present occupies our attention, the steam is first introduced into the casing F, the valve, H, of which, at each stroke, closes the passage, *d*, communicating with the second casing, E, before the piston reaches either end of its stroke. It follows, that the steam contained in the cylinder at the time of closing the passage, *d*, must augment in volume or expand, whilst its pressure will consequently decrease during the remaining advance of the piston: the engine is then said to be working *expansively*; and in this case a quantity of steam is expended for each stroke, equal only to a third, half, or two-thirds of the capacity of the cylinder, according as the introduction of the steam is intercepted at one-third, one-half, or two-thirds of the stroke; it is the ratio between the quantity of full steam-pressure introduced, and the entire capacity of the cylinder, which expresses the degree of expansion at which the engine works.

PARALLEL MOTION.

387. The rectilinear alternate movement of the piston is transformed into a continuous circular motion on the first motion shaft, L, by the intervention of the connecting-rod, J, and crank, K; but with this arrangement there is naturally a lateral strain upon the top of the piston-rod, I, and in order that its movement may be perfectly rectilinear and vertical, it is jointed to a system of articulated levers, forming what is termed a parallel motion.

This mechanism is composed of two wrought-iron rods, or radius bars, V, Figs. 1, 4, and 8, which oscillate on the fixed centres, *i*, and are articulated at their opposite extremities to the levers, X, near their middle, by means of the pin, *n*. The levers, X, are also of wrought-iron, and are jointed at one end to the cross head, e^2, Figs. 4, 8, and 9, of the piston-rod; and at the other to the racking lever, Y, attached to a cross spindle, *o*, and oscillating in bearings, in a couple of cast-iron brackets, Z, bolted to the lower part of the frame, or pedestal, S.

The head of this last-mentioned oscillating lever is detailed separately, in Figs. 21 and 22, Plate 40. It has brasses, to embrace the journal of the spindle, p^4, by which it is connected to the ends of the levers, X.

The combination of this mechanism is such, that the point of attachment, e^2, constantly moves in a straight line throughout the entire stroke. It may be designed on geometrical principles, as indicated in the diagrams, Figs. 8 and 11. To this effect we have supposed, that after having drawn the horizontal line, $e^2\,p$, and the vertical, $e\,e^3$, distances, $e\,e^2$ and $e^2\,e^3$, are set off on the latter, equal to the half stroke of the piston, or to the radius of the crank; then, with the points, e, e^3, describe an arc, with a radius, $e\,p$, equal to the length of the lever, X, which is taken at pleasure, but should never be less than the stroke of the piston. If we next lay off this distance from e^2 to p^2, the space, $p\,p^2$, will express the amount of oscillation of the rod, Y, the centre of the oscillation *o*, of which we place below, on the vertical line, drawn at an equal distance from and between the two points, p, p^2. We next fix the point, *n*, of attachment of the rods, V, to the lever, X. This point, *n*, during the movement of the parallel motion, necessarily describes a circular arc, of which it is requisite to find the centre. In investigating this problem, it is to be observed that, whatever may be the position of the lever, the point, *n*, is always at an equal distance from the extremity, *p*, or the other one, *e*. If, then, we in succession draw the lines, $p\,e$, $p^1\,e^1$, $p^2\,e^2$, $p\,e^3$, indicating the different positions of the lever, corresponding to those, e, e^1, e^2, e^3, of the piston-rod end, we shall, on each of these lines, obtain the several positions, n, n^1, n^2, n^3, by laying off on them either of the distances, $p\,n$ or $e\,n$. We can then very easily find the centre of the arc passing through these points, (10 and 11).

Fig. 12 represents the diagram of an analogous parallel motion, but one in which the rods, V, are so disposed, that their point of attachment is exactly in the middle of the levers, X; and in this case, their axis of oscillation lies in a plane passing through the vertical axis, $e\ e^3$.

DETAILS OF CONSTRUCTION.

STEAM CYLINDER.

388. The cylinder is cast in one piece with its bottom cover and lateral steam passages. As it should be bored with great care, so as to be perfectly cylindrical in the interior, a central opening is made in the bottom for the passage of the spindle of the boring tool; this opening, however, is afterwards closed by the small cover, a^3, cemented at its junction surfaces, and bolted down to the bottom of the cylinder. The upper end of the cylinder is closed by a cast-iron cover, A^1, which is formed into a stuffing-box in the centre, to embrace the piston-rod, which works steam-tight through it. The packing is compressed or forced down for this purpose by a gland, (142,) bolted to the stuffing-box, and hollowed out at the top to receive the lubricating oil. The valve-face, on the outside of the cylinder, and on which the valve, D, works, is planed very accurately, so as to be a true plane throughout. The same is done with the valve-casing at the flanges, where it is fitted to the valve-face, and all the edges of the ports and valve are carefully measured and squared to correspond to each other.

PISTON.

The piston, Figs. 8, 9, 19, and 20, is composed of two cast-iron plates, which have an annular space between them for the reception of two concentric cast or wrought-iron or brass packing-rings, c^1. These rings are cut through at one side, and are placed one within the other in such a manner, that the breaks in each are diametrically opposite to each other; their thickness gradually diminishes on each side towards the break, and they are hammered on the inside in a cold state, which renders them elastic, giving them a constant tendency to open. Since the diameter of the outer ring is equal to that of the cylinder when the two edges are brought together, the elasticity of the inner ring, combining with that of the outer one, tending constantly to enlarge them, it follows that there must be a perfect coincidence between the outside of the ring and the inside of the cylinder throughout the whole extent of the latter. Thus the contact of the piston with the sides of the cylinder only takes places through the packing-ring, and not by the plates, which are of a slightly less diameter. To prevent the passage of the steam through the break in the outer packing-ring, a rectangular opening is made in the two edges of the ring, and in this is placed a small tongue-piece, a^2, screwed to the inner ring, this piece serving to close or break the joint without preventing the play of the rings. The principal plate or body of the piston is fixed to the piston-rod by means of a key, Figs. 9 and 19. The piston-rod is made of increased diameter at its lower end. The upper end of the piston-rod is likewise fixed in a socket, I^1, Figs. 9 and 13, which terminates in two vertical branches to receive the middle of the spindle, e^2, which is held down by means of a key.

CONNECTING-ROD AND CRANK.

389. The connecting-rod, J, Figs. 8, 9, 14, and 15, terminates at its lower end in a fork, by means of which it is jointed to the crosshead e^2, on each side of the piston-rod head, sufficient space being left between them for the ends of the levers, X; brasses are fitted in either side, and secured by straps passing under them, and secured by gibs and cotters above. The head of the connecting-rod, Figs. 15 and 16, is likewise fitted with brasses to embrace the pin, f, of the crank; these brasses are tightened up by means of the pressure screw, f^1.

The crank, K, like the connecting-rod, J, is of wrought-iron, being adjusted on the end of the first motion or crank-shaft, and besides being shrunk on by heating, and then cooling it after it is on the shaft, it is further secured to it by a key. This crank is very often made of cast-iron in stationary engines, but in marine and locomotive engines it is generally forged, so as to be better suited for resisting severe strains and shocks.

The crank shaft, L, is likewise either formed of cast or wrought-iron, and besides being supported by the brasses of the pillow-block, U, it is also carried by those of a similar block, fixed, (we shall suppose), upon a cast-iron plate or box built into the wall which divides the engine-house from the workshop or factory. It should always be larger in diameter where it receives the fly-wheel, M, so as to give greater surface and leverage to hold the keys.

FLY-WHEEL.

390. The fly-wheel is of cast-iron—of a single piece in the present example, because its diameter is not more than 10 or 12 feet. When of larger dimensions, the rim and the arms are cast in separate pieces, and then bolted together. For wheels of from 15 to 25 feet in diameter, the rim is made in several pieces, and the arms are also cast separate from the boss, and all the parts are then fitted and bolted together. The arms are sometimes made of wrought-iron in wheels of small dimensions, with the view of reducing the weight near the centre, without reducing the effect of the wheel.

FEED-PUMP.

391. This pump serves to force into the boiler a certain quantity of water, to replace that which is converted into steam and expended in actuating the engine. It is a simple force-pump, consisting of a cylinder, R, in which works the solid piston or plunger, Q. The piston is not in contact with the sides of the pump cylinder, and the latter consequently only requires to be turned out at its upper part, where it is formed into a long stuffing-box and guide for the plunger, being necessarily air-tight.

On one side of the pump is cast a short pipe, to which is attached the valve-box, R^1, generally made of brass. To the lower part of this is secured the suction-pipe, T^1, communicating with a cistern of water, and having a stopcock, S^1, upon it, somewhat like the one represented in detail in Plate 17. To one side of the valve-box is likewise fitted the discharge-pipe, carrying a similar stopcock, S^2; this last pipe is generally passed through the pipe which carries off the waste steam, so that the water may take up some of the heat of this steam before entering the boiler.

It will be seen, from Figs. 9 and 23, that this valve-box contains two valves, s^1, s^2; the lower one of which, s^1, is the suction-valve, and the upper one, s^2, is the discharge-valve. The latter is much larger in diameter than the former, so that its seat may be wide enough for the lower valve to be passed through it. The upper end of the valve-box is closed by a cover, which is firmly held down by the screw, r^1, and iron-bow or bridle, q^1, see also Figs. 10, 11, and 12, Plate 11, (144.)

Both valves are made conical at the seat, as shown in Fig. 24, so as to fit more easily. The under part of the valve is cylindrical, so as to guide it; but it is cut away at the sides, to allow of the passage of the water when it rises. It is from the appearance this gives, that these valves are called *lantern valves*.

The pump cylinder is further furnished with a safety-valve, s^3, of which Fig. 25 is a detailed view. The object of this is to permit the air to escape, when it accumulates within to such an extent as to destroy the action of the pump. This valve is horizontal, and is kept in its place by the bell-crank lever, u, upon the horizontal arm of which is suspended a weight, sufficient to counterbalance the internal pressure.

The top of the plunger is surmounted by a small rod, t^1, adjustable in the socket which terminates the long wrought-iron eccentric rod, P^1, Figs. 9 and 18, the upper extremity of which is formed into a collar, embracing the circular eccentric, P.

The action of this pump is analogous to that of the pumps of which we have already given a description, in reference to Plate 37. Thus, when the machine is working, and the stopcocks, s^1 and s^2, are open, the water rises from the cistern by the pipe, R^1, the valve s^1, opening, to give it passage into the body of the pump, into which it flows as long as the piston ascends. When, however, the piston descends, the water is driven back, and, closing the lower valve, s^1, necessarily opens the upper one, s^2, and proceeds along the discharge-pipe to the boiler. The quantity of feed-water is regulated by means of the stopcocks, and may be entirely shut off by closing them; but then, in such case, as the eccentric, P, with its rod, P^1, will continue to move, it will be necessary to loosen the plunger, Q, which is done by unscrewing the thumb, v^1, by which the rod, t^1, of the plunger is attached to the eccentric rod, P^1, in such a manner that the socket, b^3, Fig. 18, at the end of the eccentric rod, P^1, will simply slide up and down the rod, without moving it.

BALL OR ROTATING PENDULUM GOVERNOR.

392. The object of this piece of mechanism is the regulation of the velocity of the machine, in proportion to the resistances to be overcome; and, accordingly, to this effect it opens or shuts a valve, c^3, placed in the steam-pipe, G, and called a throttle-valve. Just as a freer or narrower passage is left for the steam by the opening or closing of this throttle-valve, which is contained in an especial box, to facilitate adjustment, and is actuated by a rod, passing through a stuffing-box at the side—so is the quantity of steam which finds its way to the cylinder more or less; and, similarly, the consequent acceleration or retardation of the motion of the piston, as well as of the first motion shaft in connection with it.

It is composed, as seen in Fig. 1, of the vertical spindle, l, stepped, at its lower extremity, in the end of a small bracket support, k^1, and is held higher up by a second bracket, k. To its upper end are jointed two symmetrical side rods, m^1, each terminating in cast-iron or brass spheres, o^1. These side rods are also connected by means of the intermediate links, l^1, to the wrought-iron or copper socket or ring, i^1, moveable upon the main spindle.

A rotatory motion being given to the vertical spindle, and the balls being carried round with it, will have a constant tendency to fly off from the vertical line, by reason of the centrifugal force due to the rotation (251); as long as the rotative velocity remains the same, the balls will tend to occupy the mean position indicated upon the drawing, and corresponding to the *normal* velocity; that is to say, the velocity to which the apparatus is regulated. When this velocity is exceeded—in consequence, as we may suppose, for example, of some parts of the machinery being put out of gear—the balls will fly asunder, and occupy the extreme position, o^3, indicated upon Fig. 6. In this position of the balls, the socket, i^1, will be lifted up. Now this socket is embraced, at its circular groove, by the prongs of the forked lever, j^1, Fig. 9, which is connected to the vertical rod, h^1, and this by a suite of levers and bell-cranks, g^1, g^2, g^3, g^4, and g^5, communicates with the throttle-valve, c^3, drawn with its box, A^2, in Figs. 26 and 27, Plate 40. It follows, from the combination of these connections, that, as the socket rises, the valve will be shut. If, on the contrary, the velocity should be reduced below the proper point, owing to an increased resistance, the balls will approach each other, and assume the position given in Fig. 7. The socket, i^1, will descend, and, consequently, the throttle-valve will become more open, so as to allow a greater quantity of steam to enter the valve-casing, and thence pass into the cylinder. The extreme positions of the governor arms, beyond which they cannot go, are determined by the guides, m^2, fixed upon the spindle, l.

The motion of the governor spindle, l, is derived from the first motion shaft, L, by means of the grooved pullies, p^3, fixed upon the intermediate spindle, r^2, placed close to the capital of the column, T, and by the bevel-wheels, r^3, receiving their motion from the pulley, r^4, so that a constant ratio is maintained between the rate of the machine and that of the governor.

The geometrical diagram, Figs. 6 and 7, will sufficiently explain the respective positions of each of the pieces of the pendulum, and will show how the rising of the socket upon the spindle is caused by, and is in proportion to, the flying asunder of the balls, according to the number of revolutions of the spindle, and the length of the suspending arms.

MOVEMENTS OF THE DISTRIBUTION AND EXPANSION VALVES.

DISTRIBUTION VALVE.

393. We have seen that the valve, D, represented in different positions in Figs. 28 to 31, and in horizontal section, Fig. 32, is attached, by its rod, g, to the vertical rod, N^2, which is joined to the rod, N^1, of the circular eccentric, N, Figs. 33 and 34. When, as was customary until lately, the centre of the eccentric lies in a radius perpendicular to the direction of the crank, the movements of the steam piston and valve are different to each other—that is to say, when the crank passes from the left horizontal to the right horizontal position, the piston makes a corresponding rectilinear movement; the eccentric, however, passes from the lower extremity of the vertical line, drawn through the centre of the first motion shaft, to the upper extremity, or *vice versa*, and consequently gives the valve a rectilinear movement quite different to that of the piston, in such a manner that, when the latter is at the middle of its stroke, the valve, on the other hand, is at the end, and the steam-ports are consequently fully open, to give the steam the freest passage into the cylinder.

Whilst the piston is accomplishing its stroke in one direction, the valve moves up or down, and returns again to its central position, the part which it covered being opened and again shut; when, however, the crank makes two fourths of a revolution in different directions, the piston rises and falls half a stroke each way, whilst the valve makes a single rectilinear movement in one direction.

Finally, for each of these movements, whilst the velocities of the piston are increasing from the commencement towards the middle of its stroke, those of the valve are decreasing, and reciprocally. It therefore follows, that the maximum space passed through by the piston, for a given portion of a revolution of the crank, corresponds to the minimum passed through by the valve.

LEAD AND LAP OF THE VALVE.

394. Of late years, engineers have recognized the advantage of inclining the radius of the eccentric, with regard to the radius of the crank, instead of placing them perpendicular to one another, in such a manner that, at the *dead points*—that is, the extreme high and low positions of the piston—the valve shall already have passed the middle of its stroke to a slight extent; it is this advance of the valve which is termed the *lead*.

The effect of giving this lead to the valve, is to facilitate the introduction of the steam into the cylinder at the commencement of the piston's stroke, and at the same time to allow a freer exhaust of the waste steam on the other side of the piston; a greater uniformity of motion is in consequence obtained, whilst less force is lost.

In order to avoid as much as possible the back pressure due to the slow exhaust of the waste steam, it is likewise customary, in addition to the lead, to give the valve more or less *lap;* that is to say, to make the width of that part of the valve which covers the ports, *a*, *b*, Fig. 28, sensibly greater than that of the ports themselves.

In explanation of the effects due to the lead and lap of the valve, we have, in Fig. 35, given a geometrical diagram, indicating the relative positions of the crank, the piston, the eccentric, and of the valve.

Let *o* O represent the radius of the crank; with this distance as a radius, and with the centre, O, describe a semicircle, which divide into a certain number of equal parts. From each of the points of division, let fall perpendiculars upon the diameter, *o c*. The points of contact, 1, 2, 3, 4, &c., represent upon this diameter, considered as the stroke of the piston, the respective positions of the piston, corresponding to those, 2^2, 3^2, 4^2, &c., of the crank pin. It is unnecessary to take into account the length of the connecting-rod, which connects the latter to the piston, because, in the present case, the connecting-rod is supposed to be of an indefiuite length, and to remain constantly parallel to itself, so that it cannot modify the results.

With the centre, O, likewise describe a circle with a radius, O a^1, equal to that of the eccentric, N. We have assumed the point, a^1, to be the position the centre of the eccentric should have at the moment when the piston is at the end of its stroke—that is to say, at *o;* the distance of this point, a^1, from the vertical, *m n*, expresses the lead of the valve, and consequently the angle, *m* O a^1, is called the angle of lead. The position of the point, a^1, may likewise be obtained, after the following data are decided on—namely, the height of the ports, *a*, *b*, Fig. 28, the width, *r s*, of the flange of the valve, which is equal to the height of opening, *t r*, which properly expresses the amount of lead given of the port augmented by twice the lap, together with the amount to the introduction of the steam to the cylinder, and the amount of opening, s^1 t^1, expressing the lead given to the escaping steam, and which is always greater than the former, so that the exit or exhaust passages may be in communication as long as possible.

The diameter of the eccentric, N, is equal to the height of the port, augmented by the width, *r s*, of the flange of the valve, and the difference which exists between the two amounts of lead, s^1 t^1 and *r t;* it is, then, with the half of this as radius that the circle, a^1 b^1 c^1 d^1, must be described; and we then obtain the point, a^1, by setting off from the centre, O, to the right of the vertical, *m n*, a distance equal to the lead of introduction, *r t*, augmented by the lap. Starting from this point, a^1, we then divide this circle into as many equal parts as we previously divided the one into, described by the crank pin, and then through each of the points of division we draw perpendiculars to the vertical, *m n*.

We further draw the straight line, a^1 g^1, parallel to *m n*, when the distance of the several points of division from this line will indicate the successive positions of the valve in relation to those of the piston. Thus, after having drawn the horizontals, *r u*, through the extreme point, *r*, of the valve, at the moment when the piston is at the extremity of its stroke, make 1^2—1^1 equal to b^1 b^2, and the point, 1^1, indicates how far the valve has descended during the time the piston has traversed the space, *o* 1, whilst the crank has described the first arc, *o* 1^1. In like manner, set off the distances, c^1 c_2, d^1 d_2, &c., which correspond to the third and fifth divisions, reckoning from the horizontal line, *r u*, from i^2 to 3^1, and from k^2 to 5^1, on the verticals corresponding to the third and fifth positions of the piston, and consequently the positions, 3^2 and 5^2, of the crank. It will then be seen that the valve continues to descend until the moment the centre of the eccentric reaches the point, f^1, upon the horizontal line, O f^1, corresponding to the sixth position, and the valve then wholly uncovers the port, *a*, as shown in Fig. 29. During the continued revolution of the eccentric, on passing this point the distances of the points of division from the line, a^1 g^1, diminish, and the valve reascends, in such a manner as that, when the centre attains the point, *p*—that is to say, when the crank shall have performed a semi-revolution, and the piston have arrived at 18, at the other end of its stroke—the valve will occupy the position indicated in Fig. 30. This figure shows that it uncovers the lower port, *b*, for the introduction of the fresh steam, and the upper one, *a*, for the escape of the used steam. If the respective positions, 6^1, 7^1, 8^1, 9^1, &c., of the valve be determined throughout the entire stroke, by setting off upon the verticals, 6, 7, 8, 9, &c., the distances of the points of division of the eccentric from the straight line, a^1 g^1, as already explained, a curve will be formed, as at *u* 3^1 6^1 9^1 18^1, which is a species of ellipse. This diagram has the advantage of bringing into a single view the relative positions of the crank, piston, eccentric, and valve, and facilitates the determination of the position of the valve, corresponding to any position of the piston.

Thus, to obtain the position of the valve, to correspond to that of, *y*, of the steam-piston, it is sufficient to draw the vertical, *y* x^1, which will cut the curve in the point, v^1. The distance, v^1 x^1, of this point, from the horizontal, *t* u^1, passing through the upper edge of the introduction port, *a*, shows how much of this is uncovered by the valve. It will be seen, also, that the curve is cut by the horizontal, *t* u^1, in the point, y^1, which indicates the moment at which the valve closes the port. In this position the piston will only as yet have reached the point, y^2, of its stroke; and it has, consequently, to traverse the distance, y^2 18, the remainder of the stroke, before it can receive any more steam from the boiler, which shows that, with a valve which has lead and lap, we actually work the steam expansively to a slight extent. In the case before us, the steam is cut off at four fifths of the stroke.

It will be understood that, if the machine continues its action, the piston will retrace its stroke, the centre of the eccentric which had reached p will continue to ascend, and the valve will shortly attain the position indicated in Fig. 31, this taking place as soon as the centre of the eccentric reaches the point, z. In this position, the ports, a, b, are completely open—the first to the exit or eduction aperture, c, the other to the induction of the steam, whilst the valve is at the highest point of its stroke, as will also be found by continuing the curve, $u\ 9^1\ 18^1$, of which the prolongation, $18^1\ 24^1\ 30^1$, is exactly symmetrical with regard to the inclined line, $u\ 18^1$. On the same diagram, Fig. 35, we have delineated a second elliptic curve, $0^2\ 9^2\ 18^2$, equal and parallel to the first, and which indicates the respective positions of the point, s^1, of the lower edge of the cup of the valve, in relation to the port, b, so as to have, at the same time, the positions of this edge of the valve, and consequently the relation and amount of exhaust due to, or given by, the same eccentric, at any particular point in the piston or the crank's motion. This outline is obtained by setting off the constant distance, $r\ s^1$, of the valve, Fig. 28, upon the verticals, drawn through 1, 2, 3, 4, &c.

It may be remarked, that the distance between the ports, a and b, is arbitrary. It is, however, advisable to reduce it as much as possible in order to diminish the surface of the valve, and, consequently, the pressure of the steam acting on the back of it. In all cases, it is necessary that the height of the exhaust port, c, should be greater than that of the steam ports, by a quantity at least equal to the difference which exists between the lap and the lead, $t^1\ s^1$ and $t\ r$.

EXPANSION VALVE.

395. The action of the expansion eccentric, O, is analogous to that of the ordinary valve eccentric, except that the position of its centre is not regulated in the same manner.

We may observe, in the first place, that this eccentric is not immoveably fixed upon the main shaft, L, as is the case with the preceding one. It is only attached to the adjustable collar-piece, P^2, Figs. 36 and 37, by screws, u^2. This arrangement allows of its throw being increased or diminished; that is, of its centre being placed further from or nearer to that of the shaft, according to the length of stroke which it is wished to give it. To this end, its central opening is oblong in shape, and the holes for the securing screws are oblong likewise.

If the centre of this eccentric happens to be in the same direction as the crank, the expansion valve, H—the spindle, h, of which is guided by the socket-bracket, h^2, attached to the pedestal, S, and drawn more detailed in Fig. 43—is wholly open when the piston is at the end of its stroke; but we have supposed, as indicated in Fig. 35, that the centre of this eccentric is in the point, a^4, upon the circle described with the centre, O, and radius equal to L a^4, Fig. 36, of the eccentric, and that the valve does not therefore wholly uncover the entrance port, d, at this moment, so that at the time of closing it may be later on than would otherwise be the case.

As in the preceding case, we divide this circle into equal parts, starting from the point, a^4; through the point, a^4, draw a vertical line, and then set off on the various verticals, 1, 2, 3, 4, &c., the distances of the points of division from this line, measuring these from the horizontal passing through the upper edge, r^2, of the valve, H; we thus obtain a second elliptic curve, $u\ m^1\ n^1\ p^1$, the inside of which is flat—tinted with a slightly stronger shade than the ellipse corresponding to the distribution valve, so as to render the diagram more distinct. This curve cuts the horizontal line drawn through the upper edge of the port, d, in the point, n^1, which indicates at what time the valve, H, closes the entrance port, Fig. 39. It will be seen that this point corresponds to the position, 6, and 6^1, of the steam piston and crank, thereby signifying that the cut-off takes place when the piston has performed no more than a fourth of its stroke. Continuing the movement, it will be observed that the valve, H, rises higher and higher, so that it begins to uncover the entrance port a little before the piston reaches the end of its stroke; but it is evident that the steam cannot find its way into the cylinder at this point, for the distribution valve is in its turn closed, as soon as the position, $y^1\ y^2$, is passed; no inconvenience, therefore, will be caused by the fact of the valve, H, being open before reaching the end of its stroke, as indicated in Figs. 38 and 40, and as shown also in the diagram, Fig. 35.

By varying the radius of the eccentric, O, and the position of its centre relatively with the radius of the crank, it will be easily understood, that within certain limits we can alter the time when the valve, H, opens and closes the entrance port, and are consequently enabled to vary the degree of expansion.

Figures 41 and 42 show that the rod of the valve is attached to it by a T joint, which leaves the valve sufficiently free for the steam to press it constantly against the planed valve face; and a similar adjustment is adopted with the distribution valve.

The general explanations which we have given in the preceding pages, with reference to the construction and action of this engine, evidently apply to other systems, which merely differ in some of the arrangements and forms of the component pieces. Moreover, in our notes, the student will find most of the rules and formula necessary for the calculations in designing steam-engines.

RULES AND PRACTICAL DATA.

REMARKS ON MACHINE TOOLS.

VELOCITY OF THE TOOL, OR OPERATING PIECE, IN MACHINES INTENDED TO WORK IN WOOD AND METAL.

396. We now proceed to a very important practical inquiry in the construction of turning-lathes, planing, boring, and drilling machines, namely, the most advantageous cutting velocity for the different kinds of material.

It cannot, we apprehend, have escaped the observation of our readers who are in the habit of turning metal, that when a velocity exceeding certain prescribed limits is imparted to the material, the edge of the cutting-tool applied to reduce the surface of that material is brought to a soft state and rendered useless. This is an acknowledged fact, and many ingenious contrivances have been, from time to time, introduced to meet the exigency of the case, or in other words, to regulate the speed of the lathe according to the nature and diameter of the metal or other substance to be turned. It is commonly supposed that for wood the velocity cannot be too great, yet this is probably a vulgar error, since if we allow the speed to pass certain limits the tool necessarily becomes hot, loses its temper, and ceases to cut. Wrought-iron requires a slow motion, and cast-iron, above all, ceases to be cut by the edge of the tool, unless a very slow and regular motion is preserved, otherwise the tool only acts by abrasion, and actually grinds away on the face or cutting edge.

The opinion of practical men is much divided on this point—some name from ten to fifteen feet per minute as a *maximum* velocity, others allow thirty to forty feet, whilst others again regard this as the *minimum* speed which should be given to cast-iron, in order to obtain the greatest effect from the tool.

For turning or boring cylinders, or indeed any substance of which the diameter is nearly equal throughout, the velocity of the machine continues uniform; but with an increasing or diminishing diameter the speed can be easily increased or diminished by means of conical pulleys placed in opposite directions, as also by many other mechanical contrivances, which are too familiar to practical men to require any extended description. An approximate rule for the velocity of cast-iron turned, by means of the slide-rest, is about 5·5 inches per second; the metal is turned by hand-tools, the speed at its circumference is from 7 to 7·5 inches per second to rough out, and from 11 to 12 inches per second to finish.

The difference of velocity of the work, or the tool, when the turning is effected mechanically, or by the hand, is deduced from the obvious fact, that in the former case the contact between the tool and the work is constant and invariable, whilst in the latter it is intermittent.

The velocity at the circumference of the material, or of the tool for turning or planing very hard cast-iron, should be reduced to about 2·5 to 3 inches per second; and for widening or broaching cast-iron the velocity should be from about 1·5 to 2 inches per second when the skin of the metal is broken; but if a steam cylinder, for instance, is set in the boring-mill the first cut should be still farther reduced, and may be taken roughly at about 1 inch per second.

M. Armengaud, a distinguished French engineer, has given the annexed table, which indicates the average degree of speed, as well for turning as boring cast-iron, the first two columns for the former, and the second two for the latter process.

Diameter in inches.	Revolutions of spindle per minute.	Diameter in inches.	Revolutions of boring-bar per minute.
1	50	1	25
2	25	2	12·5
3	16·67	3	8·33
4	12·50	4	6·25
5	10	5	5
6	8·32	6	4·16
7	7·15	7	3·57
8	6·25	8	3·125
9	5·55	9	2·77
10	5·	10	2·5
15	3·33	15	1·66
20	2·50	20	1·25
25	2	25	1
30	1·667	30	0·833
35	1·430	35	0·714
40	1·250	40	0·425
45	1·12	45	0·56
50	1	50	0·5
60	0·834	60	0·417
70	0·716	70	0·358
80	0·626	80	0·313
90	0·554	90	0·278
100	0·50	100	0·25

The foregoing rules apply only to cast-iron; however, the maximum speed for malleable-iron in general may be obtained by multiplying the already stated velocities by ·4. If we double the speed of malleable-iron we are still within bounds for brass; and as a general rule, the softer the material the greater the speed required; for instance, soft woods require a greater speed than those known as hard wood, such as beech, oak, ash, &c. We do not consider these rules as the exact velocities, but merely give them as a near approximation to the every-day practice of those engaged in this kind of work:—the nature of the material, the shape and strength of the tool, always regulates the most economic velocity.

HYDRAULIC MOTORS.

397. The fall of a stream of water varies with the locality, and gives rise to the employment of different kinds of hydraulic motors, which are denominated according to their several peculiarities as follows:—

UNDERSHOT WATER-WHEELS.

The term undershot is applied to a wheel when the water strikes at, or below, the centre, as that shown in Fig. 1, Plate 36, (377); and the greatest effect is produced when the periphery of the wheel moves with a velocity of ·57 that of the water. To find the velocity of the water, multiply the square root of the perpendicular height of the fall in feet by 8, and the product is the velocity of the water in feet per second, which, when multiplied by ·57, the product gives the velocity of the wheel.

Example.—Required the maximum velocity of an undershot wheel, when propelled by a fall of water 6 feet in height.

$$\sqrt{6} = 2{\cdot}45$$

Then $2{\cdot}45 \times 8 = 19{\cdot}6$ feet, the velocity of water.

And $19{\cdot}6 \times {\cdot}57 = 11{\cdot}17$ feet, per second for the wheel.

BREAST AND OVERSHOT WHEELS.

Wheels that have the water applied between the centre and the vertex are termed breast wheels, and overshot wheels when the water is brought over the wheel to the opposite side, as shown in Fig. 13, Plate 36, (381). In both cases the maximum velocity at the periphery of the wheel is generally taken at about $\frac{2}{3}$ that of the water; hence, to find the velocity at the periphery of an overshot wheel, we have only to multiply the velocity of the water due to the height of head by the numerator 2, and divide by the denominator 3.

Example.—Required the velocity at the periphery of an overshot wheel when propelled by a fall of water 6 feet in height.

$$\frac{\sqrt{6} \times 8 \times 2}{3} = 13{\cdot}06 \text{ feet per second for the wheel.}$$

But one-tenth of a foot of head must be added for every foot of increase in the diameter of the wheel, from 15 to 20 feet, and ·05 more for every foot of increase from 20 to 30 feet, commencing with five-tenths for a 15 feet wheel.

This additional head is intended to compensate for the friction of water in the aperture of the sluice to keep the velocity as 3 to 2 of the wheel.

When the water flows from under, or over the sluice, we refer the reader to the formula and data given at page 124, (336).

In estimating the power of water wheels, half the head must be added to the whole fall, because 1 foot of fall is equal to 2 feet of head; call this the effective perpendicular descent; multiply the weight of the water per second by the effective perpendicular descent and by 60; divide the product by 33,000, and the quotient is the effect expressed in horses' power.

Example 1.—Given 16 cubic feet of water per second, to be applied to an undershot wheel, the head being 12 feet, required the power produced.

$$12 \div 2 = 6 \text{ and } \frac{6 \times 16 \times 62{\cdot}5 \times 60}{33000} = 10{\cdot}9$$

horses' power nearly.

Example 2.—Given 16 cubic feet of water per second, to be applied to a high breast or an overshot wheel, with 2 feet head and 10 feet fall; required the power.

$$2 \div 2 = 1 \text{ and } \frac{\overline{1 + 10} \times 16 \times 62{\cdot}5 \times 60}{33000} = 20$$

horses' power.

CONSTRUCTION OF THE WHEEL.

398. The diameter to be given to a wheel of this description has not been accurately determined, because it has not a direct influence upon the useful effect that may be obtained from it. Nevertheless, it is manifest that it should not be too small; for in that case the water would be admitted too nearly in the horizontal line passing through the centre, or even above it, which would cause great loss of power. Neither should it be too great, for in that case the exaggerated dimensions would but involve an increased bulk and weight, and, consequently, a greater load and more friction, without any compensating advantage.

In general, for a fall of from 7 to 10 feet, it is advisable to make the extreme radius of the wheel at least equal to the mean height of the fall, augmented by twice the depth of the water upon the edge of the outlet.

Thus, in the case before us, the height of the fall being limited to 10 feet, the outer radius of the wheel should not be less than 10 feet, plus twice the depth of the overflowing body of water when at its fullest.

Water-wheels on the same system, with a fall of water of about 8 feet, have often an extreme diameter no greater than this.

Supposing the fall to be 6 feet, and the number of turns per minute required to be $10 = N$; then the diameter will be $\frac{46}{10} \times \sqrt{6} = 4{\cdot}6 \times 2{\cdot}45 = 11\frac{1}{4}$ feet nearly.

If we put u to denote the velocity of the extremities of the floats, which in this case is 10 feet per second, and N the number of turns desired in a minute, the diameter will be expressed by

$$\frac{60\ u}{\pi N} = 19{\cdot}1$$

And to obtain an effect approaching the maximum, we have the diameter expressed in terms of the velocity and height of fall thus:—

$$19{\cdot}1 \times \sqrt{H} = 46$$

$$\text{And } \frac{46}{N} \times \sqrt{H} = 11\tfrac{1}{4} \text{ feet diameter of wheel.}$$

This is nearly the minimum diameter of wheel which would under any circumstances be employed; 12 feet to 25 feet may indeed be taken as the usual range; but unless the volume of water be extraordinarily great—breadth is then better than diameter—we cannot conceive of any advantage, other than may arise from some peculiarity in the nature of the machinery to be impelled, which may not be obtained with a diameter of 16 feet to 18 feet. It is, however, to be observed, that the smaller the diameter the greater is the nicety of adjustment required to make the water yield its effect upon the buckets; and possibly the errors which have been committed in this particular have led to the common opinion that a wheel of large diameter is always in practice the most effective.

To find the centre of gyration of a water-wheel, take the radius of the wheel and the weight of its arms, rim, shrouding, and float boards. Then call the weight of the rim, R, which must be multiplied by the square of the radius, and the product be doubled and then carried out. Next the weight of the arms, called A, must be multiplied by the square of the radius, and be doubled and carried out as before. Then the weight of the water in action, called W, must be multiplied by the square of the radius and carried out. If these products be added together into one sum, they will form a dividend. For a divisor, double the sum of the weights of the rim and the arms, and add the weight of the water to them. Divide the dividend by the divisor, and the square root of the quotient will be the radius of gyration.

Example.—In a wheel 24 feet diameter. The weight of the arms is 2 tons, the shrouding and rims 4 tons, and the water in action 2 tons; hence by the above,

$$\begin{aligned} R &= 4 \text{ tons} \times 12^2 \times 2 = 1152 \\ A &= 2 \text{ tons} \times 12^2 \times 2 = 576 \\ W &= 2 \text{ tons} \times 12^2 = 288 \\ &\text{Their sum} \quad 2016 \text{ dividend and} \end{aligned}$$

$$2 \times (4 + 2 + 2) = 16 \text{ the divisor.}$$

$$\text{the answer, } \sqrt{\frac{2016}{16}} = \sqrt{126} = 11{\cdot}225.$$

NUMBER AND CAPACITY OF THE BUCKETS.

399. Although the number of buckets cannot be determined in accordance with any exact rule, it is, nevertheless, of importance that their pitch should not be much greater than the depth, or thickness, of the overflowing body of water acting upon them. It is also necessary that the number of the buckets should be divisible by that of the arms of the wheel, so that the whole may be put together conveniently.

If we take, for instance, a wheel of 20 feet diameter,

$$\text{Then:—}20{\cdot} \times 3{\cdot}1416 = 62{\cdot}83 \text{ feet,}$$

We can very conveniently give this wheel 8 arms and 64 buckets; and the pitch of these last will be nearly 1 foot. With this distance between the buckets, there should not generally be a greater depth of overflow than 10 or $10\frac{1}{2}$ inches; because, at 11 inches the water would begin to choke, and not be admitted easily into the buckets, consequently it would rebound against the interior of the channel, giving rise to a continual shaking action.

Thus, then, in determining the number of buckets for an undershot water-wheel, receiving the water from an overshot outlet, it is necessary to calculate the spaces between them, so as to be about a third, or at least a fourth, greater than the depth of the water at the outlet, whilst their number must be divisible by the number of arms of the wheel.

Water-wheels of from 10 to 15 feet in diameter should have six arms for each rim or shrouding; and for wheels of 15 to 20 feet in diameter, there should always be eight arms for each shrouding; and the number of arms should obviously increase for wheels of greater diameters than 20 feet for this and larger diameters; one arm and a segment of the shrouding is generally cast in one piece.

With regard to the capacity of the buckets, and the channel taken together, it should be equal to at least double the volume of water discharge. Therefore, on this basis, we can always easily determine the depth to be given to the buckets, when the maximum discharge is known.

The distance between the buckets, however, is not the same at the inside as at the extremities, and the capacity is also further diminished by the thickness of the sides of the buckets, and by the of the machine are determined, as stated in the following expressions—it being understood that the machine has *two*, and only *two* jet orifices, and these so formed as not to cause the issuing jets to contract more than in the proportion of 97 to 100 after the fluid has left the orifices.

Width of each discharging orifice $= \sqrt{\frac{135}{1000} \frac{E}{H\sqrt{H}}} = w$

Width of each arm of machine $= 4\ w$
Diameter of the machine $= 50\ w = D$
Diameter of central opening $= 10\ w$
Number of revolutions in a minute $\frac{149{\cdot}4338\sqrt{H}}{D}$

All these rules, except the last, may be departed from with impunity; but it is impossible to enumerate the circumstances and conditions under which modifications may be safely introduced, and where they would be prejudicial. These can only be appreciated by practice and a close investigation of the action of the machine. The rules are, however, safe within a wide range of fall—in fact, for all ordinary cases.

STEAM AND STEAM-ENGINES.

403. All liquids whatever, become changed in their physical condition, by the action of two forces operating in opposite directions; and it is admitted as a general principle, that one of these forces, namely, increase of pressure upon them, tends to induce the solid state, whilst the other opposing force, namely, increase of temperature, tends to induce the gaseous state.

Under the ordinary pressure and temperature of the atmosphere, pure water is in the liquid state, and may be converted into vapour, or steam, either by diminishing the pressure, or by increasing the temperature in a certain ratio. When water contained in a vessel is raised in temperature by the application of heat, and still under the pressure of the atmosphere, it becomes converted into steam when at a temperature of about 212° Fahrenheit, which is the highest temperature, to which pure water can be raised in any open vessel; and the pressure agreeing with this temperature, will very nearly coincide with that of the atmosphere, namely, about 15 lbs. per square inch; but if the vessel be hermetically sealed, as, for example, a steam-engine boiler, the temperature and pressure may both be increased by the continuation of heat, and will press with a force equal in every direction; so that each square inch of the internal surface, exposed to the action of the steam, must be alike capable of sustaining the increasing pressure. The amount of pressure corresponding with the temperature, cannot exactly be expressed by any formula; but the expansive force of steam, as also gases in general, is nearly inversely as the volume; thus, if at the pressure of one atmosphere, steam occupies one cubic foot, the same quantity, at a pressure of two atmospheres, would only occupy one half of a cubic foot. To ascertain the amount of heat absorbed during the generation of steam, many very careful experiments have been performed; but, from the many practical difficulties connected with such experiments, no exact law has as yet been discovered; however, an approximate rule is, that a given weight of steam, contains nearly six times the heat, necessary to raise an equal weight of water, from the mean temperature to 212° Fahrenheit. This absorbed heat, cannot be measured by the thermometer alone, and is generally termed "latent heat," being more easily defined by condensing a certain weight of steam amongst a known quantity of cold water, and then noting the number of degrees the water has been elevated. In England 1000° Fahrenheit is considered sufficiently near the true amount for all practical purposes, and the practical draughtsman, will find no appreciable error by adopting this constant number.

The weight or density of steam, is exactly that of the water it contains; but, the force and weight increase in different ratios; for example, a cubic foot of steam, at the pressure of one atmosphere, weighs about 0·0364 lbs. avoirdupois, whilst steam, at a pressure of 4 atmospheres, weighs only 0·1298 lbs.

In those engines which are called low-pressure engines, the steam is produced at a temperature, very little over that of boiling water, or 212° Fahrenheit, in which case the tension of the steam, will sustain a column of mercury, 30 feet high in a perfect vacuum. It is also equal to a pressure of 1 atmosphere, or 15 lbs. per square inch. At this pressure those generally known as Watt's engines, without cut-off valves, are calculated.

There is, however, a great difference between the pressure of the steam in the boiler, and that to which the effective power of the machine is due. It is evident, that a part of the pressure will be absorbed by the back pressure due to an imperfect vacuum, as well as by the friction of the piston, the moving parts, and the leakage, and condensation in the steam passages. So that, taking into consideration these various causes of loss, the effective force may be estimated accordingly.

The rule for calculating the power of low-pressure steam-engines is—

Multiply the mean effective pressure of the steam in lbs. upon the area of the piston, expressed in square inches, and the product by the velocity of the latter in feet per minute.

The result of this calculation will be the useful effect of the engine in lbs. raised 1 foot per minute.

To find the horses power, this result must be divided by 33·000 and is expressed thus:—

$$IP = \frac{A \times P \times V}{33{,}000}$$

Where IP represents the number of horses-power; A the area of the piston in square inches; P the mean effective pressure of the steam in lbs. per square inch; and V the velocity of the piston in feet per minute.

The length of the piston's stroke is calculated by the designer, in accordance with the work the engine has to perform; so that the maximum, and minimum velocities of the piston, will agree with the following formula:—

$$V = \sqrt[2]{L} \times 128.$$

Where L represents the length of the stroke in feet, and V the velocity required.

Example.- -Let 4 = the length of stroke in feet; then the square root of 4 is 2, which, when multiplied by 128, gives 256 as the required velocity, of the piston in feet per minute. If we now divide the product already found, by the number of feet the piston must travel through to accomplish one full stroke, (which is 8 feet) we find the number of strokes per minute to be 32.

Single-acting engines, such as are employed in mines, have cylinders of the same dimensions as double-acting ones, but of only half the power. Thus, the cylinder of a low-pressure steam-engine, of 50 horses-power, and only single-acting—that is to say, receiving the action of the steam during the descent only of the piston, is exactly the same as in a machine of 100 horses-power, in which the steam acts alternately on both sides of the piston.

TABLE OF THE PROPERTIES OF STEAM, AND OF ITS USEFUL EFFECT AT DIFFERENT PRESSURES.

Pressures.		Temperatures at the given pressures in degrees of Fahr.	Weight of a cubic foot of steam in lbs. avoird.	Velocity of the steam into the atmosphere in Eng. feet.	Theoretical horse-power of 1 lb. of steam.								Pressure in pounds per square inch above that of the atmosphere.	Coefficient of the increase of volume $(1 + a\tau.)$
					Without Condensation.				With Condensation.					
In atmospheres.	In pounds. per square inch.				Without expansion.	Expansion at one-half.	Expansion at one-third.	Expansion at one-fourth.	Without expansion.	Expansion at one-half.	Expansion at one-third.	Expansion at one-fourth.		
1·00	14·70	212·00	0·0364	0	0·	–32·4	–95·2	–170·5	91·3	150·1	178·6	194·6	0·	1·374
1·25	18·38	223·88	0 0447	873	21·5	10·1	–32·3	- 87·4	95·9	158·7	190·6	209·9	3·68	1·399
1·50	22·05	234·32	0·0529	1135	36·4	39·3	10·8	- 30·6	99·3	165·2	199·6	221·1	7·35	1·421
1·75	25·72	242·78	0·0609	1295	47·4	60·8	42·5	11·1	102·0	170·0	206·2	229·5	11·02	1·438
2·00	29·40	250·79	0·0688	1407	55·9	77·5	67·0	43·2	104·3	174·2	212·0	236·5	14·70	1·455
2·25	33·08	257·90	0·0766	1491	62·8	90·9	86·5	68·8	106·2	177·7	216·7	242·4	18·38	1·470
2·50	36·75	263·93	0·0844	1556	68·4	101·8	102·4	89·6	107·7	180·5	220·5	247·1	22·05	1·482
2·75	40·42	269·87	0·0921	1608	73·1	111·0	115·8	107·1	109·3	183·2	224·2	251·6	25·72	1·495
3·00	44·10	275·00	0·0998	1652	77·1	118·8	127·1	121·9	110·5	185·4	227·1	255·2	29·40	1·505
3·25	47·78	279·86	0·1073	1690	80·7	125·6	137·0	134·7	111·7	187·6	230·0	258·7	33·08	1·516
3·50	51·45	284·63	0·1148	1722	83·8	131·5	145·6	145·8	112·7	189·4	232·4	261·6	36·75	1·525
3·75	55·12	288·86	0·1223	1750	86·5	136·8	153·2	155·6	113·7	191·1	234·7	264·4	40·42	1·534
4·00	58·80	292·91	0·1298	1774	89·0	141·5	160·0	164·5	114·6	192·8	236·9	267·0	44·10	1·543
4·50	66·15	300·47	0·1445	1816	93·2	149·6	171·5	179·4	116·2	195·6	240·5	271·4	51·45	1·558
5·00	73·50	307·94	6·1590	1850	96·8	156·5	181·4	192·0	117·7	198·3	244·1	275·6	58·80	1·574
6·00	88·20	320·00	0·1878	1904	102·5	167·2	196·5	211·4	120·2	202·6	249·7	282·2	73·50	1·599
7·00	102·90	331·56	0·2159	1945	107·0	175·6	208·4	226·5	122·4	206·4	254·6	288·1	88·20	1·623
8·00	117·60	341·83	0·2436	1978	110·6	182·4	217·9	238·4	124·3	209·7	258·8	293·1	102·90	1·644
9·00	132·30	351·32	0·2708	2006	113·7	188·2	225·9	248·5	126·0	212·8	262·7	297·6	117·60	1·664
10·00	147·00	359·60	0·2977	2029	116·3	193·0	232·5	256·7	127·5	215·3	266·0	301·4	132·30	1 681
12·50	183·75	377·42	0·3642	2074	121·5	202·5	245·5	273·0	130·7	220·8	272·9	309·5	169·05	1·718
15·00	220·50	392·90	0·4288	2109	125·7	210·0	255·6	285·4	133·4	225·5	278·9	316·4	205·80	1·751
17·50	257·25	406·40	0·4924	2136	129·0	216·0	263·6	295·2	135·7	229·5	283·9	322·2	242·55	1·779
20·00	294·00	418·46	0·5549	2159	131·8	221·0	270·3	303·3	137·8	233·0	288·3	327·2	279·30	1·804
25·00	367·50	439·34	0·6775	2196	136·3	229·1	281·0	316·2	141·2	238·9	295·7	335·8	352·80	1·847
30·00	441·00	457·16	0·7970	2226	140·0	235·6	289·5	326·4	144·2	244·0	302·0	343·1	426·30	1·884

The above table very nearly coincides with some of the most recent experiments, on the mechanical properties of steam, and, with its assistance, many important calculations will be readily obtained.

In determining the size of cylinder for any given pressure, or for any given boiler, the volume of steam is a necessary element in the calculation, and for this purpose the following rules are submitted for ordinary steam. They are based on the ascertained facts that steam of atmospheric pressure is very nearly 1700 times the volume of the water which produced it; and that for each additional degree of heat of Fahrenheit's scale, steam expands when in contact with water, ·00202 times in bulk, hence for any pressure we have:—

$$\text{Volume} = \frac{1700 \times 14{\cdot}75}{\text{pressure}} \times \frac{1 \times {\cdot}00202\ (\text{temp.}—32)}{1 \times {\cdot}00202 \times 180}$$

The following rules are very simple in their application:—

By Pambour,

$$\text{Volume} = \frac{10000}{1{\cdot}421 \times {\cdot}331\ \text{pressure}}$$

Pole gives,

$$\text{Volume} = \frac{24250}{\text{pressure}} + 65$$

Example.—Required the volume of steam of 100 lbs. pressure per square inch relatively to water as 1?

By Pambour,

$$\frac{10000}{100 \times {\cdot}331 \times 1{\cdot}421} = 289{\cdot}6 \text{ times.}$$

By Pole,

$$\frac{24250}{100} + 65 = 307{\cdot}5$$

or a difference of nearly 18 volumes of the water forming the steam.

The volume may also be found by adding 4·29 to the pressure in pounds per square inch, and deducting the logarithm of this sum from 4·4799. The natural number of the remainder will give the ratio of the volume of steam to water.

For the last example we have—

log. of 100 + 4·29 = 2·0182; and 4·4799
less 2·0182 = 2·4617

and the natural number of 2·4617 = 289·4

or nearly the same as Pambour's formula gives.

The volume of steam under expansion may be found approximately by adding 459, to the respective temperature, before and after expansion, and dividing the greater by the lesser sum. The volume due to the lowest temperature multiplied by the quotient will give nearly the volume for the highest temperature.

Steam is estimated to flow into a vacuum with a velocity equal to that due to a body of the same density, falling through a space equal to the height of a column of steam of the given pressure. For instance, it would require a column of steam about 63500 feet high, to give a pressure of 45 lbs. upon a square inch. The velocity due to the pressure, may be found by adding 4·29 to the pressure in pounds per square inch, and deducting the logarithm of this sum from the logarithm of the pressure. To one-half the remainder add 3·3254, and the natural number of this sum will be the velocity in feet per second.

Example.—Required the velocity with which steam of 100 lbs. pressure would rush into a vacuum?

Pressure 100 lbs. whose log. = 2·00000

and pressure 100 + 4·29 = 104·29 whose log. = 2·01828

and remainder $\div \frac{1}{2}$) $\bar{9}$·98172

leaves $\bar{9}$·99086

to which add 3·3254

gives 3·31626 whose

natural number = 2071 feet per second.

The difference between the velocities of any two pressures, is the velocity with which steam would flow into steam of a lower pressure. Thus, steam of 120 lbs. gross pressure would flow into steam of 20 lbs. pressure, at a velocity of 2079 — 2010 = 69 feet per second.

Such is the estimated velocity of steam, when there are no frictional obstructions to its passage; but, as these are conditions not obtainable in practice, the velocity will be reduced in proportion to the resistances it has to overcome. In locomotive engines at high velocities these resistances are very considerable; for, with a back pressure, from $\frac{1}{10}$ to $\frac{1}{4}$ of that in the boiler, narrow steam-ports, incomplete exhaustion, and rapid action of the slide, this velocity it is evident must be very materially lessened. Those who have carefully observed indicator cards, will have seen that the steam in the cylinder, only attains a force near that in the boiler, when the resistances against the piston, retard its progress until such force has accumulated; but if the resistance does not require that force, the piston moves on more rapidly than allows time for such accumulation.

The steam may, therefore, be 100 lbs. in the boiler; but, if 35 lbs. will overcome the resistances against the piston, including the back pressure, the piston will move on with a velocity to prevent a greater pressure. Yet if that piston were arrested, even for a second, the pressure would equalise itself very nearly in the boiler and in the cylinder. This indicates that the velocity of the steam, in passing from the boiler to the cylinder, is not so great but that the motion of the piston sensibly affects its accumulation there. Since 1000 per minute is an extreme velocity for a piston, it leads to the conclusion that the actual velocity of steam, in a locomotive engine, is moderate; but what the exact velocity may be remains to be determined by experiments.

AREA OF PORTS.

404. In calculating the area of the steam ports and passages, it is not necessary that the elastic force of the steam in the boiler should be considered; for although at the higher pressures the velocity of the entering steam will be greater, still the average velocity of the escaping, or exhaust steam becomes a constant, and with an average velocity of piston, $\frac{1}{20}$th the area of the steam cylinder, is considered quite sufficient, and a very general rule; however, marine engines, designed to drive the screw direct, should be somewhat increased, in consequence of the greater speed of piston. Their ports average from $\frac{1}{8}$th to $\frac{1}{18}$th of the area of the cylinder. Locomotive, and other quick speed engines, average between $\frac{1}{12}$th and $\frac{1}{14}$th of the cylinder's area; and some pumping engines, with a very long stroke, are designed so that the area of the steam valve, shall not be below $\frac{1}{10}$th that of the steam cylinder. An approximate rule for an average velocity of piston is as follows:—

Rule.—Multiply the square of the diameter of the cylinder in inches by the length of the stroke in feet; multiply this product by 11; divide the last product by 1800; and, finally, to the quotient add 8. The result is the area of each steam port in square inches.

To show the use of this rule, we shall apply it to a particular example. We shall apply it to an engine whose stroke is 6 feet, and diameter of cylinder 30 inches. Then, according to the rule, we have

30 = diameter of the cylinder in inches.

30

900 = square of diameter.

6 = length of stroke in feet.

5400

11

59400 ÷ 1800 = 33

8 = constant to be added.

41 = area of steam port in square inches.

Rule.—To find the size of the steam ports for the locomotive engine.—Multiply the square of the diameter of the cylinder by ·068. The product is the proper size of the steam ports in square inches.

Example.—Required the proper size of the steam ports of a locomotive engine whose diameter is 15 inches. Here, according to the rule, size of steam ports = ·068 × 15 × 15 = ·068 × 225 = 15·3 square inches, or between 15¼ and 15½ square inches.

After having determined the area of the ports, we may easily find the depth when the length is given, or, conversely, the length when the depth is given. Thus, suppose we know the length was 8 inches, then we find that the depth should be 15·3 ÷ 8 = 1·9125 inches, or nearly 2 inches; or suppose we knew the depth was 2 inches, then we would find that the length was 15·3 ÷ 2 = 7·65 inches, or nearly 7¾ inches.

Area of eduction ports.—The proper area for the eduction ports may be found from the following rule.

Rule.—To find the area of the eduction ports.—Multiply the square of the diameter of the cylinder in inches by ·128. The product is the area of the eduction ports in square inches.

Example.—Required the area of the eduction ports of a locomotive engine, when the diameter of the cylinders is 13 inches. In this example we have, according to the rule, area of eduction port = ·128 × 13^2 = ·128 × 169 = 21·632 inches, or between 21½ and 21¾ square inches.

Breadth of bridge between ports.—The breadth of the bridges between the eduction port and the induction ports is usually between ¾ inch and 1 inch.

AIR PUMP AND CONDENSER.

405. The stroke of the air-pump piston in many stationary and marine engines, is exactly one-half that of the steam piston, and the diameter in these cases is found as follows:—

Rule.—Multiply the diameter of the steam cylinder in inches by ·6. The product is the diameter of the air-pump in inches.

Example.—Required the diameter of the air-pump when the diameter of the steam cylinder is 30 inches, and the stroke of the air-pump piston one-half that of the steam piston, $30 \times \cdot 6 = 18$ inches.

It is not absolutely necessary that the air-pump piston's stroke should be exactly one-half that of the steam piston; and in many marine engines we find sometimes a shorter stroke, with a greater diameter, and in others a longer stroke with a less diameter; but in every case the cubical contents of the pump, must average about $\frac{1}{6}$th the cubical contents of the cylinder; and if the temperature of the injection water, on entering the condenser, be very high, a still greater capacity must be given to the pump; so that the waste water may be drawn from the condenser as it accumulates therein.

INJECTION.

The quantity of injection water, necessary to condense a given quantity or weight of steam, so that the waste water may never exceed 100° Fahrenheit, may be found as follows :—

Rule.—Add the sensible, and latent heat in the steam together, and substract the required temperature of the waste water; divide the number found, by the difference of temperature between the injection, and waste water, and the product will be the number of times, the quantity of injection water, must exceed the quantity of water in the steam.

Example.—Required how many times the water for condensation, must exceed the quantity of water in a given volume of steam, at the temperature of 220° Fahrenheit, taking the latent heat at 1000°, and the injection water at 50°.

$$\frac{1000^\circ + 220^\circ - 100^\circ}{100^\circ - 50^\circ} = 22{\cdot}4 \text{ times.}$$

The quantity of water working between these temperatures, must therefore be about 24 times, the quantity of water in the form of steam; and to admit this quantity during the time of the engine's stroke, when working at the average speed, the injection orifice should not be less than $\frac{1}{10}$th the diameter of the steam cylinder.

FEED-PUMP.

406. The cubical contents of the feed-pump, may be roughly taken at about $\frac{1}{240}$th of the steam cylinder; that is, when the cylinder is double-acting, and the feed-pump single-acting; and the following rule may be taken with safety for all sizes of engines :—

Rule.—Multiply the square of the diameter of the cylinder in inches by the length of the stroke in feet; divide the product by 30. The quotient is the content of the feed-pump in cubic inches.

Thus, for an engine whose stroke is 6 feet, and diameter of cylinder 50 inches, we have,

$$\begin{array}{rl} 50 & = \text{diameter of cylinder.} \\ 50 & \\ \hline 2500 & = \text{square of the diameter of cylinder.} \\ 6 & = \text{length of stroke in feet.} \\ \hline 30)15000 & \\ \hline 500 & = \text{content of feed-pump in cubic inches.} \end{array}$$

SLIDE VALVE.

407. The terms "lap" and "lead" of the slide are distinct terms, well known amongst engineers, and may be explained thus: When the valve face exactly equals the breadth of the port, that is in the line of the valve's motion, the valve is then said to have no "lap;" but, when the valve face is greater than the breadth of the port, the difference is then termed the amount of "lap," and if the difference is very great, the steam will be cut off from the cylinder, considerably before the termination of the piston's stroke; and to find how much "lap," or "cover," must be given to the steam side of the valve, to cut off the steam at the required point of the stroke, the rule is as follows :—

Rule.—From the length of the stroke of the piston, substract the length of that part of the stroke that is to be made before the steam is cut off. Divide the remainder by the length of the stroke of the piston, and extract the square root of the quotient. Multiply the square root thus found, by half the length of the stroke of the valve, and from the product take half the lead, and the remainder will be the cover required.

The "lead" of the slide, is the amount of opening between the steam edge of the valve, and its respective port, which, according to the size of the engine, and the speed of the piston, varies from about $\frac{1}{8}$ to $\frac{3}{8}$ of an inch, consequently we must add the lead and cover together, to obtain the exact part of the piston's stroke, at which the steam will be cut off.

Rule.—To find at what part of the stroke any given amount of cover on the steam side will cut off the steam.—Add the cover on the steam side to the lead; divide the sum by half the length of stroke of the valve. In a table of natural sines find the arc whose sine is equal to the quotient thus obtained. To this arc add 90°, and from the sum of these two arcs, substract the arc whose cosine is equal to the cover on the steam side, divided by half the stroke of the valve. Find the cosine of the remaining arc, add 1 to it, and multiply the sum by half the stroke of the piston, and the product is the length of that part of the stroke, that will be made by the piston before the steam is cut off.

Rule.—To find how much before the end of the stroke, the exhaustion of the steam in front of the piston will be cut off.—To the cover on the steam side add the lead, and divide the sum by half the length of the stroke of the valve. Find the arc whose sine is equal to the quotient, and add 90° to it. Divide the cover on the exhausting side, by half the stroke of the valve, and find the arc whose cosine is equal to the quotient. Substract this arc from the one last obtained, and find the cosine of the remainder. Substract this cosine from 2, and multiply the remainder by half the stroke of the piston. The product is the distance of the piston from the end of its stroke, when the exhaustion is cut off.

Rule.—To find how far the piston is from the end of its stroke, when the steam that is propelling it by expansion is allowed to escape to the condenser.—To the cover on the steam side add the lead, divide the sum by half the stroke of the valve, and find the arc whose sine is equal to the quotient. Find the arc whose cosine is equal to the cover on the exhausting side, divided by half the stroke of the valve. Add these two arcs together, and substract 90°. Find the cosine of the residue, substract it from 1, and multiply the remainder by half the stroke of the piston. The product is the distance of the piston from the end of its stroke, when the steam that is propelling it is allowed to escape to the condenser. In using these rules all the dimensions are to be taken in inches, and the answers will be found in inches also.

In locomotive engines it is rarely advisable to set the valve with less than $\frac{5}{16}$ inch lead, as allowance should be made for the straining and wear of the joints and other surfaces of the valve-gear, which combine to reduce the lead when in actual work. As some engineers, however, rigidly proportion the lead to the travel and lap, the standard proportions of 1 inch lap, $\frac{5}{16}$ inch lead, and $4\frac{1}{2}$ inch travel, may be reduced to general terms. Assuming the travel as unity, and dividing it into 100 parts, the lap and lead may be expressed in percentages. Thus,

Travel, $4\frac{1}{2}$ inches $= \frac{72}{16} =$ 100 parts.
Lap, 1 inch $= \frac{16}{16} =$ 22·2 parts, say 22 per cent. of travel.
Lead, $\frac{5}{16}$ inch $= \frac{5}{16} =$ 6·94 parts, say 7 per cent. of travel.
Whence the general rule

The travel of the valve being given. To find the lap and lead necessary for a maximum admission of 75 per cent., or three-fourths of the stroke.

Rule for Lap.—Multiply the travel by 22, and divide by 100; the result is the required lap.

Rule for Lead.—Multiply the travel by 7, and divide by 100; the result is the required lead.

EXPANSION VALVES.

408. The expansion or cut valve, is so arranged and wrought either by an eccentric or cam, that the valve will always be open, when the engine is on the centre; and so soon as the piston has travelled the required distance, which must be some fractional part of the full stroke, such as $\frac{1}{4}$, or $\frac{1}{3}$, the valve is then suddenly closed, by the revolution of the expansion eccentric, which is quite independent of the common eccentric for working the slide-valve, and may be altered in relation to the crank at pleasure, so that the steam may be cut off at different points. When a cam is employed for actuating the expansion valve, it is generally stepped, so as to form three or more different grades; and by shifting the actuating lever upon the different grades of the cam, different degrees of expansion are readily obtained. With the common slide-valve, it is impracticable to give as much cover or lap as to cut off the steam at $\frac{1}{3}$ the stroke; but, by the use of a separate expansion valve, any degree of expansion can be given correctly.

The "link motion," and many other modes for regulating the amount of cut off at pleasure, are fully explained in many works on the steam-engine; but we think an explanation of these in the "Practical Draughtsman" unnecessary.

GOVERNOR.

409. When the governor-balls are revolving at their mean velocity, we have only to consider the vertical height of the point of suspension, above the horizontal plane passing through the centre of the balls, and this distance may be found from the following formula:—

$$h = \frac{35186}{n^2}$$

From which we have the following rule:—

Rule.—To determine the height of the point of suspension above the plane of the balls, when moving with mean velocity.—Divide the number 35186 by the square of the mean number of revolutions per minute. The quotient is the height of the point of suspension, above the plane of the balls when moving with mean velocity.

Example.—In a particular engine the governor is so connected with the fly-wheel shaft, that when the engine is moving with average velocity, the upright spindle makes 40 revolutions per minute. Required the proper height of the point of suspension, above the plane of the balls.

In this example—

$$h = \frac{35186}{40^4} = \frac{35186}{1600}$$

$$= \frac{351{\cdot}86}{16} = 21{\cdot}99 \text{ inches.}$$

so that the proper height is about 22 inches.

BOILER.—DIMENSIONS OF THE SEVERAL PARTS OF FURNACES AND BOILERS.

410. Perhaps in none of the parts of the steam-engine does the practice of engineers vary more, than in those connected with furnaces and boilers. There are, no doubt, certain proportions for these, as well as for the others, which produce the maximum amount of useful effect for particular given purposes; but the determination of these proportions, from theoretical considerations, has hitherto been attended with insuperable difficulties, arising principally from our imperfect knowledge of the laws of combustion of fuel, and of the laws according to which caloric is imparted to the water in the boiler. In giving, therefore, the following proportions for the different parts, we desire to have it understood, that we do not affirm them to be the best, absolutely considered; we give them only as the average practice of the best modern constructors.

AREA OF FIRE GRATE.

The average practice is to give nearly a square foot for each nominal horse-power. Hence the following rule.

Rule.—To find the area of fire grate.—Multiply the cube root of the length of stroke in feet, by the square of the diameter in inches; divide the product by 86: the quotient is the area of fire grate in square feet.

Example.—Required the total area of the fire grate for an engine whose stroke = 8 feet, and diameter of cylinder = 50 inches.

Here, according to the rule,

Total area of fire grate in square feet =

$$\frac{50^2 \times \sqrt[3]{8}}{86} = \frac{2500 \times 2}{86} = \frac{5000}{86} = 59 \text{ nearly.}$$

Rule.—To find the total area of the flues or tubes in smallest part.—Multiply the number of horse-power by 11·2; the product is the total area in square inches of flues or tubes in smallest part.

Example.—Required total area of flues, or tubes for the boiler of a steam-engine, when the horse-power = 400.

For this example we have according to the rule,

Total area in square inches $= 400 \times 11{\cdot}2 = 4480$.

We may also find a very convenient rule expressed in terms of the stroke and the diameter of cylinder. Thus,

Total area of tubes or flues in square inches =

$$\frac{11{\cdot}2 \times d^2 \times \sqrt[3]{S}}{47} = \frac{d^2 \times \sqrt[3]{S}}{4}$$

Rule.—To find the total effective heating surface of marine flue boilers.—Multiply the square of the diameter of cylinder in inches by the cube root of the length of stroke in feet; divide the product by 10: the quotient expresses the number of square feet of effective heating surface.

Example.—Required the amount of effective heating surface for an engine whose stroke is = 8 feet, and diameter of cylinder = 50 inches.

Here, according to Rule 2, Effective heating surface in square feet =

$$\frac{50^2 \times \sqrt[3]{8}}{10} = \frac{2500 \times 2}{10} = \frac{5000}{10} = 500.$$

The total heating surface of flue and tubular marine boilers is about the same, namely, about 10 square feet per horse-power.

Rule.—To find the area of the chimney.—Multiply the square of the diameter expressed in inches, by the cube root of the stroke expressed in feet; divide the product by the number 5; the quotient expresses the number of square inches in the area of chimney.

Example.—Required the area of the chimney for an engine whose stroke = 8 feet, and diameter of cylinder = 50 inches.

We have in this example from the rule,

Area of chimney in square inches =

$$\frac{50^2 \times \sqrt[3]{8}}{5} = \frac{2500 \times 2}{5} = 1000.$$

Rule.—Multiply the square of the diameter of the cylinder in inches, by the cube root of the stroke in feet; divide the product by 15; the quotient expresses the number of cubic feet of steam room.

Example.—Required the average capacity of steam room for an engine whose stroke is 8 feet, and diameter of cylinder 5 inches.

In this case, according to the rule,

Steam room in cubic feet =

$$\frac{52^2 \times \sqrt[3]{8}}{15} = \frac{2500 \times 2}{15} = \frac{5000}{15} = 333\tfrac{1}{3}.$$

The constant aim of different engine makers, is to increase the amount of the duty; that is, to make 10 lbs. of fuel exert a greater effect than one horse-power; or, in other words, to make 1 lb. of coal, raise more than a quarter of a million of lbs. one foot high. To a great extent they have been successful in this.

Modern practice varies from 2 to 5 lbs. of coal for each indicated horse-power per hour; and the most economical results, have been obtained by the use of very high-pressed steam, with great expansion, in both single and double cylinder engines. Where the expansion is very great, every means of preventing any loss of heat must be carefully attended to, as it is generally understood that steam during expansion converts heat into work.

SAFETY-VALVES.

For the sake of safety, the orifices of the safety-valves are commonly ·8 of a square inch per nominal horse-power.

Rule.—Divide the length of the lever by the distance between the fulcrum and the valve, and the quotient gives the leverage.

Given the whole length of the lever 24 inches, the distance between the fulcrum and valve 3 inches, the diameter of the valve 2½ inches; required the weight put on at the end of the lever, so as to have 50lbs. per square inch upon the valve; also, to divide the lever so as to have 40, 30, 20 lbs. &c. upon the valve with the same weight.

$(2.5)^2 \times {\cdot}7854 = 4{\cdot}9 =$ area of the valve.

$4{\cdot}9 \times 50 = 245$ lbs. whole weight on the valve.

$\frac{245}{8} = 30{\cdot}625$ lbs. = the weight which must be put on at the end of the lever to give 50 lbs. per square inch.

And $\frac{4.9 \times 40}{30.625} = 6{\cdot}4$; then, $6{\cdot}4 \times 3 = 19{\cdot}2$ inches, the distance from the fulcrum the weight must be placed to have 40 lbs.

$24 - 19{\cdot}2 = 4{\cdot}8$; that is, the weight must be shifted in towards the fulcrum 4·8 inches to have 40 lbs. per inch; and for 30 lbs. per square inch, move it in 4·8 inches more, &c.

CHAPTER XIII.

OBLIQUE PROJECTIONS.

APPLICATION OF RULES TO THE DELINEATION OF AN OSCILLATNIG STEAM-CYLINDER.

PLATE 41.

411. In geometrical drawing, the planes of projection on which the objects are represented, are chosen, when possible, so as to be parallel to the faces of such objects; from which it follows, that these are expressed in their exact shapes and dimensions. It often happens, however, that certain parts of the machine or apparatus to be drawn, are inclined in regard to the other parts, so that all the surfaces cannot be parallel to the geometrical planes. The projections of the inclined parts are oblique, and, consequently, are seen as foreshortened.

It will be evident that the general method employed in projections is applicable to the delineation of oblique projections. But it will be necessary first to represent the objects as if parallel to the plane of the drawing, so as to obtain the exact proportions and dimensions, such views being auxiliary to the production of the oblique representations.

412. Thus it is proposed to represent a six-sided nut, the edges of which are inclined to both the horizontal and vertical plane.

We first of all represent this nut, in Fig. 1, as placed with its base parallel to an auxiliary horizontal plane, represented by the line, L T, Fig. 3. This gives the regular hexagon, *a b c d e f*.

If we were to make the vertical projection of this prism on a vertical plane, parallel to one of the faces, or to *a d*, we should, in this second auxiliary plane, have the projection of the edges *a b*, *c d*.

The straight line, L^1 T^1, Fig 3, indicates the line of intersection of these two auxiliary planes, when placed in their actual position with regard to the nut; and it is, therefore, the base line of the two projections. This line forms, we shall suppose, the angle, $L\ o^1\ L^1$, with the base line of the actual drawing in hand, which angle, likewise, expresses the amount of inclination of the top and bottom of the prism, with the actual horizontal plane; whilst the angle $o^2\ o^1\ o^3$, formed by the perpendiculars, drawn to each of the lines through the point, o^1, expresses the amount of inclination of the edges and axis of the prism with regard to the vertical plane. After this it is merely necessary, in order to obtain the points, a^1, b^1, c^1, d^1, to set off to the right and left of the point, o^1, on the line, L^1 T^1, the distances, o^1a^1, o^1d^1, and o^1b^1, o^1c^1, respectively equal to *oa* or *od*, and *gb* or *gc*, in Fig. 1. Drawing perpendiculars to the line, L^1 T^1, through each of the points, a^1, b^1, c^1, d^1, and limiting them by the lines, $a^2\ d^2$ and $a^3\ d^3$, Fig 2, parallel to the former, we

EXAMPLES IN OB

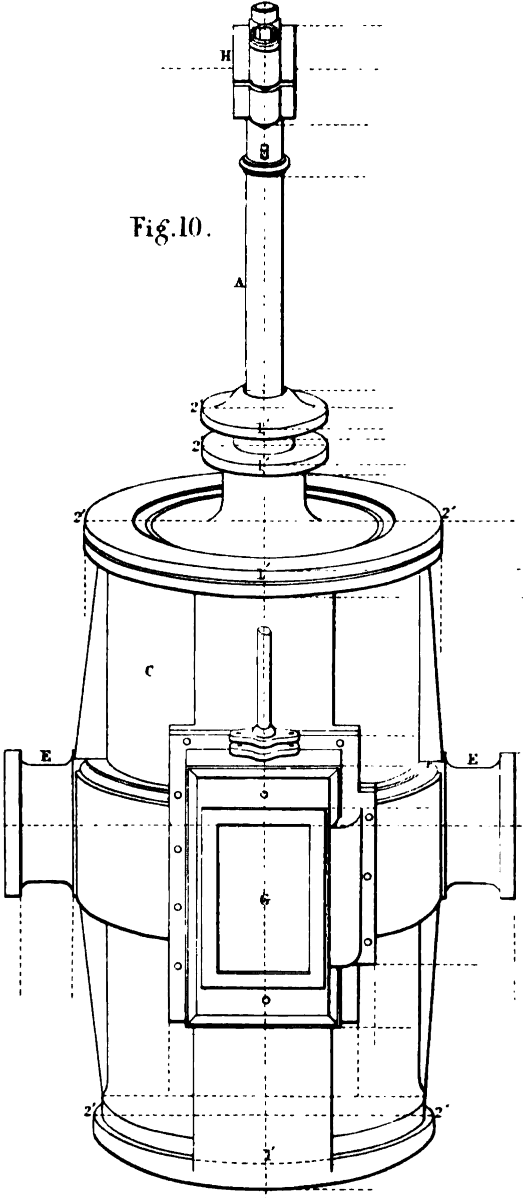


Fig. 10.

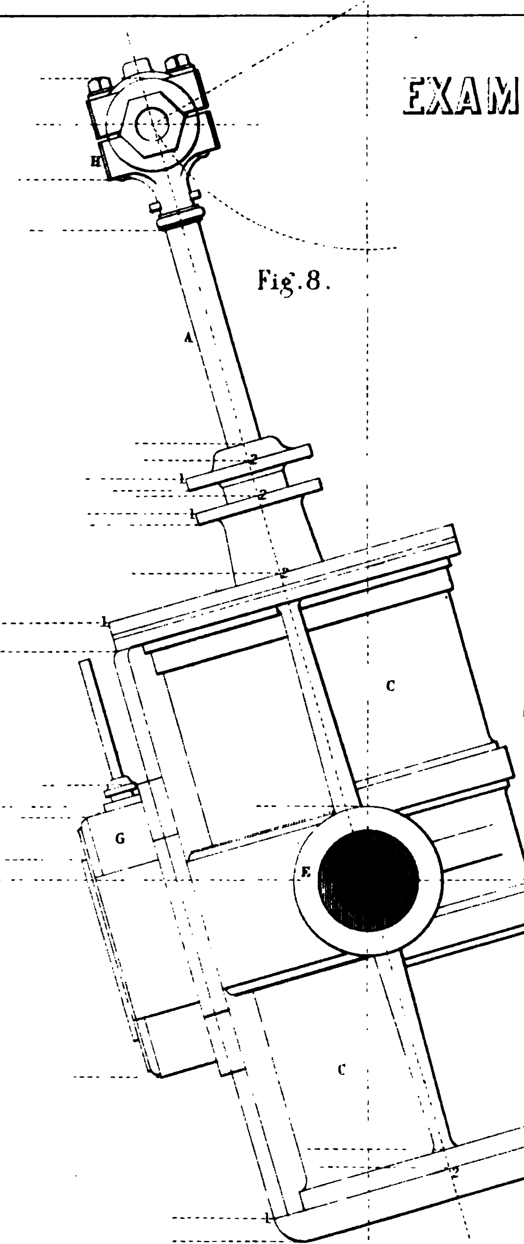


Fig. 8.

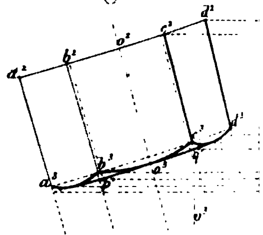


Fig. 2.

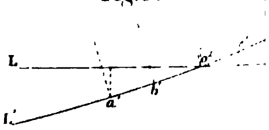


Fig. 3.

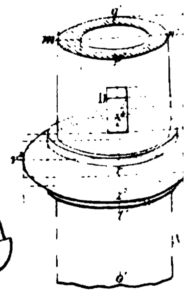


Fig. 7.

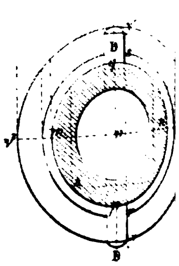


Fig. 5.

Fig. 15.

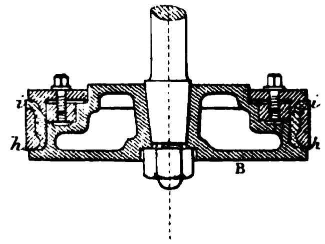
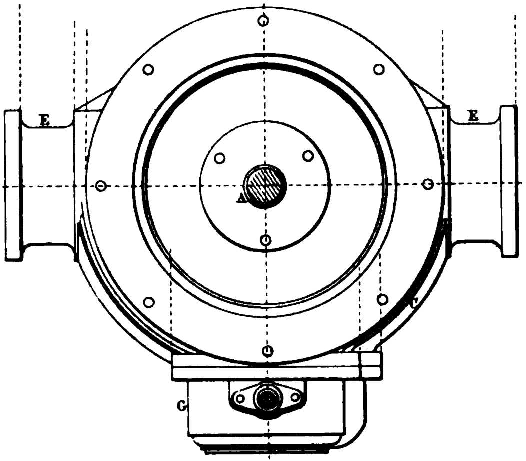
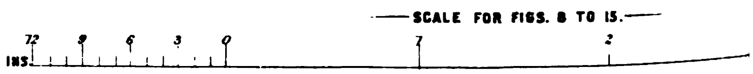


Fig. 9.





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UE PROJECTION.

Fig. 4.

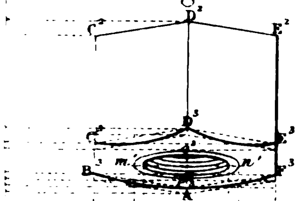
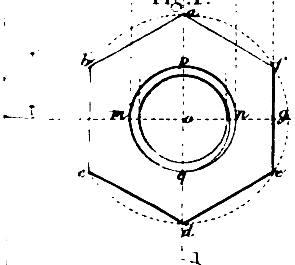


Fig. 1.



ig. 6.

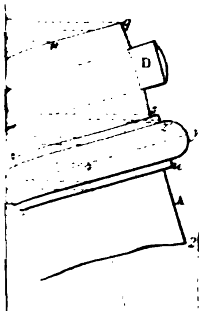


Fig. 11.

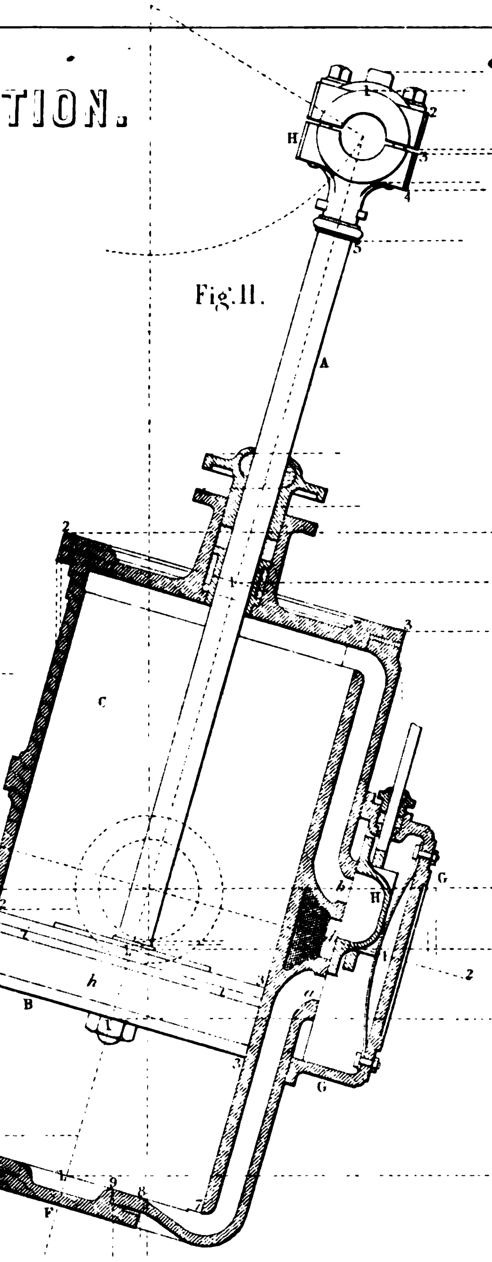


Fig. 13.

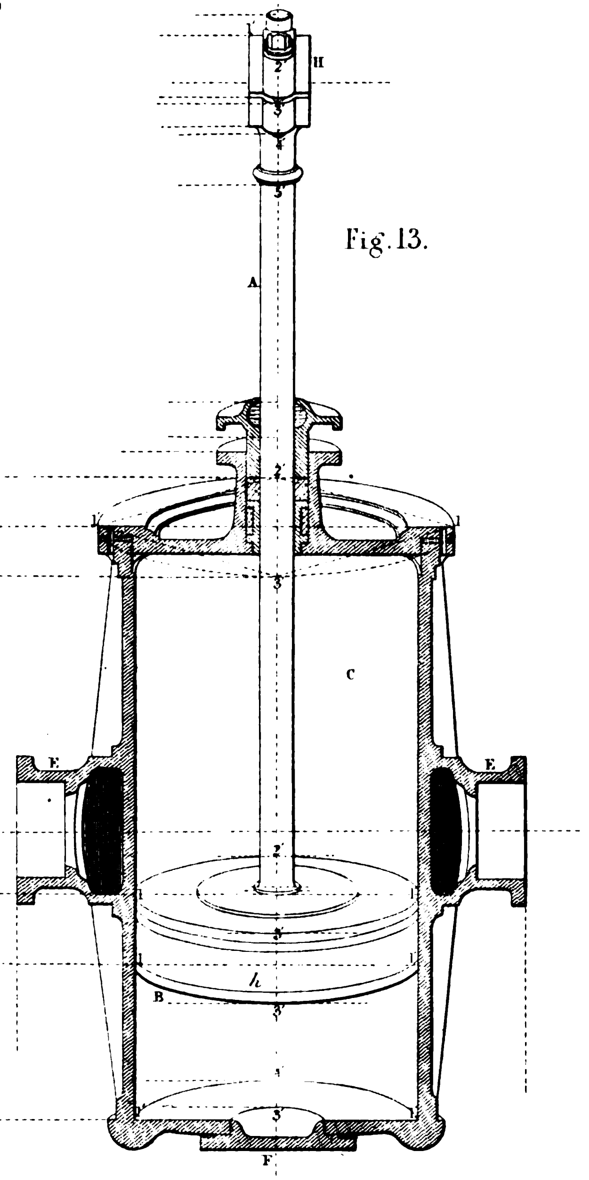


Fig. 14.

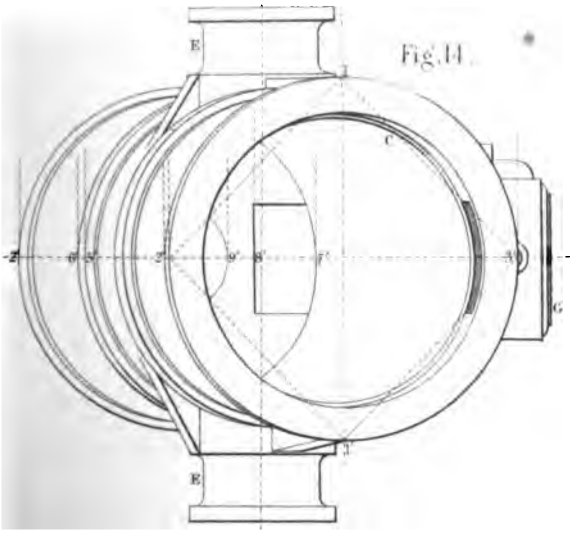
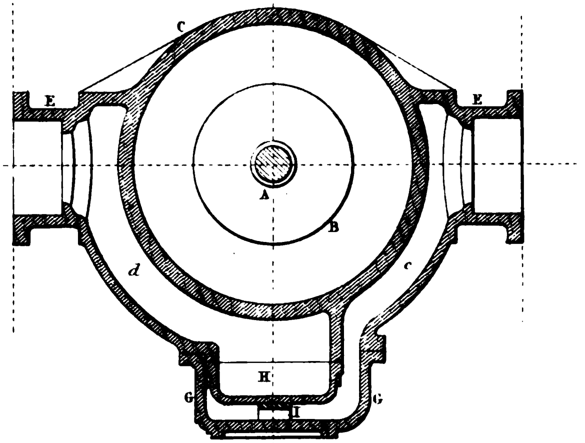


Fig. 12.



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obtain the entire vertical projection of the prism, as upon the auxiliary plane, parallel to one of the faces, as *c b* in Fig. 1. When one of the bases of the nut is rounded, or terminated by a spherical portion, which is generally the case, as already seen (183), its contour is limited by circular arcs, expressing the intersection of each face with the sphere.

We can then, by means of the two projections, Figs. 1 and 2, obtain the oblique projection, Fig. 4, upon the vertical plane, L T; Fig. 1 giving the widths, the distances of each of the points from the axial line, *a d*, which passes through the centre, *o*, and Fig. 2, defining the vertical heights or distances of the various points above the horizontal plane.

To this end, through any of the points, as *c*, for example, which is the horizontal projection of the edge, c^2 c^3, Fig. 2, erect a vertical line, and through the extreme points, c^2 c^3 of the former draw a couple of horizontal lines, cutting the vertical line in C^2 and C^3. The same operation is performed with regard to the points, *a*, *b*, *d*, *e*, and *f*; which are projected in A^3, B^3, D^2 D^3, E^2 E^3 and F^3, in Fig 4. The whole matter consists, therefore, in drawing vertical lines through each of the points in Fig. 1, and horizontal lines through the corresponding points in Fig. 2. The intersections of these lines give the projections of the extremities of each of the edges in the oblique view, Fig. 4.

If it is wished to obtain the projections of the circular outlines with minute exactness, it will be necessary to determine, at least, three points in each arc; and as we have the extremities already, we only require now to find the middle of each. It is the same for the circle representing the central opening of the nut. Its oblique projection is necessarily an ellipse, the proportions of which are obtained by the projection of the two diameters, *m n*, and *p q*, perpendicular to one another; the one, *m n*, which is parallel to the vertical plane, does not alter in magnitude, and is therefore simply projected in the line, m^1, n^1, in Fig. 4, thus giving the transverse axis of the ellipse, whilst the other *p q*, which is inclined and foreshortened, is found in the same way as the other inclined lines, from the points, p^1, q^1, in Fig. 2, and projected vertically in Fig. 4, on the line p^2 q^2, which is the conjugate axis.

413. The oblique projection of a circle is always an ellipse, the transverse axis of which is equal to the actual diameter of the circle, whilst the conjugate axis varies according to the inclination or angle which the plane of the circle makes with one of the planes of projection. The application of this principle will be seen in Figs. 5, 6, and 7. The two first of these figures represent the horizontal and vertical projections respectively, made upon the auxiliary planes, of a portion of the cylindrical rod, A, of the piston, B, working in the oscillating steam-cylinder, C; and the last, Fig. 7, is the oblique projection of this part of the piston-rod upon the vertical plane, corresponding to that of Figs. 10 and 13, but to a much larger scale.

It will be remarked, that the upper part of the fragment of the rod being limited by a plane, *p q*, Fig. 6, perpendicular to its axis, is projected vertically as an ellipse, in Fig. 7, the transverse axis m^1 n^1, of which is equal to, *m n*, or *p q*, whilst its conjugate axis p^1 q^1, Fig. 7 is equal to the oblique projection of the line, *p q*, in Fig. 6. The cylindrical fillets, *r s*, *t u*, &c., of this rod, are projected obliquely, as similar ellipses, of which portions only are apparent. For the torus, or ring, which is comprised between these two fillets, the oblique projection is a curve, which results from the intersection of an elliptical cylinder, the generatrices of which are horizontal, and tangent to the external surface of the torus. If, therefore, we wish to determine this curve with great precision, we must use the very same method adopted in determining the shadow proper of the external surface of the torus (316). In practice, however, when the drawing is on a small scale, we may content ourselves with determining the principal points in the curve, by projecting first the point, *v*, Fig. 6, situated upon the middle of the diameter, y y^1, of the torus, and drawing through it the line, v^1 v^2, equal to the diameter; and, secondly, drawing the horizontal lines touching the external contour of the torus in the points, z, z^1, Fig. 6, over to z^2, z^3, upon the axial line, q^1, o^1, Fig. 7; then draw an ellipse with these two lines, v^1 v^2 and z^2 z^3, for the transverse and conjugate axes respectively. The key, D, which passes through the rod, A, being rectangular in section, is projected in Fig. 7, by a couple of rectangles, as indicated by the dotted projection lines.

414. Proceeding upon these principles, we can in a very simple manner make oblique projections of various objects, more or less complicated in form, when we have already the projections of these objects upon auxiliary planes, making any known angle with the actual plane of the drawing. Thus, Figs. 10 and 13 are the oblique projections of an oscillating steam-cylinder, the first representing the cylinder in external elevation, whilst the second is a section made through the axis of the cylinder.

It will be seen that these projections have been obtained in the same manner as those already given in Figs. 4 and 7; that is to say, the external projection, Fig. 10, is derived from the two right projections, Figs. 8 and 9—one made upon an auxiliary vertical plane, parallel to the axis of the piston-rod, and perpendicular to the axial lines of the trunnions, and the other upon a horizontal plane, parallel to the cylinder ends, and, consequently, perpendicular to its axis. All the different parts of this cylinder are, in Fig. 10, projected by straight lines and ellipses, according as they are rectillinear or circular in contour. It is the same with the section, Fig. 13, and the horizontal projection, Fig. 14, which are both derived from the two right projections, Figs. 11 and 12, made upon auxiliary planes; one vertical, and passing through the axis of the cylinder, and through the valve-casing, whilst the other is perpendicular to this axis, and passes through the line, 1—2, Fig. 11. The dotted working lines, indicated upon the various figures, show with sufficient clearness the various constructions necessary to obtain these oblique projections. We have, moreover, applied numbers to the different parts projected, and more particularly to the axes or centre lines, which show at sight the correspondent parts in the different projections.

415. These drawings represent the cylinder of a steam-engine, different from that which we have already described. The present one is called an oscillating steam-engine, because, instead of the cylinder being vertical and immovable, it oscillates during the motion of its piston, B, upon the two trunnions, E, carried in suitable bearings in the engine-framing. This arrangement of oscillating cylinder has the advantage of dispensing with the parallel motion, and of attaching the rod, A, of the piston, directly to the crank-pin, to which its motion is transmitted, without the intervention of any connecting-rod. In the head, H, of the rod, there is, consequently, formed a bearing, which embraces the crank-pin.

The bottom of the cylinder is cast in the same piece with it, but it has a small central opening, for the passage of the spindle of the boring tool, by means of which the interior of the cylinder is turned smooth and true. This opening is closed by a cast-iron cap, F, bolted to the bottom of the cylinder. Against a planed face, upon one side of the cylinder, is fitted the valve-casing, G, which receives the steam direct from the boiler, and has within it the valve, H, which has an alternate rectillinear movement, at the same time oscillating along with the cylinder. During this movement, the valve alternately uncovers the ports, *a*, *b*, Fig. 11, which conduct the steam to the top and bottom of the cylinder. A blade spring, I, attached to the inside of the valve-casing, at the back of the valve, constantly keeps the latter well up against the planed valve face.

The steam coming from the boiler introduces itself into the casing through the passage, *c*, Fig. 12, which communicates with one of the trunnions, E, and the escape of the steam, when it has acted upon the piston, is effected through the exit channel, *d*, which communicates with the other trunnion.

The piston, B, is composed of a cast-iron body, on the outer surface of which is cut out a groove, to receive the hempen packing, *i*, partly covered by an elastic metal ring, *h*, fitting exactly into the inside of the cylinder.

Oscillating cylinder-engines have always been admired for their simplicity and beautiful action; but it is only of late years, since superior workmanship has been attainable, that such engines have been constructed of considerable size. The aptness of this arrangement for engines of the largest size was demonstrated by Penn, in the case of the *Great Britain*, and more recently by Scott Russell, in the large paddle-engines of the *Great Eastern*.

CHAPTER XIV.

PARALLEL PERSPECTIVE.

PRINCIPLES AND APPLICATIONS.

PLATE 42.

416. The name of Parallel Perspective is given to a peculiar method of representing objects according to the system of oblique projections. It differs from that described in the last chapter, inasmuch that the visual rays, hitherto supposed to be perpendicular to the geometrical planes, now form a certain angle with these planes, remaining, however, constantly parallel to each other; from which it follows that, according to this system of perspective, all the straight lines, which are parallel in the object, maintain their parellel direction in the picture. Although, in general, it is immaterial what the angle of inclination is, it is nevertheless preferable, in regular drawing, to adopt some particular angle as a matter of convention, which will have the advantage of giving the entire dimensions of the object in a single projection or view.

Let A B and A^1 B^1, Figs. 1 and 2, be the two projections of a straight line, to which we wish to make the visual rays all parallel; the vertical projection, A B, of this straight line, forms an angle, C A T, with the ground line, L T, which angle is, we shall suppose, equal to 30°, and its horizontal projection, A^1 B^1, is such, that the distance of the point, A^1, from the point, A, where it touches the horizontal plane, is equal to twice the length, A B, of its vertical projection, the point, B, being that at which it touches the vertical plane.

We shall proceed to show, by means of the various figures in Plate 42, that, in taking the above straight lines as directrices for the visual rays, a single projection will be quite sufficient to express all the dimensions of any object. Instead of making the directrices of the different objects, represented in this plate, to coincide with the actual projection of the straight lines which we have just indicated, we have, by preference, chosen the mere setting of these same lines round at an angle of 30°. Thus, the lines, e^1 d^3 k^1 i^2, Fig. 5, are the straight lines perpendicular to the planes of projection, set round to the angle in question; whilst, on the contrary, the straight line, *y z*, represents the projection of these lines properly parallel to the horizontal projection, A^1 B^1, in Fig. 2.

417. The finished view, Fig. 3, is the representation in *parallel*, or, as it is sometimes called, *false* perspective, of a prism, E, with a square base, resting upon a plinth, F, also prism-shaped and square. In the first place, we suppose this prism to be represented in the horizontal projection, Fig. 5, by two concentric squares, a^1 d^1 *e f* and h^1 i^1 *k l*: and in the vertical projection, Fig. 4, by the rectangles, *a b c d* and *g h i j*. These projections are made upon the supposition, hitherto acted upon, that the visual rays are perpendicular to the geometrical planes.

If now, on the contrary, the visual rays make with the planes of projection an angle equal to that of the given straight line, Figs. 1 and 2, each of the faces parallel to the vertical plane continues to be parallel to this plane, and is represented by a figure equal to itself, whilst all the faces perpendicular to the two planes are in projecting rendered oblique, in such a manner, that the lines horizontally projected become parallel to A^1 B^1, Fig. 2; and those vertically projected, to A B, Fig. 1. Consequently, if through the points of the projecting angles, *a*, *h*, *i*, *j*, Fig. 4, are drawn straight lines, parallel to A B, they will express the directions of all the edges perpendicular to the vertical plane. Since then, as we have already stated, the length of the projection, A B, Fig. 1, is equal to one-half the perpendicular, A A^1, Fig. 2, if from the points, *a*, *h*, *i*, *j*, and on each side of them, we measure off, upon the oblique lines just drawn, the distances, a a^2 and a f^2; h h^2 and h l^2; i i^2, i k^2, &c., respectively equal to half the lengths, a^1 m and h^1 n, &c., Fig. 5, we shall, in Fig. 4, have the perspective representation of the various straight lines perpendicular to the vertical plane; and as all the other edges are parallel to this same plane, such lines as are actually vertical are represented as vertical, whilst all lines parallel to the base line remain horizontal. Thus, the edges, *a b*, *h g*, *d c*, *i j*, being vertical, are in the parallel perspective represented by the verticals, a^2 b^2, h^2 g^2, i^2 j^2, d^2 c^2; and likewise the edges, *a d*, *b c*, *h i*, &c., which are parallel to both planes of projection, are rendered by the straight lines, a^2 d^2, b^2 c^2, h^2 i^2, &c., parallel to the base line.

EXAMPLES IN PAR

Fig. 3.

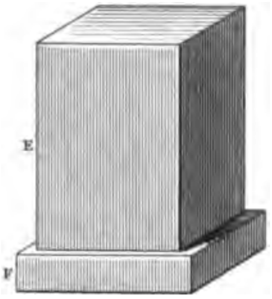


Fig. 6.

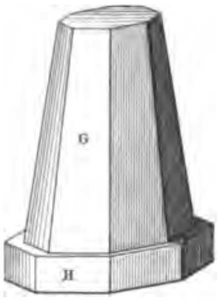


Fig. 4.

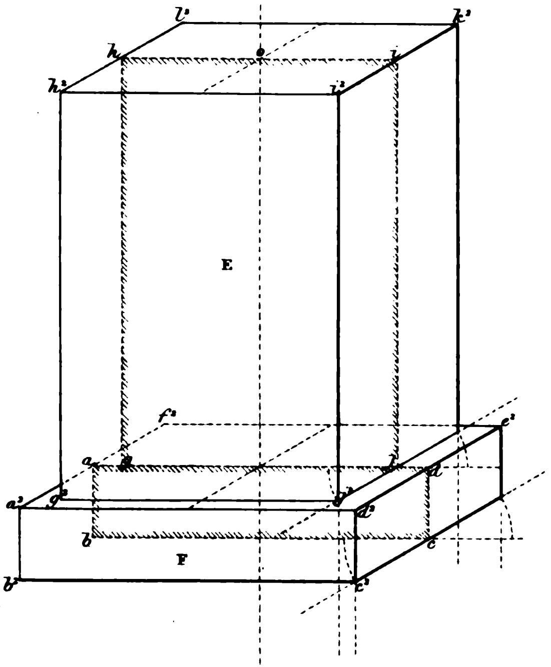
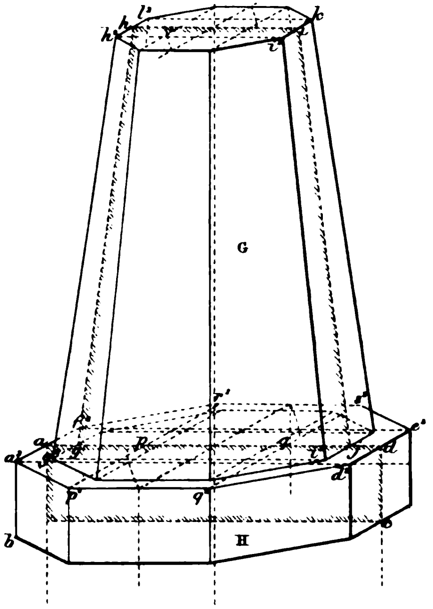


Fig. 7.



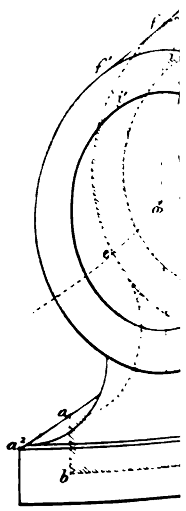


Fig. 5.

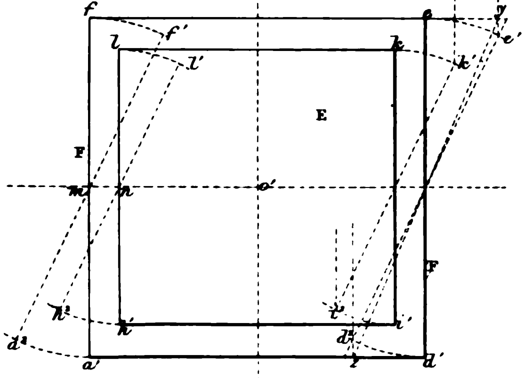
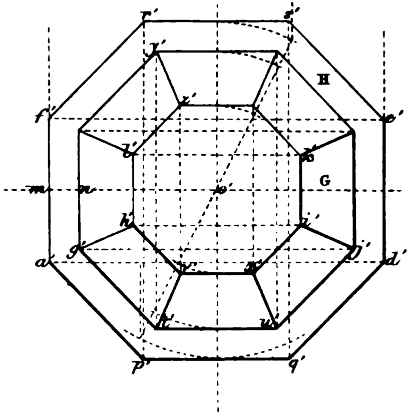


Fig. 8.



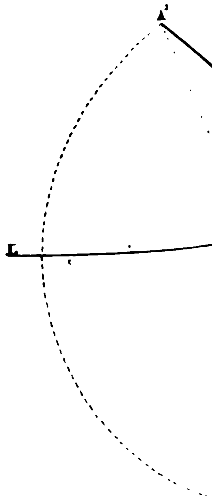


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Fig.9.

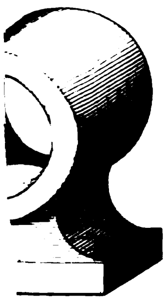


Fig.11.

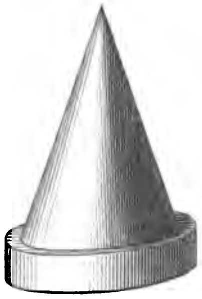


Fig.14.

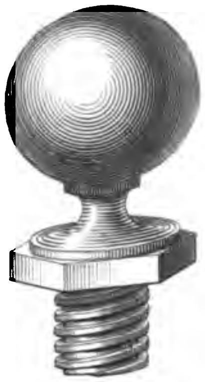


Fig.10.

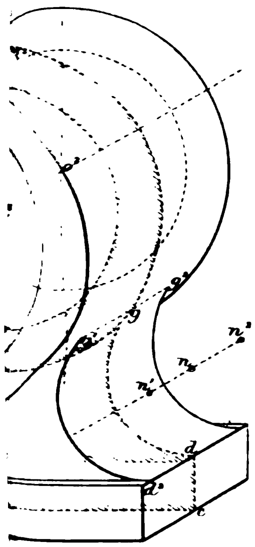


Fig.12.

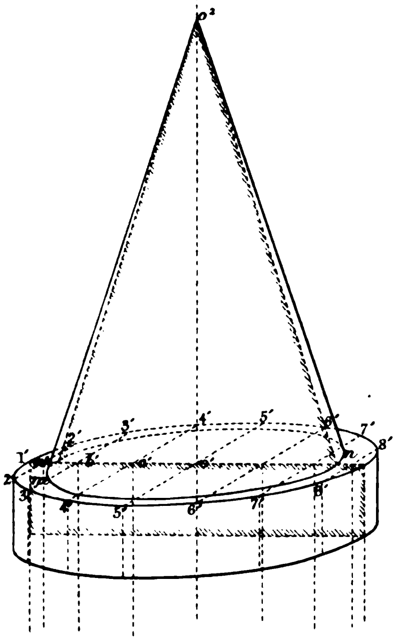


Fig.15.

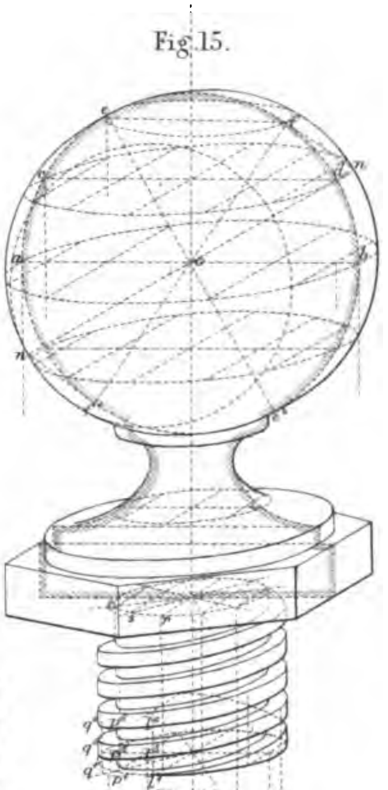


Fig.17.

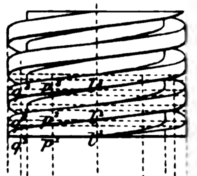


Fig.16.

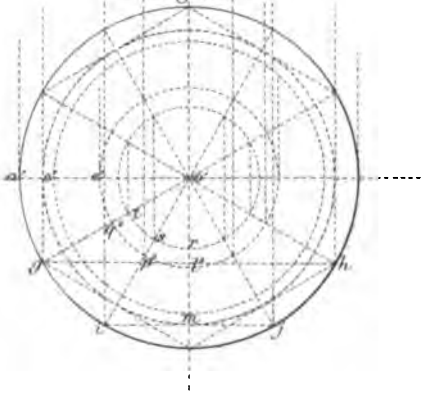


Fig.13.

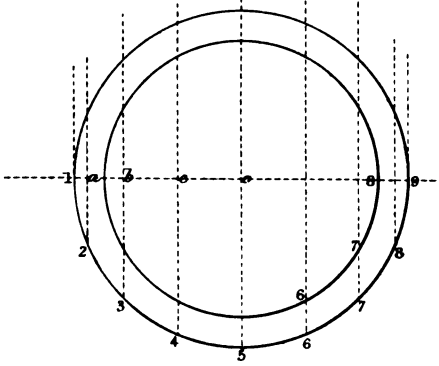
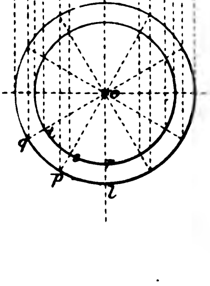


Fig.18.



g.1.

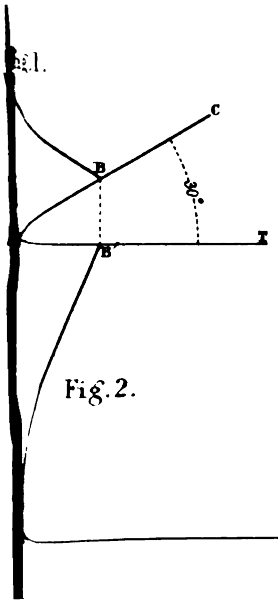


Fig.2.

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It will be easily seen, that by adopting the angle we have indicated for the direction of the visual, or more correctly termed, representative ray, that the one single view in parallel perspective is sufficient to make known all the dimensions of the object; for, on the one hand, we have the exact widths and heights of those faces which are parallel to the plane of the projection, as if the perspective view did not differ from an ordinary geometrical projection, in which the representative rays are supposed to be perpendicular to the plane; and, on the other hand, the oblique lines representing all the edges actually perpendicular to the vertical plane, and which are exactly equal to half the actual lengths of the latter.

We may here observe, that the base of the prism being square, the sides, $k\ i^1$ and $l\ h^1$, are equal to the sides, $h^1\ i^1\ l\ k$. Consequently, in order to construct the perspective or oblique projection, Fig. 4, the plan, Fig. 5, is not needed, since it would have answered the purpose equally well to have made the lines, $d^2\ e^2$ or $i^2\ k^2$, equal to the half of $a\ d$ or $h\ i$.

418. The shaded view, Fig. 6, represents a frustum of a regular pyramid, G, resting upon an octagonal base, H, the vertical projection of which is indicated by dotted lines in Fig. 7, and in horizontal projection in Fig. 8.

According to the principle thus laid down, the parallel perspective view, shown in sharp lines in Fig. 7, is obtained, in the first place, by drawing all the lines which are perpendicular and parallel to the base line in Fig. 8, and passing through the opposite angles of each of the octagons, representing the upper and lower bases of the pyramidal frustum, and of the plinth. Of these lines, all, $a^1\ d^1$, $f^1\ e^1$, $h^1\ i^1$, &c., which are parallel to the base line, as well as the sides, $p^1\ q^1$, $t^1\ u^1$, $v^1\ x^1$, which are likewise parallel to the former, remain horizontal in the perspective view, Fig. 7, whilst, on the other hand, all the straight lines, such as $p^1\ r^1$, $t^1\ y^1$, $v^1\ z^1$, as well as the sides, $a^1\ f^1$, $h^1\ l^1$, $i^1\ k^1$, which are perpendicular to the base line, become inclined at an angle of 30° from this line, as shown in Fig. 6, or, in other words, parallel to the straight line, A B, Fig. 1.

If now, through the points, a, g, p, q, &c., and the points h, o, i, of the two bases of the pyramidal frustum, we draw parallels to the straight line in question, and then mark off from each of those points, and on each side of them, the distances, $a\ a^2$, $g\ g^2$, $p\ p^2$, $h\ h^2$, &c., respectively, equal to the semi-lengths of the corresponding straight lines, $m\ a^1$, $n\ g^1$, &c., of Fig. 8, we shall have the parallel-perspective representation of all these lines; and consequently, by joining the extreme points of each of them, we shall also have all the lines representing the contours of the two bases; and further, by joining the angles of these bases, we define the lateral faces, and complete the view, as shown in outline in Fig. 7, and shaded and finished to a smaller scale at Fig. 6.

419. The parallel-perspective representation of a cylindrical object, of which the axis is perpendicular to the vertical plane, as shown in the finished example, Fig. 9, may be determined without the assistance of a horizontal projection; that is, when the length of the cylinder is known, as well as that of any other part which may be perpendicular to the vertical plane.

Let $a\ b\ c\ d\ g\ f\ e$, Fig. 10, be the vertical projection of this object, the perspective of its base, $a\ b\ c\ d$, will be parallel to A B, Fig. 1. The circles which have their centres at o, being parallel to the vertical plane, are represented in perspective by two circles equal to themselves; and their position is obtained by drawing through the point, o, the straight line, $o^1\ o^2$, also parallel to A B, Fig. 1, and marking off a distance, lying equally on both sides of the point, o, equal to half the length of the cylinder, measured in the direction of the axis, perpendicular to the vertical plane. Then with the points, o^1, o^2, as centres, describe circles with the radii, $o^1\ f^1$ and $o^1\ i^1$, respectively equal to $o\ f$ and $o\ i$; straight lines, $f^1\ f^2$ and $i^1\ i^2$, drawn tangentially to these circles, and parallel to the axis, $o^1\ o^2$, express in parallel perspective the generatrices of the two cylinders forming the contour of the object. The cylindrical pieces which join the cylinder to the base are determined in the same manner by means of the line, $n\ n^2$; drawn through the centre, n, of the circle, $d\ g$, parallel to A B, Fig. 1, or $o^1\ o^2$, and by the distances $n\ n^1$, $n\ n^2$, together equal to half the actual length of these cylindrical surfaces. The base being drawn as in the preceding example, completes the parallel perspective of this figure, as shown shaded and in outline in Figs. 9 and 10.

420. The example, Fig. 11, represents a cone resting upon a cylindrical base, both cone and base having the same axis perpendicular to the horizontal plane. This cone and cylinder are projected in elevation in dotted lines at Fig. 12, and in plan or horizontal projection in Fig. 13.

The circles in Fig. 13, representing the bases of the cone and cylinder, are to be divided into a certain number of equal parts; and through the points of division, 1, 2, 3, &c., perpendiculars are drawn to the base line, and are prolonged as far as the horizontal line, $a^1\ o^1$, which is the vertical projection of the two bases. Through the points, a^1, b^1, c^1, o^1, are drawn straight lines parallel to A B, Fig. 1; and on each of these are set off the distances, $a^1\ 2^1$, $b^1\ 3^1$, $c^1\ 4^1$, $o^1\ 5^1$, &c., Fig. 12, respectively equal to half the lengths of the perpendiculars, 2 a, 3 b, 4 c, 5 o, &c., Fig. 13, which operation gives the points, 2^1, 3^1, 4^1, 5^1, &c., Fig. 12, through which an ellipse must be traced, to represent the perspective of the base of the cylinder. In the same manner we obtain the points through which passes the ellipse, representing the perspective of the base of the cone.

The heights of the cone and its base remain precisely what they really are, in consequence of their common axis being parallel to the vertical plane; but this is not the case with their bases, which, being horizontal, are projected obliquely, in the form of the ellipses we have just drawn. The apex of the cone, at the upper extremity of the vertical axis, does not change, and for the generatrices, or sides of the cone, it is simply necessary to draw through the apex the straight lines, $o^2\ m$ and $o^2\ n$, tangents to the ellipse representing the base of the cone, whilst the generatrices of the cylinder are tangents to the two ellipses representing its upper and lower bases.

421. The example, Fig. 14, is the parallel perspective representation of a metal sphere or knob, attached to a polygonal base, by a circular gorge, forming altogether an ornamental head for a screw. In Figs. 15 and 16 are represented the vertical and horizontal projections of this piece respectively, in dotted and sharp lines, as in the other figures.

We must, in the first place, remark that the sphere, the radius of which is $o\ a$, Fig. 15, may be determined in its perspective representation in several ways. First, by imagining the horizontal sections, $a\ b$, $c\ d$, $e\ f$, which give in plan the circles, with the radii, $a^1\ o^1$, $c^1\ o^1$, and $e^1\ o^1$, and the perspectives of each of these circles may be obtained by operations similar to the preceding, which will give a series of ellipses, to be circumscribed by another ellipse, tangential to them all. Second, by drawing the planes, $g\ h$, $i\ j$, parallel to the vertical plane, and which are projected in perspective as circles, with the radii, $l^2\ g$, $i\ m$, the centres being upon the oblique line, $n\ n^1$, parallel to the line, A B, Fig. 1, and passing through the centre, o, of the sphere. The external curve, drawn tangential to all these circles, will be elliptical, as in the preceding method, and represent the perspective of the sphere. Thirdly, by at first drawing through the centre, o, an oblique line, $n\ n^1$; then a perpendicular, $e\ e^2$, passing through the same point. Then set out from the centre, o, and on each side, the distances, $o\ e$, $o\ e^2$, equal to the radius, $o\ a$, of the sphere, which gives the conjugate axis of the ellipse. To obtain the transverse axis, it will be necessary to draw tangents to the great circle of the sphere, parallel to the oblique ray of projection, A B, Fig. 1, as brought into the vertical plane. This line is brought into the vertical plane, as at A^2 B, Fig. 1, in the following manner:—At the point, A, a perpendicular to A B is erected, and the distance, A^2 A, is made equal to A A^1, when A^2 B is joined.

These tangents touch the sphere in the points, f, f^1, Fig. 15, which may be obtained directly by drawing through the centre, o, the line, $f\ f^1$, perpendicular to A^2 B. These tangents, further, meet the line, $n\ n^1$, in the points, $n\ n^1$; and the distance between these points is, consequently, the transverse axis of the ellipse, which represents the perspective of the sphere, and which may be drawn according to any of the known methods.

This last method is evidently the shortest and simplest of the three for obtaining the parallel-perspective of the sphere, but it is confined in its application for any other surface of revolution, as, for example, the gorge, which unites the sphere to the hexagonal base, cannot be defined in this way. In cases where the axis of the surface of revolution is vertical, as in this example, it will be necessary to adopt the first general method, which consists in taking horizontal sections. When, on the other hand, the axis of the surface of revolution is horizontal, we must employ the second process, which consists in taking sections parallel to the vertical plane, or perpendicular to the axis. The perspective of the thread of the screw, of which the sphere is the ornamental head, is determinable in a manner analogous to that of a circle. It is sufficient, in fact, first to draw the two geometrical projections, Figs. 17 and 18, of one or two convolutions of the thread, and to find the perspectives of the very points which have served for the construction of the helices. Thus, for example, we put the circle, $l\ p\ q$, Fig. 18, into perspective, as at $l^4\ p^4\ q^4$, Fig. 15, retaining for this purpose the same points, l, p, q, &c., Fig. 18, which were employed in delineating the screw, Fig. 17. Through these points, l^4, p^4, q^4, &c., in Fig. 15, draw vertical lines; and upon them set off the distances, $l^1\ l^2$, $l^1\ l^3$, $p^1\ p^2$, &c., Fig. 17. Then through the points, l^4, l^5, l^6, &c., draw the curve, which will be the parallel-perspective of the outer helix of the screw-thread. By going through the same operation for the inner circle, $r\ s\ t$, Fig. 16, we shall obtain the similarly perspective outline of the inner helix.

An examination of the lower part of Fig. 13, will further show that the heights on the vertical lines are precisely the same for both helices, for they are taken upon radii common to the two circles, $l^2\ p^2\ q^2$, and $r\ s\ t$, considered as bases.

We have deemed it unnecessary to enter further into the development of this species of perspective, of which we have already given a good general application and highly-finished example, in the boring-machine, represented at Fig. 1, Plate 35, in which are collected most of the various forms which present themselves for delineation amongst mechanical elements and machinery.

In that example, as well as in the finished figures forming the top row in Plate 42, which we have just been studying, we have supposed the representative or visual rays to be in all cases parallel to each other, and to be inclined at an angle of 30°, with the ground or base line in the vertical projection, in such a manner that a single view serves to express all that two, or even three, geometrical projections can do, showing not only the external contours of the objects, but also whatever may be upon their surface.

The utility of this system of giving, at a single view, a general and precise idea of the actual appearance of any object will be apparent. It is a method of representation often more intelligible to the generality of people than a series of geometrical projections, and in many cases it will greatly simplify the process of sketching buildings or machinery.

CHAPTER XV.

ISOMETRICAL PROJECTION.

PRINCIPLES AND APPLICATIONS.

PLATE 43.

422. Another mode of representing such objects as are related to the rectangular parallelopiped, so as to bring three of their sides into the picture, is by drawing them according to the rules of Isometrical Projection (or Isometrical Perspective, as it is sometimes termed); and this mode of projection or drawing has the advantage of allowing every line to be measured by one scale. Professor Farish of Cambridge was the first to show, that in projecting solids comprised under rectangular faces, that if the projecting rays are parallel to the diagonal of a cube having its faces parallel to the faces of the solid to be projected, the representation of such solids may be made by a common scale without the trouble of actual projection, in such a manner that all right angles will be represented in the drawing, either, by angles of 60°, or 120°; and that all the right lines in the drawing would bear the same proportion to each other which those of the original object bore to each other respectively. This mode of representation he termed Isometrical, a word signifying "equally measuring."

423. To proceed to examples: Figs. 1, 2, 3, and 4, represent the isometrical projection of a solid cube, as that shown in plan and elevation in Figs. 5 and 6, in four different positions, each showing three sides of the cube, the other three opposite and parallel sides being invisible. In Fig. 1, the upper horizontal face, A, and the

EXAMPLES IN ISO

Fig.1.

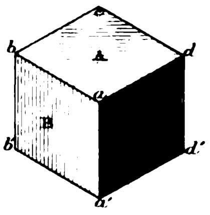


Fig.3.

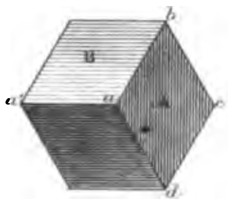


Fig.13.

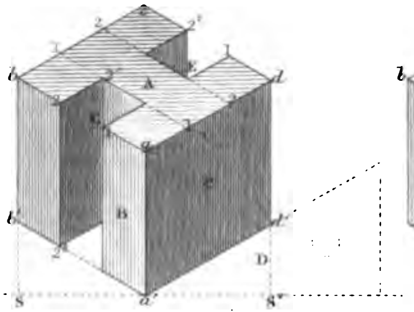


Fig.14.

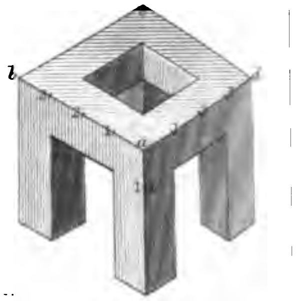


Fig.2.

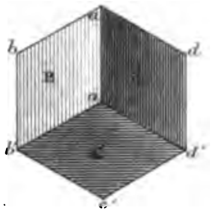


Fig.4.

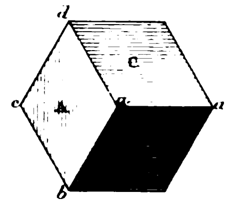


Fig.15.

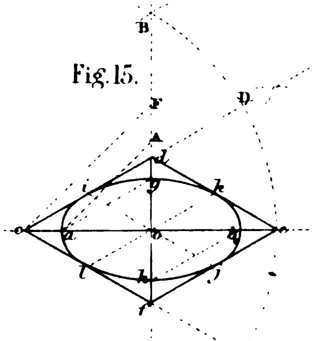


Fig.18.

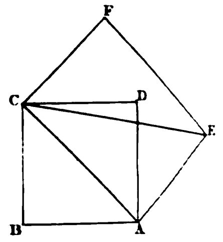


Fig.5.

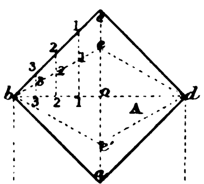


Fig.7.

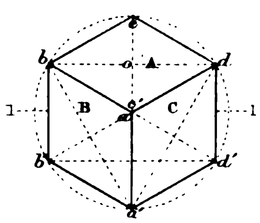


Fig.16.

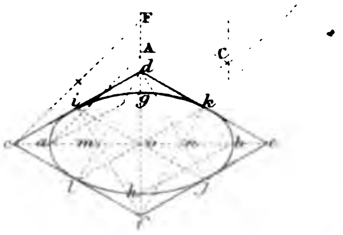


Fig.19.

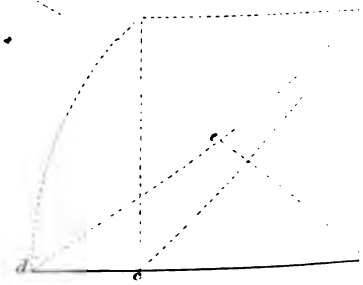


Fig.6.

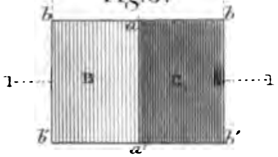


Fig.9.

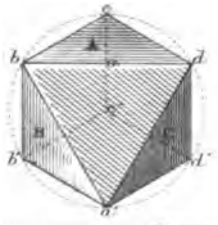


Fig.8.

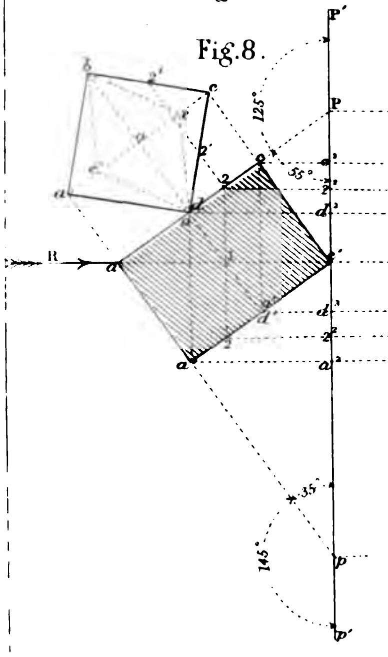


Fig.10.

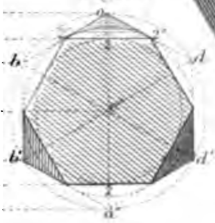


Fig.12.

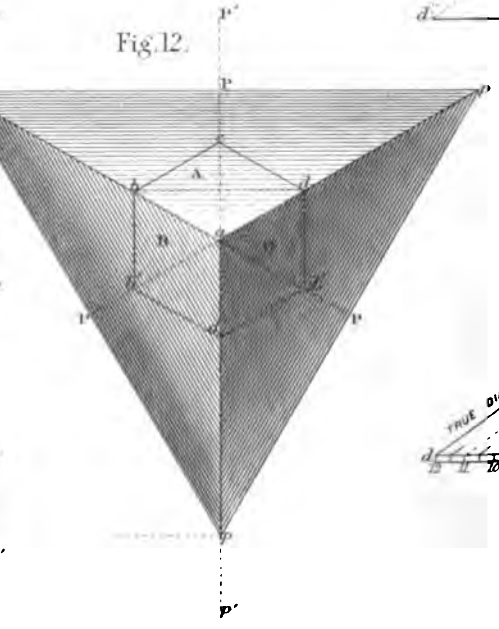


Fig.20.

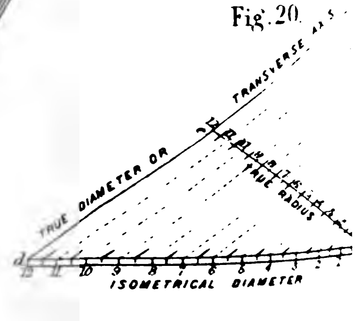
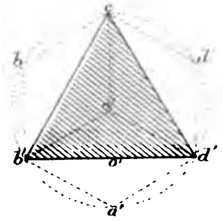


Fig.11.



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TRICAL PROJECTION.

Fig.17.

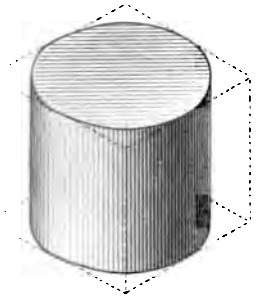


Fig.25.

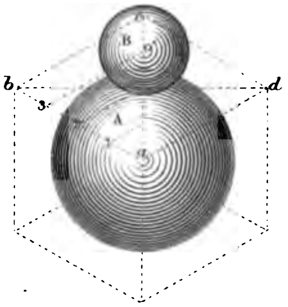


Fig.28.

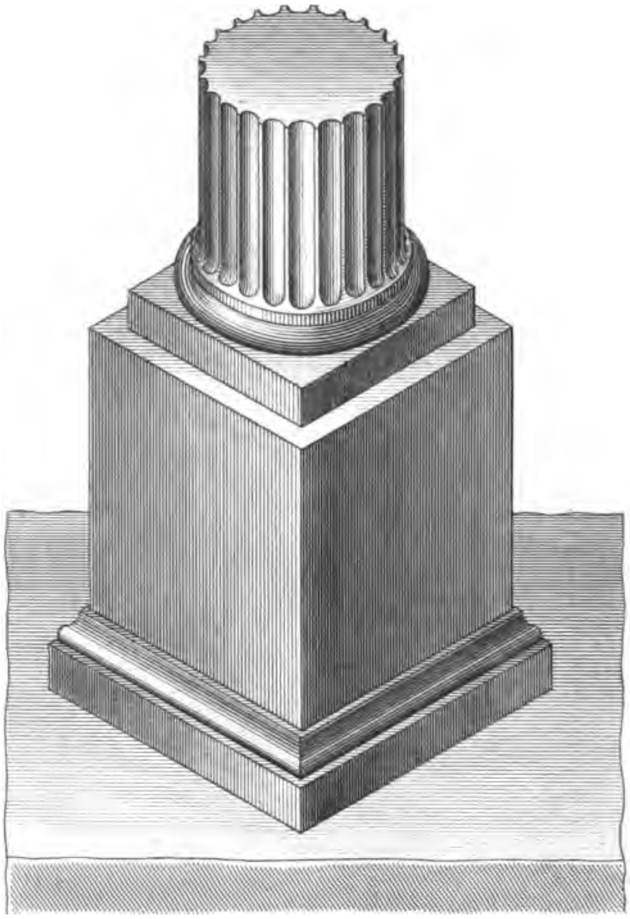


Fig.21.

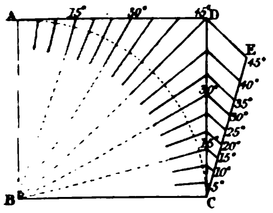


Fig.26.

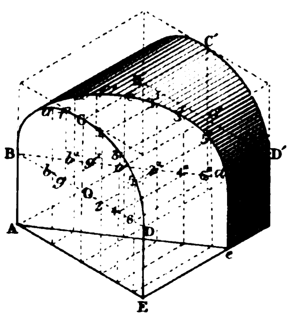


Fig. 22.

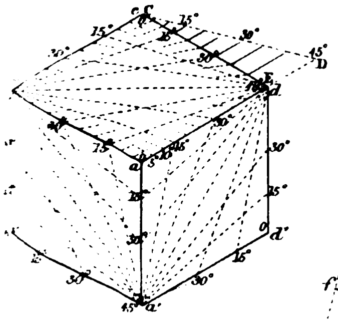


Fig.23.

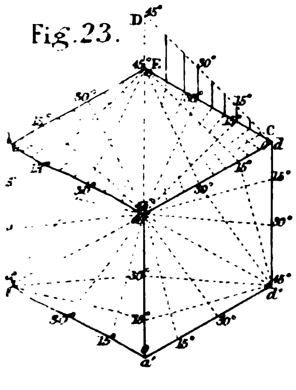


Fig.24.

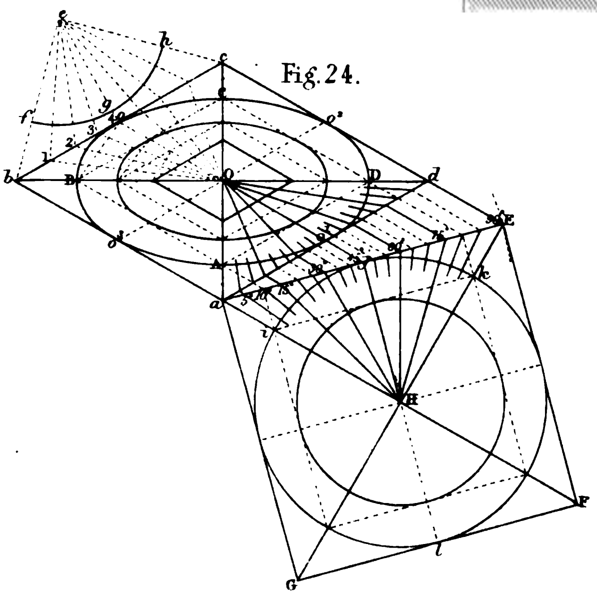
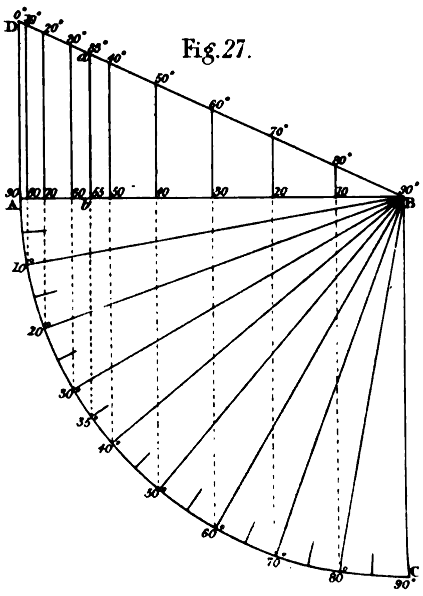


Fig.27.



two front vertical faces, B and C, are visible, while in Fig. 2, it is the under face, A^1, along with the same two vertical sides, which are seen, or, as it were, having the same projection of the cube, as that shown in Fig. 1, turned half round, or 180°, on the diagonal $a\ c^1$, as an axis. Figs. 3 and 4 are lettered, and supposed to be turned round, in this way, one-fourth of a circle, or 90°, to the right and left respectively, showing the two front sides, B and C, as horizontal, which was vertical in the two first figures, and the side, A, as vertical which was horizontal. These four projections of a cube, though shaded as solids, can also be viewed, and stand for the projection of three sides of the interior of a hollow cube. With a slight effort, the eye can make these figures either seem to project out from the surface of the paper, or recede beyond it, according as the eye is focused for the corner, *a*, of the representation of the cube, at a little distance, on the near, or far, side of the paper on which it is drawn.

It will be seen on reference to Fig. 7, which shows the construction and outline of the isometrical cube as projected in Fig. 1, that all the boundary lines of the figure are equal, as well as all the sides, that is, both the three anterior faces which are seen, and the three posterior ones which are only dotted in, are equal, as well as their boundary lines. A cube drawn on this principle is easily delineated, as it is simply a hexagon, $a^1\ b^1$, $b\ c$, $d\ d^1$, inscribed in a circle, (Fig. 19, Plate 1; 16) completed by drawing the three radii $a\ a^1$, *a b*, and *a d*, to the alternate angles, and these will represent a vertical, and two of the upper horizontal edges of the cube, as shown in Figs. 1 and 7. It will be evident, on comparing the several lines in Fig. 7 with their corresponding ones in Figs. 5 and 6, of which they are the isometrical projections, that they are all foreshortened, except such lines, or dimensions, as lie and are measured on a plane parallel to the plane of projection, which are in this case *equal*, both in the isometrical projection and in the actual cube. This will be seen from a comparison of the lines and measurements, *b o*, *b d*, $o\ a^1$; 2—2, 2^1—2^1, and $b^1\ c$, $b^1\ o^1\ d^1$, $c\ o^1$, taken on the three sectional surfaces in Figs. 9, 10, and 11, which agree with, and are equal to, the corresponding measurements in the plan and section Fig. 8, from which these isometrical sections are taken, through the lines $o\ a^1$, 2—2, and $c\ o^1$, respectively, of the largest rectangular-sectional elevation of the cube parallel to the plane of projection, P *p*.

From the foregoing, it will be seen that, when a cube is isometrically projected, as in this case, having one of its longest diagonals, $a\ c^1$, at right angles to the plane of projection, P *p*, and as shown in Figs. 1, 2, 3, and 4, as one of its greatest rectangles, $a\ c\ c^1$, a^1, passing through that diagonal either vertical or parallel to the horizontal plane, all the lines forming the outline of the isometrical square faces are foreshortened, and equal to the shortest diagonal of the equilateral parallelogram, while that of the major axis, or diagonal, is equal to the diagonal across the face of the original cube, of which it was the isometrical projection. This will be quite clear on referring to Fig. 8, which shows the plan of the cube, Fig. 5, reproduced, with its greatest rectangle projected and turned round on 1—1, Figs. 6, 7, and 8, as an axis, until the two opposite diagonals, $a\ c^1$, are at right angles to the plane of projection, P *p*, which here shows as a line, the whole figure being brought into the picture as at right angles to Figs. 1 and 7, and shown vertical for the convenience of squaring or projecting the several points over to Fig. 10, which makes no difference, it being borne in mind that *a c*, the top face of the cube, is parallel to the horizontal plane, and *a d*, the front edge, parallel to the vertical plane, and that the eye of the spectator is supposed to be looking in the parallel direction indicated by the arrow R. The line $c^1\ a$, which is the diagonal of the cube at right angles to the plane of projection, P *p*, is produced in the direction of the arrow R, this line R c^1, shows the direction of the projecting rays of light, which are drawn through all points of the cube parallel to this line, and therefore, also, perpendicular to the plane, P *p*. The points *c*, *d o*, a^1, and $d^1\ o^1$, being thus projected parallel to $a\ c^1$, on P *p*, in c^2, d^2, a^2, and d^3, respectively, it will be seen that the diagonals *a c* and $a^1\ c^1$, of the upper and the lower faces of the cube are foreshortened into $c^1\ c^2$ and $c^1\ a^2$, which are also the projection of the edges, $c\ c^1$, $a\ a^1$, while the edge $d\ d^1$, or centre line $o\ o^1$, is foreshortened into $d^2\ d^3$, on the plane of projection P *p*, and it will be seen at once that, by the simplest problems of the first book of Euclid, these lines being all the parallel projections of parallel lines, intersected by parallels, are equal; that is, as before stated, the isometrical projection $d^2\ d^3$, Fig. 8, or $d\ d^1$, Fig. 7, of the edge of a cube, is equal to $c^1\ c^2$, $c^1\ a^2$, Fig. 8, or $c^1\ c$, $c^1\ a^1$, Figs. 7, 9, and 11, the minor or foreshortened diagonal of the same cube; that is, to the radius of the circle, and to all the lines forming the sides of the isometrical square inscribed in it, as squared over from the plane of projection P *p* in Fig. 8, to the isometrical section, Fig. 10, being indicated by the same letters of reference in this and the foregoing figures. Fig. 10 shows the section of the cube through its centre, parallel to the plane of projection, (cutting the six sides symetrically,) and producing a hexigon, the radius of the inscribed circle being equal to the line $c^1\ d^2$, projected on the line P *p*, whilst its sides and the radius of the circumscribed circle are each equal to 2^1—2^1, Fig. 8.

To find the isometrical radius or side of a square corresponding to an edge of the original cube or square, and *vis versa*, the side of the actual square from the isometrical square. Let *a b c d*, Figs. 5 and 8, be a superficial square, or one face of a cube; on it draw the two diagonals, *a c* and *b d*, crossing each other at *o*, making all the angles at the intersection equal, that is, four right angles, and from the points, *b* and *d*, on the latter diagonal, draw the lines, *b e*, $b\ e^1$; and *d e*, $d\ e^1$, making angles of 30°, with the line, *b d*, and they will intersect each other in *e* and e^1, on the other diagonal *a c*, the lozenge or equilateral parallelogram, $b\ e\ d\ e^1$, is the isometrical representation of the square, *a b c d*, and any of its sides, as *b e*, will be the radius of the circumscribed circle in which to draw the isometrical cube; then, (for the converse), let $b\ e\ d\ e^1$, Fig. 5, be the given isometrical square, and as before, draw the two diagonals, *b d*, and $e\ e^1$, and produce the latter in the direction of *c*, make half the minor diagonal produced equal to that of half the major, that is, make *o c*, equal to *o b*, and join *b c*, then *b c* is equal to the actual side of the square, of which, *b e*, was the isometrical projection. The proportion of the isometrical radius or side of the square to that of the original line or side is, (that is, *b e*, is to *b a* or *b c*), as ·8164 to 1·0; or in whole numbers, nearly, as 9 to 11.

If the side, *b c*, Fig. 5, of the original square be taken as a scale of yards, feet or inches, and divided off accordingly, lines drawn through such divisions, parallel to *c o*, as 1, 2, 3, will divide *b e* in the same proportion, or into the same number of aliquot parts, and make it an isometrical measure or scale of parts corresponding to that of the original for all isometrical lines making angles of 30° with a line parallel to the plane of projection, as the three longer diagonals, $b\ a^1$, *b d*, and $d\ a^1$, of the faces of the cube in Fig. 7, and on surfaces making similar angles as those of the sides of the cube, (namely, 55° and 125), with the plane of projection; that is, on all the lines which represent the boundary of the vertical and horizontal planes of the cubes in Figs. 1, 2, 3, 4, and 7, and of all lines parallel to them, on the same or parallel planes to those of the sides of the cubes in these several figures, whether taken as solid or hollow cubes.

If the draughtsman has a plan and elevation of the object to be put into isometrical projection, to anything near the scale he desires it to be drawn, a pair of proportional compasses may be used for taking all measurements on the regular boundary lines of the vertical and horizontal planes, or lines parallel to them, instead of the isometrical scale just described.

In practice, however, it will frequently be found unecessary to convert the true dimensions into isometrical ones, as the isometrical drawing being made on the same scale as the plan and elevation on which it is founded, saves much labour, and the deviation from strict correctness is attended with little or no inconvenience if the actual state of the case is kept in mind.

At Fig. 12 is shown the isometrical projection of the cube, Fig. 8, with that of its three planes or faces, till they meet the plane of projection, by producing the lines, *a c*, and *a* a^1, Fig. 8, which represent the upper horizontal face, A, and the centre vertical angle or radius, *a* a^1, in these two figures until they meet the plane of projection, P *p*, in P and *p*, respectively. When these points, and the other sides, B, C, and edges or radii, *a b*, *a d*, are projected into the picture, as at Fig. 12, a triangular pyramid is represented, having an equilateral base, *p p p*, of which P *p* is the perpendicular, each side of the base is parallel to the longer diagonal of a side of the cube, and at right angles to the lesser one, and to one of the central radii, which are perpendicular to the plane of projection. and corresponding to the plane of section in Fig. 8.

The lines *a* P and *a p*, which represent the upper horizontal face of the cube, and vertical radius or front angular edge, and which are respectively parallel to the horizontal and vertical planes of projection, Figs. 8 and 12, make the angles *a* P *p*, and *a p* P, with the plane of projection P *p*, respectively equal to 55°, nearly,[*] and 35°, with their complements *a* P P^1, and *a p* p^1, respectively equal to 125°, and 145°, as marked in Fig. 8. It will be observed that the angles are the same as regards the projecting rays, only it is the sides which make angles of 35° and 145°, with them, and the angle edges which make those to 55° and 125°, with the regs.

Isometrical drawing is greatly facilitated by the use of triangles, "angles," or "set squares," one which is much employed is that marked D, Figs. 6 and 16, of Plate I., having one right angle, and of the other two angles the one is 30°, and the other 60°; another set square used is that of an isosecles triangle, having two of its angles each 30°, and the other 120°; a third angle is sometimes used, which is that of an equilateral triangle, having its three angles, each equal to 60°. The use of these instruments will be readily seen; they are to be got from opticians, and have usually isometrical scales, cubes, and circles, laid off on one of their faces.

As simple examples in isometrical drawing, and in the foregoing rules, under cubes, we give Figs. 13 and 14, the inspection and reproduction of which, as well as that of the other figures, it is hoped will enable the student to draw many rectangular and similar objects. These examples are necessary to a small scale, but in practice it would be better to draw these diagrams to two or three times the size of those given.

In drawing Fig. 13, it is not necessary to go through much of the construction described. Draw the vertical line, *a* a^1, equal to the isometrical line or side of the square, and let S S^1 be a horizontal base line or edge of a T— square, then with the angle, D, Fig. 6, Plate 1, laid on it, make the angles S a^1 b^1, and S^1 a^1 d^1, equal to 30°, and make the lines a^1 b^1, and a^1 d^1, each equal to *a* a^1; or make a^1 S and a^1 S^1 each equal to the half of the diagonal of the original square, then through these points S, or b^1 and S^1 or d^1, draw perpendiculars parallel to *a* a^1, and through the point *a* draw *a b*, *a d*, parallel to a^1 b^1, a^1 d^1, respectively, which will intersect the verticals in *b* and *d*, through which draw *b c*, parallel to *a d*, and *d c* to *a b*, which will complete the outline of the cube. The two slots shown E E, being one-third of the side in width and depth, and in the centre of the two opposite sides, the two edges *a b*, and *a d*, are trisected in 1 and 2, through which points, lines are drawn parallel to the boundary lines of the top horizontal face A, also with the square and angle D, as all horizontal boundary lines are generally drawn with the angles of 60° and 30°, to the vertical and horizontal lines of the picture, when it is the longest diagonal of the cube, which is perpendicular to the plane of projection, as in this and the foregoing instances. The intersection of these lines 1—1, 2—2, give the outline of the end of the slots and perpendiculars drawn through; these intersections at 1, 2, on the left hand vertical side, B, and also a short one through 2^2, with a short horizontal through 2^3, parallel a^1 d^1, completes the whole of Fig. 13. Fig. 14 is drawn much in the same way as the last figure, only the edges *a b*, *a d*, are divided into four equal parts, one of each being taken as the dimension of the four corner pillars, and also as the thickness of the top slab, which has a concentric square taken out of it equal to two of these parts, or one-fourth of the whole area of the top surface.

424. In drawing isometrical circles (which are always ellipses), on planes parallel to the isometrical sides of the cube, as A, B, and C in Figs. 1 to 4, with a view to the delineation of columns, wheels or cylinders, &c., it is usual either to have the actual diameter of the circle required, or the isometrical square in which the circle is required to be inscribed, given. Taking the first case, then, the following is one of the most simple constructions given by Mr Bradley as a means of finding the various isometrical points required:—Let *a b*, Fig. 15, be given as the actual diameter of the required circle, or as the side of an original square in which such a circle could be inscribed; bisect *a b*, in *o*, by a perpendicular to it, and produce it on both sides of *a b*, and make *o* A on one side equal to *o a*, or *o b*; join *a* A, and set off the length of the diagonal, *a* A, each way from *o*, along *a b*, produced to *c* and *e*, from *c* or *e*, as a centre with *c e*, as a radius describe a segment of a circle, and from the opposite points, *e* or *c*, with the same radius intersect it in B and C, (B and C will fall on the perpendicular, *o* A, produced); then, bisect the arcs, *e* B, *e* C, by lines from *c*, they will cut, *o* B and *o* C, in *d* and *f*, respectively; then join *e d* and *e f*, and *c d e f* will be the isometrical square, equal to, or corresponding to, the actual square, having a side equal to *a b*, in which to inscribe the isometrical circle, having an isometrical diameter equal to *a b*. Through the centre, *o*, draw parallels, *i j*, *k l*, to the sides, *d e* or *c f*, and *d c* or *e f*, respectively, and they will each be isometrical diameters; so that their extremities, *i*, *j*, *k*, and *l*, will give four points in the isometrical circle, while a line drawn parallel to any one of the sides, as *c d*, through the points, *a* and *b*, will intersect *d f* in *g* and *h*, the extremities of the minor axis, *g h*, of the ellipse, of which *a b* is the major axis, and which may now be by any of the methods given (55 to 59), or inscribed through the eight points, *a*, *i*, *g*, *k*, *b*, *j*, *h*, and *l*, as was required.

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[*] The angles are given in the nearest whole numbers of degrees, as it is, though exact enough for general practice, 55°, is some minutes and seconds too much, and 35°, as much too little.

If, however, it is the isometrical square, *c d e f*, Figs. 15 and 16, which is given to inscribe the ellipse or isometrical circle, then the lines, *i j*, *k l*, drawn parallel to the sides of this square through the centre, *o*, of the intersection of the two diagonals, *c e*, *d f*, will be two diameters, and give four points of the ellipse; make *o* F on half the lesser diagonal produced equal to *o c*, half of the larger diagonal, join *c* F, and make *o a* and *o b* each equal to a half of *c* F, *a b* will be the major axis of the ellipse, of which *a* and *b* will be two more points. Lines drawn through *a* and *b*, parallel to the sides of the square, will intersect the shorter diagonal, *d f*, in *g* and *h*, which gives *g h* as the minor axis, and its extremities are other two points of the ellipse, which may now be inscribed in the given square through the eight points, *a*, *i*, *g*, *k*, *b*, *j*, *h*, and *l*, which have been found, by isometrical elliptical curves, or moulds, (which are to be had from opticians made for the purpose)[*] or it may be drawn, approximately, from four centres with the compasses; thus, as seen in Fig. 16, join *f i*, *f k*, and *d l*, *d j*, these lines will intersect the major axis in the two points, *m* and *n*, which are two of the centres for drawing the lesser arcs at the ends of the major axis; that is, from the points, *m* and *n*, with *m i*, *m l*, or *n k*, *n j*, as a radius draw the arcs, *i a l* and *k b j*, then with the other two points, *d* and *f*, from which the greater arcs of the ellipse is drawn, with *d j*, *d l*, or *f i*, *f k*, as a radius describe the arcs, *j h l*, and *i g k*, joining the lesser arcs at the extremities of the two diameters, *i j* and *k l*.—This completes the ellipse, which comes very near to the true curve and proportions of an isometrical ellipse. Fig. 17 is an illustration of an isometrical cylinder, drawn according to the latter method, and shown as inscribed within a cube, which is shown in dotted lines.

It can be proved that the minor axis, the isometrical diameter, and the major axis, of a circle isometrically projected, are in the same ratio as the side, A B, Fig. 18, the diagonal of the face, A C, and the diagonal, C E, of a solid cube. That is, taking A B $= \sqrt{1}$, then A C $= \sqrt{2}$, and C E $= \sqrt{3}$, as is obvious from the figure A C F E, being a section through the cube by a plane through the parallel diagonals of two parallel faces, A E, being made equal to A B.

The proportions of the major and minor are to the isometrical diameter as follows:—

Major axis.	Minor axis.	Isometrical diameter.
1·225	·707	1
2·449	1·414	2
3·674	2·121	3

It will be seen from the above three ratios given that, having got the first, any other may be found, as the others are simply multiples of the first by the number of units in the isometrical diameter.

The following mode of constructing circles isometrically, without going through much of the foregoing construction, or previously making the isometrical squares or cubes to inscribe them in, will be found useful. Draw the line *a b* and *a d*, Fig. 19, at right angles to each other, and from *a*, with *a b* as a radius, describe a quadrant cutting *a d*, in *c*, join *c b*—then with *c b*, from *a*, cut *a d*, in *d*, making *a d* equal to *c b*, and join *d b*:—Now, if *d b* measure the diameter of a circle to be represented isometrically, or side of a square, in which such a circle could be drawn, it will be the major axis, and *a b* will be the minor of the ellipse, while *c b*, or *a d*, will be the isometrical diameter, or equal to the side of the isometrical square, in which the ellipse, corresponding to the original circle, having *d b* as its diameters. Bisect *d b* in *e*, and join *a e*, then *a e* is equal to half the radius *d b* of the original circle. In this way a geometrical scale may be constructed, as represented in Fig. 20, from which any two of these three measurements may be found, any one of which being given. To construct the scale, draw the triangle *a b d*, Fig. 20, the same as that in Fig. 19; then if it is from the actual sizes or dimensions of the elevation and plan of the objects desired to be delineated in isometrical projection that the draughtsman is working, it is the transverse or major axis *d b*, or *a e*, equal to the half of that axis, which is divided into the same number of equal parts, or to the same scale as that of the original drawing on which the isometrical representation is founded. Through these divisions on *a e*, draw lines parallel to the hypothenuse *d b*, they will intersect the perpendicular *a b*, and the horizontal line *a d*, in equal parts, proportional to the original whole lines. To find the isometrical radius and minor axis of an ellipse which would represent an actual circle of a given diameter, take half that diameter, or the radius of the given circle, with the compasses, and find a measure to which it is equal on the line *a e*, from the point *a*, then suppose, in this instance, that this radius is equal to 10 of the parts or divisions, that is, to *a* 10, on the *a e*; then the measurement *a* 10, on the line *a d*, will be the isometrical diameter, and *a* 10, on the line *a b*, will be the minor axis of the ellipse, or isometrical circle. Then, again, if *a* 10, on *a d*, is equal to the given isometrical diameter, then *a* 10, on *a e*, will be equal to one-half of the major axis, or the hypothenuse 10—10, will be equal to the whole major axis, and *a* 10, on *a b*, will be the minor axis of the ellipse as in the former instance.

425. To set off angles on the faces of an isometrical cube, or on rectangular planes situated at the same angle as these surfaces, as regards the plane of projection, let A B C D, Fig. 21, be the face of a cube, divide any of its right angles, as A B C, into degrees, producing the lines to the two sides, A D, and C D, each of these lines will thus form a scale of tangents to 45°, terminating at D; draw C E, making any angle with C D, and equal to a side of the isometrical square; join D E, and through the division in the line of tangents, C D, draw lines parallel to D E, they will divide the line, C E, into a line of tangents, for the isometrical square, in the same proportion as C E of the original line. Figs. 22 and 23 show the tangential lines, C D and C E, transferred to one edge, *c d*, of the cube so as to divide it tangentially. The first shows the acute angle at *b* of the top face divided, and the second that of the obtuse angle at *a* of the same face, by lines drawn from their respective apex to the divisions on the subtending tangents. The three radii of the inscribed cube being respectively divided into tangential scales of degrees, commencing at their outer extremities, and at their inner extremities or centre. Any of the twenty-four isometrical right angles of the cube may thus be divided into degrees, and any angle set off on the faces of the cube with reference to their sides, and from either of their ends. Instead of going through this construction, and laying off the sides of each face tangentially, a pair of proportional compasses may be used, and the measurements taken with one end of the compasses from the original line of tangents, C D, Fig. 21, and the compasses being previously set

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[*] Such elliptic curves, or templets, are easily constructed by building a small cone out of thin slips of wood veneer, and glueing them together with paper between them, so as to admit of their being taken apart, *when truly turned*, which must be done, taking great care that the axis of the cone is at the proper angle with the plane of the slips of wood, so that the conjugate and transverse axis of the ellipses will have the true isometrical proportion to each other. This method of making curves also applies to those of the hyperbola and parabola, taking care to have the axis of the cone parallel to the layers of wood in the one case, and the side of the cone in the other.

to the isometrical proportion, then the size on the other ends are transferred to the isometrical side of the square. An ordinary sector may be used, laid along the boundary line with which the angle is required to be made. The parallel rulers being laid, to the line of the angle required, can then be carried parallel along the surface to the particular point through which the angle is required to be drawn.

From the foregoing it will be seen that all measurements made on lines, making the same angles with the boundary lines, on the same face, or any plane making the same angle with the plane of projection, bear the same proportion as the original measurements; that is, a line of a particular length in the one case is equal to a line of the same length in the other; and also, that all similar or equal angles on such similar or parallel planes, as regards the plane of projection, are equal, and represent equal angles in isometrical projection.

426. In order to divide the circumference of an isometrical circle into any number of equal parts, we may proceed thus:—From the middle of one of the sides, *b c*, of the isometrical square, *a b c d*, containing the circle, A B O C D, whose circumference is to be divided, Fig. 24, erect the perpendicular, *o e*, bisecting *b c*, and making *o e* equal to *o b*, or *o c*, and join *e b*, *e c;* then from *e* as a centre with any radius, as *e g*, describe the arc, *f g h*, cutting *e b*, *e o* and *e c*, respectively, in *f*, *g*, and *h;* then the arcs, *g f* and *g h*, are each equal to one-eighth of a whole circle, and the angles, *g e f*, and *g e h*, each equal to 45°. Divide the arc, *g f*, into one-eighth of the number of parts required in the whole circumference of the circle, A B O C D, and from *e* draw lines through these divisions, as shown in the figure intersecting the line, *b b*, in 1, 2, 3; then from these points draw lines to the centre, O, which will divide the arc, *o* B, corresponding to one-eighth of the isometrical circle into four equal parts. Then to complete the division of the whole circle, it is only necessary to transfer the divisions on *b b*, or the whole side, *b c*, to the other three sides, commencing at the corners, and having the central division coinciding with o, o^1, o^2 and o^3, respectively; then, as before, draw lines from all these divisions to the centre, O, and the circumference of the inscribed isometrical circle will be divided into 32 equal parts or angles. By this method, the teeth of wheels, &c., may be very readily drawn in isometrical projection. In a similar manner, such isometrical circles or squares may be divided into angles of degrees, and any angle found round the centre, O, with any isometrical radius. Divide the arc, *f g h*, into 180°, and lines drawn through these will divide the two lines, *o b* and *o c*, into two tangential lines of 45° to the radius, *o* O; and these being laid off on the other three sides, as in the last case, will, by drawing lines to the centre, O, divide the whole circle into degrees.

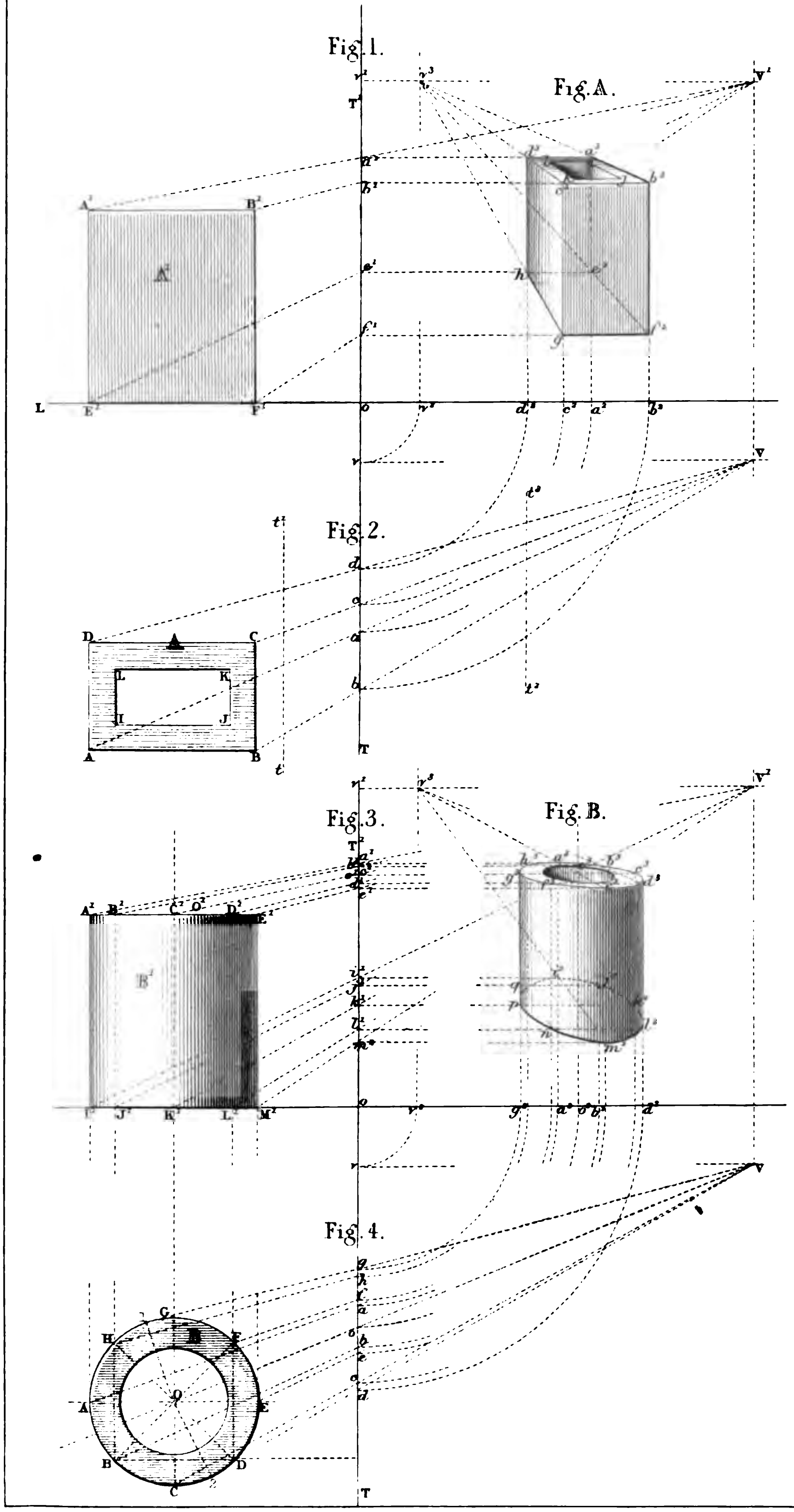
This division of the isometrical circle into degrees may be also effected by taking the side of the original square as the tangential line. Thus, produce the side, *c d*, Fig. 24, in the direction of E, then from *a* as a centre, with a radius equal to the side of the original square, draw an arc intersecting *c d*, produced in E, join *a* E, and upon it construct the original square, *a* E F G, of which *a b c d* is the isometrical representation. Within the square describe the circle of which the ellipse, A B C D, is the isometrical representation, draw also the diagonals, *b a*, produced in *a* F and E G, then the intersection, H, of these lines is the centre of the square and circle, and *a* H E, is a right angle, and the arc, *i j k*, the quarter of the whole circle, *i j k l*. Let the quadrant, *i j k*, be divided into degrees, and carried out, as mark in 5°, to the line, *a* E, then parallel to the sides, *c d* E, and *b a*, of the isometrical square, draw lines through these divisions to *a d*, they will divide the side, *a d*, into an isometrical scale of tangents corresponding to *a* E, O o^1, and H *j*, being their respective radii, while lines drawn from the degrees on the line, *a d*, to the centre of the ellipse, O, will divide its quadrant, A o^1 D, into the 180° contained by it. In this way any angle may be set off on the isometrical circle corresponding to that of the original circle.

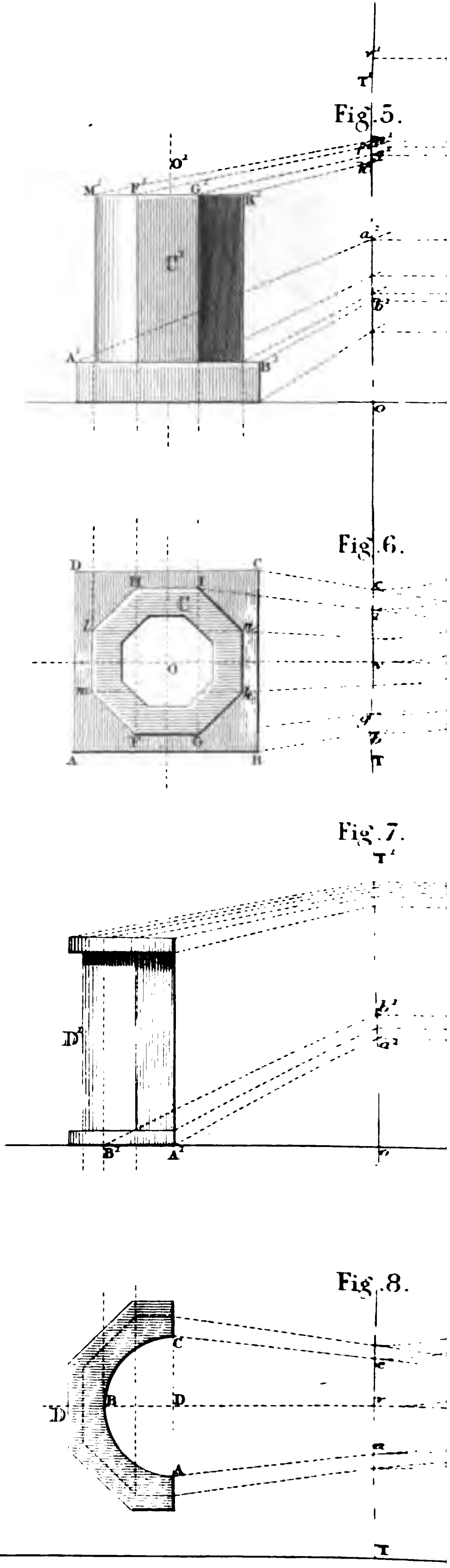
427. From what has been said on the isometrical projection of cubes and circles, and from the definition of a sphere (103), it will be evident that the isometrical projection of a sphere or globe is a circle, whose diameter is equal to that of the original sphere, of which it is the projection, or equal to one side of a face of the original cube in which it could be contained or inscribed, and as shown isometrically projected in a dotted cube in Fig. 25. Then, to represent a ball or globe, B, as resting on the top of the first ball, A, or in the centre of the upper surface of the cube containing it, as shown in this figure, produce the vertical diameter of the ball, A, or the minor diagonal, *a c*, of the top surface, *a b c d*, of the cube, and draw the other diagonal, *b d*, the point O, of the intersection of the two diagonals, will be the point of contact of the ball, B, with that of A, or the face of the cube:—Suppose the diameter of the second ball, B, is half that of the first, A, then from O, on *a c*, make O *o* equal to half the isometrical diameter of a circle equal to that of the globe, B, to be drawn, which will, in this case, be one-fourth of *a b* or *a c*, these being equal to the isometrical diameter of the globe, A, which is twice that of B. From *o* as a centre, with a radius equal to half that of the globe, A, describe a circle; this circle will represent the globe, B, resting on the top of the dotted cube, or the inscribed sphere, A, in isometrical projection, as was required.

428. If it is required to represent in isometrical projection the section of an arc, as cut by a perpendicular plane, making an angle of 30°, with the end, proper, of such arc, let A B C D E, Fig. 26, be the end of such arc, isometrically inscribed in a dotted cube; then project the horizontal plane, B O D D^1 B^1, through the end horizontal diameter, B O D, and axis of the arc, O O^1, and on the line, D D^1, of this plane, as a line of tangents, set off D *d*, equal to 30° (or two-thirds of the whole line, D D^1, which represents 45°); join B, *d*, and B *d* will be the isometrical horizontal diameter of the arc on the required plane of section, corresponding to, and at the required angle with, that of B O D, on the end, proper, at right angles to the axis. A vertical plane passing through the axis of the arc, O O^1, will intersect the original end plane, and the plane of section in two straight lines, O C and *o c*, respectively equal to the vertical radii on these two planes, giving the point, *c*, in the crown of the arc, on the plane of section, corresponding to C, on the original end plane. In a similar manner, by taking vertical planes parallel to the axis, on any vertical lines, as 1—2, 3—4, &c., cutting the roof of the arc, and its horizontal diameter on the original plane, will be obtained similar or corresponding intersections or perpendiculars, 1^1—2^1 and 3^1—4^1, &c., on the sectional plane; thus giving the points, 1^1 3^1, &c., in the roof of the arc on the latter plane, corresponding to 1, 3, &c., on the original plane. A line drawn through such points, thus found, on the plane of the section, as, B, a^1, *c*, 1^1, 3^1, *d*, *e*, will give the isometrical projection of the given arc, A B C D E, on the plane, A B *c d e*, making an angle of 30°, with the end face of such arc as was required.

This method of projecting sections, on other planes than the isometrical plane, from the projections on the latter plane, is applicable to all kinds of parallel cylindrical objects, and also to sections of the sphere, whether hollow or solid.

EXAMPLES





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PERSPECTIVE.

Fig. C.

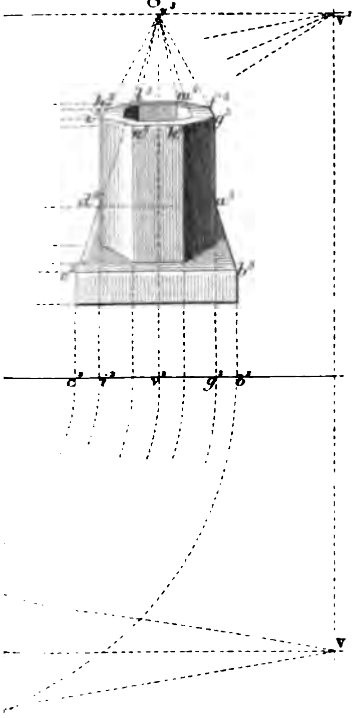


Fig. D.

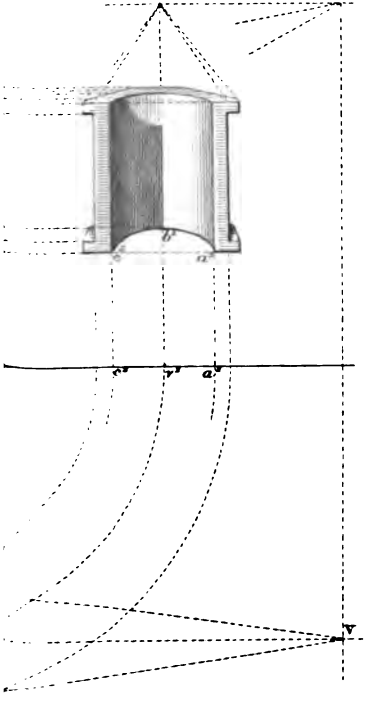
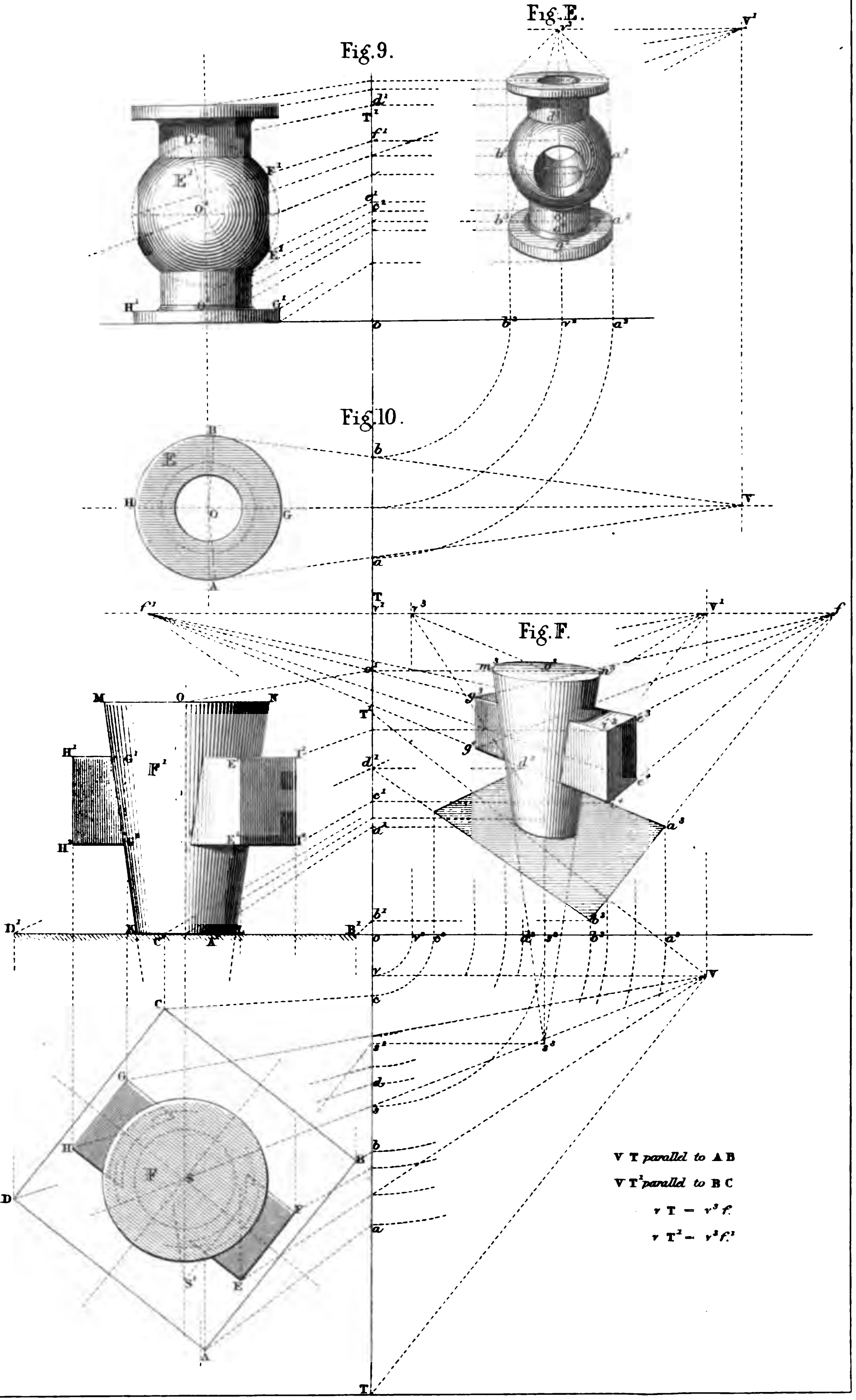


Fig. 9.

Fig. 10.

Fig. E.

Fig. F.



V T *parallel to* A B

V T² *parallel to* B C

v T = v³ f.

v T² = v³ f.²

Mackay & [illegible]

429. Let A B, Fig. 27, be the section of a plane on which any projection is made, and let it represent any definite measure, say ten feet, if a line, as B C, be perpendicular to A B, it can only be represented by a point on the plane of projection. If the quadrant, A C, be divided into degrees, then the dotted lines parallel to B C, divide the line, A B, into a scale of sines. This scale represents the lengths of lines equal to A B at every tenth degree of inclination from A to C. If D B be the length of a line parallel to the plane of projection, then the several divisions on that line show the measure of the projection of any line of the same length, making an angle therewith of any degree numbered thereon. The measure of the length of the projection being taken from B, or if A D be the length of a line parallel to the plane of projection, then the several perpendiculars, parallel to A D, are the lengths of the projections at the several angles, the line, *a b*, being the length of the isometrical line. In the two former scales, *a* B, and *b* B, are the lengths of the isometrical projections of the lines, D B, and A B respectively.

Fig. 28, on Plate 43, is an example of the application of isometrical projection to the elevation of the pedestal and lower portion of a fluted column, and affords a good example of the proper mode of treating and shading such an object.

From what we have said on this subject, and the examples given in Plate 43, the student will be enabled to draw a great variety of subjects in isometrical projection. This method of delineation, although not so generally useful as the ordinary systems of projection, will nevertheless be found particularly applicable to the delineation of plans, or partial elevations of buildings, farm structures, models, fortifications, and other similar purposes. Drawings of this kind may be prepared with accuracy and rapidity, and to those who are unaccustomed to architectural sections and plans, they are far more easily understood. We may therefore regard the time devoted to this branch of study by the student as by no means ill bestowed.

CHAPTER XVI.

TRUE PERSPECTIVE.

PRINCIPLES AND APPLICATIONS.—ELEMENTARY PRINCIPLES.

PLATE 44.

430. Perspective, properly so called—but here defined as true or exact perspective—differs from parallel or false perspective, in its being founded upon the laws which regulate vision; that is, instead of being parallel to each other, the visual rays converge to a point. An object is said to be drawn in perspective when it represents the object as having the very appearance which the object itself has, when seen from a particular point of view, or point of sight, as it is termed.

The visual rays travel from the object in straight lines, converging to a point at the eye, and forming a cone of rays. Let us suppose this cone to be intercepted by a transparent plane, or diaphragm, of any form—then, noting the points where the rays from the various parts of the object pierce this diaphragm, let us paint upon it the outline, and complete the picture with colours, which to the eye shall have the exact appearance of those of the object itself, modified, as they may be, by distance, position, form, and by their being in the light or shade. After doing this, we may remove the object, but the picture upon the diaphragm will make the same impression upon the eye that the object would itself. It is to the representation of objects in this exact and natural manner, that the art of drawing in perspective is devoted.

The fixed point, to which the rays converge, is called the point of sight; and in diagrams explanatory of perspective, it is shown, together with the converging rays, as projected into the plane of the picture. To determine the perspective delineation of an object mathematically, we must have given us the horizontal and vertical projections of the object; also those of the point of sight, and the position of the plane of the picture; and the general problem of perspective then reduces itself to conceiving the visual rays as passing from the various points of the object to the point of sight, and to find the intersection of these rays with the plane of the picture.

FIRST PROBLEM.

THE PERSPECTIVE OF A HOLLOW PRISM.

431. Let A^1 and A, Figs. 1 and 2, be the vertical and horizontal projections of the prism which we wish to delineate in perspective, the point of sight being projected in V and V^1, and the plane of the picture, in T and T^1, being supposed to be perpendicular to the planes of projection, and vertical, as is generally the case, to the horizontal plane, or base line, L M.

Through the point, V, in the horizontal projection, Fig. 2, draw visual rays from each of the points, A, B, C, D, appertaining to the external contour of the prism. The intersection of these points with the plane of the picture, T T^1, determines the points, *b*, *a*, *c*, *d*, which give, upon the horizontal projection, T *o*, of the latter, the perspective of the points, A, B, C, D.

In like manner, through the point, V^1, in the vertical projection, Fig. 1, draw the visual rays, A^1 V^1, B^1 V^1, E^1 V^1, F^1 V^1, intersecting the vertical plane, T^1 *o*, of the picture in the points, a^1, b^1, f^1, and e^1; these last being, consequently, the vertical projections of the perspective of the points, A^1, B^1, E^1, F^1.

From the position given to the plane of the picture, the perspective of the object is not visible, since all the points are situated in the vertical line, T T^1, the plane being represented on edge, we must therefore imagine this plane to be turned over upon the vertical line, T^1 *o*, as an axis on to the plane of the drawing; whilst we must suppose the line, T *o*, the horizontal projection of the picture-plane, as turned about the point, *o*, as a centre, through a right angle, bringing it to coincide with the horizontal or ground line, L M. When this is done, the points, *a*, *b*, *c*, *d*, will describe arcs of circles, and, finally, coincide with the points, a^2, b^2, c^2, d^2, on the ground line. Next, upon these last erect perpendiculars, to meet the horizontals, drawn through the points, a^1, b^1, e_1, f^1; the points of intersection of these cross lines will be the perspectives, a^3, b^3, c^3, d^3, of the corners, A, B, C, D, of the top of the prism. We have, likewise, the points, e^2, f^2, g, h, for the perspectives of the corners of the bottom of the prism, which is parallel to the top; consequently, by joining all these points together in couples, as shown in Fig. A, we obtain the entire perspective of the external outline of the prism. As the prism is hollow, we shall see in the perspective view the outline, *i j k l*, corresponding to the edges of the hollowed out part, I J K L, in Fig. 2, and obtained in the same way as the outer contour, *a b c d*, just described.

The point of sight, of which V^1 and V are the geometrical projections, is projected upon the picture-plane in the points, v, v^1; and when the picture-plane is turned over, and round as before explained, the point will be found at v^3, which is the position of the point of sight upon the perspective drawing.

It must be observed, that in this example the lines, $a^3\ b^3$, $a^3\ d^3$, and $b^3\ c^3$, which express the perspective of the corresponding lines, A B, A D, and B C, are the intersections with the plane of the picture, of planes passing through these lines, and through the point of sight. Now, since the intersection of two planes is always a straight line, the following conclusions may be drawn; that:—

432. First, *The perspective of a straight line upon a plane is a straight line.*

It may also be remarked, that the verticals, such as, $b^3\ f^2$, $c^3\ g$, $d^3\ h$, are the perspectives of the vertical edges, projected in the points, B, C, D; whence we deduce that:—

433. Secondly, *The perspectives of vertical lines are verticals, when the plane of the picture is itself vertical.*

It will further be seen, that the horizontals, $b^3\ c^3$, $d^3\ a^3$, $f^2\ g$, $e^2\ h$, of the perspective view, correspond to the straight lines projected horizontally in B C and D C, which are parallel to the plane of the picture; whence it may be gathered, that:—

434. Thirdly, *The perspective of any straight horizontal line, parallel to the plane of the picture, is itself horizontal.*

Further, it follows from the two preceding principles, that all lines parallel to the plane of the picture are represented, in perspective, by lines parallel to themselves.

Finally, the straight lines, $a^3\ b^3$, $d^3\ c^3$, $e^2\ f^2$, and $h\ g$, which all converge to the same point, v^3, the projection of the point of sight upon the plane of the picture, correspond to the edges, A B, D C, E F, which are horizontal, but perpendicular to the plane of the picture; whence it follows, that:—

435. Fourthly, *The perspectives of lines which are horizontal, but perpendicular to the plane of the picture, are straight lines, which converge to the point of sight, and are consequently foreshortened.*

It will be seen from Figs. 1 and 2, that the whole width of the perspective representation, Fig. A, is comprised between the points, *b* and *d*, which lie in the outermost visual rays, drawn in the horizontal projection; and that its height is limited by the two points, a^1 and f^1, which correspond to the extreme visual rays in the vertical projection. The angle formed by the extreme visual rays is termed the *optical angle*. In the present example, this angle, *b* V *d*, in the horizontal projection, differs from the angle, $a^1\ V^1\ f^1$, in the vertical projection.

The positions of the object and point of sight being given, the dimensions of the perspective representation vary according to the position of the plane of the picture. It will thus be seen, on referring to Fig. 2, that if this plane be removed from $T\ T^1$ to $t\ t^1$, nearer to the object, the limits to the perspective representation by the extreme visual rays will be enlarged; whilst, on the contrary, if we remove the plane of the picture to the position, $t^2\ t^3$ nearer to the point of sight, the limbs will be narrowed. This plane of the picture, T^1 T, is generally placed (as in the several figures on this plate) between the actual projections of the object and the point of sight; but when the perspective view is desired to be larger than the projections, it is placed on the other side of them, and the projection or visual rays produced beyond them to it.

Again, if, in place of moving the plane of the picture, the point of sight is removed further off, or brought nearer to the projections of the object, the size of the perspective outline will thereby be augmented or diminished. It may therefore be concluded, that:—

436. Fifthly, *The dimensions in the perspective representation do not wholly depend, either on the actual size of the object, or on the distance from which it is observed, but also on the relative distances of the point of sight, and of the object from the plane of the picture.*

Thus the sides, D A and C B, Fig. 2, are actually equal, but the former is further from the plane of the picture than the latter; so that whilst this is represented by the space, $c^3\ b^3$, in the perspective view, Fig. A, that is limited to the much smaller space, $d^3\ a^3$, in the same view.

SECOND PROBLEM.

THE PERSPECTIVE OF A CYLINDER.

FIGURES 3 AND 4.

437. To obtain the perspective outline of a vertical cylinder, such as the one projected vertically in B^1, Fig. 3, and horizontally in B, Fig. 4, we proceed, as in the preceding example, to draw through the point of sight, V^1 V, a series of visual rays, extending to the various points, A^1, B^1, C^1, D^1, E^1, and A, B, C, D, E, taken on the upper end of the cylinder, by preference at equal distances apart. These lines intersect the plane, T *o*, of the picture in the points, *d*, *c*, *a*, *g*, &c., in the horizontal projection, and in the points, a^1, c^1, d^1, &c., in the vertical projection.

By bringing the plane, $T\ T^1$, of the picture, into that of the diagram before us, or what is equivalent to it, by finding the points, g^2, a^2, &c., by means of arcs, drawn with the centre, *o*, on which the plane is supposed to turn, and drawing the horizontal lines through the corresponding points in the vertical projection of the picture-plane, $o\ T^1$, we obtain the points, c^3, d^3, a^3, g^3, &c., which are points in an elliptic curve representing the top of the cylinder in perspective, which is visible in consequence of the point of sight being above it.

The same points, A, B, C, D, of the horizontal projection, squared up to the base of the vertical projection in $I^1\ J^1$, K^1, L^1, M^1, when transferred over to the perspective view, gives the perspectives, $i^2\ j^2\ k^2\ l^2\ m^2\ n\ p\ q$, of the bottom of the cylinder, of which, obviously, only a part is visible.

The two vertical generatrices, $d^3\ l^2$, $g^3\ p$, being drawn tangential to the upper and lower ellipses, complete the perspective outline of the cylinder, Fig. B. The greater the number of points taken in the plan, Fig. 4, the more perfect will the ellipses be obtained in the perspective view, and if from O, the centre of the cylinder perpendiculars, $O^1\ O^2$, were let fall on the extreme boundary or tangential rays, giving the two extreme points, 1 and 2, at which the visual rays touched it, and these two points were transferred to the perspective view, along with the other eight, a very perfect ellipse could be traced through them, while the two perpendiculars drawn through the two points, 1 and 2, would give the extreme generatrices of the perspective cylinder, and the longest axis of the ellipse.

As this cylinder is hollow, an operation similar to the preceding will be called for in delineating the upper visible edge of the hollowed-out portion.

It must be observed that, in taking an even number of divisions, at equal distances apart, upon the horizontal projection of the cylinder, and setting them off from the diameter, $c\ g$, parallel to the plane of the picture, we have always a couple of points situated upon the same perpendicular to the plane, and of which the perspectives are, consequently, situated on the same straight line, drawn through the projection, v^3, of the point of sight. This occurs with the points, B, D, the perspectives, $b^3\ d^3$, of which, are situated upon the line, $v^3\ d^3$, so that we have a means of verifying the preceding construction.

THIRD PROBLEM.

THE PERSPECTIVE OF A REGULAR SOLID, WHEN THE POINT OF SIGHT IS SITUATED IN A PLANE PASSING THROUGH ITS AXIS, AND PERPENDICULAR TO THE PLANE OF THE PICTURE.

FIGURES 5 AND 6.

438. Let V and V^1 be the projections of the point of sight, situated in the vertical plane, V O, passing through the axis, O, O^1, of the solid, C, C^1, and perpendicular to the plane, T T^1, of the picture. It will be seen at once, that in the perspective view, Fig. C, this point must be projected on the vertical line, $v^3\ v^3$, representing the axis of the object, and in relation to which all the lateral edges of the object are symmetrical. Such are the sides, A B and C D, which are perpendicular to the plane of the picture, and which are represented in perspective by the lines, $a^3\ b^3$ and $d^3\ c^3$, both directed to the point of sight, v^3. It is the same with the edges, F G, H I, of which the perspectives, $f^3\ g^3$, $h^3\ i^3$, likewise converge to the point of sight, v^3.

As for the vertical edges, they retain their vertical position in the perspective, and the horizontal lines, D A, L M, N K, C B, parallel to the plane of the picture, are rendered in the perspective view by parallels, such as $d^3\ a^3$, $l^3\ m^3$, $n^3\ k^3$, $c^3\ b^3$.

It may be gathered from the solution of this problem, that whenever the point of sight is in a plane, passing through the axis of a regular solid, and perpendicular to the plane of the picture, the perspective representation will be symmetrical with reference to the centre line, and that it is, therefore, quite sufficient to go through the constructive operations for one side only of the figure.

FOURTH PROBLEM.

THE PERSPECTIVE OF A BEARING-BRASS, PLACED WITH ITS AXIS VERTICAL.

FIGURES 7 AND 8.

The point of sight being situated, as in the preceding example, in a vertical plane, passing through the axis of the object, and perpendicular to the picture-plane, the perspective will likewise be symmetrical in reference to the centre line, $v^2\ v^2$; and it is unnecessary to notice this property further. The inside of the brass being cylindrical, and being terminated by horizontal semicircular bases, the perspective of these bases will be rendered by two regular semi-ellipses, of which it will be sufficient to determine the axes. The transverse axis, $a^3\ o^3$, is equal to the perspective of the straight line, A C, or $a\ c$, on the line, T o, which is horizontal and parallel to the picture-plane; and the semi-conjugate axis, $b^2\ d$, is equal to the perspective of the line, B D, or $B^1\ A^1$, which is also horizontal, but is perpendicular to the plane of the picture, and is, consequently foreshortened, whilst it coincides with a line passing through the point of sight, v^3; that is, $a^1\ b^1$, on the line, $T^1\ o$, squared over to $d\ b^2$, on the line, $v^2\ v^2$. It will be remarked, that the transverse axis of the ellipse, corresponding to the upper end of the semi-cylinder, is equal to $a^2\ c^2$, but that the conjugate axis is foreshortened to a greater extent than that, $b^2\ d$, of the lower one, in consequence of being at a less distance below the point of sight. The effect of the perspective, in this case, is well rendered in Fig. D.

FIFTH PROBLEM.

THE PERSPECTIVE OF A STOPCOCK, WITH A SPHERICAL BOSS.

FIGURES 9 AND 10.

439. In carrying out the general principle, it will be seen that, in order to obtain the perspective representation of a sphere, we should draw through the point of sight a series of rays, tangential to the outer surface; but in order to ascertain the points of contact of these tangents, it will be necessary to imagine a series of planes as passing through the sphere, producing circular sections, and then to find the perspective of each of these circles. These being found, a curve, drawn to circumscribe them, will be the perspective of the sphere.

In the example, Figs. 9 and 10, the point of sight being still chosen as before—that is, as situated in a plane passing through the centre of the sphere, and perpendicular to the picture-plane, the perspective of the sphere will, on this account, simply be an ellipse, having for conjugate axis the base, $a\ b$, of the optical angle, A V B, on the line, T o, of the horizontal projection; and for transverse axis, the base, $c^1\ d^1$, of the angle, $C^1\ V^1\ D^1$, on the line, $T^1\ o$, in the vertical projection; because the right cone, formed by the series of visual rays tangential to, and enveloping the sphere, is obliquely intersected by the plane of the picture. If, however, the point of sight were situated upon a horizontal line, passing through the centre, O^1, of the sphere, the perspective of the latter would evidently be a circle.

The spherical part of the stopcock is traversed by a horizontal opening, for the reception of the key, and the edge, $e^1\ f^1$, of this opening, being situated in a plane parallel to the picture-plane, will, in the perspective, Fig. E, be rendered by a circle, of which the diameter is $e^1 f^1$, on the plane, T o, Fig. 9.

As to the cylindrical flanges on either side of the stopcock, for forming its junction with a line of piping, the semicircle, A G B, is represented by a portion of an ellipse, having the horizontal line, $a^2\ b^2$, corresponding to A B; and the vertical, $g^2\ o^2$, being the perspective of $G^1\ O^2$. The circle, A H B G, the horizontal projection of the surface of the uppermost flange, is represented in the perspective view by a perfect ellipse, as is also the upper visible edge of the inner tubular portion. But this is the same as the last case, or when the line, O V, is not perpendicular to the axis and the plane, T^1 T, it is the same as that described in problem two, and the ellipses may be formed in a similar manner.

The perspective representation of a circle, however situated otherwise than parallel with regard to the plane of the picture, is always a perfect ellipse; but the transverse axis of the ellipse does not coincide with, or is not the representation of, any diameter of the circle; for it is evident, that the more distant half of the circle must be more foreshortened in the perspective, and must occupy a less space than the anterior half, whilst the ellipse is equally divided by its transverse axis.

When the point of sight is in a line perpendicular to the circle, the rays from the latter will form a right cone; and if the plane of the picture is not parallel to the circle, the section determined by it will be an ellipse, as is well known. Again, if the circle is not perpendicular to the central visual ray, the cone of rays will be elliptical, and the sections of such cone will be ellipses, of various proportions, that parallel to the circle, however, being a circle.

The transverse axis of the perspective ellipse is the perspective of that chord in the original circle, which is subtended by the arc, between the points of contact of the extreme visual rays, as projected in the plane of the circle.

SIXTH PROBLEM.

THE PERSPECTIVE OF AN OBJECT PLACED IN ANY POSITION WITH REGARD TO THE PLANE OF THE PICTURE.

FIGURES 11 AND 12.

In each of the preceding problems, we have supposed one or other of the surfaces of the objects to be parallel with, or perpendicular to, the plane of the picture; but it may happen that all the sides of the object may form some angle with this plane. It is this case which we propose to examine in Figs. 11 and 12.

Let A B C D be the horizontal projection of a square, of which the sides are inclined to the plane, $T\ T^1$, of the picture, and of which it is proposed to determine the perspective. The point of sight being projected in V and V^1; if we employ the method adopted in Figs. 1 and 2, we shall find the points, a, b, c, d, to be the horizontal projections, and a^1, b^1, c^1, d^1, the vertical projections of the corners of the square; and when we have brought the plane of the picture, $T\ T^1$, into the plane of the present diagram, as before, we shall find the actual positions of these points to be at a^3, b^3, c^3, d^3. If we join these points, we shall have a quadrilateral figure, of which the two opposite sides, a^3, b^3, c^3, d^3, converge to the same point, f, whilst the other two sides converge to the point, f^1. These two points are termed *vanishing points.* They are determined geometrically, by drawing through V, the horizontal projection of the point of sight, the straight lines, V T and $V\ T^1$, parallel to the sides, $a\ b$ and $b\ c$, of the given square, and prolonging these lines until they cut the line, $T\ T^1$, representing the plane of the picture. Having drawn through V^1 the horizontal, $V^1\ v^1$, termed the horizontal line or *vanishing plane*, set off the distance, v T, from v^3 to f, and the distance, $v\ T^1$, from v^3 to f^1, and f and f^1 will be the required vanishing points.

It follows from the preceding, that:—

When the straight lines which are inclined to the plane of the picture are parallel to each other, their perspectives will converge in one point, situated on the horizontal line, and termed the vanishing point.

When several faces or sides, situated in different planes, are parallel to each other, their perspectives all converge to the same vanishing point, which allows of a great simplification of the operations.

Thus, the edges of the horizontal faces, $H^1\ I^1$ and $H^2\ I^2$, of the quadrangular prism, being respectively parallel to the sides of the square, A B C D, are represented in perspective by the straight lines, $i^3\ e^3$, $i^4\ e^4$, converging to the first vanishing point, f, and the straight lines, $i^3\ g^3$, $i^4\ g^4$, converging to the second point, f^1.

The cone, F^1, F, which is traversed laterally by the prism, has its apex projected horizontally in the point, S, Fig. 12, and vertically in the point, S^1, which, with its axis, appertains to Fig. 11. The perspective of the point, $S\ S^1$, on the plane of the picture, is found, in the usual way, to be at s in the horizontal projection, and at s^1 in the vertical projection; and when the plane of the picture is brought into the plane of the diagram, these points are represented by the points, s^2 and s^3, upon the same vertical, $o^3\ s^3$, which is, consequently, the perspective of the axis, $O\ S^1$, of the cone, and the point, s^3, is the perspective of the apex of the cone.

If we draw the perspectives of the two bases, K L and M N, of the frustum of a cone, according to the method already given, it will only remain to draw through the point, s^3, the two lines, $s^3\ m^3$ and $s^3\ n^3$, tangential to the ellipses, representing the bases, which will complete the perspective of the entire object, as in Fig. 𝔉.

We might amplify, to a considerable extent, the subject of perspective, but this would, possibly, be attended with an evil that we are most desirous of avoiding—that of deterring the student from entering on the study of perspective from an apprehension of its difficulty and complexity. We have, therefore, confined the subject to six problems, which, however, involve all the practical details which the student will require. In our example plate, 𝔅, we have given a good application of true perspective to the combined working mechanism of a Turret Clock, showing how applicable this mode of representation is for such subjects, in a pictorial sense. Though we have only been enabled to show one general application of perspective drawing, representing this machine as viewed from a point somewhat above it, and having its sides at a considerable angle with the plane of the picture, thus showing the top and two sides of the whole machine, which is the usual way of viewing or representing such objects. We may, however, observe, that for the representation of general or complete views of objects, its power is unlimited, and perhaps even for showing detail also, did it not add so considerably to the labour of drawing them, without a corresponding advantage. A careful perusal of this chapter, accompanied with a practical working of the several examples, will enable the intelligent student to delineate in perspective objects of a more complex nature, when called upon to do so. Perspective drawing is particularly applicable for showing the exterior views of large subjects such as bridges, towers, mausoleums, cathedrals, mansions, and villas, as well as that of the interiors of large architectural structures, in which it is required to know the effect the designs will have to the eye of the spectator when erected or constructed. While, to come to smaller subjects, in this eminently pushing and advertising age, when the manufacturer of every kind must have a first-class perspective view of all the fine designs they get up, it has now become almost a necessity for the practical draughtsman not only to be acquainted with its principles, but even to be an expert perspective artist. So that he may be enabled to delineate pictorially examples of architecture, machinery, or even more common objects, is an accomplishment which the draughtsman will, at times, find of the utmost importance to him, particularly where it is desired to bring such subjects under the notice of the general public, who are unacquainted with mechanical matters, and unable to follow out the connection of parts, as delineated in ordinary mechanical drawings. Artistic drawings of this class would also find a place on the walls of our exhibitions; where treated in any other way, they would be inadmissible, and inventors would be benefited thereby to an extent little dreamt of.

CHAPTER XVII.

ON ARCHITECTURAL DRAWING.

PRINCIPLES—SYSTEMS OF DRAWING, AND APPLICATIONS.

PLATES 45 AND 46.

THE architectural draughtsman, in the higher branches of his profession at least, is compelled to carry his operations into a different sphere from the mechanical draughtsman, because the works upon which the former is often engaged are connected with *art*, using that term in contradistinction to the more strictly defined *science* of the engineer or of the mechanician. Architecture, indeed, is one of the noblest methods by which man is able to express his aspirations after the ideal; and the beauty of his productions is as essential an element of his efforts, as their logical fitness, or the strict mathematical or scientific disposition of the materials he employs. The architectural draughtsman (so far as he is a designer), must then be able to express ideas, to excite sympathy, and, as it were, to carry the spectator beyond the objects immediately before him to their hidden meanings, quite as much as he must be able to represent upon paper the modes of execution to be followed by the workmen in carrying out his design. Emphatically, the highest branch of architectural design is a branch of pure art, and as such its principles escape, to a great extent, from critical definitions, or absolute rules of a technical or practical nature; for beauty, which has aptly been described as "the splendour of truth," cannot be thus limited by laws, which, being from their very nature limited, cannot be universally true. No doubt the ordinary architectural draughtsman is, unfortunately, seldom required to exercise the higher qualities of his profession; yet the every-day requirements of practical life may, when rightly estimated, serve to induce him to acquire the faculties necessary for his higher aims, and thus insensibly to improve his own means of performing ordinary duties. It will, therefore, be our object, in the following short essay on architectural drawing, at the same time that we endeavour to explain its technical processes, to show, directly or indirectly, how those processes may eventually be raised from the rank of merely mechanical operations to the nobler one of the exponents of a highly-refined art.

Like all other systems of drawing, architectural drawing, to adopt Mr Digby Wyatt's subdivision, may be said to consist (in its general acceptation) of two distinct operations, the imitative one and the conventional one. The imitative part of architectural drawing is concerned in the representation of a visible object, and its principal use and interest consists in the means it furnishes for the record of the works of antiquity, and in the facility it confers of visibly expressing the anticipated effect of the design the artist may have conceived. The conventional part of architectural drawing is the one by means of which the various technical processes to be employed in the construction of a building are explained to the working men entrusted with its execution; and it is never intended to convey, by the figures so drawn, the probable effect the works when executed would be likely to produce upon the eye of the spectator. It is requisite that the perfect architectural draughtsman should be master of both these classes of operations; but as the former of them belongs nearly as much to the painter as to the architect, it is far too often neglected, and attention is directed almost exclusively to the latter—a fact which may explain the common want of pictorial effect in the works of so many of the best practical architects.

Representative drawing comprises (still to follow Mr D. Wyatt's remarks) the various branches of the outline, of shade, and of colour. Of these, the former is the most requisite in architectural drawing, because the class of objects it deals with habitually depend for their effect mainly upon their outlines, and only subsidiarily so upon their light and shade, or upon their varieties of colour In the finished drawings of buildings of importance, however, it is customary to unite all the various sources of effect, and even to follow strictly the laws of perspective in so far as they modify the practical appearances of proportion in the details. Many of the most eminent architectural draughtsmen, in fact, seem to have made their designs at once in perspective, and the student cannot be too strenuously urged to accustom himself to that practice, on account of the strange modifications of outline produced by the ordinary laws of optics, all of which may be provided for, or guarded against, by carefully-executed perspective drawing. As to the power of representing outline, it is to be acquired the most readily, and perhaps the most satisfactorily, by a previous course of what is technically called "free-hand drawing"—that is to say, by cultivating the habit of sketching from natural objects, from models of architectural ornament, or from the human figure, avoiding carefully any resort to the use of compasses, straight-edges, or mechanical facilities of any description. The efforts of the student should, firstly, be directed to acquiring correct notions of proportion in the objects he attempts to represent; and he should only dwell upon details when he has mastered the general characteristics of his model. Simple geometrical forms are, for this reason, the most desirable objects of study for the beginner; and it is also desirable that he should accustom himself to draw freely and firmly straight lines, triangles, polygons, and circles, before he attempts to represent the more ornate details comprised within those outlines. On the same system, it is recommended that the student should make himself thoroughly master of the power of representing plain surfaces, before he attempts to represent solids; and in proceeding to the latter operation, he should be careful to render to himself a correct account of all the new phenomena he then is brought into contact with. Proceeding in this manner, from elementary to composite forms, the student would do wisely to accustom himself to sketch buildings actually executed, before devoting much time to the attempt to embody his own ideas; and as these various steps in his education, as an architectural draughtsman, can be, and ought to be, passed through at an early period in his career—at a period when the mind, the eye, and the hand can easily be moulded to any habit—it is desirable that this "free-hand drawing" should be acquired before attention is turned to the more practical conventional portion of his pursuit. Any one who has examined the difference between the conventional drawings executed respectively by men who are able to sketch freely, or by those who can only work by the aid of tools, must be convinced of the absolute necessity for the possession of the former faculty by any one who aspires to the title of an architectural draughtsman.

There is no royal road to the acquisition of the power of free-hand drawing, nor can any of the books hitherto written upon the subject be considered to dispense with the necessity for long practice, under the guidance of an experienced tutor. . Harding's elementary books on drawing, Ruskin's Elements of Drawing, Frank Howard's and Burnet's works upon the elements of design and colour, Piranesi, Tatham, or Vulliamy upon architectural ornament, may all in turn be consulted for advice and instruction as to the use of sketching materials, or even for models of the details of handling to be adopted. But the lessons of a good master, and unwearied perseverance on the part of the student, are the real constituents of excellence in this art; and though the first stages of its acquirement may be tedious, it will eventually be found that the pleasure arising from the possession of the faculty of sketching —of the power of representative architectural drawing—is so great, that it alone will constitute an exceeding great reward. So fascinating is this art, in fact, when it has been mastered to a certain extent, that there is danger that the student may neglect the study of the more mechanical division of architectural drawing, viz., the conventional one; for the latter offers but little play to the imagination, and tends rather to curb than to develop fancy or sentiment.

Conventional architectural drawing, in fact, is that portion of the architect's pursuits in which he represents, in a manner adopted by a species of tacit convention amongst the practical builders of the locality where he may practise, the various works he desires to have executed. This manner is by no means necessarily uniform throughout the civilized world; and, indeed, there are very distinct differences between the styles adopted respectively in Paris and in London in the preparation of working drawings, so far, at least, as the finishing the drawings may be concerned. Engineers and architects also treat their drawings in a somewhat different spirit from one another; for the former express in more minute detail than the latter habitually do in their drawings, the separate parts of their designs, whilst in ordinary cases, architectural draughtsmen content themselves with general indications of current works, and leave much to the written descriptions of their specifications. The difference between the English and the French modes of architectural design lies mainly in the fact, that the French colour up their sections, and make the whole of their drawings more decidedly in the representative style than English architectural draughtsmen habitually do, and that they indicate, on a very small scale, the details we usually reserve for working drawings. The French do not make many working drawings, but they are accustomed to set out their works at full size upon boards, or upon wall surfaces prepared for the purpose—a duty usually left, in England, to the clerks of the works, or to the foremen of the various trades employed. Now, there are advantages and disadvantages in each of these respective methods of preparing architectural drawings, some of which, perhaps, it would be advisable to combine, or to avoid, in forming a kind of eclecticism in this branch of art education. Thus, the French method of colouring sections, in which projections are indicated by the depths of the cast shadows, tends to convert those detailed drawings rather too much into pictures; and it is to be observed that, from their very nature as conventional drawings, sections are but ill adapted to such pictorial representations. At the same time, a section shaded and coloured somewhat upon this system, enables persons who may not have received a professional education to understand the probable effect of the finished work more easily than a strictly conventional drawing would enable them to do. Again, the very elaborate manner in which engineers figure their drawings certainly ensures accuracy of execution, but it often complicates the drawings to such an extent as to make them confused, and it destroys the artistic feeling of the ornamental details in the most effectual manner. In fact, the object of architectural drawing of a conventional character, is to represent, as clearly as possible, the nature of the work to be executed, and to indicate the methods of technical execution it may be requisite for the architect to prescribe. Conventional drawings, bearing somewhat of a pictorial character, often convey the most successfully the spirit in which the design has been conceived. Rigidly conventional drawings convey the most distinctly the mechanical and practical details to be followed. The true architectural draughtsman will possess, by dint of practice, even if not by natural feeling, the tact to know to what extent he should follow either of these systems; the exaggeration of either of them is, perhaps, the only thing to be avoided.

The conventional drawings made for architectural purposes consist of the plans, elevations, sections, and details. The plans represent usually, in what is called a "block plan," the general relations of the intended structure to the buildings or property around it. This block plan is accompanied by plans of the different stories of the building, in which the relative positions of the walls, partitions, staircases, doors, windows, passages, openings, &c., are indicated to exact scale, on the supposition that a horizontal plane has been drawn through the building, the spectator being supposed to look upon this plane from above, and to see every object thereon represented without any interference on the part of the laws of perspective with the geometrical forms so drawn. The scales adopted in these drawings should all be integral parts of the legal unit of measure of length adopted in the country where the building is to be erected; and in England it is generally found that the scale of $\frac{1}{16}$ of an inch to the foot (1 in 192) is small enough for block plans, whilst detailed plans are generally drawn upon the scale of $\frac{1}{8}$ or $\frac{1}{4}$ of an inch to the foot (1 in 96, or 1 in 48). English workmen, in fact, usually carry "rules" subdivided on the edge to these scales, and they are so accustomed to their use, that there is found to be inconvenience in the adoption of more arbitrary ones. Otherwise there can be no doubt but that decimal scales—or such scales as are based on the multiples or submultiples of an unit, counted by decimals—are the most generally understood. The decimal scales are used by almost all continental nations, and occasionally by English engineers, who, in that case, adopt the yard as their unit, instead of the foot. It is desirable, under these circumstances, that the student should accustom himself to the use of the two classes of scales—of the "decimal" and of the "duodecimal" scales, as they are sometimes appropriately called. The habit will be found of use when he is obliged to consult foreign technical books on construction, from which so many valuable lessons, and so much information are to be derived.

It is customary to prepare the drawings of the Elevations of a building upon the same scale as the one adopted for the plans; and in the case of the drawings to be followed in execution, to indicate everything in its geometrical dimensions. Attention has been previously called to the advantages attending the practice of designing important structures in perspective; and, indeed, it is customary, in competitive designs for such purposes, to represent one or more of the elevations in such a manner. In the geometrical elevations, however, the whole of the general features of the external design must be represented clearly and intelligibly; the nature of the different materials to be employed must be indicated by the use of conventional colours; the foundations, and the constructive details below the level of the ground-line, should be shown in dotted lines, the works not intended to be thus covered up should be shown in full lines; and at times it may even be desirable to indicate the projections by means of cast shadows, though, as a general rule, working men are likely to be confused, rather than assisted, by the introduction of such shadows. All external openings, all door and window dressings, all strings, plinths, cornices, columns, pilasters, parapets, balustrades, and other ornaments, must be indicated in their true proportions, and in their general outlines; observing, nevertheless, that upon the small scale usually adopted in elevations, it is not desirable to treat these details in any very elaborate manner, but rather to leave them for the working drawings. This is essentially the case with respect to Gothic elevations, because the profiles of the mouldings are often of so complicated a nature as not to admit of their being represented geometrically without the introduction of so many dotted lines as to confuse the elevation, and to some extent, also, it occurs in classical elevations. If, therefore, the character of the ornamental details be generally indicated upon the coloured, and the perspective elevations, very slight indications of them upon the working elevations, with references to the working drawings, will suffice for all practical purposes in the hands of an intelligent foreman, or clerk of the works. Almost every architectural draughtsman, however, has his own practice in this matter.

The general Sections are drawn habitually upon the same scale as the plans; that is to say, to the scales of ¼ or ⅛ of an inch to the foot, and they should indicate the relative heights of the different floors, their modes of construction (when the latter do not involve anything out of the ordinary practice of trade), the heights of the doors and windows, the rise and position of the stairs, the position and modes of support of the internal cisterns, the nature of the roof, when the latter is not of any remarkable span or mode of construction. Should the cornices, the wall decorations, or ornamental joinery to be introduced, require any structural arrangements, these should be indicated upon the general sections; but, for the reasons before stated, it seems hardly advisable to represent, on the small scales to which they are drawn, the ornamental details themselves. It is desirable that the sections and the plans should all be coloured upon precisely the same system, and for this purpose certain colours have been adopted, conventionally to represent certain kinds of materials. We propose to return to this branch of the subject, upon which so much of the intelligibility of architectural drawings, to be put into workmen's hands, depends. It is of course understood that everything shown on a section is represented in its geometrical dimensions without any attempt at perspective effect.

The scale and the character of Working Drawings must depend entirely upon the nature of the works to be executed, and their greater or lesser degree of novelty of design. Thus in carpenter's work, if a floor, or a roof, be represented, the general arrangements are usually shown, on scales varying from ¼ to ½ an inch to a foot; whilst any peculiar joints, any new system of framing, any iron ties, bolts, stirrups, or plates, are shown on larger scales—sometimes even "full size," that is to say, of the size they are actually intended to have in execution. Sometimes these details are even given in what is technically called "isometrical perspective," a species of conventional perspective in which the parallel lines do not "vanish;" but although this style of drawing may occasionally illustrate some classes of outline of a peculiarly complicated form, it is likely to mislead ordinary workmen, and should therefore be resorted to on exceptional occasions only. The details of joiner's work rarely require to be drawn out to scales larger than one inch to the foot, and that scale will almost always suffice to convey all the necessary indications of the working details of smith's work, masonry, or of the other trades connected with the execution of the solid parts of a building. Occasionally, it is true, that highly-decorated wood pannelling, in the style of the wainscoting of the Elizabethan or the Renaissance period, requires to be indicated with an amount of detail which a workman would not be able to understand, if drawn at less than full size; but in such cases it is preferable to indicate the general arrangements by means of drawings on a rather large scale, and to leave the details "to be set out on rods," as workmen say, of the actual size of execution. All foliage decorations, such as the capitals of columns, ornamental friezes, bands, flowers, &c., and all profiles of mouldings, must be drawn by the architect at full size; for these details require in their treatment so subtle a knowledge of the laws of optical effects, that they never should be left in any manner to the discretion of uneducated workmen. It is precisely in the treatment of these details that the architectural draughtsman will find that the faculty of free drawing, and the scientific knowledge of the conditions which regulate picturesque effect, are of the greatest value. The distance from whence the detail in question has to be examined, the manner in which it receives or reflects light, the character and colour of the objects surrounding it, all combine to modify the effect it is able to produce in an infinity of ways which it is impossible to describe *a priori*. A long experience, and deep study, are requisite to complete this portion of the education of the architectural draughtsman, in the higher branches of his profession at least.

It may be as well here to observe, that at the present day a very influential school of artists has endeavoured to introduce a fashion for the imitation of natural objects in architectural ornamentation, and that we ourselves consider that there is so much danger to the interests of true art, in the exaggeration of this principle, that we feel it to be desirable to dwell awhile upon what we hold to be the correct principles of such architectural designs. Ornament is introduced in the details of a building principally for the purpose of obviating the disagreeable effects which might otherwise be produced by plane surfaces; and it is worthy of remark, that precisely in proportion to the sublimity of an architectural composition is the latter devoid of ornament. The grandest monuments of architecture are, in fact, totally devoid of foliage ornament, and they depend for their effect upon the rythm, and the harmony of their proportions, irrespectively of their details. The attempt, then, to introduce a system of ornamentation, based upon reproductions of natural objects, appears to be founded on the mistake of supposing that those objects might, at some period, have been used constructively in the positions they are made to occupy; whereas, all true architectural ornamentation, must be of a purely conventional character. It is, no doubt, desirable to study the manner in which Nature displays the beauty of leaf, stem, and flower; but there is so much of contradiction between the ephemeral grace of natural plants, and the fixed rigidity of stone or of metal ornaments, that a literal rendering of such natural objects must be out of place in a building. In this instance, as in so many others, exaggeration has been allowed to interpose in the adoption of a good elementary principle. The architectural designer should carefully master the spirit in which Nature has treated the outlines, and the details of foliage, so as to guard against any violation of the laws regulating them; but it would be ridiculous to introduce imitations of actual plants in positions wherein plants could never grow. In addition to these remarks upon the treatment of foliage ornamentation, it is necessary to warn the draughtsman against the introduction of such work in places wherein it would appear to interfere with the solidity of a construction. Profusion of ornament has always accompanied the introduction of a debased style of architecture; and at all times the aphorism, that "construction should be ornamented, not ornament constructed," is true. The distinct marking of the rustic joints, in a large mass of plain masonry is, for instance, far more logical than the diapering of the wall surfaces so common in the transition mediæval architecture; and the draughtsman should always bear in mind that repose is very frequently as essential an element of beauty in an architectural design as elaborate ornamentation can be, especially when the latter has been designed upon principles which are likely to overshadow the ideal of the structure it is intended to adorn.

When the building upon which the architectural draughtsman is employed involves novel principles of construction, or details of that nature beyond the usual sphere of working practice, it is essential to explain them by elaborate working drawings. Roofs of large span, groined arches, stone or brick arches of large dimensions, especially require to be so treated; and in many cases the temporary scaffolding, the centres, and other methods of practical working, must be clearly shown. In treacherous strata the methods of foundation must be indicated with the greatest care; and if any particular precautions are required in the beds and joints of the masonry of the superstructure, as in the case of lighthouses, in the tying-down wells of suspension-bridges, &c., they must be shown in detail, nay, often at full size. The construction of floors, and the dimensions of girders, may, without inconvenience, be indicated by working drawings on a small scale. Iron railings, being, in fact, architectural decorations, often require to be drawn full size.

Although it has been assumed, throughout the preceding observations, that the drawings, whether of plans, elevations, sections, or details, have been made correctly to scale, and although the architectural draughtsman may have added scales to all his drawings, it is desirable, wherever it is possible so to do, that the real actual dimensions the works are intended to present should be indicated in figures. It is generally understood that the figures so given are to be followed in preference to the dimensions ascertained by the use of the scale, and that the dimensions marked or shown on detailed drawings are more binding upon the constructor than are the figures or dimensions on general plans. When the drawings are—as is usually the case—accompanied by a written specification or description, it is also understood that the indications therein given are to be followed before, or in preference to, those given on the drawings, whether general or detailed. It behoves the draughtsman, therefore, to exercise great care, in order to avoid any discrepancy between the various documents thus placed in the workmen's hands; the figures placed upon the drawings must likewise be so placed as not to confuse or interfere with the effect of the latter. The only essential condition to be observed in this matter is, that no room should be left for misapprehension of the draughtsman's intention.

We have already given examples of the conventional colours ordinarily used, in Plates 10 and 33; but in reference to architectural drawing more particularly, we may add the following:—

1. *Ground*, unless when it is desired to indicate the special nature of the strata traversed, is represented by a shaded and variagated tint of sepia, or of Indian ink, with occasional plays of pink, yellow, or blue. The upper surface should be uniform, the lower parts of the section may be lighter, and with more of a picturesque effect, provided it do not interfere with the important parts of the drawing.

2. *Concrete* is represented by a uniform tint of burnt umber of rather light tone, dotted with a much darker tone of the same colour tolerably evenly over the surface.

3. *Brickwork* is represented, in section, by a tint of carmine; in elevation, by a light tint of burnt sienna, shaded by light tints of Indian ink, or of a warm neutral colour.

4. *Stone* is represented, in elevation, by a flat tint of yellow ochre, and in section by a tint of carmine, hatched with inclined lines of a darker tint of the same colour.

5. *Wood*, in elevation, is represented by a light tint of gamboge; in section, by a tint of the same colour, hatched with lines in dark burnt sienna.

6. *Cast-iron* is represented by a tint of indigo.

7. *Wrought-iron* is represented by a tint of Prussian blue.

8. *Slate* is represented by a tint of Payne's grey, or by a neutral tint having in its composition a dash of carmine.

9. *Lead* is represented by a cold neutral tint, of rather a darker shade than the slate.

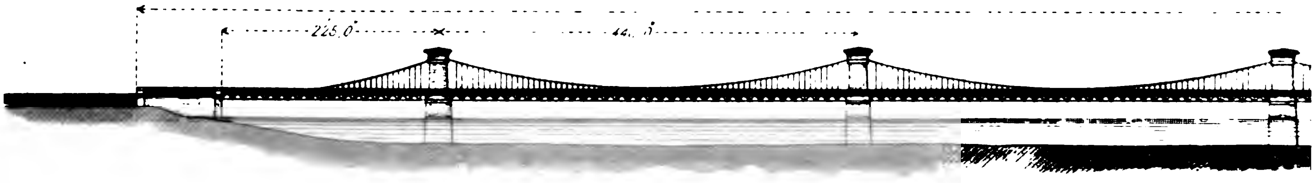
10. *Plaster* is sometimes represented by a very light, cold, neutral tint, but, perhaps, more often by a light tint of burnt umber laid uniformly over the surface.

Two Plates, 9 and 29, have been given for the purpose of illustrating the style in which the simpler, and the more complicated, architectural designs are usually represented in uniform colour, though of slightly different tones. Plate 45 is given as an example of the mode of treating bridges. Although bridge architecture is susceptible of the greatest possible degree of artistic beauty, it is a melancholy fact, that the majority of modern structures of that description are as ugly as it is possible to make them. The Kieff Bridge has, therefore, been selected "to point the moral" of this censure. It is a work which consistently expresses the ideal qualities we are entitled to expect in a bridge, and it does not suggest any ideas of another character; at the same time its outline is elegant, its details simple and effective; indeed, it is essentially *true* throughout. This is the greatest praise that can be awarded to any artistic production; and truth should be the great object of the architectural draughtsman, whether he adopt as his mode of visible expression the representative or the conventional branch of his art.

The suspension bridge over the river Dnieper, at Kieff, Southern Russia, by C. Vignalles, engineer, is represented by our Plate 45. Fig. 1 is a general elevation of the complete bridge; Fig. 2 an enlarged elevation of one of the river piers; Fig. 3 a transverse section at the centre of one of the spans; and Fig. 4 a partial plan

SUSPENSIO

Fig. 1



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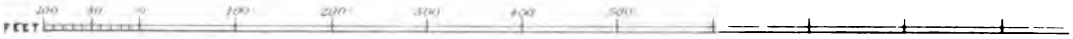

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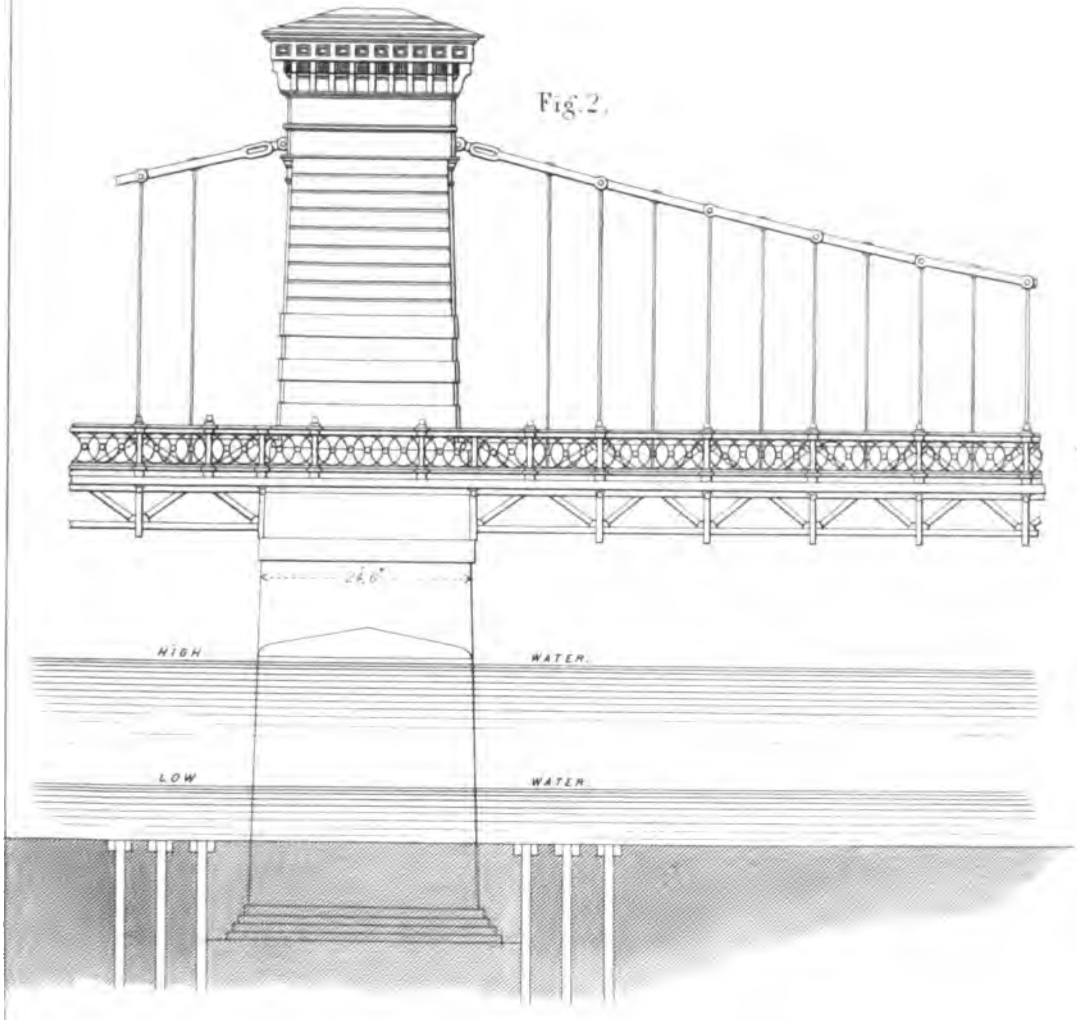
Fig. 2.



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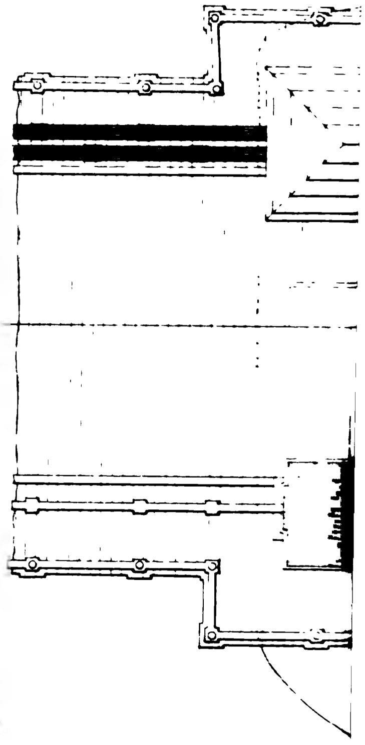

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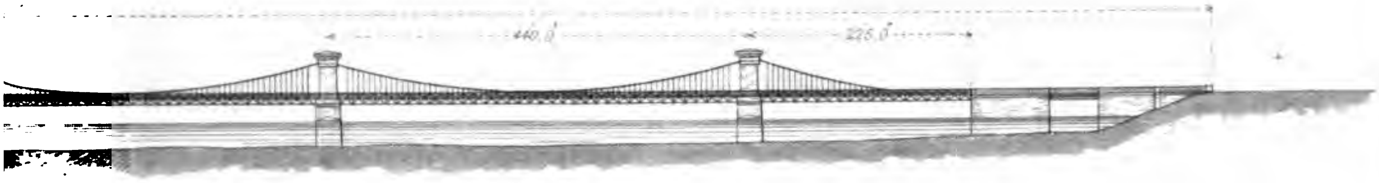
BRIDGE.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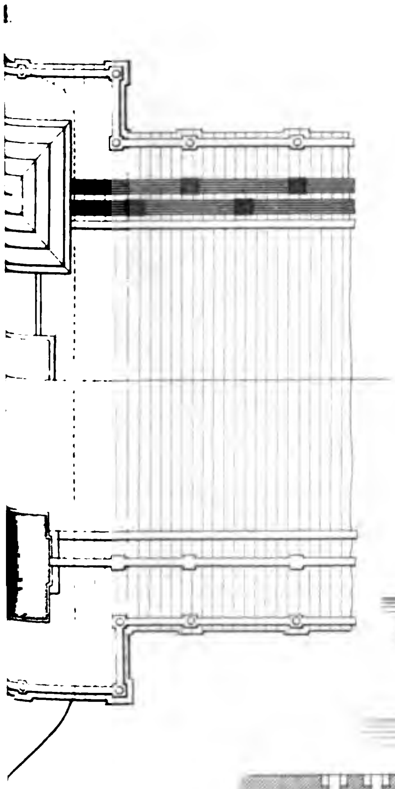
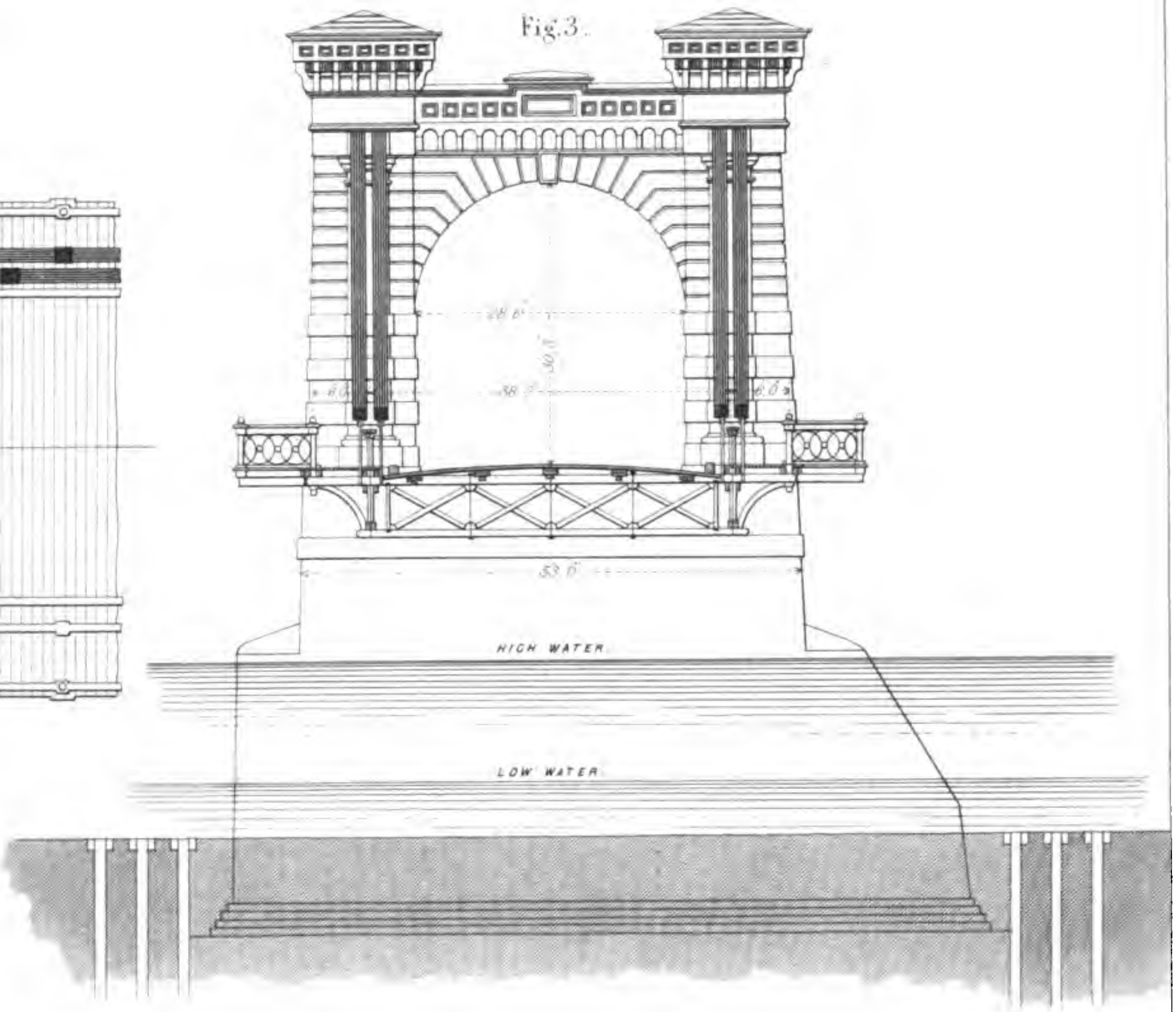


Fig. 3.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6.0
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HIGH WATER
LOW WATER

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ENGINE & B

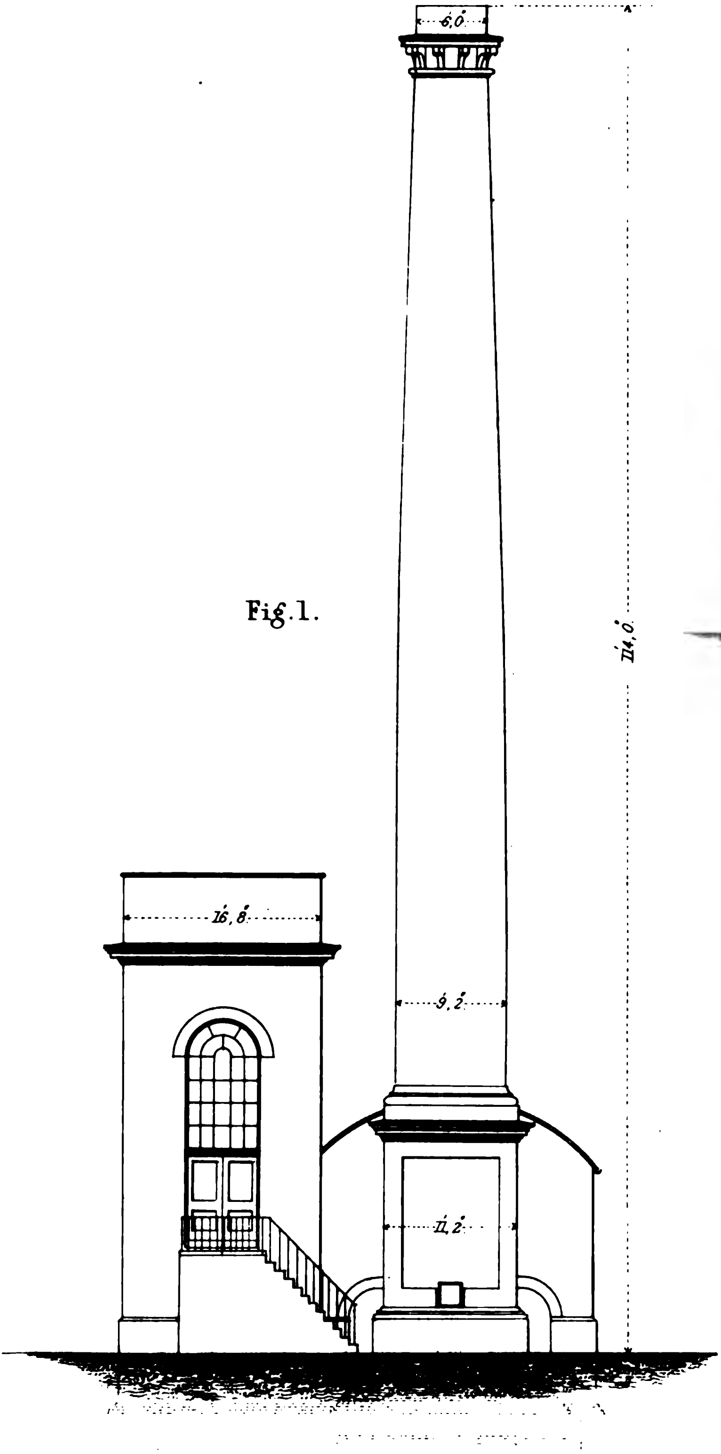


Fig. 1.

Fig. 4.

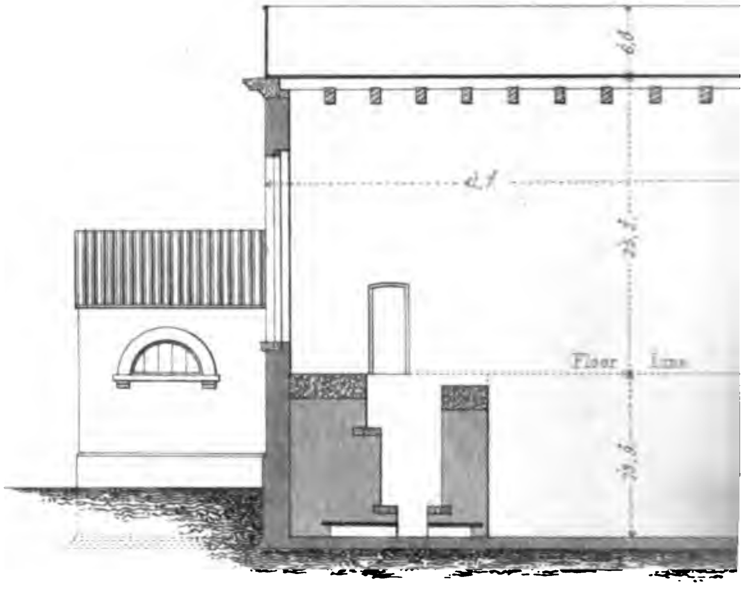
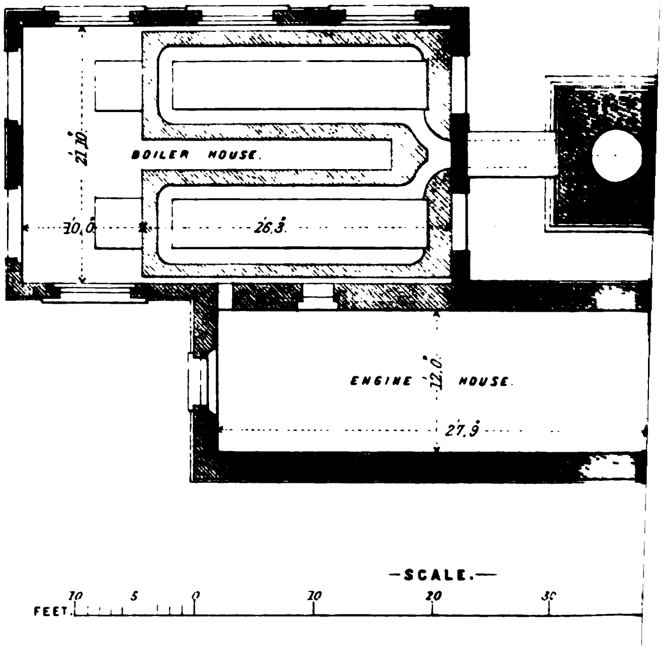


Fig. 5.



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LER HOUSE,

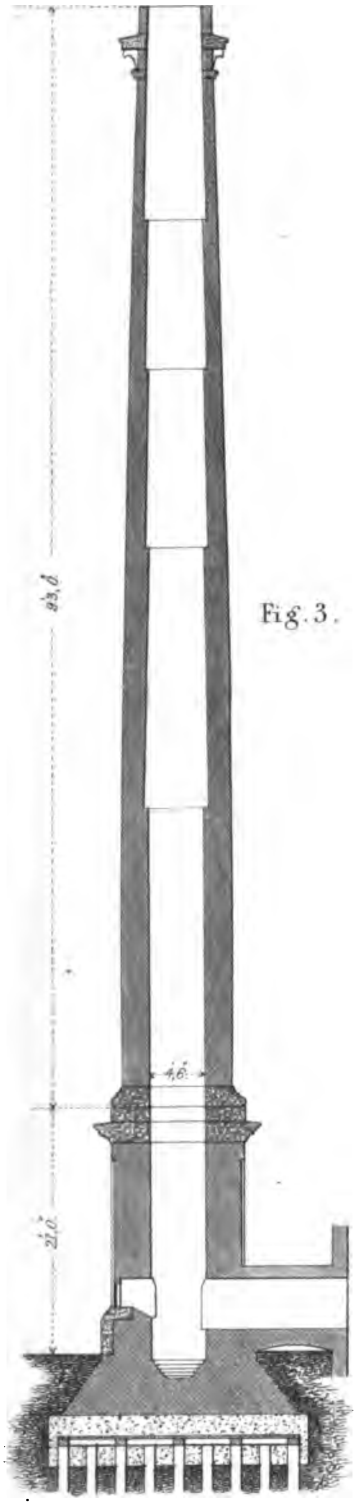

Fig. 3.

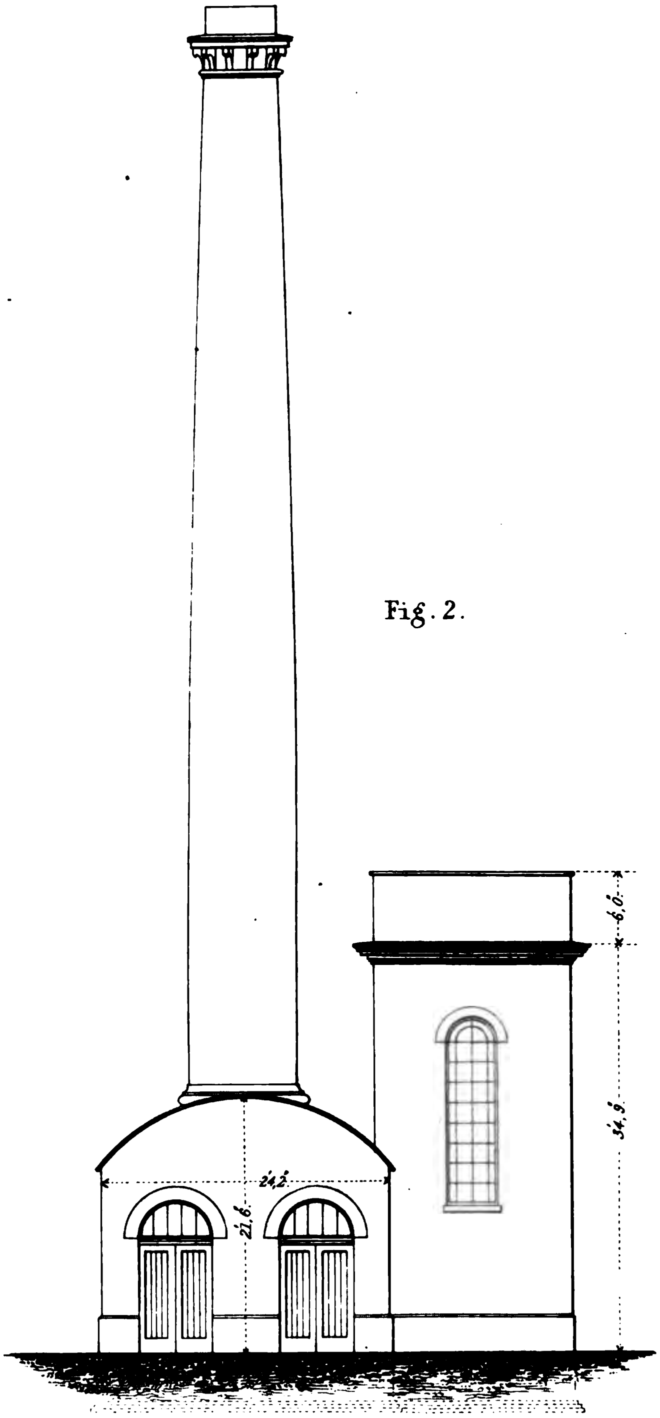

Fig. 2.

Mackay & Kinmond Glasgow

and horizontal section of the same. This bridge has six openings, in the suspension portion of its length, consisting of four large openings of 440 feet, from centre to centre of the piers, and two smaller side openings of 225 feet, measuring from the faces of the abutments to the centres of the piers; there is also a swivel-bridge on one side, of 50 feet span, for the purpose of navigation. The total length of the bridge is 2562 feet, and the clear water way is 2140 feet at the highest floods. At ordinary summer level, the platform is 30 feet above the water line, and its extreme width is 52 feet 6 inches. The floods in spring usually rise 20 feet, and in summer the greatest depth of water in the navigable channel is 40 feet. The extreme height, from the deepest foundation to the top of the caps of the suspension towers, is 112 feet. The forms of the cut waters are more particularly shown in the enlarged plan, Fig. 4.

The chord of the chains of the large openings, clear of the towers, is 416 feet, and their versed sine is 30 feet; their minimum sectional area, excluding pins and overlaps, is 328 superficial inches. They are four in number, two on each side of the roadway. The total weight of the platform and rods, clear of the towers, is 2350 tons; the total weight of the chains is 1578 tons, of which 1076 tons are suspended clear of the towers, so that the total suspended weight is 3426 tons. The test weight, or proof load, was fixed at 63 lbs per foot superficial of the bridge; but as the sand used for that purpose had become saturated with water, the load actually applied was not less than 3000 tons.

The Kieff Bridge was commenced in April, 1848, and was opened on 10th October, 1853. It cost £432,000.

ENGINE AND BOILER-HOUSE.

PLATE 46.

To illustrate the manner in which architectural drawings are generally finished, we give an example in our Plate 46, which represents an engine and boiler-house erected on one of the affluents of the Thames. Figs. 1 and 2, Plate 46, are west and east elevations of the engine-house, boiler-house, and chimney-shaft and connecting flue. Fig. 4 is a longitudinal section of the engine-house, and Fig. 5 a complete plan of the whole.

There is nothing remarkable in the structure thus illustrated, so far as its æsthetical merit is concerned; but the practical details it involves embraces, on account of the peculiarity of the site, almost every branch of the constructive arts essential for the stability of a building exposed to violent dynamical efforts, and erected upon a bad foundation, which in this case consisted of compressible peat and alluvial matters of about 30 or 40 feet in thickness. The engine-house encloses a double cylinder 40 horses-power engine (working nearly up to 80 H.P.); and the chimney was built to a great height on account of the peculiar form of the valley in which it is situated. The system of foundation adopted for the chimney and engine-house is piles and planking embedded in concrete, whilst the boiler-house is established upon a solid bed of concrete without piles. Under the chimney the piles were driven as follows:—Swedish fir-baulks, 10 × 10, were placed so as to form nearly a close sheeting round the edges of the proposed bearing surface, and, after these piles had been driven, the intermediate piles were inserted at the distances indicated in Fig. 3, Plate 46. Cross-sleepers were tenoned upon, and across the head of the piles, and upon which 2½ planking was firmly spiked. The heads of the piles being surrounded with concrete before the sleepers were placed, as also being well supported by concrete, carefully rammed round them, before the planking was laid; then the whole of the wood work was finally embedded in hydraulic concrete so as to prevent the access of moisture to it, as well as to maintain the perpendicularity of the piles under the superincumbent load. The engine-house was piled on the same system, and in both cases care was taken to drive the outer rows of piles first so as to compress the ground around those of the interior—the length of each was about 22 feet. Under the boiler-house, the ground was found rather firmer than that under the seating of the chimney and engine-house, it was therefore decided that a bed of concrete, 5 feet thick, would suffice to distribute the weight with safety, and these foundations have succeeded admirably.

The superstructure was designed as simple, but as substantial as possible, the only attempts at ornamentation being the execution of the arches in bricks of a lighter colour than the ordinary walling, and the introduction of stone cornices to the engine-house, the base and capital of the chimney, and the sills of the windows. The outline of the chimney was made to present the regular entasis of a Roman Doric column, so far as the relative height would allow; but perhaps it would be worth while here to remark, that the system followed in this instance, of making the interior diameter as nearly as possible uniform, has a tendency to retain the smoke about the outlet. The construction and form of the tank over the engine-house is sufficiently indicated by the elevation and details, which also show the position of the hoisting beams inserted for the purpose of facilitating the erection and repairs of the steam-engine. The floor supporting the tank is Russian timber, and the scantlings chosen with reference to the ordinary trade dimensions of the respective materials.

In the working drawings usually prepared by architects, it is not customary to indicate so many of the details as are shown in this plate; but long experience has, however, convinced us that their careful preliminary study, and their clear conventional indication, ultimately saves both trouble and time; and we therefore earnestly recommend the student to follow the system on which this design has been worked out.

FLOUR MILL.

PLATE 47.

Flour mills being one of the most important and universal applications of machinery and gearing generally, we have thought it necessary to give this Plate and description of a large and very complete mill of this class as a sequel to the mechanical department of the body of this work. This mill is an excellent example of French engineering and millwright work, and shows many good, and, to the English student, novel arrangements connected with such mills. This Plate, 47, is the representation of the machinery of a flour-mill, in geometrical projection, driven by belts, and as fitted up by M. Darblay, at Corbeil.

The construction of flour mills has latterly undergone very important improvements, as well in reference to the principal driving machinery, as to the minor movements, and the cleaning and dressing apparatus. As such machinery belongs to a most important class, we have selected a mill, as an illustrative example of the subject before us, giving all the recently improved modifications now at work, both in this country and on the Continent.

Before the introduction of what is known as the American system, very large uncovered millstones, of upwards of six feet in diameter, were employed; and these gave what were then considered very good and economical results. These flour mills were worked by water-wheels or wind-mills; but as improvements have gradually been introduced into this class of machinery, not only was the entire internal mechanism changed, but also the motor, and the description of stones. The American flour mills, commonly known on the Continent as English mills, differed from the older mills in the employment of smaller stones, with furrowed surfaces, and in their being driven at a much greater speed, requiring, in consequence, more wheel-gear to bring up the speed. A mill on the old system, with large stones of six feet in diameter, ordinarily goes at the rate of 55 to 60 revolutions per minute, being moved, we shall suppose, by a water-wheel, making 10 or 12 turns in the same time. Such a mill will only require a large spur-wheel and a lantern-wheel; or better, a large bevel-wheel and a bevel-pinion, in the ratio of 5 or 6 to 1.

But a modern mill, in which the stones are generally 4 feet in diameter, should make 115 or 120 revolutions per minute; whilst it may be impelled by an overshot water-wheel, making only 3 or 4 revolutions per minute; so that it is necessary to employ multiplying gearing between the power and the work. When this multiplication is obtained by gearing, two or three pairs of wheels are generally employed. The essential features of this gearing, consisting of a large horizontal spur-wheel, driving a spur-pinion on the mill-stone spindle, have recently been superseded, in many instances, by belt and pulley gearing. This arrangement has the advantage of rendering the motions more smooth and steady, with less vibration and noise, whilst it also admits of an easy arrangement for the stoppage of a single pair of stones, without stopping the prime mover and the whole mill, which is a very essential point, more particularly in a large and important mill, where many pairs of stones are at work.

Plate 47 represents a mill of this description, driven by belts, comprising 10 pairs of stones, placed in two parallel rows. The establishment contains several sets of stones, arranged in this manner, and each set of mills is driven by a hydraulic turbine, on Fourneyron's system.

Fig. 1 is an elevation, prolonged as far as the vertical shaft of the turbine, and showing part of the gearing at that end in vertical section. Fig. 2 is a transverse vertical section, taken at right angles to the horizontal driving-shaft, or to that of Fig. 1, and corresponding to it. Fig. 3 represents the plan of a portion of the principal gearing, and one of the rows of stones, and corresponds also to Fig. 1.

At A, in Fig. 1, is represented the upper end of the vertical wrought-iron shaft, upon which the turbine is fixed lower down. This shaft is supported by a step-bearing at its lower extremity, and in a brass collar bearing, *a*, at its upper end; this bearing being in two pieces, adjusted in the top of the hollow pillar-frame, B, resting upon the foundation-plate, and also bolted by a bracket-piece to the cast-iron pedestal, D, of the bearing, which carries the end journal, *b*, of the main driving-shaft, E.

This shaft, E, has fixed upon it, in the first place, the bevel-pinion, F, with strong thick cast-iron teeth, driven by the horizontal cog bevel-wheel, G, and keyed upon the top end of the turbine-shaft, A. This shaft is connected by a coupling-box, *c*, to a wrought-iron shaft, C, which passes up to the higher floors of the building, where it serves to drive the various accessory apparatus of the mill, such as the sack-hoists, pressing, washing, dressing machines, endless-chain elevators, &c. At each floor, the shaft is supported by a collar-bearing, like that shown in section at *d*, in this figure.

The horizontal shaft, E, which drives the two rows of stones, is in several lengths, joined together by cast-iron couplings, as at *e*, and it is supported at different points of its length by the pedestal-bearings, *f*, seen in all the figures, each with its oil-receiver at the top, is bolted down to the bases, or strong sole-plates, of the arched cast-iron standards, H, all of which are firmly secured to a strong wall of masonry below. These standards, H, are formed into receptacles at their tops, to receive the brass footstep-bearings and steel pivot-pieces, to support the lower case-hardened extremities, *g*, of the vertical shafts, I. An adjusting-screw, *i*, is introduced from below, to raise the bearing when necessary; and the upper journal of the shaft revolves in the inverted cup-bearing, J, the bracket of which is bolted to the cross beams of the ceiling.

Each of the vertical shafts, I, has keyed upon it a bevel-pinion, K, as well as two horizontal pulleys, L, of the same diameter. The pinions gear with the wooden-teethed bevel-wheels, K^1, keyed upon the horizontal driving shaft, E; and the pulleys are put in communication by means of the leather belts, *h*, with other similar pulleys, L^1, of the same diameter. These last are each keyed upon the shaft or spindles of a pair of stones, M. A tension-pulley, N, upon a short vertical spindle, supported by the two arms of a second vertical spindle, O, serves to stretch the belt of each pair of stones to the requisite degree of tightness. For this purpose, a lever, K, is fitted to the vertical spindle, O, and to its free extremity is attached a cord, passing over a couple of guide-pulleys, *l*, sustaining a small weight, *m*. Thus, in the position given to each of the levers, *k*, in the drawing, the weights are supposed to be acting; and the belts are, consequently, in a stretched state, and the motion of the pulleys, L, is communicated to the pulleys, L^1. But if the weights be lifted up, so as not to act, the levers, *k*, will be set free, and also the tension-pulleys, N; and the belts will be slack, so that the motion will no longer be communicated, and the pulleys, L^1, and consequently the pairs of stones, will be stopped. The vertical spindles, O, are supported in bearings, carried by cast-iron brackets, P, bolted to the under side of the cross-beams. The tension-pulleys can thus assume various positions, whilst their supporting-arms vibrate upon the vertical spindles, O. In order that the belts may not fall when they are slack, iron-guides, *n*, are placed at intervals, attached to vertical rods, *o*, depending from the ceiling.

As the millstone shaft, or spindle, may be either made of cast or malleable-iron, its lower end is fitted with a case-hardened or steel step, which revolves upon a steel pivot-piece, *q*, adjusted in the bottom of a brass footstep-bearing, shown detached and enlarged at Fig. 4, which is itself contained in a cylindrical cast-iron cup, *r*, carried by the box, n^1, formed in the casting, p^1, which surmounts the solid masonry, o^1, upon which the entire framing of the mill is supported. Screws are introduced through the sides of the footstep-bearing receptacle, by means of which the centre of the shaft is adjusted; whilst the shaft is adjusted vertically, and, consequently, the distance between the stones, by means of the screwed spindle, *s*, Fig. 2, which has a small spur-wheel, *t*, keyed upon it, with which a small pinion, *u*, Fig. 1, is in gear, this last being actuated by the handle, *v*, upon its vertical spindle. By turning this handle to the right or the left, the small wheels are set in

FLOUR MILL.

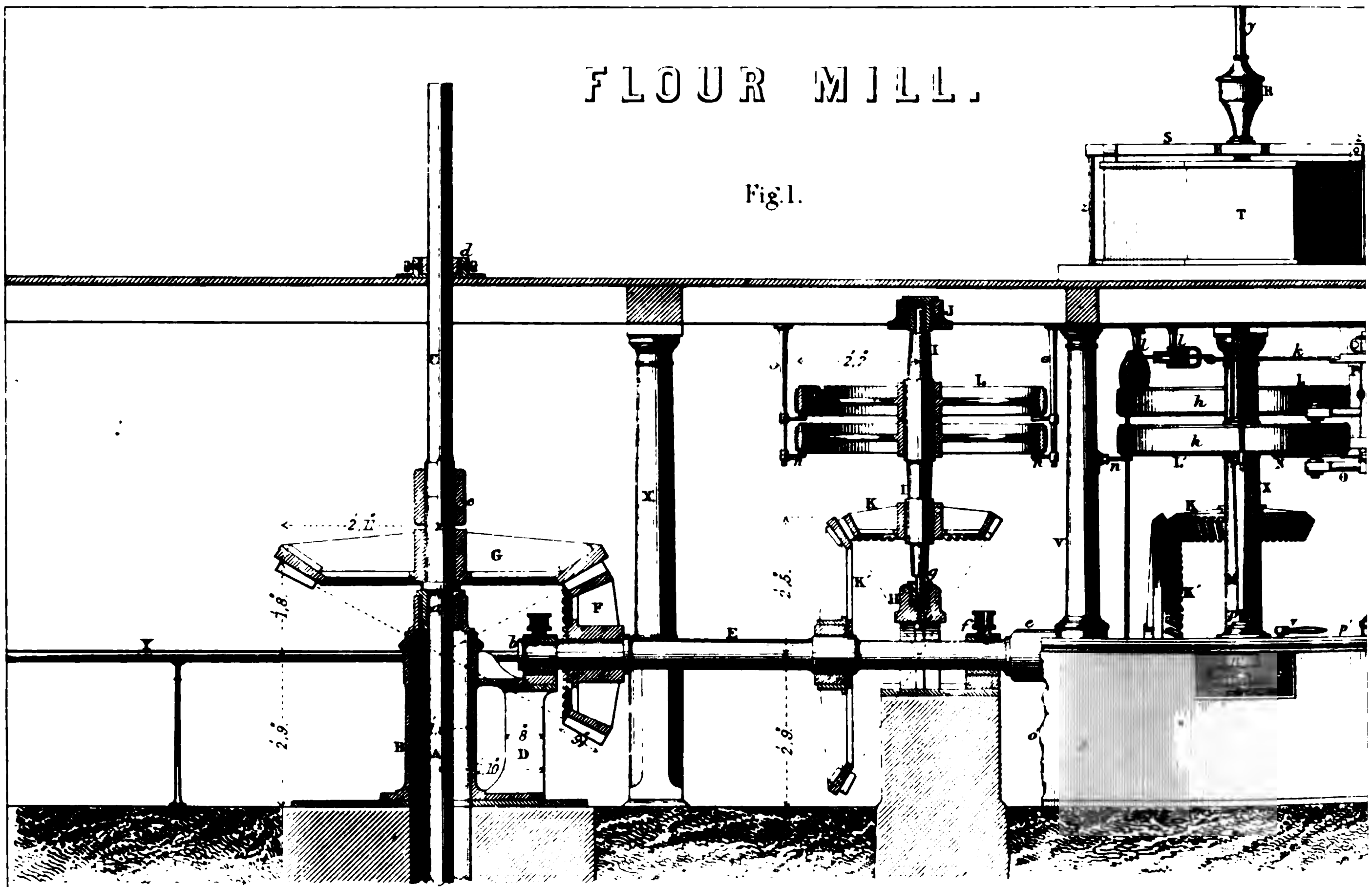

Fig. 1.

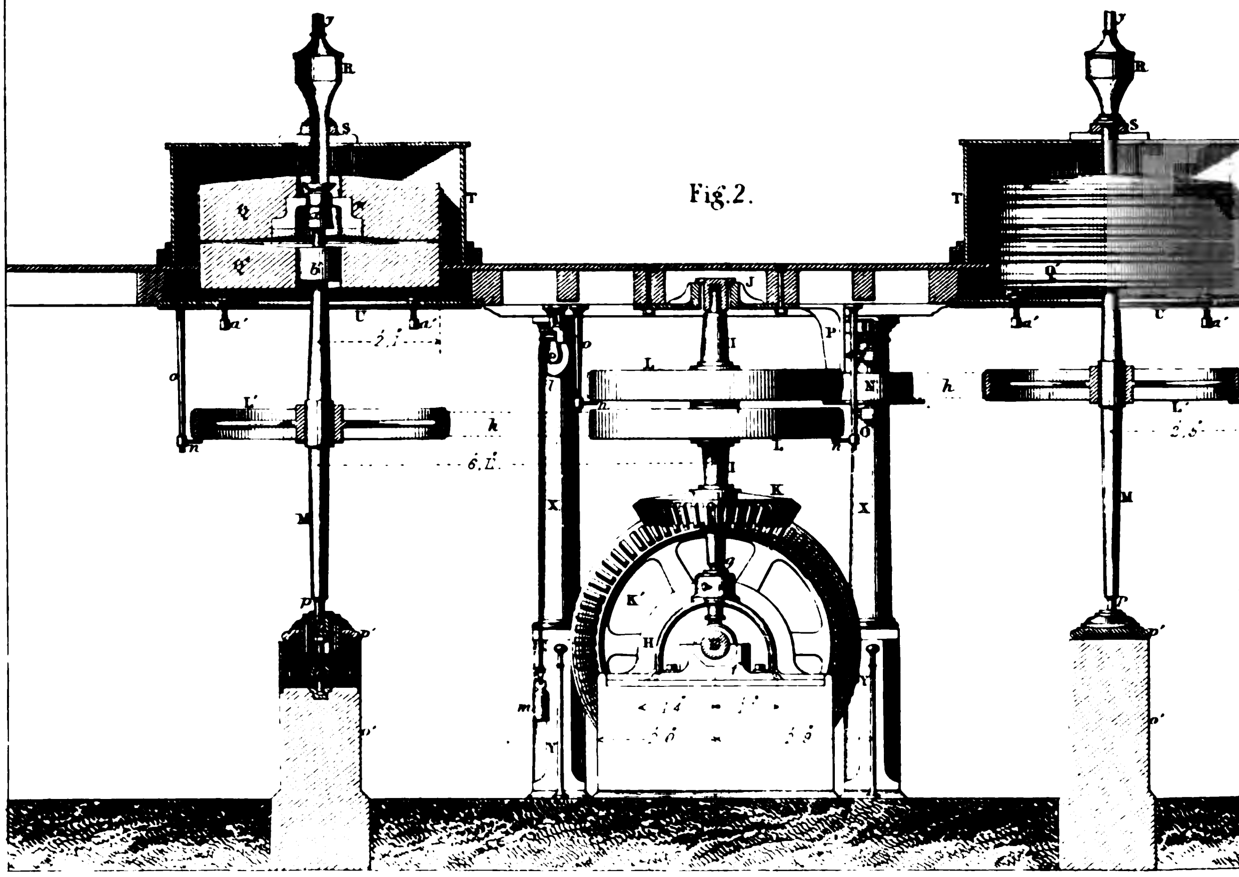

Fig. 2.

Fig. 4.

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PLATE 47.

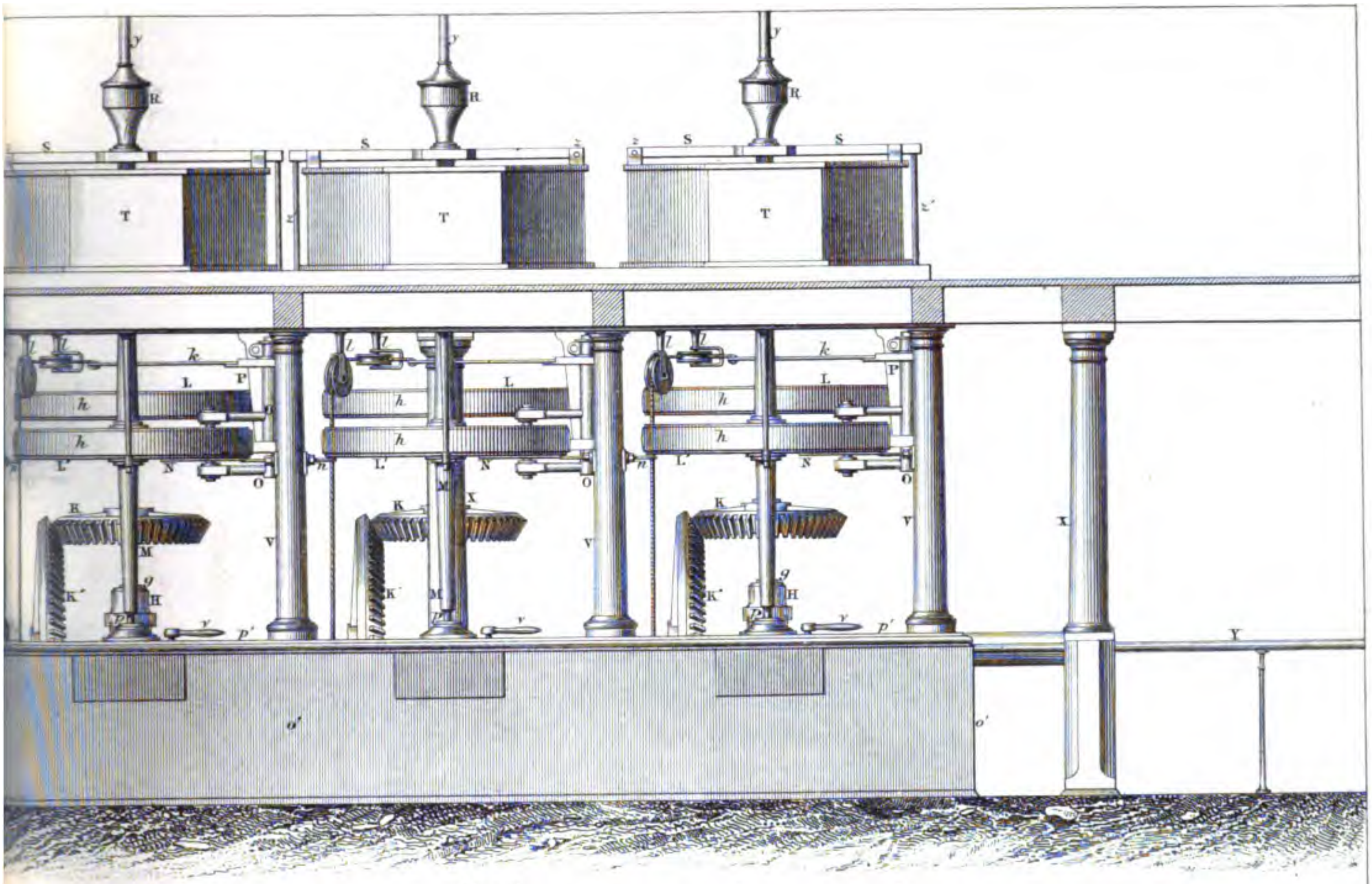
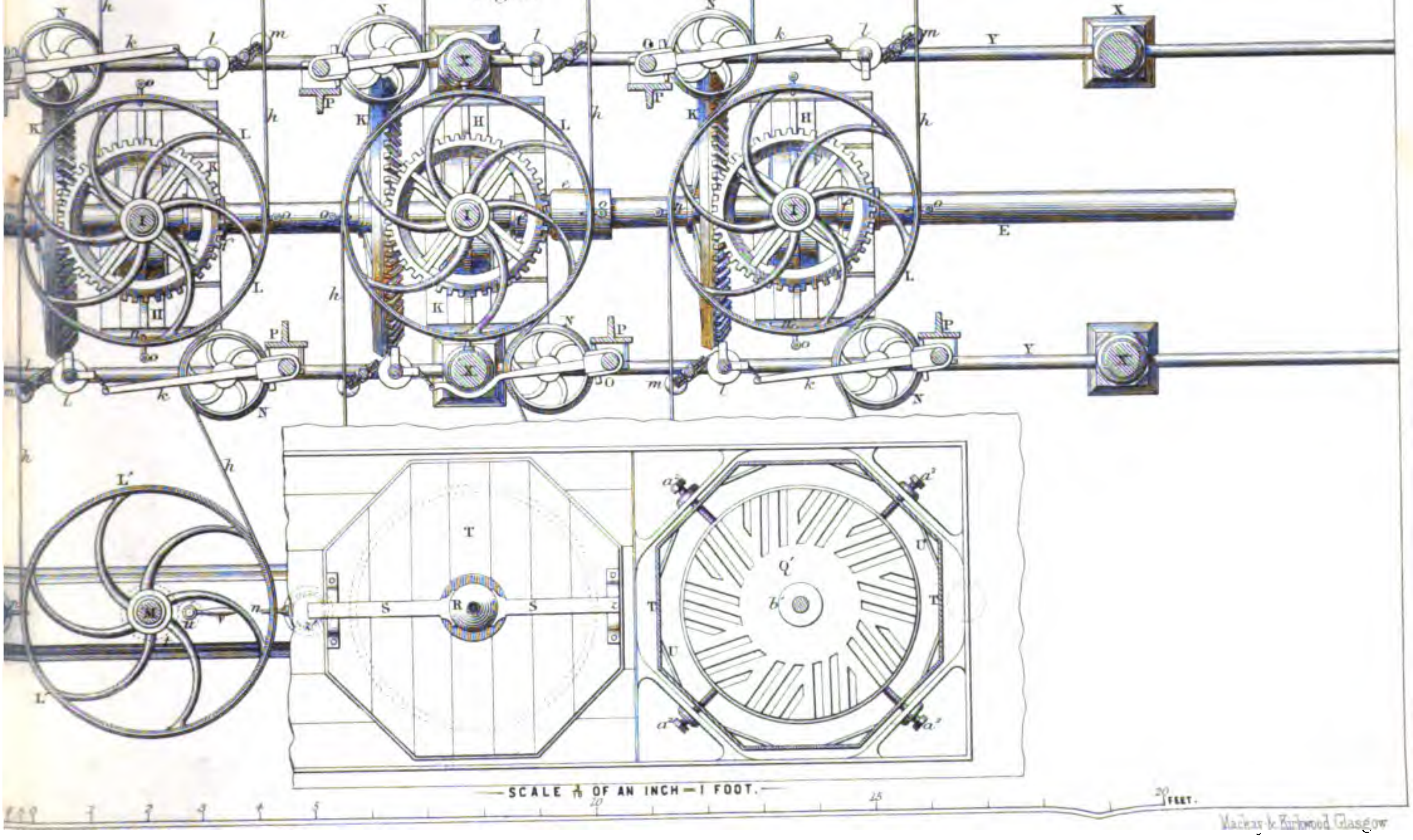


Fig. 3.



—SCALE ⅛ OF AN INCH—1 FOOT.—

10 15 20 FEET.

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motion; and as the screwed portion of the spindle, *s*, cannot otherwise turn, it is forced to rise or fall, and with it the bearing and footstep of the millstone shaft. It is in this manner that the pitch, or distance, between the two stones is adjusted with all desirable precision, according to the kind of work required from the stones.

The upper end of the millstone shaft is also steeled case-hardened, and is entered a certain distance into the boss of the centre-piece, or rhind, *w*, Fig. 3, which is fixed across the eye of the upper stone, or runner, Q, and firmly imbedded into the stone at either side. On the top of the boss of the centre-piece, *w*, is a species of metal saucer, into which dips the lower end of the pipe which conducts the grain down from the funnels, R, generally made of copper. These funnels, which receive the grain, communicate by the pipes, *y*, with a single hopper above, and they rest upon the wooden cross-pieces, S, held down on the one side by a hinge, *z*, and on the other by a vertical iron rod, z^1, by means of which they are raised or lowered at pleasure, so as to set the bottom of their pipes at a greater or less distance above the bottom of the saucer below. The object of this arrangement is to allow more or less grain to enter between the stones. The supports of the cross-pieces, S, are fixed upon a wooden casing, T, which covers each pair of stones, a space being left inside all round the stones, into which the produce of the grinding falls, as it issues from between the stones. It is thence conducted, by suitable channels, either to receiving-chests, or to the elevators, by which it is carried to the upper part of the building, to undergo the subsequent processes.

The lower immoveable stones, Q^1, of the same diameter as the runners above, are fitted with metal eyes, *b*, furnished with brasses, which embrace the shafts of the runners, and assist in preserving their perfectly vertical position. These stones are grooved, as indicated in the plan of one of them, Fig. 3; that is to say, shallow channels are cut out of their working surfaces, so as to present on one side a sharp edge, and act with the runner like scissors, cutting each grain as it comes upon them. The fine close-lined dressing, which is given to the surface between these channels, completes the fracture and crushing of the grain. These lower stones rest upon the cast-iron plates, U, but with the intervention of the three adjusting screws, a^1, which allow of the obtainment of an exact level; whilst four lateral screws, a^2, entered through the lateral cast-iron frame, serve to adjust with accuracy the centre of the stone.

The base plate and side frames are not only bolted to the cross beams of the building, but they are also supported at intervals by cast-iron columns, V, placed between each pair of stones, and resting upon the plates, p^1, and the solid masonry, o^1, below them. The ceiling is additionally supported by the solid wooden columns, X, placed at the ends and between the two rows of stones. An iron railing, Y, is placed on each side of the driving-gear, to prevent accidents which might arise from persons passing too near the heavy wheels. Cavities are constructed in the masonry, for the reception of the mechanism for adjusting the footstep-bearings of the runner-shafts, already described. These openings are usually covered by suitable doors.

The best millstones used, in this or any other country, are made of French materials. The stones from the valley of the Marne have the credit of performing more work, and turning out better and whiter flour, than any others. They are purely siliceous in composition, and slightly tinged with ferruginous matter; they are now exported to every part of the world where the British system of grinding is followed. Formerly, great pains were taken to extract enormous stones from the quarry beds, to be used either as monolith grinders, or two or three only, combined into one, in a very rude manner, open faces being selected to grind upon, instead of the modern artificial furrows. The grand improvement of cutting furrows in the grinding faces, in such a way as to improve the grinding action without interfering with the centrifugal effect, is an English invention, introduced only forty years ago. Hence the uncertainty attending the use of porous or partially cellular stones was removed; and the French makers gradually improved upon the idea, by building up together small fragments of stone of equal hardness, so as to insure a good grinding surface throughout—this being unattainable in large masses, where softer and more porous parts frequently occur alongside the harder areas. The makers thus contrived to get composite stones, each increment of the surface of which was of the same grain, hardness, porosity, and colour; and as the manufacture grew up to be an important branch of industry, the niceties of suiting the materials to the peculiarities of the country where the stones were to be used, the special system adopted by the millers, and even the character of the grain to be reduced, were all carefully attended to. Thus it is, that the millstone manufacture has become a precise art. In building such stones, the workman selects a solid centre-piece, or eye-stone; and round this he sets his choice-selected masses, previously bound together, and fixes the whole with plaster of Paris, such accuracy being observed in the fittings, that the entire structure hardly exhibits a joint. The smith then encircles the stone with a retaining hoop of wrought-iron, put on hot, so as to fit tightly on cooling. The dresser then reduces the yet uneven surface to a plane; and the furrow-cutter follows the dresser, first setting off, and then cutting out, the grooves which are to produce the sharp cutting edges. The eye is then completed, and the running stone balanced, to be of equal weight all round, cavities being left for the insertion of lead, when the stone is started in work, so that it may run with perfect steadiness. A second hoop of cold iron is then added to give further strength, and the stone is left to dry. M. Roger, a French maker of repute, produces annually some 500 millstones, and an immense number of the inferior or burr stones—all excavated from the valley of the Marne.

The ingenious and important contrivance of the "antifriction curve," by Mr Schiele, has found an important application in the grinding surfaces of millstones, in which it has introduced a striking departure from old-established principles. The inventor was led to the consideration of this system of working surfaces by the irregularities of wear in the common conical plugged stopcock. Considering the truncated cone of the stopcock plug to be divided into a series of infinitely short lengths, he proposed to take a more obtuse cone for each larger portion, and in such progression, that it would require equal pressure for every portion of the surface to cause a uniform sinking of the plug in the course of wear. The contour thus obtained is of a peculiar curve, as shown in Fig. 1. The main feature of the generating curve for such a surface, is the equality of all tangents drawn from the curve surface to the axis; hence the use of the simple instrument illustrated by Fig. 2. This contrivance consists merely of a straight brass wire, A, jointed at one end by a pin to the upper surface of a small wooden slider, B, which is hollowed in the centre to receive the tip of the finger in drawing a curve. A small drawing-pen, C, ingeniously formed out of a slip of steel bent over the wire, A, and screwed to a brass bush, so as to form two broad nibs, is arranged to slide from end to end of the wire, being adjustable at any point by stiff friction, caused by a spring, which, fitting a groove in the wire, retains the vertical position of the pen. In drawing a curve, the rod or wire carrying the pen is set a right angles with the slider, B, which is drawn in a right line along the edge of a ruler, whilst the wire carrying the pen is left to find its way from its initial angular position to that of a line in the same plane as the slider; and, in doing this, the pen describes the curve we have represented.

Fig. 1.

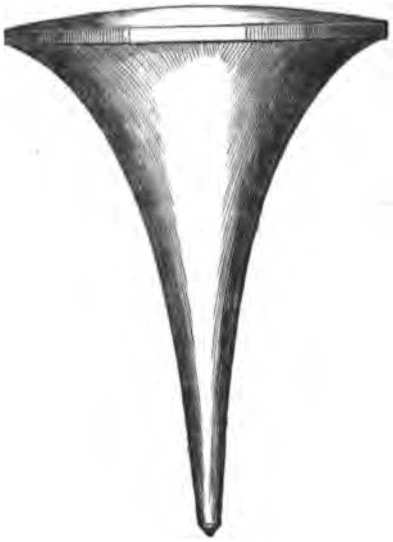
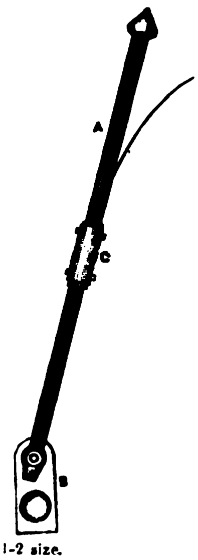


Fig. 2.



1-2 size.

Fig. 3.

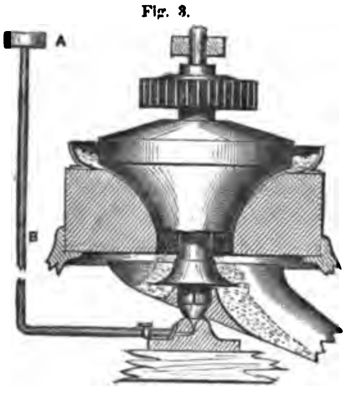


Fig. 3 is a vertical section of a millstone arrangement on this system, showing how the gradual variation of the curvature, in relation to the increasing distance of the parts from the centre of motion equalizes the rubbing pressure in the most perfect manner. The same sketch also shows the adaptation of the principle to footsteps, together with a new system of lubrication of these surfaces so liable to extreme abrasion. The oil supply is kept in an elevated vessel, A, whence a pipe, B, proceeds downwards to the footsteps, upon which a pressure is thus constantly kept by the oil column, a stopcock being introduced to regulate the supply.

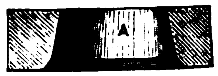
Mr Schiele now makes independent or self-contained flour-mills of this kind, of such simplicity and compactness, that four complete mills, or sets of stones, placed together, may be worked in a room, 10 feet square; a single shaft driving the set, from the centre, by means of a horizontal band-pulley, from which endless bands pass to corresponding pulleys on the spindle above the upper stone. In mills of this kind, when by wear the runner has sunk three inches, the adjusting screws of the steps arrive at the end of the 3-inch traverse allotted to them. The runner is then lifted from its seat, and the thin end is shortened to this amount. This plan of renovation may be repeated twice, thus allowing for 12 inches wear in a 26-inch runner; and the stones, so reduced, are still valuable for smaller mills.

The peculiar portability of these mills is a valuable feature of improvement. No fixtures are required, as the weight of the parts insures steadiness in working. Perfect uniformity of wear in the grinding surfaces is attained by the use of the curved face; and the expensive dressing necessary in flat stones is here entirely obviated, as the occasional grinding of hard surfaces roughens the faces to an extent sufficient for grinding all the softer materials, which gradually smooth down the faces.

Any of the materials ground in common mills, and many which the latter cannot properly act upon, are capable of reduction in these mills. For flour and other finely-ground substances a few air-channels are formed down the face of the runner. Their best speed is only half that of common stones; and the inventor states that his experimental results go to show that a two-feet runner produces as much flour as a four-feet flat millstone, the power required being a minimum. If the stones run empty, no contact can take place, therefore there is no firing, nor does a variation in the feed or speed cause any difference in the relative position of the stones, on account of the firm and steady revolution on the curved pivots. The antifrictional qualities of these pivots are pretty well elucidated by the fact of the very minute consumption of oil upon them.

The "Ring Millstone," invented by Mr Mullin of Gilford, Ireland, is proposed as the means of securing four special advantages—economy in manufacture, simplicity and effective ventilation, increased production of meal, and a saving of labour in repairs. Fig. 4 is a vertical section of the stone, and Fig. 5 is a corresponding plan. The "eye," A, is made excessively large in proportion to the stone's diameter, exceeding, indeed, half the latter dimension. This increased area admits a greater volume of air than is usual, and this air, coming in contact with the more rapidly revolving portion of the stone, is passed between the working surfaces by the action of the centrifugal force. Besides, the increased circumferential length of the eye admits of the formation of three or four times the ordinary number of leading furrows, for the distribution of the air over the grinding surfaces, and thus more grain is ground, without any risk of overheating by friction. Finally, by getting rid of the great area of superabundant stone at the centre, the operation of dressing is obviously simplified very considerably.

Fig. 4.



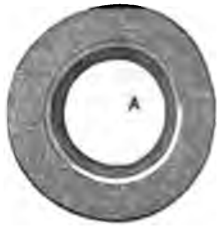


Fig. 5.

In this country, Mr Fairbairn of Manchester, perhaps, takes the foremost position as a flour-mill engineer. Mr Joyce, of the Greenwich Iron Works, has also produced some excellent work of this kind. The Union Corn Mills, Birmingham, and the Bone Mills of Mr Lawes, at Deptford, may be mentioned as working examples of the respective performances of these makers.

Fig. 1.

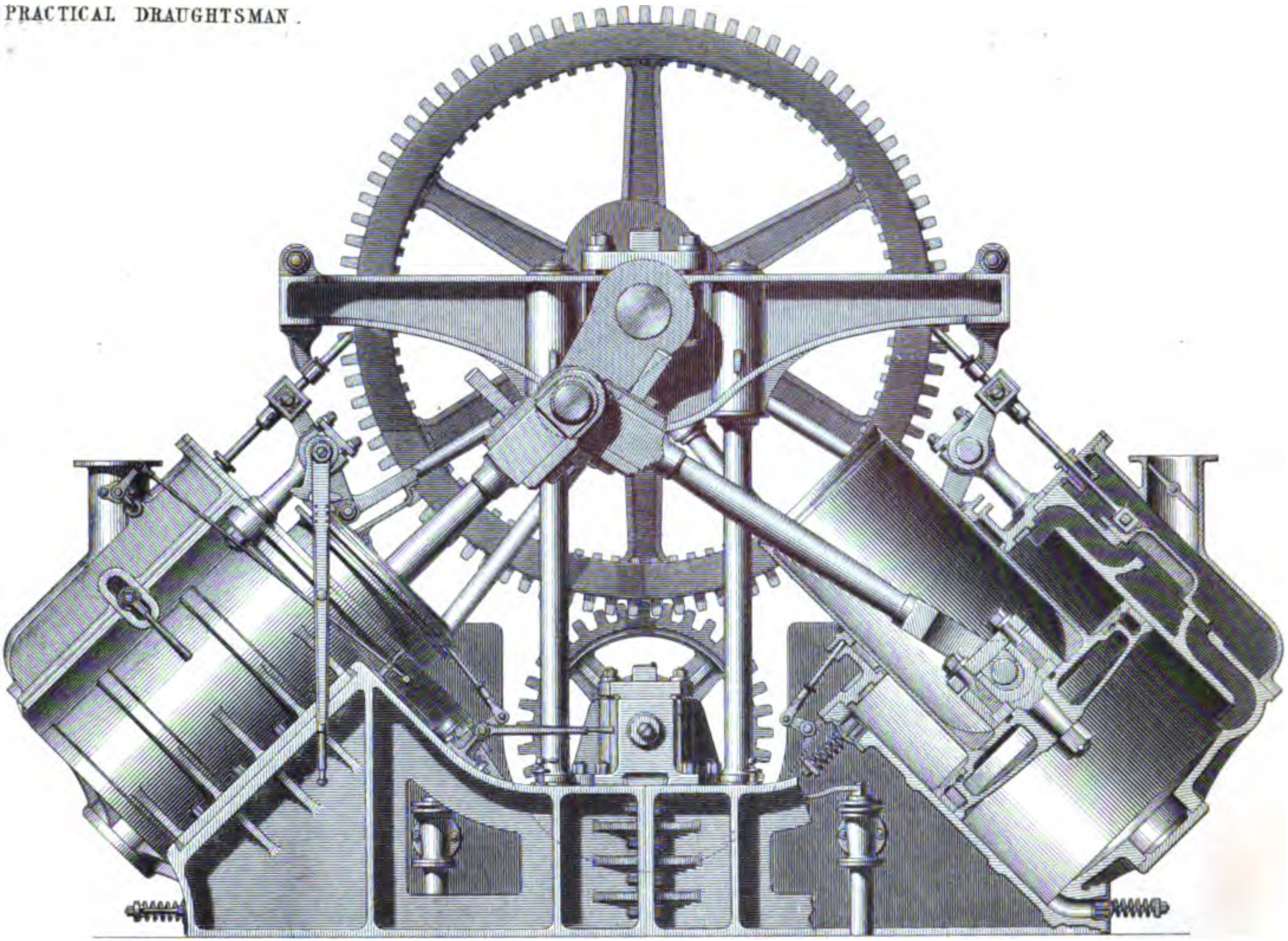
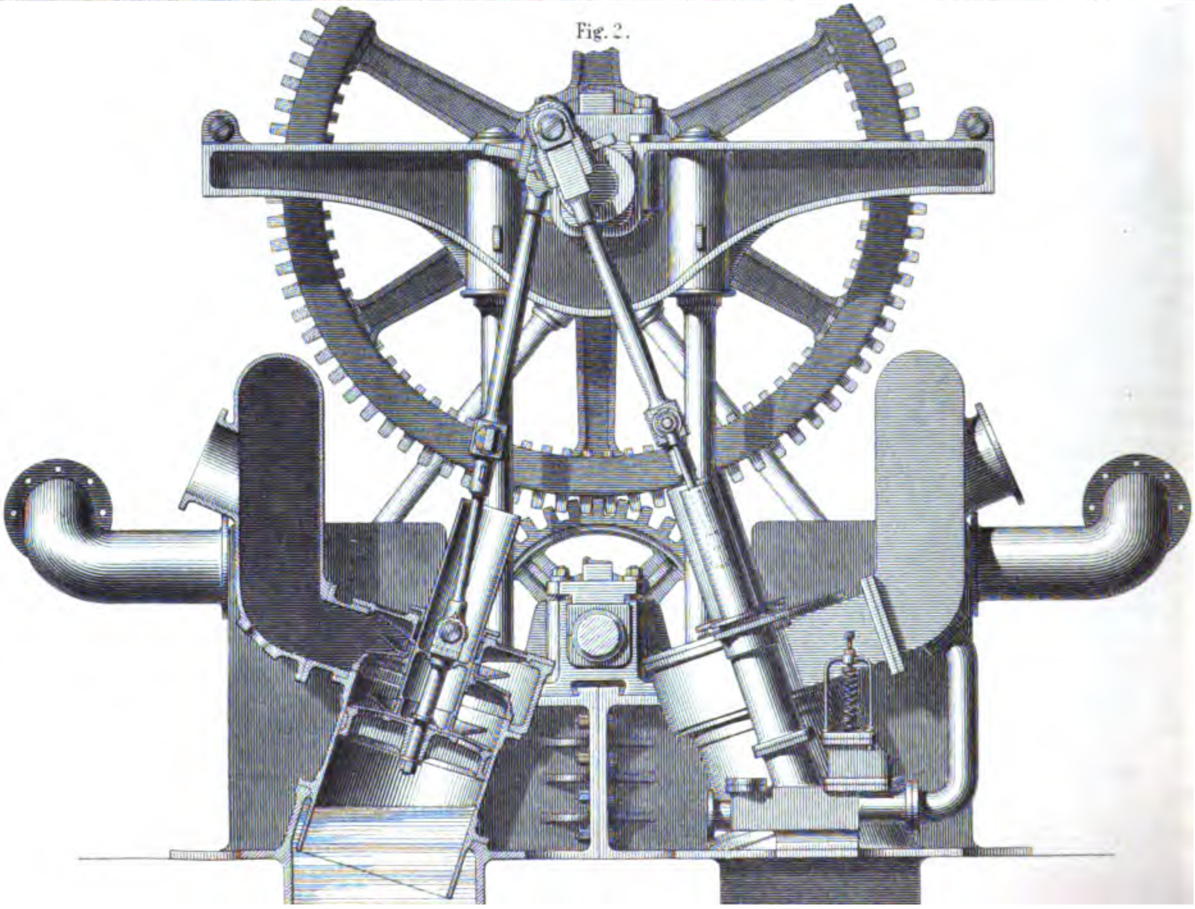


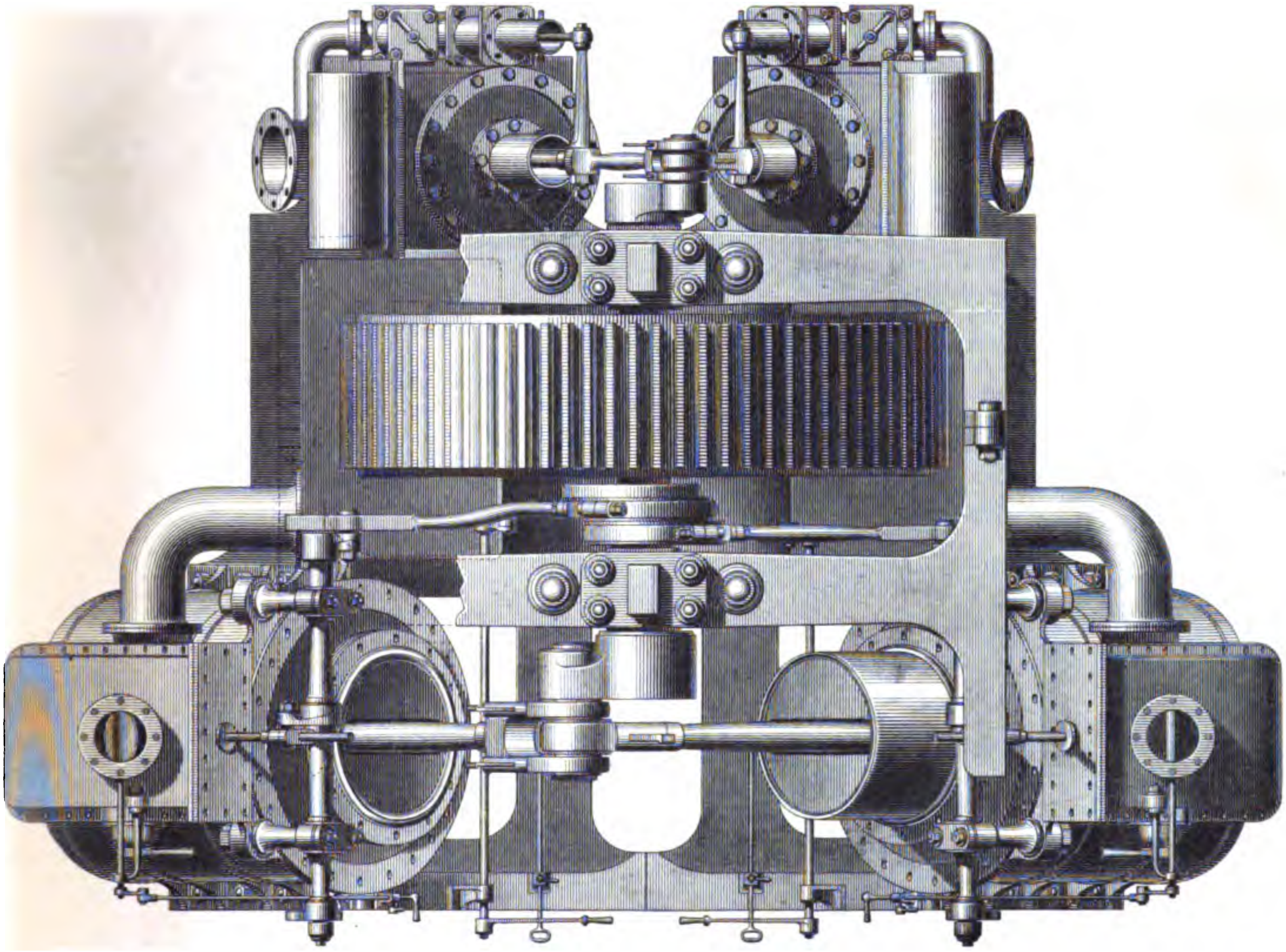
Fig. 2.



EXAMPLE PLATE . A .

ARED TRUNK MARINE ENGINES,

Fig. 3.



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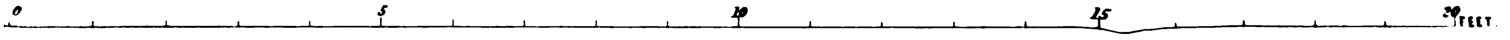

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CHAPTER XVIII.

EXAMPLES OF FINISHED DRAWINGS OF MACHINES.

DESCRIPTION OF THE MACHINES AND GENERAL REMARKS ON THE PRODUCTION OF SUCH FINISHED DRAWINGS.

GEARED TRUNK MARINE ENGINES.

EXAMPLE PLATE, A.

In accordance with the promise set forth in the preface and title-page of the present edition of this work, we have given from time to time, along with the other plates, a number of example plates. These plates are given, as a series of illustrations, of finished drawings of some of the most useful and generally employed machines of the day.—Being all taken from actual machines, constructed by some of our most eminent engineers, since the publication of the first edition of this work, so that they may be taken also as the most recent achievements of the engineering skill, in their architecture and design, as well as of mechanical construction up to the present time.

This chapter, which is devoted to the description of these finished plates, will, it is hoped, make an appropriate conclusion to our labours for the education and instruction of the practical draughtsman in industrial design; and if he has attended to what has gone before, up to the present, it will enable him to produce such works of art with his own hands, and do much towards the cultivation of the designing and constructing qualities of his mind.

To this end, then, we have selected as the first, and now proceed to describe, Example Plate, A, which represents a pair of Geared Trunk Marine Engines, as designed and constructed by Messrs. Tulloch and Denny, Engineers, Dumbarton. These engines combined are of 135 nominal horses-power, and were made in duplicate sets for the steam-ships *Cottingham* and *Empress*. The latter vessel was employed by Government in the transport service in 1855 and 1856—her average speed during that period being 10½ knots per hour, with a consumption of 19 tons of coal per day.

DESCRIPTION OF ENGINES.

The three elaborate and highly-finished views of these engines, which forms the first of our new series of specimen plates, afford an excellent example of compact heavy geared engines of the diagonal class. They are represented in vertical end elevation, in Fig. 1, as looking towards the stern of the vessel, on the crank and cylinder end, and as having the port-engine cylinder, valve and steam casing, shown in vertical section. Fig. 2 is a corresponding end elevation looking forward, on the air-pump end of the engines, showing the crank and other connections for working these pumps, as well as those of the bilge and feed. The screw-shaft is shown in cross section and the air-pump of the port-engine, with its bucket, valves, trunk, and hot-well, are shown in vertical section. These two views, when making the drawings, ought to be drawn on the same horizontal base line, while Fig. 3, which is a complete plan or horizontal projection, corresponding to Fig. 1, ought to be drawn exactly below that figure, having the vertical centre line produced downwards, to form the horizontal axis of the main-crank shaft of the engines. These views should be thus arranged on one large sheet of paper for the convenience of squaring over, or projecting the particular parts in the several views, and not as placed in the plate to suit the size of the paper.

The foundation of these engines consists of a pair of large strong frame castings bolted together at their centre by flanges, and to the foundation, formed on the floor of the ship, by flanges cast all round the sole plate. These frames are formed, at the forward part, with strong outward projecting angular flanges, for carrying the two steam cylinders, and to which they are firmly bolted.

The two cylinders are disposed athwart ships, at right angles to each other, the apex of the angle being the centre of the crank-shaft. These cylinders have pistons of the "trunk" kind for the internal play of the direct connecting-rods, which have forked lower ends for jointing to pedestal pieces, bolted through the pistons at the bottoms of the trunks. The upper ends of these connecting-rods pass upwards to the same overhead crank-pin on the first motion or crank-shaft, carried in bearings on an entablature frame, formed in two pieces, and bolted together, and supported from the foundation by four wrought-iron pillars and diagonal stays. This overhead crank-shaft carries the large main driving mortise spur-wheel in direct gear with the pinion, fast on the screw-propeller shaft.

The steam-valves are of the common single chamber slide kind, having a longitudinal strengthening feather in the centre of the chamber. They are disposed with their casings on the upper sides of the steam-cylinders, and are worked from a pair of eccentrics on the first motion-shaft. The eccentric-rods are respectively jointed to cranks or rocking-levers on the ends of rocking-shafts, carried in short pedestal bearings bolted through sockets formed on the upper flanges of the cylinders. The central portions of these rocking-shafts have short levers jointed to the valve-spindles, which is best seen in the starboard engine in Fig. 3, where a port of the entablature is shown broken away for that purpose; whilst the opposite ends of the shafts carry the hand-gear lever and details for the operations of the attendant engineer in starting and stopping the engines.

The air-pumps are disposed, at the opposite end of the engines, to that of the steam-cylinders, and at a slight angle of inclination as regards the vertical centre line. They are bolted to, and inserted a short distance into, wells or chambers in direct connection with their respective condensers, and formed in two extended portions of the bottom or main basement frame of the whole engine and cylinders. The condensers, which are cast along with the bottom bed-plate, are disposed between the cylinders and air-pumps, and have a semi-circular space, formed in the centre between, so as to leave room for the spur-pinion of the screw-shaft to work.

The angular arrangement of the air-pumps permit of their being disposed in the same transverse vertical plane, so that the screw-shaft can pass between them, while their buckets, which are of the trunk class, can be worked from a single overhead crank on the after-end of the main first motion-shaft—the two connecting-rods being jointed to the same crank-pin, as in the case of the main crank connecting-rods at the other end of the shaft.

The discharge and bucket-valves, which are formed of discs, or broad rings of vulcanized india-rubber, are secured to their seats by their annular guards—the former being screwed down by the air-pump covers, and the latter by the trunks, to which they are attached—while the trunks themselves are firmly screwed to the buckets by large central bolts, the heads of which are formed into double joints to receive the lower ends of the connecting-rods. These trunks are enabled to be much smaller in proportion than those of the cylinder, from the greater lengths of connecting-rods in relation to their respective length of stroke. The connecting-rods are formed in two pieces, for the convenience of disconnecting, but rigidly attached by a socket-joint at the mouth of the trunks, where there is also formed an enlargement to receive a long projecting lateral stud for working the feed and bilge-pumps from the same motion. These pumps are of the trunk-plunger or stoup-class, and fixed to the outer end of the sole plate, disposed close to, and parallel with, the air-pumps.

We annex a few details of the dimensions of the engines, as notes for practical reference:—Diameter of steam-cylinders, 50 inches; diameter of steam-cylinders trunks, 31 inches; effective diameter of piston, equal to 45 inches; length of stroke, 42 inches; diameter of air-pumps, 26 inches; length of stroke, 24 inches. The mortise spur-wheel on the first motion-shaft is 9 feet diameter, 24 inches broad on the face of the teeth, 4 inches pitch, and has 85 teeth in it. The spur-pinion is 3 feet 8 inches in diameter, 24 inches broad on the face of the teeth, 4 inches pitch, and has 34 teeth in it.

The screw-propeller is of cast-iron, and is three-bladed, 11 feet diameter, 13 feet pitch, increasing to 14 feet, 3 inches pitch.

The boilers are two in number, containing 372 brass-tubes, 3½ inches outside diameter, and 6½ feet long. The fire-grate area forms in the aggregate 119 square feet.

TO COPY THESE ENGINES OR DRAW THEM FROM SKETCHES.

Such highly-finished drawings of engines and complete machines as those represented in our series of example plates are usually executed to small scales, either from large working drawings or sketches, for the purpose of illustrating the several designs in a coloured picture, which would show every part of the machine to the spectator as well as if he saw the machine itself, or to be sent to the engraver to be faithfully transferred to the steel or copper-plate for a like purpose; hence the draughtsman cannot devote too much pains to the accurate delineation and artistic effect of his drawing, as a whole, and in all its parts.

The mere delineation of such finished drawings, as our example plate, A, represents, afford a great many examples of the application of the rules and instructions which we have given during the progress of this work, in outlining, shadow, lining, and shading, as well as in rectangular and oblique projection. We now propose to point out to the young beginner the practical application of these rules, and give a resumé of the mode of procedure in the laying down and delineating of such a sheet of drawings as this plate represents.

In laying down these finished drawings, whether from the large detached working drawings, or from sketches taken from the complete machine, or, which is more usual, from its several parts while they are in process of manufacture in the various parts of the workshops, the draughtsman proceeds much in the same way, by setting off the principal centre lines and measurements of the whole machine, which on the paper will, as it were, represent its skeleton or osseous system, which may now be clothed or completed by filling in all the minor measurements and details.

In addition to what we have already given (375), in reference to Plates 35 and 36, on the taking of sketches for the production of such drawings as the one under consideration, we may here remark, that it is not necessary to sketch the whole machine in such instances as the present, where two sides of it are alike and symmetrical on each side of the main centre line, as a sketch of the one engine on either side of that centre line, with all the sizes marked on it in plain figures, will suffice. As formerly stated, (375) these general sketches are usually executed in the form of elevations and sections, taken at right angles to each other, as also one horizontal view, along with as many minor sketches of the details and written notes as will enable the draughtsman to complete the finished drawing.

A perspective sketch also is sometimes of great use, as, besides saving much time in making several detached superficial ones, at right angles to each other, it admits of the three measurements—height, length, and width, being all marked on the same sketch, and therefore of being seen at a glance, without requiring to turn to the other views to mark on the sizes, or find them again when required.

The draughtsman, in proceeding to lay down the complete view of these engines on the paper, first draws a horizontal base, or floor line, upon which the bed-plate is supposed to rest. Upon this base-line two perpendiculars should be erected, at a considerable distance apart, which will form the main centre line of Figs. 1 and 2 respectively. From these base and perpendicular centre lines all the principal measurements are taken; thus the height of the centre of the screw-shaft is set off from the base on the perpendicular, as also the centre of the main first motion-shaft from that of the screw-shaft, according to the diameter of the wheels. Then from this centre, two lines drawn to the right and left, making angles of 45°, with the centre and base lines, will give the centre lines of the two steam cylinders, on, and from which, most of the chief measurements connected with the cylinders should be marked, such as their diameter and length, as well as all sizes connected with the piston, trunk, and valve. We may here impress upon the mind of the younger draughtsman the great importance of working from his principal centre lines, and taking as many of his measurements from them as possible, particularly all chief measurements, as the non-observance of this in practice leads both to the multiplication and complication of errors as regards the sizes and positions of the various parts throughout the whole drawing. Whilst the observance of this simple rule or practice prevents the multiplying of one error of measurement from another, it also increases the chances of detecting any casual wrong measurement which may have been taken (by the eye) from its disproportion to some previous or subsequent one taken from the same centre, which can consequently at once be corrected without leading to more, or perhaps greater, errors than the first.

We may also here observe, that in taking or transferring measurements from one view of the drawing to another, by compasses or a pencil on the edge of slip of paper, the draughtsman ought always to take them from the original view and centre line of the drawing which was laid off from the scale; this mode of procedure lessens the chances of little inaccuracies getting into the work, and ensures the consistency of all the views with each other.

Having decided at what position to show the pistons, say, as in Fig. 1, the port-piston at half stroke, then having drawn it in that position with the lower joint of the connecting-rod, a radius drawn from the centre of this joint, with a radius reaching to the centre of the crank-shaft, will intersect the circle of the crank's motion, in the centre of the crank-pin, and thus give the proper position of the crank and pin, in relation to the supposed position, the port one being at half stroke, and the starboard one near the bottom of its stroke. Lines drawn from the centre of the crank-pin, thus found, to the centre of the joint of attachment near the bottom of the trunks, will give the centre lines of the two connecting-rods, which can now be drawn with the proper sizes, symmetrical to these lines, as well as the crank behind them, the centre line of which is a line drawn from the centre of the crank-pin as found, to the centre of the crank-shaft. After these principal parts and centres are drawn in, the pillow-blocks of the screw-shaft, and bearings of the crank-shaft, as well as the lower frame and entablature, with its supporting malleable-iron pillars and diagonal stays, should be all penciled in; but, as a general rule, the details, and all the front or nearest objects, ought to be inked in first. In drawing radial, or rocking-levers, such as those on the wiper or rocking-shaft, through which the slide-valve is wrought, it is usual to fix or find their position by supposing them to be at right angles to the direction, or line of motion, when in the position of half stroke. For example, if the slide-valve, and eccentric which works it, is at half stroke, that is, in its central position covering all the ports, then the wiper or rocking-lever, to which the eccentric-rod is attached, ought to be at right angles to a line drawn through the centre of the crank-shaft and pin or stud of the lever, and the lever which is connected to the valve-spindle should be at right angles to the axis of that spindle in this position. This position of the parts also decides and gives the lengths of the rods connecting them; that is, the length of the eccentric-rod and valve-spindle, while the several centres and lengths of the connections being thus found for one position, can now be removed to, or drawn in, at any point of their stroke or motion to suit the disposition of the other parts of the engine, such as the proper relative position of the slide-valve and its cam or eccentric to that of the crank and piston, as very fully gone into in reference to Plates 25, 38, 39, and 40. As stated in relation to the cam and eccentrics in these plates for working steam-valves, they are generally made to have a "throw," or give a direct lateral motion equal to that required by the valve, so that both the gab and valve-levers are in this case of the same length. But this is not absolutely necessary, as considerable compensation can be obtained by varying the lengths of these two levers. In general, however, it is better to make the eccentric have more throw than the valve, as by that means the gab-lever will require to be longer than the valve-lever; and thus should any play arise from the wearing of the joints, links, or gab, it will have a less injurious effect in disarranging the position of the valve itself. Rocking-levers of all kinds, when at the central position of their stroke, are usually made half the perpendicular of their arc of motion, that is, half the line bisecting the cord and arc of the motion of the lever, projecting longer, (or "overhung") past the direct rectangular line of motion. All these points, by a little observation on the part of the student, will be well seen in relation to the slide-valve gear of the port cylinder in Fig. 1 of this plate.

Great nicety and care is required in delineating the details and thicknesses of the metal in the sectional parts of drawings, when executed on small scales, such as the present. It is also of great consequence that the circles and curves of the feathers of the framing, and of the other parts, as those of the ports, casing, &c., ought to join each other, and the straight lines into which they run, at a tangent to each other—rules for most cases of which were given, in reference to the several figures on plate 3.

All the front objects, cylinders, framing, &c., being drawn in, the draughtsman now finishes the elevation of the spur wheel and pinion, as shown in Fig. 1, of which he only at first penciled in the pitch-lines, or circles, on setting off the centres of the shafts, in accordance with the instructions given in reference to plate 20. The driving-wheel is a fine large example of heavy gearing, of the class known as mortise, or cog-wheels, having wooden teeth let into mortises, formed in the rim of the wheel to receive them. Little further requires to be said, as regards their delineation, or construction, than to refer the reader to Part 201, where such wheels are fully described, except that we may note, that, in such large broad wheels of this class as the one shown, the arms and rim have usually two or more face-arms, or webs, joined by strong feather-arms, and which form intermediate rings, or face-webs, in the crown or web, dividing the mortises and teeth into several divisions across the width of its face; thus enabling the cogs to be put in short widths across the rim of the wheel, which is a great desideratum, in many respects. In such heavy and broad spur wheels as those to which we are now referring, the several rims of mortises and cogs, in the breadth across the face, are frequently stepped round a proportional part of the pitch, in relation to each other, and the teeth of the cast-iron pinion are set round its rim in the same manner, so as to work properly in gear, with the teeth of the cog-wheel thus arranged. This way of placing the teeth of such wheels equalizes the strain on them at their tangential point of action, and insures a more steady, and less vibratory action.

The delineation of the back elevation, Fig. 2, is proceeded with much in the same way as that described in reference to Fig. 1, except that a great many of the heights are simply projected, or squared over, from Fig. 1 to Fig. 2, such as the centres, bearing-block, and framing supporting the two shafts, while the lateral measurements of most of the same parts, can also be transferred, by measuring from the centre line of the first, and transferring them to the same distance off the centre line in Fig. 2; as, for instance, the distance of the centre line of the malleable iron stays supporting the entablature, from the main centre line of the engines, and all similar dimensions of objects which are the same in both views. The centre lines of the air-pumps are set off at a predetermined angle, on both sides of the main centre line, and having the centre of the crank-shaft overhead as the apex of the angles. All the concentric parts of the pumps, buckets, valves, trunks, &c., are all measured from the centre lines, as well as all the similar dimensions of the feed and bilge-pumps, whose centres are in the same line in this view. The position of the centre lines of the crank and connecting-rods for working the air-pumps, as well as their several lengths, are found in the same way as that described in reference to the crank and connections of the working steam cylinders in Fig. 1. Several of the parts, such as the attachment of the bottom frame below the thrust-block of the screw shaft in Fig. 1, and many of the sizes of that block, with their bolts and nuts, &c., are the same as those of the back part of the frame, and the pillow-block, which rests on it behind the pinion of the screw shaft, shown in Fig. 2, are simply copied from the one to the other, as well as those parts of the wheel and pinion which are visible behind the other parts, which will complete the outline delineation of Fig. 2. We may, however, remark, before leaving this figure, that the angle of the air-pumps is made as little as possible, premising, however, that, consistent with the design of these engines, of working them from the same overhead crank-pin, they must stand in the same transverse vertical plane, and at such a distance apart as to admit of their fitting in separate wells, in direct connection with the respective condenser of each engine, and at the same time being capable of removal from their place, as well as their covers, valves, or buckets, without interfering with the screw shaft, which passes to the stern of the ship, over and between them. It is usual also, in such arrangements for working the air-pumps as that shown, to set the crank of the pumps diametrically opposite to that of the crank of the steam cylinders, for the more perfect balancing of the engines.

In proceeding to draw the plan view, Fig. 3, of this plate, the important centre lines are first put down, as stated in reference to the other figures. The first important one, which is the axis of the crank shaft and screw shaft, which fall in the same line in this plane, is simply squared down from Fig. 1 to Fig. 3, supposed to be exactly beneath, and corresponding to, it. At a sufficient distance from Fig. 1, on this line, to prevent any part of the plan interfering with the base line of the elevation, Fig. 1, erect a perpendicular to the base, which will represent the transverse vertical plane passing through the axis of the two steam cylinders. Then, from this centre line, let the draughtsman set off his several measurements, in the direction of the length of the engines, either from his previous ideas, or calculations for the design, or from his sketches of the actual engines, as the case may be. That is, set off the distance of the centre line of the first journal from the centre line of the cylinders, and draw a line through that point, parallel to the centre line of the cylinders, or at right angles to the axis of the crank shaft; this line will be an important centre line, being that of the first journal entablature frame, and vertical stays supporting it, and the crank-end of the first motion-shaft. Then, in like manner, set off the distance of the centre of the second journal from the first, and a line drawn through it, as before, will give the centre of the other journal bearing, entablature frame, and stays supporting it, and the other end of the crank-shaft. From this latter centre set off the transverse centre line, representing a vertical plane passing through the air-pumps, and another in like manner parallel to it, indicating the centres of the bilge and feed-pumps.

The next thing is to ratify these measurements before the draughtsman goes further, by adding up the intermediate distances on a scrap of paper, or in his note-book, and see that they collectively agree with that of the whole distance. Or, if he is drawing from sketches, let him at once fill in the several sizes, as the width of connecting-rod end, at the crank-pin, and the depth of the eye of the crank. This will prove at once if the first bearing measurement was right. By doing the same at the air-pump end, a like confirmation will be the result. In like manner, by filling in the width of the large spur-wheel, and the eccentrics, with their balance discs, or weights, with a proper clearance between each of the parts, the whole intermediate distance between the two bearings of the crank-shaft will be ratified; and in this way the greater number of the principal measurements, in the direction of the length of the engines, being laid down correctly before much of the work has been done—thus avoiding the chance of working away from some centre-line, which might, after much work had been drawn, be found to be incorrect. Nearly all the lateral, or transverse measurements, of the various objects in Fig. 3 are found, and their outline completed by squaring them down from Fig. 1. Thus the horizontal axis of the main crank-pin, the axis of the wiper-shafts, the centres of the stays, and the bolts of the bearings of the first motion-shaft, as well as the outline of the circumference of the large wheel, and that of all the cogs seen on the upper surface, also many other sizes, are all simply squared down from Fig. 1, and it is hoped will be seen and thoroughly understood, from the instructions laid down on geometrical projection in Chapter IV., and, from time to time, subsequently enforced during the progress of this work.

The horizontal projection of the cylinder and trunks afford a good application of the instructions given on oblique projection in Chapter XIII. All the sizes in the projection of these cylinders are obtained from Fig. 1, the conjugate axes of all the ellipses being obtained by directly squaring down the extreme points of oblique edges of the circles to the centre line indicating the horizontal projection of the axis of the cylinder, while the transverse axis of the ellipses are transferred from the actual sizes taken at right angles to the axis of the cylinder in Fig. 1, to the centre line bisecting the conjugate axis of the projection of the same circle, at right angles to the horizontal centre line of the cylinders in Fig. 3. The ellipses drawn by any of the methods given (Parts 55 to 59) by arcs or curves gliding tangentially into each other through the four points thus found, will produce the projection of the several circles very simply, as shown in this example of horizontal oblique projection of the cylinders.

The horizontal oblique projection of the air-pumps are projected in a similar way from the back elevation, Fig. 2, and also the hot and cold water pumps adjacent to them, except that these pumps are at so much smaller an angle, that the ellipses are almost circles. The nuts are here shown in the covers and stuffing-boxes; but, in the cylinders and steam-casing, it is only the bolt-holes which are shown to indicate their position and number, and still save the labour of showing so many small nuts in oblique projection, this practice being not uncommon with draughtsmen in such circumstances.

In regard to the shading and finishing given to such drawings as this plate represents, we must refer the student back to the rules and instructions given on this subject in Chapters IX. and X. with reference to Plates 26 to 32, and to this and our other example plates, which are executed with great care by first-class draughtsmen and engravers. We may, however, observe, that by having good strong light and shade on all near objects and details, the effect is to bring such parts of the drawing distinctly out in relief, which is of considerable importance in the production of good and effective drawing. As regards harsh cast shadows, it is usual, in complete drawings of machines such as our plate represents, to be very sparing of them, and only to put them in, at such parts as give effect and force to the drawing, without obscuring any of the important or working parts of the machine.

As a general rule, particularly in such drawings as that which our example plates represent, of machinery, where every part is desired to be distinctly seen and understood, the best general effect is produced by strong shading with good bold lights, and effectively softened shadows, where they do not obscure other objects, while it leaves every part of the machine clear and distinct.

THE DELINEATION OF THESE ENGINES WITH A VIEW TO THEIR DESIGN AND CONSTRUCTION.

Amongst the chief duties of the practical draughtsman, is to be able to design and draw original machinery speedily, and correctly. This can only be acquired by long experience and careful attention to every detail, so that no confusion may afterwards arise, during the execution of the work he has drawn. A good and quick draughtsman, may be able to form in his mind, an idea of the principal parts of the machine he is about to draw, but the minor parts can only be arranged whilst his drawing is in progress. It is therefore necessary that a complete drawing of the machine, should show every important detail in its proper place, and duly proportioned as to strength and *clearance* in every position of its motion.

When an engineer agrees to construct a pair of marine engines for a certain vessel, he must be supplied with drawings, or tracings of the hull, showing the longitudinal and midship sections. From these, the practical draughtsman arranges and designs the machinery to correspond in every particular, and in many cases also to agree with a previously prepared specification, giving the chief dimensions, such as the diameter of cylinder, the length of the stroke, the area of the steam-ports, &c.; but, when the practical draughtsman has no specification, it becomes his duty to calculate the chief sizes necessary to produce a certain power, for a given tonnage, before proceeding to pencil in the leading lines of his drawing.

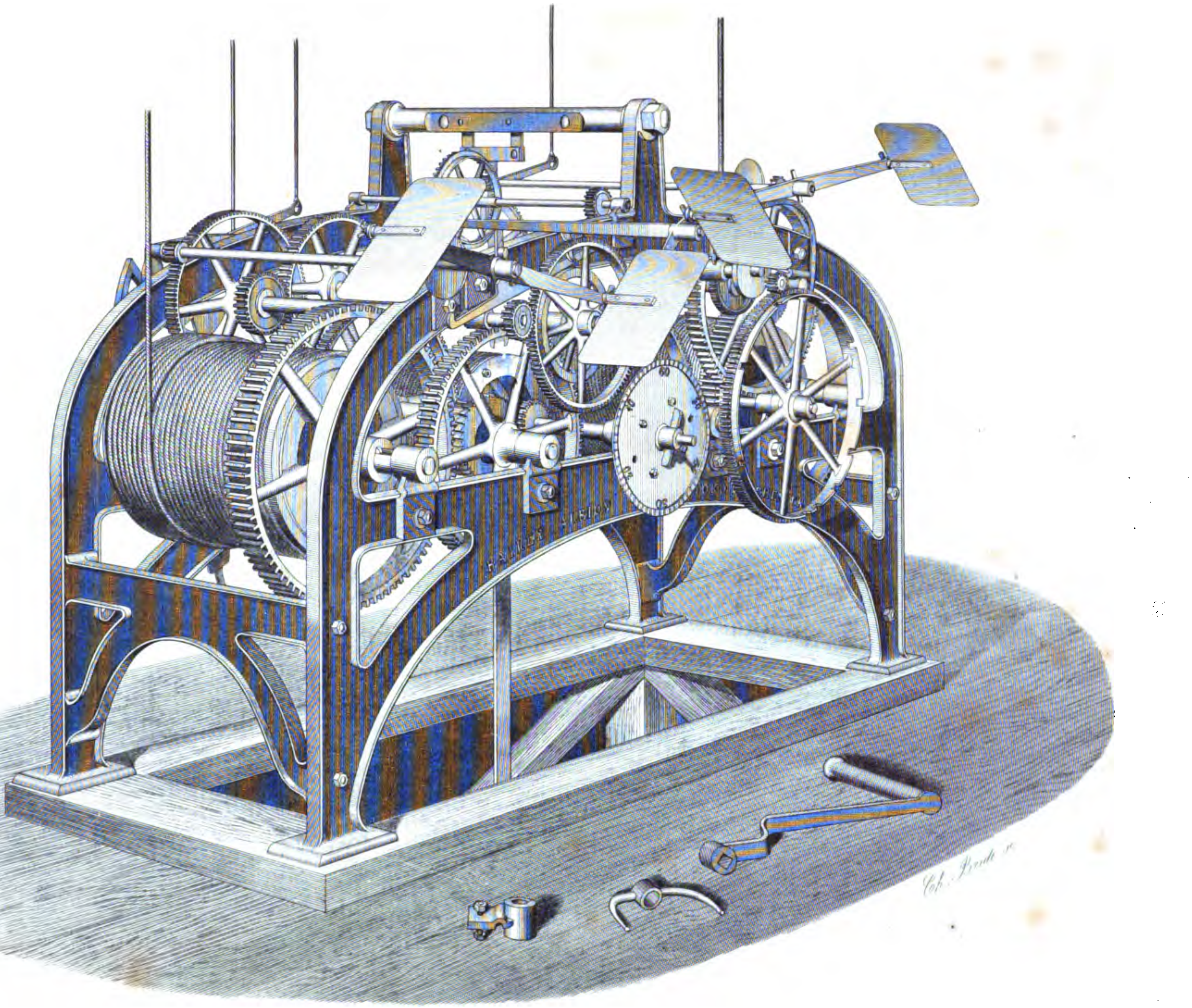
In the example plate, we have given a first-class specimen of a pair of "geared marine trunk engines" for driving a screw propeller. These engines have cylinders 50 inches diameter, the length of stroke 42 inches; the diameter of the trunks are 31 inches, and consequently the mean effective area of the piston is reduced to 1586 square inches. Supposing we have now to design a similar pair of engines, our first steps would be to ascertain the dimensions of the hull, which can be got from a cross section through the engine space. From the floors of the vessel to the screw shaft centre, deduct the height of plates necessary to form a steady, suitable engine-seat, and commence the drawing with the seat line for a base, upon which the whole design is to be erected. Upon this base line erect a perpendicular line; for the centre of the screw-shaft, crank-shaft, and hull, must in this case all coincide. Upon the perpendicular line drawn, we have now to calculate and mark the height of the crank-shaft, in accordance with the scale we employ. Between the crank-shaft and the seat, we find that cylinders having a stroke of 42 inches cannot be drawn, without introducing the idea of making a trunk engine; and even then, the cylinders must be inclined. We therefore consider it advisable to try the cylinders at an angle of 45° with each other, so that one crank would serve to connect both steam cylinders. Draw the hypothenuse of a right angled triangle, from the crank-shaft centre to the base line, on each side of the perpendicular line, and from the crank-shaft centre describe the circle of the crank-pin's path, which is, with a radius of 21 inches, measured on the scale in use for this particular drawing. It would be impossible to give any rule for the length of the connecting-rod, as every maker knows how often every rule of this kind must be departed from, to suit circumstances of greater importance; but a connecting-rod, of the length given in our plate, can be conveniently arranged by adopting the trunk, which should be made of a diameter capable of clearing the connecting-rod when at the point of greatest oscillation, and no more. The position of the cylinders on each side of the centre, should now be measured off and penciled in. One cylinder need only be drawn in section, as both are in this case alike; however, it is absolutely necessary that the sectioned cylinder should be drawn with the piston at about half stroke, when the greatest angle of the connecting-rod, and the diameter of the trunk by measurement, can be accurately ascertained. At a tangent to the crank-pin circle, draw the centre line of the connecting-rod until it meets the hypothenuse or centre line of the steam cylinder in section, which, in this instance, is on the port side. On each side of the connecting-rod centre line, draw two parallel lines, measured to correspond with the rod's greatest diameter. Two parallel lines, showing the greatest depth of the piston, should now be drawn across the whole diameter of the cylinder, at a sufficient distance from the oscillating centre of the connecting-rod, which distance, is merely to give room for the requisite strength around the oscillating centre of the rod, and its respective pillow-block. From the upper side of the piston, and on the cylinder centre line, measure off the whole length of the stroke, and add to this measurement the depth of the stuffing-box when the gland is in its highest position, also say one inch for clearance. These together, is the proper length of the trunk, which line may now be drawn to intersect the cylinder centre line at that point, and extended through the connecting-rod lines. Where these lines intersect each other, at the lower side of the rod, draw a line parallel to the cylinder centre line, from the upper side of the piston to the end of the trunk, and this line gives the correct diameter of the trunk. When the starboard cylinder is represented at full stroke, the corresponding connecting-rod centre is in a direct line with the crank-shaft; but the port cylinder is not then exactly at half stroke, because its connecting-rod is then at an angle, and consequently shortened; so that the distance between the piston and crank-pin is variable, and least when the angle of the connecting-rod is greatest. A radius drawn from the oscillating centre of the rod, through the centre of the crank-shaft to the crank-pin, gives the difference due to the angular position of the rod. We may now measure the distance between the crank and screw-shafts, and calculate what diameter of wheel and pinion will be required. To give two revolutions of the screw-shaft for one of the engine, these must stand to each other in the relation of 2 to 1; but in this instance the spur-wheel is 9 feet diameter, and the pinion only 3 feet 8 inches diameter, which is nearly 2½ to 1; however, draw the pitch lines in pencil of both the wheel and the pinion, and begin now and design the front part of the foundation, which, in the present arrangement, is made in two, and bolted together in the centre. It is very essential that any of the patterns, or loam work required, should be of the simplest and strongest form, as these are the most expensive details to correct, during the progress of the work; and should any oversight appear, the practical draughtsman is always considered accountable, as also for any unnecessary expense or extra weight, where it is not required, especially in marine engine work. We must therefore design the foundation in accordance with our practice; and although no two engineers might produce the same shape, still a casting of nearly the same weight and strength would be required for a similar pair of engines. The front part of the foundation frames, upon which the cylinders are bolted, is lowest in the centre, and raised gradually to what has been considered a sufficient bearing surface for the proper fixing of the cylinders thereto, and at the centre the screw-shaft can be conveniently drawn on end when required.

Before proceeding further with the elevation, Fig. 1, it becomes a necessity to draw as much of the plan as will enable us to ascertain the position of the entablature, or upper framing, supporting the crank-shaft, and whether the malleable iron columns can be in or out of the centre line of such framing. Immediately underneath Fig. 1 extend the perpendicular centre line sufficiently far so that no part of the plan can have any chance to touch the elevation. At right angles to this line, draw another line for the centre line of the steam cylinders; upon, and where these lines intersect each other, measure off the length of the crank-pin, equally on each side, so that two connecting-rods can have each a proper bearing surface thereon. Then on the perpendicular line set off with the scale the depth of the crank, and length of bearing at that end of the shaft. Through the centre, or middle of the journal measurement, draw a line parallel to the line of cylinder centre, and this line runs through the centre of the two supporting malleable iron columns shown in Fig. 1 of the "example plate." There is nothing, we find, to prevent these columns being drawn in the centre of the entablature, we may therefore turn to our first view, and pencil in the shape of the entablature, with the two supporting columns. The student might ask, Why not have only one column in the centre instead of two? One reason is, that it would stand directly in the line of the screw-shaft, and prevent its being drawn on end, whereas with two, the shaft can be conveniently drawn on end through between the two. Another very important reason is, that when any strain on the entablature would tend to cause side oscillations, the greater the distance between the two columns, the more direct the end strain upon them. There is nothing in the present design to prevent these columns from being placed farther apart than they really are; but no doubt the designer has found, by previous experience, that they are sufficiently far apart to ensure a steady support, when diagonal stays are also used in combination with the vertical columns. We may now turn our attention to the cylinder steam-ports, and slide-valve, which is represented in the drawing to have a certain amount of lap or cover; but, before proceeding further, we refer the reader to the instructions on the designing, and constructing, of steam engines already given, in connection with Plates 38 to 40, as well as Parts (403 to 409) of practical data, on steam and steam engines. The area of the steam-ports should bear a strict relation to the steam cylinder's diameter, the steam pressure, and the amount of cover or lap, necessary to cut off the steam at the proper moment. Different manufacturers of marine steam engines, adopt rules varying from $\frac{1}{18}$th to $\frac{1}{20}$th the area of the steam cylinder, for the area of steam-ports, making the exhaust-port to agree with the cover on the steam side of the valve, which port generally becomes about double the area of the steam induction-ports, but wholly depends upon the cover or lap given to the slide-valve, and this port must be sufficiently open at all times, to allow the steam issuing from the cylinder, a free passage to the condenser. After calculating the area of the steam-ports, the amount of lap or cover to give the required cut off, and the length of the slide-valve's stroke, set off a line through the centre of the cylinder, at right angles to the engine's centre line, and begin to measure off the steam-ports, upon another line drawn parallel to the cylinder, and at the same time sufficiently far apart from the cylinder itself, so as to admit the exhaust-port and connecting-pipe of the proper area. This parallel line is termed the cylinder face, upon which the length and the position of the slide-valve, must be set off and drawn in pencil lines. The depth of the cup in the valve, is generally rather more than the width of the steam-ports, and the length of the valve-cup can only be set off when the valve is in the position of giving lead. If say $\frac{1}{8}$th of an inch is sufficient lead on the steam-side of one port, $\frac{3}{4}$ths of an inch would suffice for the opening on the exhaust side of the other steam-port, and may be marked and drawn as one end of the valve-cup in that position. From the end line now drawn, measure off the lap or cover of the valve, and draw the short line to represent the extremity or leading edge, and afterwards make the other end of the valve exactly similar to the end already found. We may now begin to design the valve-casing, which may be of any dimensions, so that a proper supply of steam, at all positions of the valve, may be regularly maintained. Many practical draughtsmen prefer making a diagram of the position of the steam-ports and slide-valve on a separate piece of paper, and then transfer the measurements therefrom to the chief original drawing, which may now be cleanly and correctly penciled in, and the drawing in no way destroyed by the many measurements required at that particular part.

The port cylinder in section may be now considered designed, and the accompanying cylinder in elevation may be measured and drawn in by measurements transferred from the one to the other. Having so far designed the first view, we must turn again to the plan, which, if the drawing-board and square are correct, may be squared down, and the transverse and conjugate axis of the ellipse of the cylinders drawn correctly, and in accordance with what we have already given in the former Part, whilst treating on the delineation of these engines from sketches. The transverse axis of these ellipses must be drawn and measured off equally on each side of the chief centre line. Every point or projection in the first view or elevation must be squared down and marked on the plan, and those points nearest the eye should always be first drawn, whether in pencil or in ink lines. On the chief centre line, the connecting-rod ends are nearest the eye; and these, if carefully squared down, may be correctly delineated by marking every projection upon that line; then measure off the different thicknesses, and draw the lines parallel and equally distant therefrom. The crank, from its position in the elevation, will be correspondingly foreshortened in the plan; but the practical draughtsman will find no difficulty in delineating such a figure, when properly squared down or measured from the elevation. The entablature is represented as partly broken away, so that the details, immediately underneath, may be more clearly drawn in and defined. One-half of any figure is quite sufficient in an original drawing, if the other half is exactly the same, and in every case should be left out, if more important details can be drawn in its place on the sheet. Upon the vertical line, forming the axis of the crank-shaft in the plan, measure off the space required for the eccentrics, and draw a line parallel to the chief centre line (say two inches clear of the eccentrics), and horizontal so as to represent one side of the cog spur-wheel. From the elevation square down the diameter of this wheel, and measure off the breadth in accordance with your previous calculation, and draw another horizontal line to represent the aft side of the wheel. Clear off the wheel now penciled in, draw the front part of the aft crank-shaft journal, and transfer the measurement from the front journal to the other, and draw in the entablature so as to give perfect clearance of the spur-wheel teeth.

The cylinders, crank-shaft, and spur-wheel gearing, is now arranged, as the plan and elevation agree so far as these details are connected; but we have now to design condensers, and air-pump or pumps. Some draughtsmen might, to economise room, think of putting the condensers between the two cylinders, instead of the

TURRET CLOCK.



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foundation already pencilled in; but, on noticing the space unoccupied on each side of the spur-wheel, he may also venture to try the condenser in this space; and on considering the simplicity of the casting, and the accessible way of working the air-pumps, by an overhung crank with its necessary connections, he at once pencils part of this arrangement, and cautiously looks over his design as a whole. One air-pump for both cylinders is not uncommon; but, should an accident occur to that pump or its connecting details, where is the remedy? Another pump or spare gear, we reply. Then why not construct two pumps, so that an accident, happening to any part or detail of one engine, the other may be still capable of working into some port?—slow, but perhaps sure. He therefore considers it advisable to make separate pumps, and distinct separate condensers; so that one, or both in combination, may be wrought *ad libitum.*

We may now design two air-pumps, and for simplicity of construction, &c. we endeavour to get them arranged on the same line athwartships; so that one crank-pin, with two connecting-rods, will serve the reverse purpose of that adopted whilst arranging the steam cylinders. We may next try to arrange them as nearly vertical as circumstances will admit of, and we consider single-acting pumps most advisable. Then how are these pumps to be wrought? If we adopt guides for the true rectilinear motion of these pumps, an extra expense will be incurred; and since the trunk system is adopted in the steam cylinder connections, we may now try the air-pumps on the same principle, and find that a mere fractional addition of the air-pumps' diameter will give an equal area to that lost by the trunk. We then calculate the diameter without any trunk, and try what diameter of trunk will be necessary. On another corner of the sheet, and say in a line with the elevation, begin to draw a view of the engines looking forward. Set the square, and extend the base line to that corner of the sheet where the view, Fig. 2, in our example plate is to be drawn. Notice how much space on the sheet will be necessary for this view, and in the centre of that space erect a perpendicular; upon which square across the centre of the crank-shaft, and around that point as a centre describe a circle equal to the stroke of the air-pumps, and on each side of the circle squared across from the screw-shaft begin to design the pumps, so that they will in no way prevent the removal or alteration during the repairs of any other detail. On each side of the screw-shaft, and from the crank-shaft as a centre, we now construct the acute angle, the base line being one side of that angled triangle, and upon these lines we now begin to set off the position and diameter of the air-pumps in a similar way to that which we recommended in pencilling in the cylinders. The feed and bilge-pumps are arranged on the aft side of each air-pump, and are wrought by a projecting arm or crosshead from the connecting-rods of the air-pumps in a convenient and simple manner, but might be modified, and even greatly improved, by connecting the trunk of the air-pump directly with the plunger of the bilge and feed-pumps, and in this case the plunger might then be solid, instead of hollow trunks, as shown in our example plate. The practical draughtsman will observe that the throttle-valve, injection, and starting gear-handles, are all arranged conveniently in the front of these engines, so that the attendants can readily work either when required. The escape and blow through valves, and several of the minor details, may be greatly modified to suit the circumstances of the case, and if simple and accessible, may be equally efficient to those shown in our "example plate," a lengthened description of which we consider quite unnecessary.

"TURRET CLOCK."

EXAMPLE PLATE, B.

This plate affords an excellent example of the application of true perspective to the correct representation of mechanism. So far as effect is concerned, there is undoubtedly a great preference to be given to true perspective; but, on the other hand, a drawing so made is all but useless to the practical workman, from the fact of there being no means of readily measuring off the *dimensions* of the various parts. The application of true perspective is of great utility, however, where a clear and comprehensive idea of a piece of mechanism is to be imparted without a tedious verbal description. Although the plate before us will no doubt appear at the first glance to be no light undertaking for a young draughtsman, yet to the student who has carefully perused and mastered the instructions contained in our previous pages, and in Chapter XVI., on True Perspective, no insurmountable difficulty will be presented in its delineation. Perhaps the most important gift required to make such a drawing is *patience;* for although the multiplicity of parts renders the figure complicated in appearance, still each part is little more than a repetition of its neighbour; and the draughtsman who can draw a single shaft and spur-wheel in correct perspective will find very little more difficulty in delineating the entire clock. The "horizontal line," which corresponds to the height of the eye, and upon which both vanishing points must of course be fixed, passes in this figure through above the centre of the upper transverse stay connecting the two side standards. The respective positions of the two vanishing points having been decided upon, the rest is comparatively plain sailing, and with *patience* and care in delineating the smaller details, may be completed without much difficulty.

The perspective delineation of mechanical subjects has been greatly facilitated of late years by the introduction of photography. In the subject before us, however, the more minute details would hardly be obtained by photography with that deeree of sharpness and distinctness requisite for the engraver, although the general disposition of the main lines of the figure would be readily fixed by the aid of that process, leaving the small details to be inserted by the hand of the draughtsman.

The clock which forms the subject of our plate was made recently by Messrs. John Bailey & Co. of the Albion Works, Salford, Manchester, for the new church, All Souls, Halifax. The various parts were made by steam-power and self-acting tools from suitable templates, thereby ensuring accuracy and good workmanship, which, through the economy of time and labour effected, the cost of construction of the clock has been proportionately reduced. The clock indicates the hour upon two dials 8 feet 6 inches in diameter. The arbor carriers are attached to the frame by means of bolts, and steel steady pins, in such a manner that any single wheel may be taken out without disturbing the frame. One of these carriers is represented detached from the clock. The hours are struck upon a bell of 25 cwt., and the quarters upon smaller bells. The clock is provided with patent wire ropes, a set dial to set the hands, maintaining power, and a compensating pendulum 156 ·55 inches long, the bob of which weighs 2 cwt. The main or great wheels are 18 inches in diameter; the pendulum springs are very short, in order to enable the point of suspension to be determined with accuracy. In the striking trains, the ordinary pin-wheel for lifting the levers is dispensed with, and a snail cam with an anti-friction roller substituted, thereby effecting a considerable reduction in friction and wear and tear of the moving parts.

"TWIN DREDGER OF 30 HORSES-POWER."

EXAMPLE PLATES C & D.

THE DELINEATION.

We give here two plates, as specimens of extraordinary skill and accuracy in minute—we might almost say, microscopic—delineation of machinery. We would especially draw the attention of our young disciple to the bevel and spur gearing, and to the chain work in plate C. It is only by long practice and perseverance that the student can attain the skill and degree of accuracy requisite for the production of drawings, such as those before us, for the engraver; but let him not, therefore, be disheartened, neither let his patience weary at the slowness with which his work progresses, even when he is fully competent to perform it. Plate D contains some good specimens of projection and bevel-wheel gearing, on a slightly larger scale.

The one plate represents a longitudinal vertical section of the dredger, and the other is a transverse section of the same, taken through the engine-room, and, consequently, showing the engine more clearly than in the first plate.

In laying down these two views, the shell, or hull, of the dredger is, of course, first delineated, after which the principal centre lines are produced, both vertically and horizontally. Plate C is what we may call a compound figure, as it contains a number of separate objects, each complete in itself, and therefore requiring a set of centre lines of its own. For example, we may point out the two "crabs" on the deck, the steam-crabs, and the boilers. The main vertical centre-line in plate C will coincide with the axis of the vertical driving shaft, which transmits the motion of the engine to the bucket-chain, seen at the left hand end of the figure. There is no general horizontal centre-line, but several minor ones will, of course, be required for the lines of shafting and piping. In Plate D three main vertical centre-lines will be required, namely, one bisecting equally the entire figure, and passing through the air-pump of the engines, and the other two, (fixed from the first,) corresponding to the axes of the two vertical shafts for driving the bucket-chain gearing. In this figure we have need of two main horizontal centre-lines, drawn respectively through the upper tumbler-shaft for actuating the bucket-chain and through the main crank-shaft of the engines. The engines, of course, will require their own special set of subordinate centre-lines, the two chief ones of which will be drawn vertically through each cylinder. One of the pair of chains is removed in Plate D, and a portion of the shaft bearings and tumbler shown in section, whilst the sister chain and its tumbler are in complete elevation, the buckets being projected from those in plate C—a vertical centre-line being, of course, first laid down, upon which the buckets are to be projected.

The representations of water and earth in section, are very effective in these plates, but this particular mode of treatment comes more within the scope of the engraver than the drawing-pen of the draughtsman, as these two elements are usually represented by the aid of colours. The true tint for water, in our estimation, is a pale bluish green, and for earth, sepia and burnt sienna; but here a little artistic taste and skill will be requisite to give a pleasing effect.

A few words on the subject of our plates may not be out of place here. This dredging-machine was constructed about six years ago for the Commissioners of Leith Harbour, the designs furnished by the maker being approved of by Mr Rendel, the engineer. It has been a most successful machine, its performances being at the rate of 1840 tons discharged per day of ten hours. The hull is composed of plate iron. It is 90 feet long by 30 feet wide, and 8 feet 6 inches in depth. A central well is made in the hull for the passage of the endless chain of buckets, and for the introduction of the ladders, or bucket frames, upon which the endless chains are supported. These ladders are upheld at one end by dead-eyes, cast in a piece with the pillow-block sole-plate, through which the horizontal tumbler shafts pass, which drive the buckets, whilst their lower ends are slung in chains, which enable the buckets to be elevated or lowered, as desired. The engines are of the "oscillating cylinder" class, each cylinder being 25 inches in diameter. These cylinders are supplied with steam from a pair of boilers, placed one on each side of the central well, near the longitudinal centre of the vessel. We need not refer to the transmission of motion from the engines to the bucket-chains, and to the several crabs, as the drawing pretty fully explains itself.

THE DESIGN AND CONSTRUCTION.

In beginning to design and draw a double dredger, similar to that represented by the engravings we have given as example plates, the approximate weight of the hull, and machinery, and the draught of water required, must first be obtained.

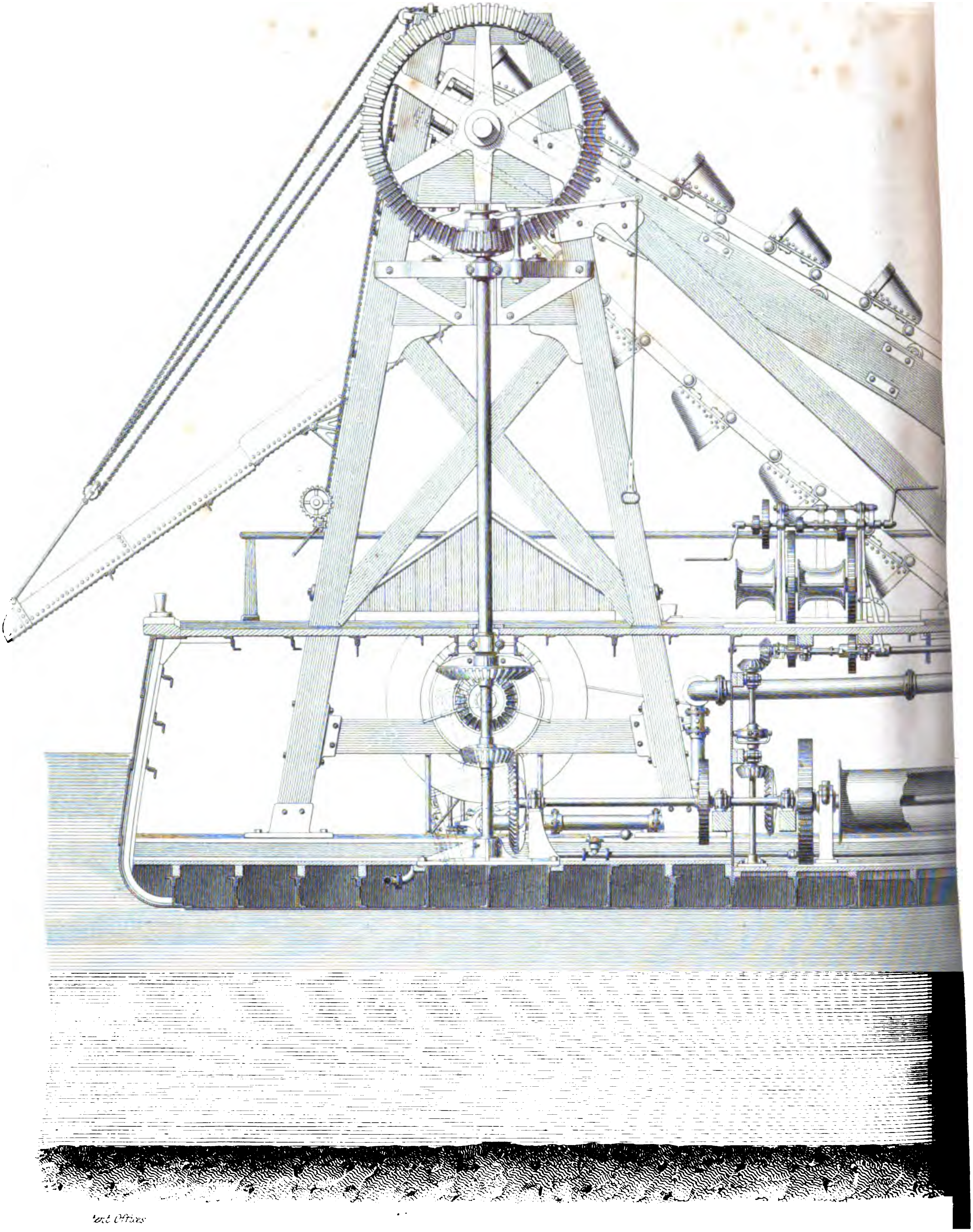
With this data, the length and breadth of the hull, to displace an equal quantity of water, may be readily calculated and noted. These, and many of the most important dimensions, should be fixed before even a line is drawn on the sheet, and most commonly the draughtsman is furnished with a previously prepared specification, wherein the different sizes are fixed and stated.

With such a specification, the sheet may be measured, and the largest scale upon which he can complete the drawing should be adopted; and, in case of error, it should be the only scale beside the draughtsman, as he is liable to be betrayed when his mind is fully engaged with some important detail. On the lower edge of the sheet, a scale may be drawn without any inconvenience, and upon this scale every measurement should be taken either with the compasses, or, with a strip of paper, whereon the exact measurement is marked with pencil before being transferred to the required part in the drawing.

With these precautions, the draughtsman may begin by drawing a horizontal line in pencil for a base, upon which the whole is to be erected. This line should extend slightly beyond the greatest length of the hull. Upon this line set off the exact length from stem to stern of the vessel, and at these points erect two perpendiculars, that is, one at the stem, and one at the stern.

In the centre of the base line, measure and set off the height from deck to top-plating, and square that distance on to the perpendiculars at each end, and mark thereon the midship depth. If the deck is to have, say six inches of sheer, measure and mark that height upon the two end lines, and draw the curve cutting these three points for a deck line.

The weight of the machinery must be equally distributed throughout the whole length of the hull, and as the bucket-frame is the most important detail, its length from centre to centre, and its position in the vessel, must now be decided. On noticing the longitudinal section, Plate C, it will readily be perceived that the machine discharges at one end only, and the nearer the centre of the upper tumbler to that end, the less the height required; we therefore erect perpendiculars for the centre of the supporting frame, which in the present case is made of wood. This should be a strong well-bound frame, to prevent the vibrations consequent upon the tumbling motion of the buckets as they empty.



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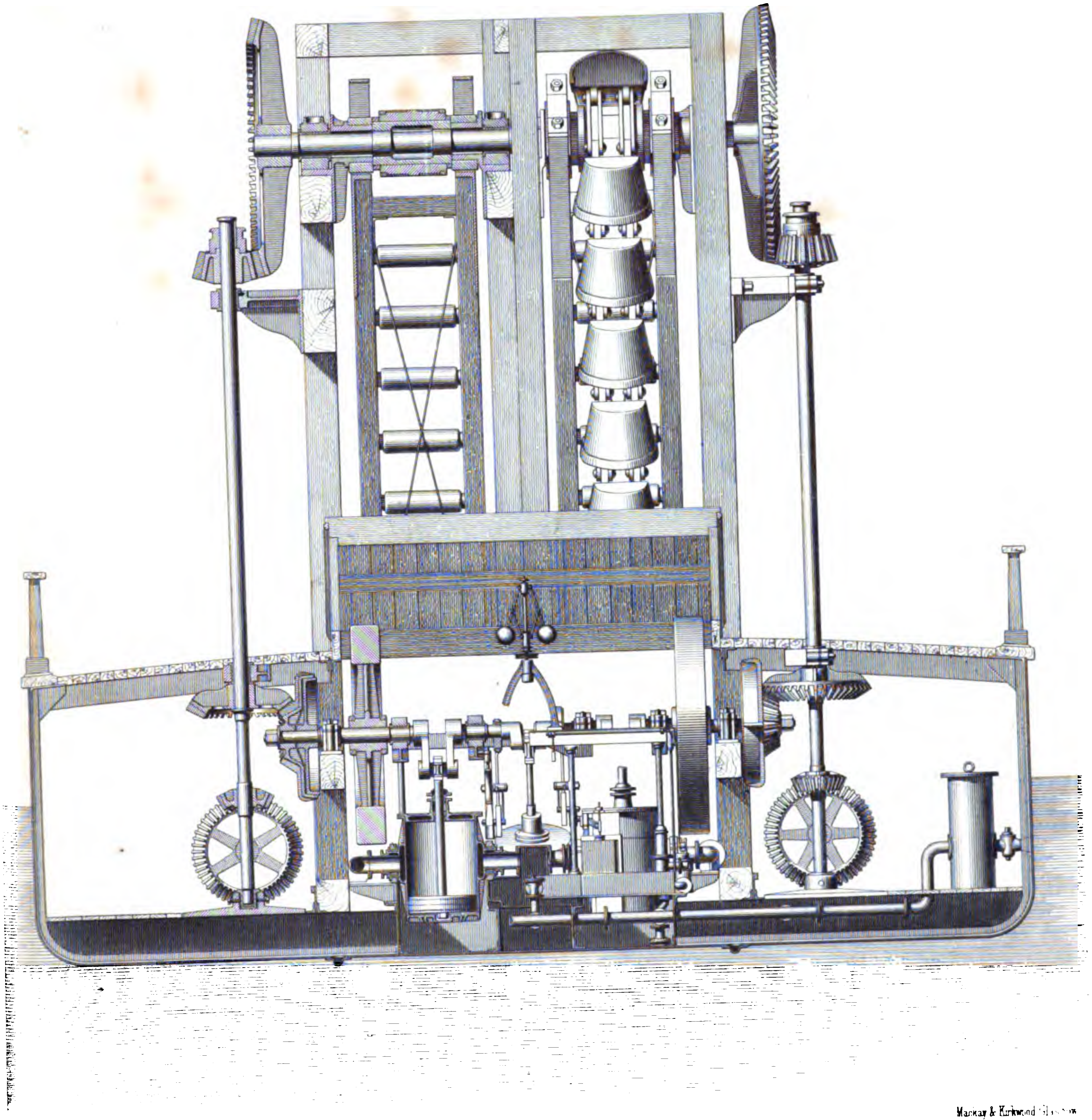
TWIN DREDGER OF 30 HORSES POWER,



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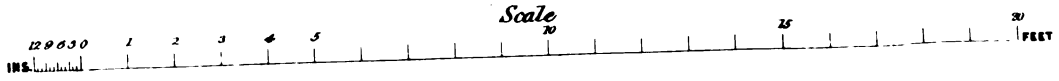


The height of the upper tumbler-shaft, above the water line, depends upon the size and shape of the punts to be used for conveying the dredged material away from the dredger, say that the height is decided to be so many feet, then measure off the exact distance as a centre, and describe a small circle equal to the diameter of the shaft around that centre, and begin to pencil in the supporting frame with the diagonal stays running right down to the keelson.

The length and angle of the bucket-frame may now be fixed to agree with the dredging depth line; and if made of wood, as in the present case, it must be strengthened by a malleable-iron stay, running nearly the whole length of the frame. A small portion of this stay is shown in our example plate, under water, the rest is hid by the diagonal stays of the main framing; but by combining the malleable-iron stay rod with the wood, a strong beam is produced, and the greater the depth of the central support, which, in this case, is made of cast-iron, the greater the strength of the bucket-frame, which, although capable of bearing a greater weight, must not vibrate, or deflect, as a great deflection or vibration would throw the buckets and the bucket chain completely out of their course. Experience, in this instance, as well as many others, goes before any rules we could offer; and if without any extra weight, the strength can by any means be increased, this detail of the dredger cannot be too strong.

The shears for raising the bucket-frame is another important frame, which must be not only strong enough to raise the bucket-frame, when the buckets are full, but when the buckets get a catch of heavy stones, or fixed pieces of wood, although the engine is unable to drive the buckets, the hoisting-gear may be connected, and, by the raising of the bucket-frame, the obstruction may be quickly removed, without damaging the dredger.

These shears must be of a height capable of raising the bucket-frame above deck, so that a bad bucket may be taken off, and a new one replaced. The strength of these two frames, as shown in the plates, are pretty fair specimens of what is required; and the draughtsman, in designing a new dredger, would lose nothing by examining what other engineers have done before him. The frames of the hull are spaced off at about two feet apart, which is a very general distance for this class of vessel, and the floors forming the engine-seat, &c., are the frames from which the others must be spaced off. Immediately under the upright shaft, and in the very centre of the engine, one frame is decided, and the rest must now be spaced off to agree. The height of the floors may be considerably modified, to suit the length and breadth of the vessel, but in no case should they be less than 10 or 12 inches, running up the bilges also to strengthen that part of the hull.

The boiler, or boilers, is the next most important detail we have to consider; and as the greatest weight is towards the stern, we must endeavour to put the boiler and coal-bunkers as far forward as practicable, to neutralise or distribute the weight throughout the vessel, so that the draught of water fore and aft will be nearly equal.

On examining our example Plate 𝔇, which is an end-section, through the engine-room, it will be seen that this dredger is double, that is, having two bucket-frames, and two row of buckets, working in one well in the centre of the vessel. The gearing for communicating the motion from the engine to the upper tumbler, consists of an upright shaft, driven by a bevel pinion, with a friction-nave, one keyed on each end of the engine crank-shaft. The pitch and strength of the teeth, to overcome the vibratory motion of the tumbler-shaft, must be somewhat greater than that given in many books as a rule, per horse-power of the engine. On the upper end of the upright shafts, a loose pinion gears into the bevel-wheel keyed on the end of the tumbler-shaft. This loose pinion is connected and disconnected by means of a sliding coupling, or clutch, which can be readily thrown out of gear by the attendant on deck, when anything serious occurs. Should the attendant be out of the way when the buckets are firmly held by any substance, such as a large stone, the friction-nave in the bevel pinion on the end of the engine-shaft slips round, so long as the buckets are held; but, although the liability to a break-down is prevented by this means, still the attendant's first duty is to disconnect the clutch-coupling on the upper end of the upright shaft, so that the friction-nave may not be destroyed.

The engines are of the oscillating cylinder class. The port cylinder in Plate 𝔇 is shown in section, and, as a general rule with ordinary sized buckets, averages from 24 to 26 inches diameter. As the length of the stroke decides the number of revolutions the engine will make per minute, this must be reduced, so that the upper tumbler-shaft shall make about six revolutions per minute. This is easily accomplished, by simply calculating the relation the one bears to the other, and then by arranging the wheels and pinions in the same proportion.

On the lower end of the upright shaft the hoisting-barrel is conveniently driven by a horizontal shaft, with a spur-pinion on the fore end, gearing into a spur-wheel, keyed on the hoisting-barrel shaft, so proportioned that the barrel must average about two revolutions per minute. The barrel, in this instance, is loose on the shaft, with a friction-break cast on one end of it, as also a friction coupling-nave, for connecting or disconnecting the barrel at pleasure, to meet the requirements of the bucket-frame. The connections for working the hoisting gear must all be on deck, so that the frame attendant can readily raise or lower the buckets, to suit the ebbing or flowing of the tide, as also the proper dredging depth, so that he may, if possible, get full buckets, and no more.

The hoisting-gear may be greatly modified. For instance, instead of a friction clutch for connecting the barrel, a common four-pronged clutch is very general, with a friction-nave in any of the driving wheels. Both methods serve nearly the same purpose, and the designer may choose either, in accordance with his judgment. We prefer the friction-nave, with the four-pronged clutch, as being most durable, and efficient in every way. We may now space off the position of the frames of the vessel, beginning at the centre of the engine, and running fore and aft therefrom; and, as the deck-beams always agree, they may also be squared up and marked in pencil; the position of the hatches, and the proper position of the crabs, can then be decided. The bow crab, in this instance, is set exactly over two deck beams, the main spur-wheel working through the deck between the two beams, and gearing into a pinion keyed on the cross shaft immediately underneath. This cross shaft, through the intervention of bevel wheels and pinions, is driven, by a lying shaft underneath the deck, the aft-end being in gear with a mitre-wheel, on a short upright shaft in connection, and receiving motion from a bevel-wheel on the end of the hoisting-barrel shaft. Between the bow crab and the engine, it is necessary that there should be a friction-coupling, arranged so as to slip when too much strain is exerted on the bow chain. This coupling is placed on the short upright shaft near the hoisting-barrel, but would be equally efficient on any part between the bow crab and the driver. In the arrangement shown, one friction-coupling serves for both bow and stern crabs, as the motion for driving the stern crab is taken off the lying shaft, beyond the friction coupling; but for head dredging it is not necessary that the stern crab should have a friction-coupling. The pitch of the bucket-links is generally about 22 inches from centre to centre, and the sides of the tumbler, whether square or pentagon, are about $\frac{1}{8}$ or $\frac{3}{16}$ inches less, minus the depth of the link. The upper tumbler is generally made square, and the lower tumbler pentagon, or five-sided. The bucket-chain has a certain amount of slack, which is regulated by means of the screws at the upper end of the bucket-frame; but, in many cases, filling-up blocks of different thicknesses are used. These are taken out, or put in, and keyed up to the pitch most suitable for the kind of ground to be dredged, and in many cases is preferable to the screws, which, when they remain perhaps for months in one position, they become rusted and so fixed as to be quite useless. The dead eyes, which hang the upper end of the bucket-frame, are generally cast in a piece with the bed-plate of the upper tumbler pillow-blocks, and firmly bolted to the sides and top of the main framing. The dead eyes are made in two halves to admit of the tumbler-shaft's removal, and these along with the pillow-blocks should be angled nearly equal to the angle of the bucket-frame at the mean dredging depth. This admits of the tumbler being freely taken out, and put into its place without drawing the bucket-frame on end; and we might add, that it is the first duty of a designer to study how any detail of a machine can be most easily removed or repaired. The wood work, although not of the greatest importance, may also be of some service to the designer, such as stanchions, flooring, and deck. The stanchions are generally made of oak, and extend three or four feet below the deck firmly bolted to the ship's side. Holes are cut in the stringer-plates at the required pitch, and the stanchions are bound by a gravat of angle iron, where they pass through the stringer-plate. On the top of the stringer-plate, the water ways are bolted, and these water ways are from 5 to 6 inches thick, curved on the inside to the flush of the deck planks, and rounded on the outside to form a bead all the way round the vessel. These water ways are generally made of teak. The flooring in cabin and store-rooms are 1¼ or 1½ inches thick, fixed to battens bolted on the top of the floor-frames, as shown in our example plate, G. The deck planks are generally made of pine, about 6 inches broad, and from 3 to 3½ inches thick. These are bolted to the deck beams, and caulked and payed in the usual way. The stoke-hole and engine-house floors are made of chequered malleable iron plates, bolted or screwed to the keelsons or floors. The bulkheads in the forepart of the vessel run right across athwartships, and those dividing the boiler and engine spaces are riveted to each side of the well. These bulkheads are generally made of ⅜ths inch iron, with angle irons riveted thereon, to strengthen them. These bulkheads are distinct partitions, caulked all round, so that the bilge-pump pipes must be led through the bulkheads, and secured by flanges and bolts, with suitable cocks arranged so that the engineer can at any time pump from either compartment. The deck pump pipes must also be similarly arranged, so that water in any of these compartments can be readily pumped out. The designer will therefore have to decide the position of these cocks, which, in every case, should be accessible in the engine-room. The cabin fittings in the forepart of this vessel are of the usual class of joiner work, and need not be dwelt upon here. Where any shaft passes through the deck, a gravat, standing up from the deck an inch or two, should encircle every opening, so that any wash of water on deck would, by this means, be carried past the openings, and over the side of the vessel through the scuppers, in the outside water ways. The height of the funnel depends somewhat on the description of boiler, but, generally speaking, is designed more to please the eye than any great advantage towards increasing the draught. The rollers in the bucket-frame, which support the buckets in their upward path, are spaced off, so that the number of rollers equals the number of buckets resting thereon, that is, one for each bucket. These rollers work in chilled cast-iron bushes, with wooden covers, to prevent the mud or sand from entering too freely. These journals average from 1½ to 2 inches diameter. The single links on the bucket chain are steeled, and the coupling-pins are also steeled. Their average diameter is from 2 to 2½ inches for large dredgers, and about 1½ inches in diameter for the smaller class of dredgers. Great care should be taken in fitting and riveting the double links on to the bottom of the buckets, for, although every precaution to prevent their working loose has, from time to time, been tried, still rough ground tells upon all, and, of course, incurs great expense to repair. To this particular detail we recommend the designer to be very careful. To describe many of the smaller, or less important, details would occupy, we think, unnecessary space, still, from what has been said, we doubt not that a pretty fair knowledge may be gained from the engravings and description just given.

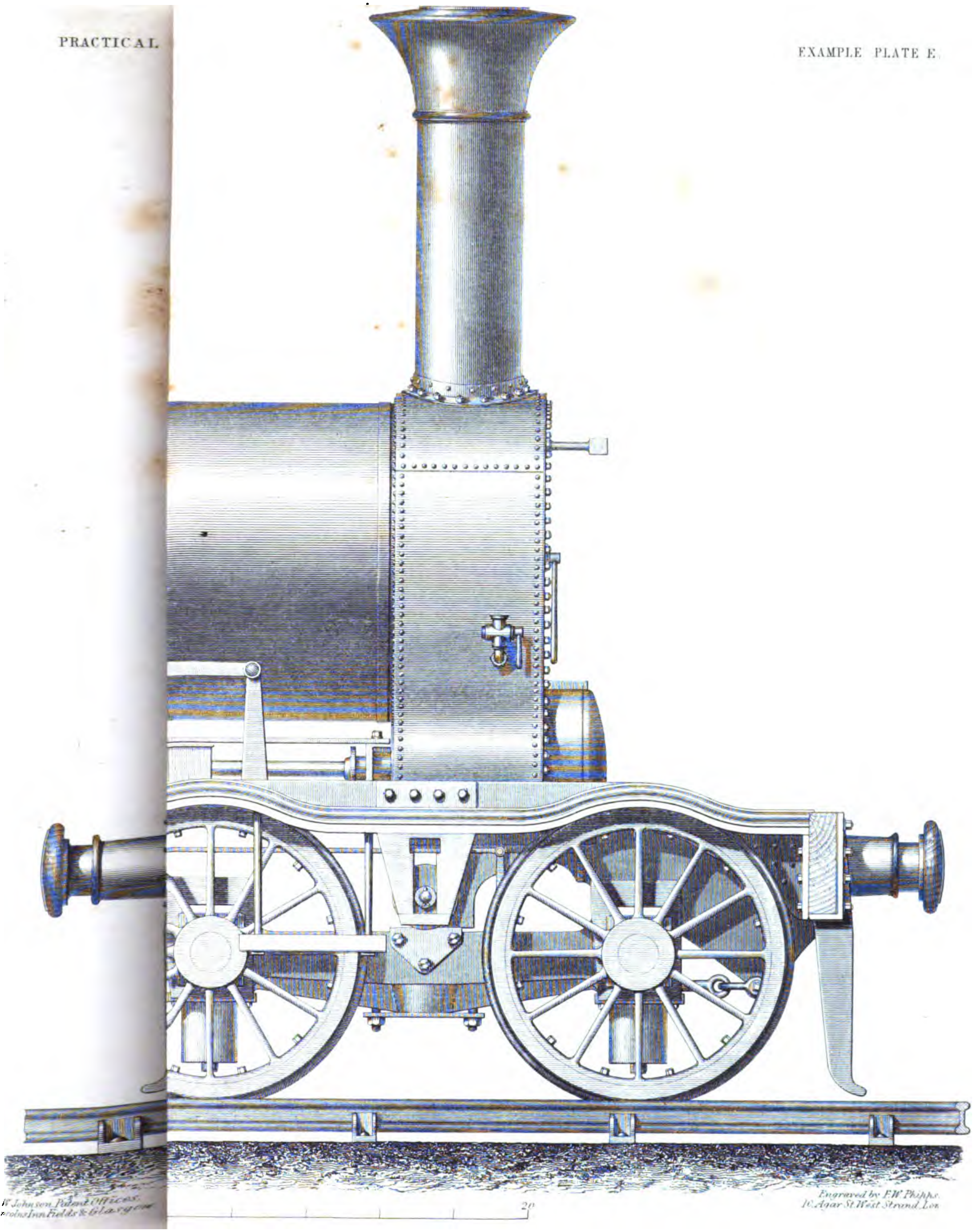
TEN-WHEELED DOUBLE BOGIE TRUNK PASSENGER LOCOMOTIVE (BROAD GAUGE.)

EXAMPLE PLATE, E.

As regards beauty and effect in shading and tinting, this plate can hardly, we think, be excelled. It forms an admirable study also of shadows, which, in this specimen, are well filled in, and largely distributed over the entire subject, giving a bold, and, if we may apply the term to a locomotive, life-like effect. In the laying down of a locomotive drawing, the first points to fix are the centres of the several supporting wheels, by drawing vertical centre lines from the surface of the rail, at the proper distances apart, and then measuring from the surface of the rail along each centre line the heights of the centres of the wheels. In this engine, the smaller wheels are all one size, and therefore one vertical measurement will answer for all, another measurement being taken for the centre of the large driving-wheels. The horizontal centre line of the barrel of the boiler may be next produced, measuring from the surface of the rail, after which the outside framing and details may be filled in, those parts which are most prominent or nearest the eye, being drawn first; and this remark applies generally to the delineation of all mechanical subjects.

This locomotive is a specimen of one of our modern and most improved engines. It was built by Messrs Rothwell & Co., Union Foundry, Bolton-Le-Moors, for the Bristol and Exeter Railway Co., under the superintendence of James Pearson, Esq., Bristol, Locomotive Engineer of the line. The adaptation of the American "bogie" carriage at each end of the frame enables the engine to pass easily and safely over the sharpest curves. These "bogies," or swivel-frames, are made entirely of wrought-iron, each frame being in one solid piece, weighing 29 cwt. when in its rough forged condition. Each bogie has, of course, its own set of four running wheels, 4 feet in diameter, the entire series of 8 being of the same size and construction. The central driving-wheels are of the extraordinary diameter of 9 feet! so that even when the engine is travelling at a

EXAMPLE PLATE E.



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Engraved by F.W. Phipps.
10 Agar St West Strand Lon.

CHECK LOOM,

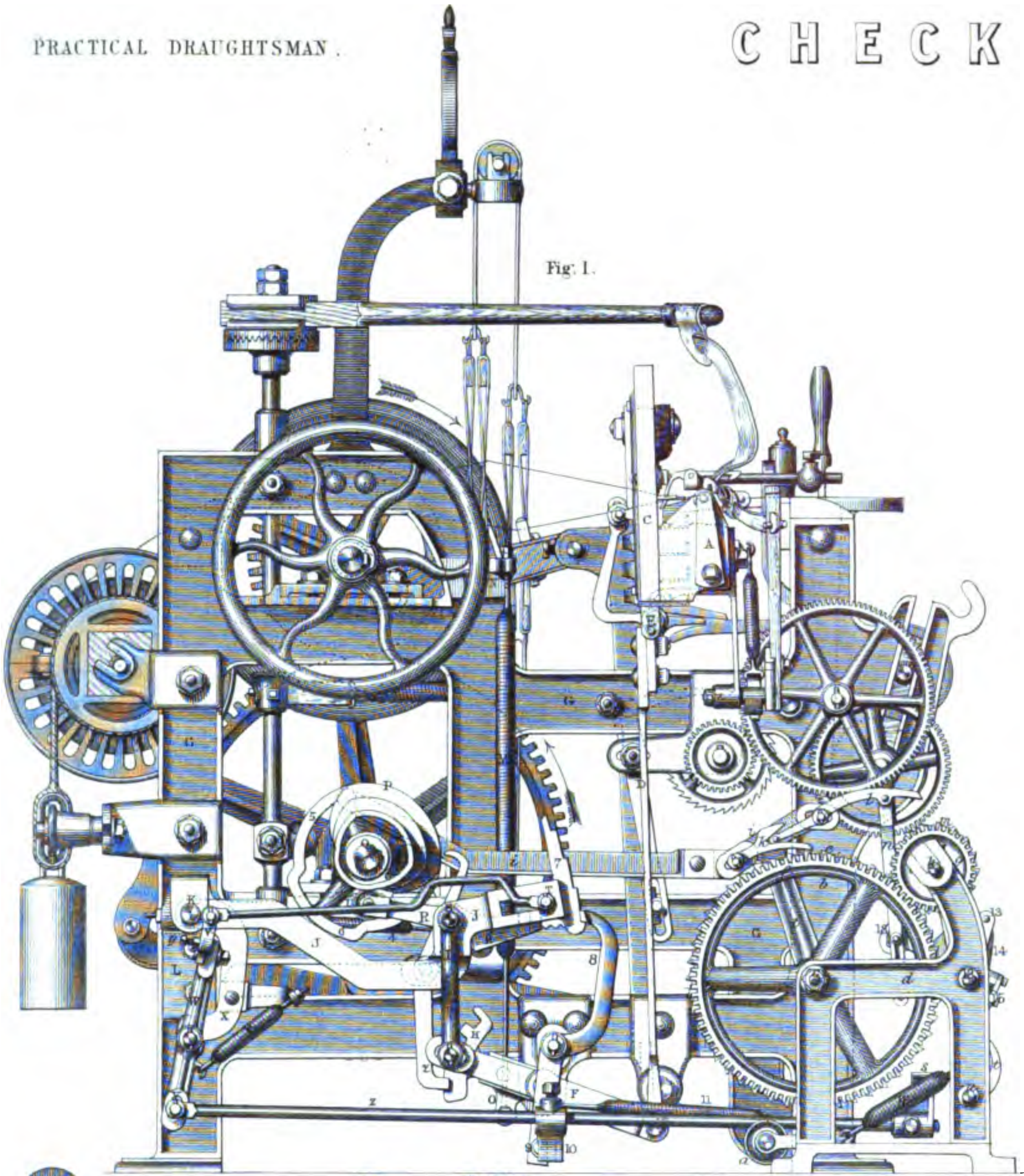
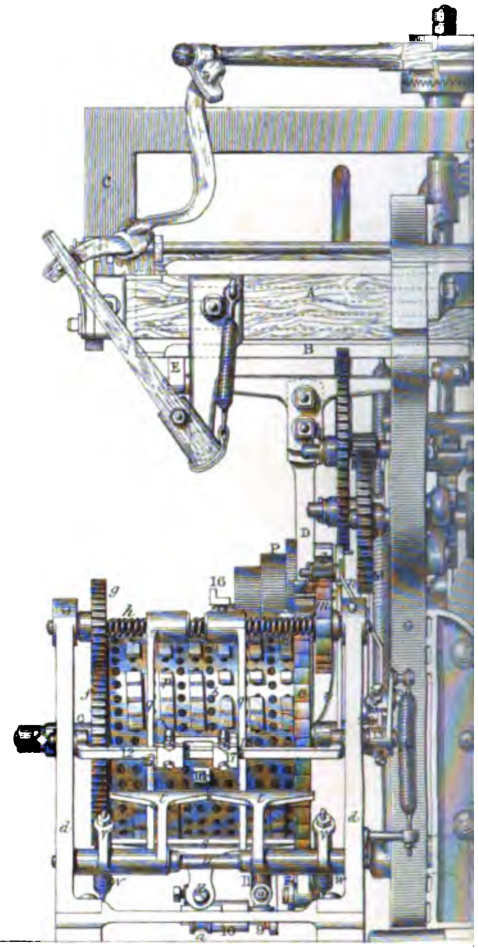


Fig. 1.



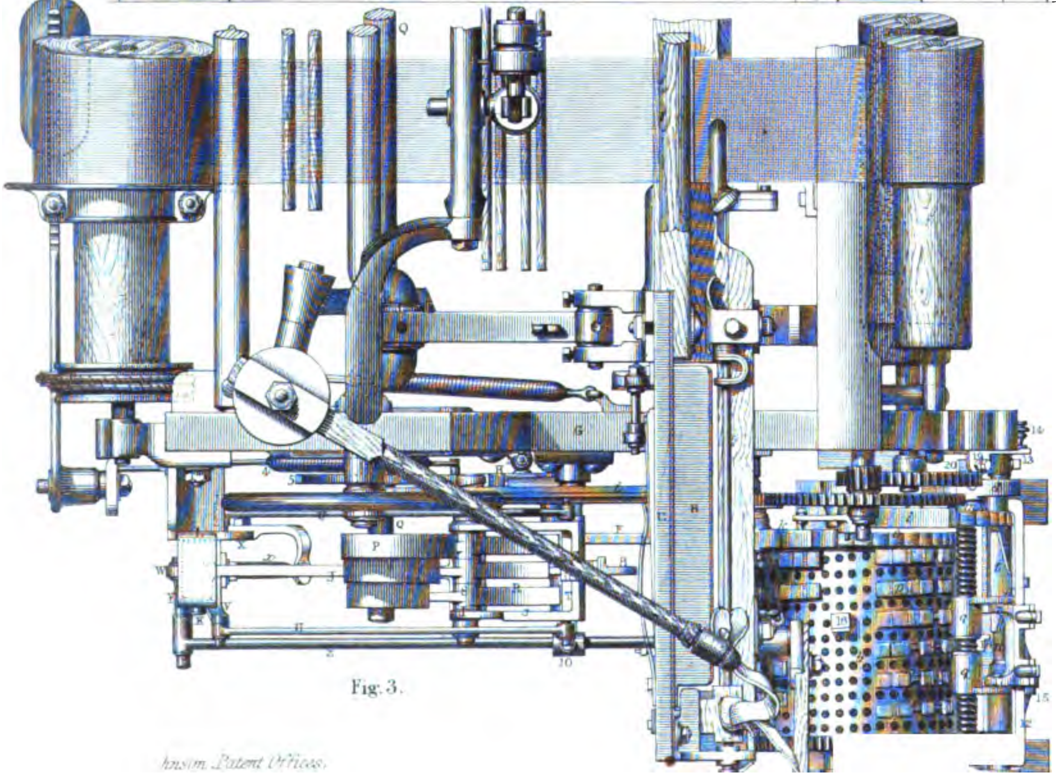
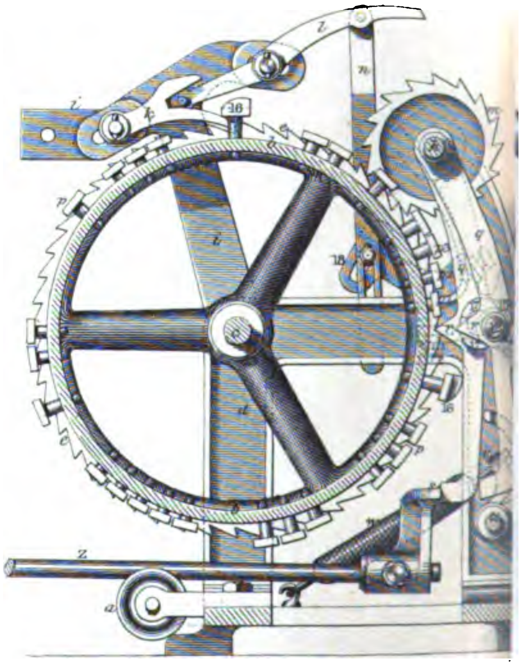


Fig. 3.

Fig. 4.



Johnson Patent Offices.

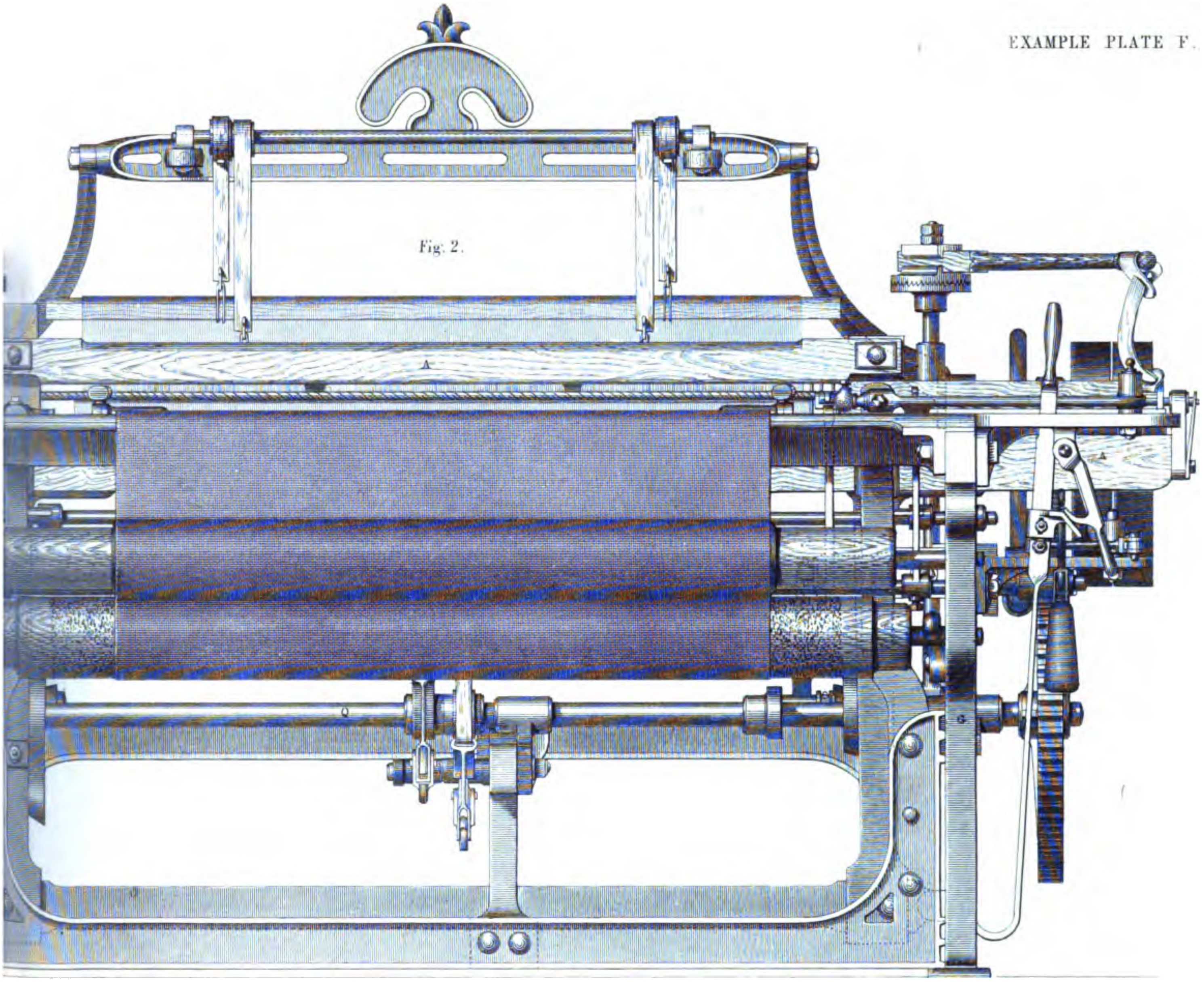
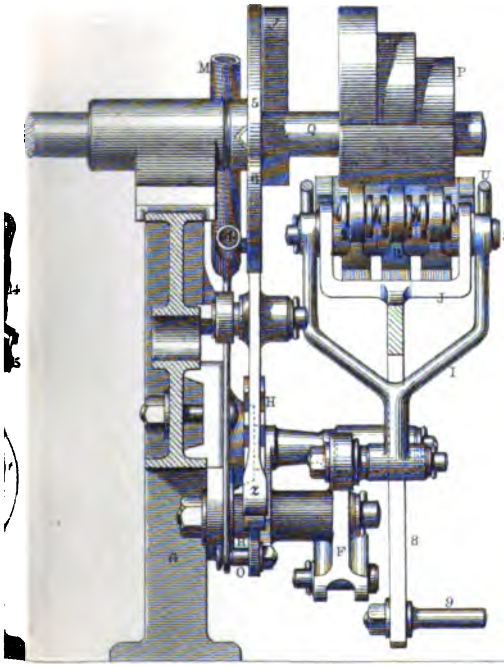


Fig. 2.

Fig. 5.



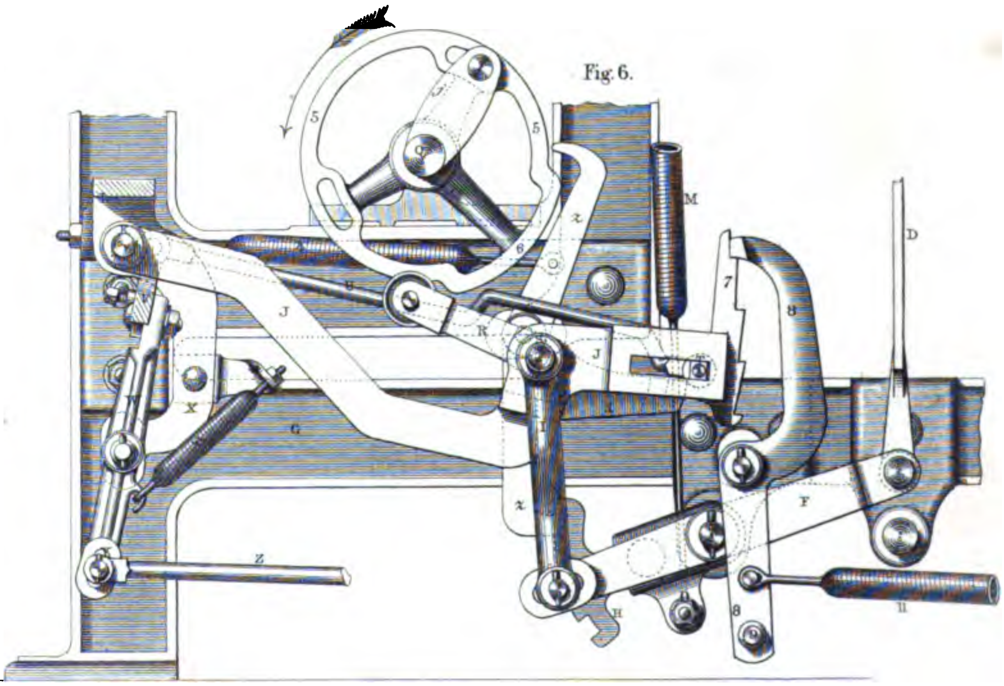


Fig. 6.

SCALE FOR FIG. 1.2.& 3.

very high speed, the pistons and working connections move only at a very moderate rate, thus greatly reducing the chances of derangement usually due to rapid reciprocatory movements of heavy mechanical details. An ordinary-sized driving-wheel, say of 6 feet diameter, draws its train forward 18·84 feet at each revolution, the pistons having to make no fewer than 280 double strokes for each mile run. But with Messrs Rothwell's 9 feet wheel, each revolution conveys the train 28·27 feet, so that the pistons make only 186 double strokes per mile, even by the train. By this means the wear and tear of the parts is kept down, although the engine has frequently attained a rate of 80 miles an hour!—the freedom from oscillation at that speed being somewhat remarkable. The boiler is 10 feet 9 inches long, with an internal diameter of 4 feet and ½ an inch; it contains 180 brass tubes of an internal diameter of $1\frac{1}{16}$ inches. The cylinders were 16½ inches diameter, with a stroke of 24 inches, and the working steam pressure is about 130 lbs.

There are two underneath tanks in addition to the tender tank of the ordinary description by which the water supply is taken in. One of these tanks is placed directly beneath the fire-box, and a second one is bolted by flange pieces to the underside of the boiler barrel between the crank axles and the hind axle of the front bogie. These two suspended tanks are connected together by a stuffing-box jointed pipe, which is carried along so as to open into the bottom of the tender tank, and put all three into communication.

The whole of the springs are of the india-rubber disc kind, those of the driving axle presenting some remarkable peculiarities. They are double, an elastic connection being formed between the boiler and the axle-boxes by large plate brackets projecting from the barrel of the boiler, and carrying centre-studs for a short double-armed lever, each end of this lever having a separate spring-box attached to it by a long link. The inner spring-box works down behind the disc plate of the driving wheel splasher, whilst the outer one, which alone is seen in the engraving, works parallel to it outside the driving wheel. The springs for the other wheels are all beneath their respective axles, and present a very compact and neat appearance. The break action is confined to the after bogie, all four wheels being acted upon simultaneously by the break blocks. The sliding bars, which carry the break blocks, are actuated in reverse directions by a screw spindle, carrying a winch for the driver's hand. The regulator-valve is a slide worked in a simple and very convenient manner by a short lever set on a pillar stud on the front of the fire-box, and passing through a slot in the end of the slide spindle. This is a far more effective plan of working the regulator than the ordinary rotatory handle. A somewhat similar arrangement is applied to the alarm whistle in lieu of the old form of steam-cock. The bogie movement is greatly aided in its snake-like powers of sinuosity, when passing severe curves, by the absence of tire flanges on the driving wheels. The bogie frames can thus pass round quick bends without causing any lateral frictional pressure upon the insides of the rails, as would inevitably arise were not the plain tires of the wheels free to work laterally over the rails. The weight of the engine, complete with coke and water, is 42 tons.

CHECK-LOOM.

EXAMPLE PLATE, F.

This plate is given as an illustration and example of the way to treat and draw a piece of textile machinery, both as a whole machine and in detail, in a practical draughtsman-like manner, so as to show the perfection of his art in producing an accurate and pleasing picture of a complicated machine. We also give this plate as a good example and study in the drawing of minute objects, as it embodies the application of a great many of the rules and instructions given throughout this work in the combination and delineation of some of the very objects taken for illustration in the course of such instructions, drawn with the greatest care and accuracy, to a very small scale. We have further been induced to select this plate, as one of our concluding series, for the education of the practical draughtsman in industrial design, not only for the example it presents in clear and artistic delineation, but also as a complete study in the combination and application of so many mechanical movements, all beautifully arranged and designed for the obtainment of specific purposes and motions, at particular times, in relation to each other, all of which are accomplished with the utmost accuracy. The power-loom is not the invention of a day, but the growth of the engineering skill of half a century; and to attempt to describe, as we have done in the case of some of our first example plates, or show how to draw or design this complex machine, would be quite impossible within the prescribed limits of this work. We must therefore be content with a few general remarks on the delineation, and in pointing out the application of some of the main subjects which we have from time to time had occasion to discuss. In consideration of the importance of the cultivation of the mind of the student in the arrangement and design of mechanical movements for specific ends, which is the great aim of this work, we shall give a description of the new mechanical arrangements of this loom to which we have alluded, and to illustrate which these drawings were specially got up; although, even to do this, occupies much more space than we could have wished. As is well known, there are three principal and essential movements necessary in every loom for the production of cloth, namely, shedding, picking, and felling, or beating up of the weft which has been picked or shot through the shed of the warp. The successful working of the power-loom at high speeds has rendered a great many other automatic and intermittent movements also necessary, such as the take up motion of the cloth when woven, and in some instances the warp before being woven is also given off by a positive motion as required. Along with several movements for stopping the loom when anything goes wrong with the picking motion, or proper action of the shuttle, when the thread goes done, or breaks; also in some cases when a warp-thread breaks, or the shed is not properly formed, the loom is also made to stop itself, and thus prevent the production of imperfect cloth. Arrangements are also introduced in some looms for throwing the lathe out of action, as also that of the reed out of its place, when anything interferes with the lathe, or beat-up action of the cloth, as well as to regulate its force and action at the fell, or part where the cloth is formed, so as to make close and open stripes of different density across. As regards the three prime and essential motions, and the modes of obtaining them, in the power-loom by the different makers, and for the production of the various kinds of fabric, or cloth to be produced, they are so various that we cannot pretend to give even a recital of them here. The loom which we have illustrated in the several views of our example Plate, F, is from the factory of one of our best makers, and is a fair type of all that appertains to the general arrangements of the power-loom, and shows to what perfection mechanical engineers have brought this class of machines, whilst it also embodies some of the most recent improvements of Messrs Muir & M'Ilwham, of the Anderston Foundry, Glasgow, for working the "drop-shuttle box" of stripe, check, and other pattern looms of this class. When a draughtsman is about to commence such a drawing as that before us, he first considers what views will best show the improvements or important parts of the machine he is desirous of most fully and clearly illustrating. Now, as the improvements for which this drawing was got up referred to none of the movements to which we have as yet alluded, but to a special class of loom for weaving plain cloth, striped with various colours, it was not necessary to draw various complete elevations, plans, and sections, to show the well-known plain shedding, picking, and beating-up actions. But as the mechanism to be shown, in this case, all refers to the action of a change or drop-shuttle box at one end of the lathe, it was only necessary for the combined or general views, to show an end and front elevation as well as a partial plan of that end of the loom where the improvements were arranged. These general views show the relation, connection, and action of the new parts to the whole machine, but as the new parts are necessarily too small to be properly seen in their minutiæ and action, enlarged detached views of them are drawn to much larger scales, so as to make them more intelligible.

Fig. 1 being the complete side or end elevation of the loom, in working order, looking on the new pattern-working mechanism; Fig 2 is a complete front elevation, corresponding to Fig. 1; and Fig. 3 is a plan of the end of the loom, corresponding to Fig. 1, and the left hand end, as seen in Fig. 2. Figs. 4, 5, and 6 are enlarged detached views of the pattern-working mechanism, Fig. 4 being a transverse section through the pattern-barrel, showing the action of the pattern-pins, *p*, on the levers, *g*, and the action of the palls, *k* and *l*, for turning the barrel in reverse directions. Fig. 5 is a sectional view, corresponding to Fig. 6, as taken at right angles to that view just behind the cam, P, through the frame, G, and lever, J, and looking from the left in Fig. 6, showing the action of the cam, P, on the anti-friction rollers of the catch-levers, R. Fig. 6 is a side view, corresponding to Fig. 1, as looking on the end of the second motion, or tappet-shaft, Q, but having the cam, P, removed, as well as part of the bracket, L, to show the attachment and working of the levers, Z and W, respectively.

As is usual in such cases as this, it is only the new parts of the machine, and a few of those of the old, which are necessary to the intelligible description of the improvements which are lettered and figured, or numbered, and with this in view all the letters and numbers are marked on the several objects to which they refer in all the views, each letter indicating the same object in every figure.

The first view that the draughtsman proceeds to lay down is the end elevation, Fig. 1, by erecting or pencilling in the side frame, G, of the loom, upon the base, or floor line, as this frame is seldom altered for a great variety of looms. This done, he now defines the position of his two main, or crank and tappet-shafts, pencilling in the pitch-lines of their connecting wheels to see that he is correct. These are usually made two to one. The next step would be to put in the breast and back-beam, or rail, and draw a line across them, which would indicate the line or chain of warp, which must be so high above the crank-shaft that the warp-threads, when at their widest shed, will have plenty of clearance. He may now pencil in the centres of his feeding and cloth beams in front, and the diameters and positions of all the teeth-wheels and ratchet-wheels for the take-up motion, so that none of the new parts may interfere with them, but may be designed and arranged to be quite clear of all that is necessary to the working of an ordinary loom.

The next thing is to pencil in the lathe, A, with the shuttle-race, and bottom of the top shuttle-box, in a line with it at such a distance below the line of cloth that it will allow the reed to beat up the weft, and at the same time, when it is drawn back to the position for the shuttle to be thrown, that the lower threads of the warp —which should, by this time, be quite full shed, as shown in Fig. 1—will rest on the race, and allow the shuttle to glide over them without injury.

Everything in a loom has to be drawn, or set, in proper position with the cranks, so that, at this stage, it now becomes necessary to decide at what position the draughtsman will place it. For various reasons the position indicated on the drawings was chosen, viz., it shows the shed fully formed, the lathe, A, nearly vertical, yet the crank has moved round, as indicated by the arrow, about ⅛th of its circle, taking the lathe back with it after having beat up the first pick of the particular coloured weft, which is in the top division of the drop-box, B; the cams on the tappet-shaft, Q, being set in proper position in relation to the crank, is shown in Figs. 1, 2, and 3, as just in the act of commencing to move the shuttle from the right hand side, or making the second pick, which brings the shuttle home into its top box again, allowing time for the shifting and steady fixing of the levers and drop-box prior to the first pick of a new colour being given. Another advantage of the position chosen is, that it shows the lathe, A, and shuttle-box, B, clear and distinct, away from other objects, and all the catches and levers for working the latter, in a normal position, and not in the act of shifting; the duplex cam, P, being turned in the upward direction, is clearly seen.

The position of the crank, and, consequently, that of most of the other cams, levers, and catches, to produce their required effect at the proper time, during the crank's revolution, must now be pencilled in accordingly. Instead of the length of the connecting-rod, however, in this case being decided by the lever or lathe at half-stroke, as is usual, it is best found by supposing the lathe, A, to be up at the "fel," or the reed in the position of striking the cloth, and the crank at its farthest point forward, or in a direct line towards the centre of attachment to the lathe. The distance from the centres of the crank, and that of the pin in that position, will be the length of the connecting-rod.

There are four shuttle-boxes arranged vertically in the drop-box, B, as close together as the shuttle and driver can be got to work freely. This distance between the heights of the boxes determines or regulates the various dimensions of the shift of the levers and notched catches which work and hold the drop-box at these various heights, as well as the height of the steps of the cam, P, on the end of the tappet-shaft, Q, which raises the shuttle drop-box through the levers, J and F, connected to each other by an open link piece, I, the other end of the lever, F, being jointed directly to the lower end of the oscillating rod, D, of the drop-box.

In drawing such an arrangement of levers as that now before us, the draughtsman proceeds very much in the same way, whether doing it from sketches or designs, so that, in this case, it is unnecessary to do more than go through the construction, to find the centre of attachment, oscillation, as well as the points of contact of the cams, and the position of these levers in relation to the other parts and movements of the loom. As is very common in the movements of machines of this class, the arrangement for working the drop-box is duplex—namely, one mechanism, or a combination of parts, is for producing the physical or direct movement of the shuttle-box, whilst the other, as it were, performs the mental function, and decides, or brings into play, a little catch, T, when the first is to come into action and move the shuttle-box. As it is desirable that the lathe, A, and shuttle-box, B, with its rod, D, should oscillate as near as possible from the same centre, a line drawn parallel to the base-line through the centre of the journal of the rocking-shaft of the lathe, will give the proper centre-line of the lever, F, when in the centre of its motion; on a vertical line through the same centre set of the four points or positions, that is, three divisions, each equal to the height between two of the shuttle-boxes, so that 1½ of that distance above and below the centre-line will give the two extreme positions of the centre of the joint of attachment of the lever, F, with the lower end of the rod, D, proper to the top and lowest shuttle-boxes, as seen in these two positions of the parts in Figs. 1 and 6 respectively.

The draughtsman, at this point, has to consider the best arrangement for working this lever, F, the position of which has been so far decided upon. Cams of the construction described in reference to Figs. 2 and 3, Plate 25, (228,) are most applicable; and as the tappet-shaft is quite convenient, he immediately sees how the whole thing can be conveniently wrought; and, after a little reasoning and some rough trials of sketches on a piece of paper, he decides it will be very good to have a three-stepped cam, as P; and as the steps would be too high to give them the whole throw of the shuttle-box at each time, and knock and shake everything, he draws in the cam (228), as shown in Fig. 1, to come into action, and have the drop shuttle-box change just before the left hand picking-cam, on the tappet-shaft, Q, Fig. 2, comes into action, and while the cranks are turning the front part of their revolution. The levers, R and J, must now be drawn in, but as the short levers, R, are made to come under the different steps of the cam in their respective planes, and fixed by the little catch-bar, T, when the lever, J, is to be moved, we will speak here of the whole as one lever, J. For steady working, the convenience of getting all the attachments applied, and a small radial angular action, this lever, J, ought to be a good length, so that its fulcrum is fixed at K. The extreme back edge of the main frame, G, in a bracket, L, and the little anti-friction pulleys of the levers, R, being drawn in on the vertical line below the cam, P, and touching the concentric centre web, or eye, from which all the steps spring, we draw a line from K through the centre of these pulleys. The centre of the little bar, T, may be fixed at any convenient point in this line, which would leave length for the action of the loose lever, R. Then, with K as a centre and a radius reaching to the centre of the anti-friction pulleys, draw an arc, on which set off three divisions. from the centre of the same pulley downwards. Lines drawn through these points to the centre, K, will give the position of the lever, J, at the three other positions proper for the three lowest shuttle-boxes. In Fig. 1 it is shown in the highest position of its motion, and at Fig. 6 in its lowest. It has been here thought best to connect the lever, J, by the same joint-pin, S, on which the levers, R, oscillate to the lever, F, through the open link, I. A vertical line through the centre of that joint-pin, S, gives the centre-line of the link, I, and it will intersect the horizontal centre-line of the lever, F, in its central position. The distance from that point of intersection to the centre, S, when the lever, J, is also at the centre of its motion, will be the length of the link, I, with which the positions of the front end of the lever, F, can now be found; consequently, having the two extremes and central position of this lever, F, its fulcrum may now be found by drawing its centre-line at any of these extreme inclinations, as shown in Figs. 1 and 6, which would intersect the horizontal centre-line at half stroke in its fulcrum, or centre of oscillation, thus giving all the principal centres and measurements of the first mechanical arrangement referred to. The distance between the notches at 7 and H, for the catches, 8 and Z, for holding the levers, J and F, are found, since the positions and motion of these levers have been obtained by producing the radial or centre lines in their various situations, till they intersect the arcs, drawn at the proper depth of the notches, to suit the motion given to the catch-levers, 8 and Z. The shape and configuration of these levers and catches have to be made to suit the requirements of the locality where they are placed, to clear the adjacent objects; as, for instance, the bend in the lever, J, to clear the cam, P, and give room for the short lever, R, to work below them, also the bend in the catch, 8, to clear the notched segment, 7, when it is in the position shown in Fig. 6.

Space will not admit of going through the drawing and construction of the second arrangement of this duplex mechanism, which consists of a pattern-barrel, *b*, and its levers and catches, as we have done the first; but from the description of the whole arrangement and action, along with what has been said of the first part, the draughtsman will, we hope, form a good idea of how to proceed in delineating this and similar subjects.

The pattern-barrel, *b*, is a good example of the helices, Figs. 1 and 2, Plate 15, (270,)—the holes and pattern-pins around its periphery forming a regular pitched single-threaded right-handed spiral, as well as the fine square-threaded left hand screw, *z*, for carrying the hanging levers, *g*, across the surface of the first. The small-teethed wheels and ratchets, for working the pattern-barrel and cloth-feeding rollers, are all good illustrations of such as have been discussed in reference to Plates 8 and 20, and present no new feature requiring special notice, except that they are drawn to small scales, and afford a finer example of smaller work than anything given in the plates of the body of the work.

For all further understanding of the delineation of this example plate, we must leave the student to perceive the several requirements necessary to be gone through in order to make the different limbs of the machine perform the functions for which they are required, and which will be best understood from the description which we have thought it best to give.

In the particular loom to which this new arrangement of parts has here been applied, the shuttles are changed at one side only of the loom lathe, A; and the shuttle drop-box, B, which is in this instance constructed for four shuttles, is arranged in the ordinary way, to slide up and down in a frame, C, upon the side of the lathe, being shifted by means of the rod, D, jointed through a cross-head at its top end to lugs, E, on the under side of the drop-box. The lower end of the rod, D, is jointed to the front end of a lever, F, which oscillates upon a stud-pin carried in a bracket bolted to the lower cross-bar of the side frame, G, of the loom. The lever, F, is formed with an elongated boss, which has formed upon it, near the framing, a notched segment, H, lying directly or nearly behind the back portion of the lever, F; this segment, H, serving to lock the shuttle drop-box, B, in its various positions. The back end of the lever, F, has jointed to it a forked link, I, the two upwardly directed legs of which are jointed to the opposite sides of an open frame lever, J, arranged to vibrate on a centre, K, in a bracket, L, bolted to the back portion of the side frame, G. The parts being thus connected, it will be obvious that the shuttle drop-box, B, will rise as the frame lever, J, falls, and *vice versa*. A powerful spring, M, is applied so as to give the lever, J, and connections, a tendency to rise whenever they are free to do so, and thus lower the shuttle drop-box; the spring being attached at its upper end to a bracket, N, bolted to the upper part of the side frame, G, and at its lower end to a pin, O, on the arm of the segment, H; this pin being set in such a position, with reference to the centre of motion and the movement of the parts, that the spring acts with proportionately less leverage the more it is stretched, so that its action is rendered more uniform than it otherwise might be. When the shuttle drop-box is to be raised, this movement is effected by the depression of the frame lever, J, such depression being caused by one or other of the three stepped cams, P, formed by preference in one piece, and keyed upon the second motion or tappet shaft, Q, of the loom; this shaft being prolonged laterally beyond the side frame, G, of the loom for the purpose. Each step of the cam is proportioned so as when in effective action to bring a particular cell of the shuttle drop-box on a level with the race. Thus the cam surface of smallest radius brings the second cell from the top up to the race; that of the next larger radius brings the third cell up, and the one of the largest radius the fourth or lowest cell. These several steps of the cam, P, do not act directly upon the frame lever, J; but each one acts upon a pulley carried by a separate lever or catch-bar, R, which swings loosely on a horizontal spindle, S, fixed across the open part of the frame lever, J, and vibrates with the rotation of the cam, without imparting any motion to the frame lever, J, unless locked by the action of the pattern mechanism, so as to be incapable of yielding to the depressing action of the cam, without causing the frame lever, J, to descend also. The locking is effected by means of a transverse horizontal bar, T, which slides in slots in the sides of the frame lever, J, near its front end. The catch levers, R, are made with tails projecting in front of the spindle, S, the tails being of different lengths, to correspond inversely with the cams; the catch-lever acted on by the cam of smallest radius having the longest tail, and the front tails of the others shorter in succession. When, however, the locking bar, T, is at the front end of its slot, it is quite out of the way of the longest-tailed catch-lever, R, and when the cams, P, come round, they depress the back ends of the catch-levers, R, causing their tails to rise, but without moving the frame-lever in any way. But when the locking bar, T, is moved one degree back in its slot, it comes over the tail of the longest catch-lever, R, corresponding to the cam of smallest radius; and on the cams coming round, the tail of this catch-lever being prevented from rising, its back end cannot be depressed without carrying down with it the frame-lever, J. The frame-lever, J, is consequently depressed to the extent corresponding to the cam of smallest radius, and the shuttle drop-box is raised in the manner hereinbefore explained, so as to bring the second cell on a level with the race. If the locking-bar, T, is shifted a degree further back, it comes over the tail of the second or middle catch-lever, R, so that on the cams coming round, the frame-lever, J, will be depressed to the extent corresponding to the second cam, and the third shuttle-cell of the drop-box will be raised to a level with the race. Similarly, by shifting the locking-bar, T, still further back, so as to come over the tail of the shortest catch-lever, R, the cam of largest radius will be caused to give an effective stroke, and so raise the fourth or lowermost shuttle cell to a level with the race. The locking bar, T, is shifted as the weaving action goes on, in accordance with the requirements of the pattern, by two rods, U, jointed to it, one on each side, outside of the frame-lever, J, through the slots in which the bar, T, projects. The two rods, U, which are curved to clear the spindle, S, are jointed at their back ends to the two arms of a crosshead, V, bolted to the upper end of a lever, W, carried on a stud-pin fixed in a curved arm, X, formed upon and projecting down from the bracket, L. The lever, W, with its crosshead, V, is so set that the points of connection of the rods, U, may always be as nearly as possible in a plane which passes through the centres, K, and the locking-bar, T, when the frame-lever, J, is in its middle position, in order that the movements of the frame-lever, J, may affect the position of the locking-bar, T, as little as possible. The lower end of the lever, W, carries an adjustable stud, Y, upon which is jointed a rod, Z, passing towards the front of the loom, and resting near its front end upon a roller, *a*. It is this rod, Z, which is acted upon by the pattern mechanism, and transmits all the movements governing the action of the cams, P. The pattern mechanism consists of a barrel, *b*, carried loosely upon a horizontal spindle, *c*, fixed in the side standards, *d*, of a small frame placed at the side of the loom to carry the pattern mechanism. Washers of leather are applied to the bosses of the barrel, to prevent it from being jerked too far when moved. The barrel is formed with a ratchet-wheel, *e*, upon one end, and a spur-wheel, *f*, upon the other end, and the spur-wheel is in gear with a pinion, *g*, upon a parallel screw-spindle, *h*, carried in bearings in brackets formed upon the standards, *d*. The barrel, *b*, is formed with holes disposed helically or spirally round it; the pitch of the helical line of holes on the barrel, and the pitch of the screw-spindle, *h*, together with the sizes of the wheels, *f* and *g*, being so proportioned that a nut-lever, *q*, carried by the screw-spindle, *h*, as it were, follows the helical line of holes upon the barrel being moved longitudinally a distance equal to the pitch of the helical line of holes at every turn of the barrel. The pitch of the teeth of the ratchet-wheel, *e*, corresponds to that of the holes on the barrel, as taken circularly, and at each revolution of the tappet-shaft, Q, the barrel is moved round to the extent of a hole by the action of a lever, *i*, centred upon the spindle, *c*, and acted upon by a cam, *j*, Fig. 6, on the shaft, Q. The pall, *k*, which is carried by the lever, *i*, and which acts upon the ratchet-wheel, *e*, is forked so as to be lifted out of gear by a pin on the back end of another pall, *l*, also carried on a pin on the lever, *i*. The second pall, *l*, is for causing the barrel, *b*, to turn backwards, or in the opposite direction to that in which it is driven by the pall, *k*, when required; the backward motion being produced by the action of the pall, *l*, on a ratchet-wheel, *m*, fast on the screw-spindle, *h*; the reversal of the motion being effected by simply lowering or raising the pall, *l*, by means of the link, *n*, the manner of actuating which link will be hereinafter described. The holes in the barrel, *b*, are contrived to receive pins or buttons, *p*, which have various amounts of projection from the barrel surface, according to the number of shuttles used. Thus in the present instance, in which arrangements are made for working four shuttles, there are pins, *p*, of three different sizes, corresponding, in fact, to the three cams, P, already described; the absence at any point of any pin, and the consequent inaction of all the cams, P, corresponding to the use in the pattern of the top shuttle, and the pins and cams corresponding severally to the other three shuttles, making four in all. The screw-spindle, *h*, carries two nut-levers, *q*, the bosses of which are tapped to work upon the thread of the spindle, the levers consequently moving longitudinally back and forwards, as the screw-spindle turns alternately in opposite directions. The nut-levers are formed with bevelled pointed projections, *r*, which come in contact with the pins, *p*, on the barrel, *b*, each lever assuming a position determined by the amount of projection of the particular pin, *p*, acting upon it at any one time. The nut-levers are, however, never both in action at once, but are thrown into and out of gear alternately. The lower ends are prolonged to act upon a horizontal crosshead, *s*, fixed upon the front end of the rod, Z, which transmits the pattern action to the actual shifting details, as already explained. Each nut-lever, *q*, is always pressed towards the barrel, *b*, by the crosshead of a lever, *t*, turning loosely on a spindle, *u*, fixed across the framing, *d*; the lever, *t*, being formed with an elongated boss, which has another short lever, *v*, upon it, acted upon by a powerful spring, *w*, in such a manner as to cause the lever, *t*, to press the nut-lever, *q*, towards the barrel. The rod, Z, has imparted to it, by the lever, W, which is acted upon by a spring, *x*, a constant tendency to push the crosshead, *s*, forward against whichever nut-lever, *q*, happens to be in action. With these arrangements, it follows that when an unoccupied hole on the barrel, *b*, is opposite to or against the projection, *r*, on the nut-lever, *q*, in action, the spring, *w*, will force back the nut-lever, and with it the crosshead, *s*, and rod, Z, as far as they can go; thereby causing the locking-bar, T, to occupy the front end of its slots, clear of the catch-levers, R, so that the revolution of the cams, P, at this period do not depress the frame-lever, J; and the shuttle drop-box, A, consequently remains in its lowest position, with its top cell on a level with the race. If, at the next shift of the barrel, *b*, a pin, p^1, Fig. 4, of the smallest or first size acts on the nut-lever, *q*, it will push it forward against the pressure of the spring, *w*. The spring, *x*, will then shift the lever, W, and rod, Z, a corresponding distance, the lever, W, bringing the locking-bar, T, back over the tail of the first catch-lever, R; the effect of which will be to lift up the shuttle drop-box, so as to bring its second shuttle cell on a level with the race in the manner already explained. If pins, p^2 and p^3, Fig. 4, of the second and third sizes, follow in succession upon the barrel, *b*, the nut-lever, *q*, will be pushed further forward at each shift, and the lever, W, in following it will shift the locking-bar, T, further back, so as in succession to act on the second and third catch-levers, and through their means cause the corresponding shift upwards of the shuttle drop-box. If the shuttle drop-box is to remain unaltered during more than one shift of the barrel, *b*, a pin of the size corresponding to the position of the drop-box is repeated in as many successive holes on the barrel as is necessary; or, if the drop box is in its lowest position, the proper number of holes are simply left unoccupied. The projection, *r*, of the nut-lever, *q*, is by preference never allowed to press against the surface of the barrel, *b*, at the places where the unoccupied holes occur; but is prevented from doing so by the lever, W, the crosshead, V, of which is fitted with an adjustable screw-pin, *y*, contrived to come against the bracket, X. This pin, *y*, is set so that the lever, W, cannot allow the rod, Z, to move so far backwards as will admit of the nut-lever touching the surface of the barrel; whilst the extent of movement actually allowed is sufficient to shift the locking-bar, T, quite clear of the catch-levers, R. The pin, p^3, is followed by another pin of the same size, in consequence of which the lowest shuttle-cell will be kept on a level with the race during two shifts of the barrel, *b*, that is to say, during two turns of the tappet-shaft, Q, corresponding to four single picks of the shuttle. The next hole on the barrel is, however, left unoccupied, which will have the effect of lowering the shuttle drop-box to its lowest position. But before proceeding to describe what takes place when the shuttle drop-box is lowered, it will be well to explain, to a certain extent, the action of the locking mechanism which holds the shuttle drop-box in position between the shifts.

As already mentioned, a notched segment, H, is formed upon or attached to the boss of the lever, F, for the purpose of locking the shuttle drop-box in position. This segment, H, is formed with four notches corresponding to the four positions the shuttle drop-box is in this instance constructed to assume, and the locking is effected by the entrance into one of these notches of the lower bent end of a locking-lever, *z*, carried upon a stud-pin attached to the side frame, G, of the loom. The upper arm of this locking-lever, *z*, is pulled backwards by a spring, 4, to give its lower end a tendency to enter one of the notches in the segment, H, when permitted to do so by the action of the cam, 5. The cam, 5, is fixed upon the tappet-shaft, Q; and it may here be mentioned that it has bolted to it the cam, *j*, which acts on the lever, *i*, giving motion to the barrel, *b*. The periphery of the cam, 5, is formed of two concentric surfaces, the one of smallest radius permits the lower end of the lever, *z*, to be kept in a notch of the segment, H, by the action of the spring, 4, during the greater part of a revolution; but when the projecting portion, 6, comes round at every revolution, it keeps out of the notch in the segment, H, against the power of the spring, during the time that the shuttle drop-box is or may be being shifted. In connection with the locking mechanism there is a stepped catch-piece, 7, formed or fixed upon the front end of the frame-lever, J, and acting with a catch-lever, 8, to regulate the descent of the shuttle drop-box. The catch-lever, 8, is carried on a stud upon the bracket supporting the lever, F, and its lower end carries a pin, 9, which is acted upon by an adjustable stop, 10, fast upon the rod, Z, against which stop the lever is always kept by a spring, 11. Supposing that at the next change of shuttle the drop-box is only required to be lowered one step, in which case, before the lever, F, can be moved, the locking-lever, *z*, must be shifted clear from the segment, H. This is effected by the coming round of the projecting portion, 6, of the cam, 5; and it is obvious that the locking-lever must at all times be quite clear before the cams begin to depress the frame-lever, J, should it have to be depressed (which is not the case, however, when the parts are in the position shown in Fig. 6). In other words, in all cases, whether the frame-lever, J, is about to be depressed or to be raised, the unlocking is effected, as far as regards the segment, H, some time before the most prominent portion of the cam, P, which next comes into effective action actually bears upon the frame-lever. This would allow time for the spring, M, to pull up the lever, F, further than it might be required to go, were it not for the catch-piece, 7, and lever, 8. When the shuttle drop-box is in its highest position, and the locking-bar, T, is at the back end of its slots in the frame-lever, J, as in Fig. 6; the bar, Z, being in its most forward position, allowing the spring, 11, to pull the lever, 8, so that its top end is thrown over the top catch of the piece, 7. Then if, on the next shift taking place, the shuttle drop-box is to fall, for example, to the extent of only one shuttle cell, the rod, Z, is moved one degree backwards by the action of the barrel, *b*, and turns the lever, 8, off the top catch of the piece, 7, but not further than will allow the second catch of this piece to come up to and be caught by it. This takes place before the locking-lever, *z*, is free from the segment, H, so that when the spring, M, comes into action it cannot lift the frame-lever, J, higher than the catch-lever, 8, will allow it to go. Then, as the cams, P, come round, the largest one does not again depress the frame-lever, J, to its previous position, for the locking-bar, T, is by this time moved forward clear of the tail of the catch-lever, R, corresponding to this cam. If the shift down of the shuttle-box had been to the extent of two or three cells, instead of only one, the corresponding movement of the rod, Z, would have shifted the catch-lever, 8, so as to hold the frame-lever, J, at the corresponding catch of the piece, 7. The position of the catch-lever, 8, remains unaltered, when no change of the shuttle drop-box is required, and prevents the frame-lever, J, from moving when the segment, H, is released. It is found desirable in practice to make the teeth of the segment, H, or the solid portions between the notches, to project to different extents; the top tooth projecting the most, the next tooth slightly less, and the two next ones slightly less still. The object of this arrangment is to insure the entrance of the locking-lever, Z, into the proper notch when a shift up to the extent of two or three shuttle cells takes place, the increased projection of the upper teeth catching the end of the locking-lever, Z, and preventing the comparatively more rapid movement from carrying the segment, H, and the parts connected with it, too far. If the pattern to be woven is of the class in which the second half is precisely like the first half, but with the various portions in an inverted order, a single nut-lever, *q*, upon the screw-spindle, *h*, will be sufficient, one half of the pattern being woven whilst the barrel turns in one direction, and the other half whilst the barrel turns in the opposite direction; the nut-lever being correspondingly carried longitudinally first in one direction and then in the other, so that, at the end of each complete repeat of the pattern, the parts are in the proper positions for starting a fresh repeat, a mere reversing contrivance being required at the end and at the beginning of the half pattern. In cases, however, where the pattern is not repeated, inversely in this way, two nut-levers, *q*, are required, as shown in the several figures of our example plate, F. These two nut-levers, *q*, are disposed so that their projections, *r*, upon which the barrel pins, *p*, act, are about half the length of the barrel apart; and contrivances are applied for throwing them alternately into action, the two never being in action together. The entire extent of pattern which is to be set out upon the barrel by means of the pins, *p*, whether this extent comprehends repetitions of complete repeats or of portions of repeats of the pattern, or forms in itself one complete repeat of the pattern, is divided into two portions, which are set out, one upon each half of the barrel, but so that one portion runs in one direction and the other portion in the opposite direction. Then, when the barrel is turning in one direction, that nut-lever, *q*, is in action, which is acted upon by the portion of the barrel which has its pins arranged to suit the portion of the pattern as running in the corresponding direction; and the other nut-lever, *q*, will come into action when the first one has traversed over the whole of its portion of the pattern; the barrel and nut-levers being simultaneously reversed to work over the portion of the pattern running in the opposite direction. Thus, on the second portion of the pattern being completed, the first nut-lever, *q*, which has been out of action in the meantime, will have returned to the point from whence it originally started, and on the reversal of the movements, this first nut-lever will come into action again, and commence a fresh repeat of the pattern. The reversing of the motion of the pattern barrel, *b*, and accompanying details, as well as the throwing the nut-levers into and out of gear, is effected by means of a tumbling-bar, 12, formed with end spindles which turn in bearings in the frame, *d*. On one of the end spindles which projects through the framing, *d*, there is fixed a lever, 13, to a pin on which is connected a spring, 14, attached below to a fixed bracket. The lever, 13, and spring, 14, are so disposed, that when the tumbling-bar, 12, is midway between its extreme positions, the spring pulls directly across the centre of the spindle; and on the tumbling-bar being shifted beyond this point towards either side, the spring immediately comes into effective action, to complete the shift of the bar. The nut-levers, *q*, are made with catches, 15; that of one lever being above the tumbling-bar, 12, and that of the other lever below it, so that when the tumbling-bar is raised it holds back one nut-lever, *q*, out of action, and when it is lowered it holds back the other, either nut-lever being in action when not thus held back. At the end of the helical line of pins, *p*, or holes on the barrel, *b*, corresponding to each portion of the pattern, there is inserted in the barrel a reversing pin, 16, which projects further from the surface of the barrel than the other pins, and is formed with a lateral projection. This lateral projection comes in contact at the proper time with one or other, as the case may be, of two catches, 17, capable of adjustment upon the tumbling-bar, 12. The reversing-pin, 16, by this means, turns over the tumbling-bar, 12, the latter portion of the movement being assisted by the spring, 14, as already explained. At the same time the pin, 16, also lifts the nut-lever, *q*, which has just been in action, away from the barrel, and into such a position that the tumbling-bar, 12, locks it out of action by means of the catch, 15; the movement of the tumbling-bar releasing the other nut-lever, *q*, which is immediately forced into action by the corresponding spring, *w*. The lever, 13, is formed in one piece with an arm or lever, 18, carrying a pin working in a slot in the link, *n*, of the pall, *l*, of the ratchet movement, by means of which the pattern mechanism is driven as already hereinbefore described. Thus, at every shift of the tumbling-bar, 12, the motion of the barrel, *b*, is reversed by bringing into gear the pall, *l*, and throwing out of gear the pall, *k*, or *vice versa*, by the downward or upward shift given to the link, *n*, by the lever, 18. The stroke or movement, of the tumbling-bar, 12, is limited each way by means of adjustable screw-pins, 19, tapped through lugs formed upon the lever, 18, and arranged so that one of them comes in contact with the top side of the lug, 20, cast upon the framing, *d*, when the lever, 18, moves down, whilst the other one comes in contact with the under side of the same lug, 20, when the lever, 18, moves up.

In constructing the apparatus, each of the springs, *w*, must be made powerful enough to overcome the springs, *x* and 11, as it has to move back the rod, Z, against the action of these springs, whilst the spring, *x*, has only to shift the locking-bar, T, backwards, and to move the rod, Z, forward to keep its crosshead in contact with the nut-lever, *q*, when the action of the barrel, *b*, forces forward this nut-lever against the action of its spring, *w*; the spring, 11, serving also at the same time merely to keep the pin, 9, of the lever, 8, forward against the stop, 10, on the rod, Z. The springs, *w* and *x*, are fixed to points in such positions that they act with less leverage in proportion as they become more stretched, being placed so that they act more nearly at right angles when not stretched, and gradually less so as they become stretched.

The variable movement of the frame-lever, J, may be obtained by means of a single cam, P, of one large step, and a single catch-lever, R, the tail of the catch-bar being formed with steps like the catch-piece, 7, fast on the frame-lever itself. In this case the cam is made of such a size as to give the greatest shift of which the shuttle-box is capable, and any lesser shift is obtained by shifting the locking-bar, T, forward more or less, so that, as the cam comes round, it does not depress the frame-lever J, until one of the steps of the catch-lever comes up and catches the locking-bar, T.

In practice, however, it is preferred to adopt the arrangement represented in the several figures of our example plate, and hereinbefore described, as in it advantage is taken of the smaller spaces

WASHING MACHINE

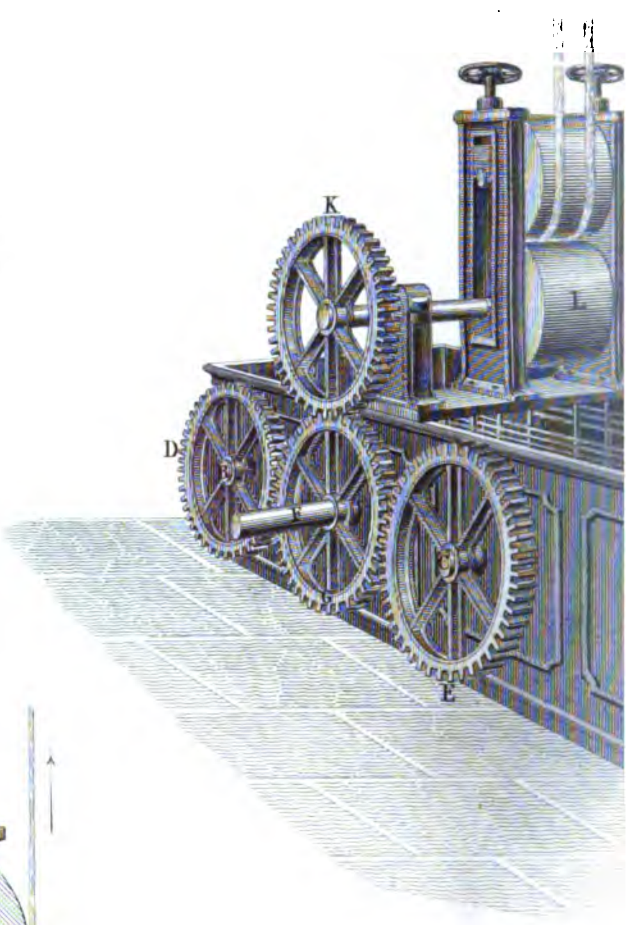
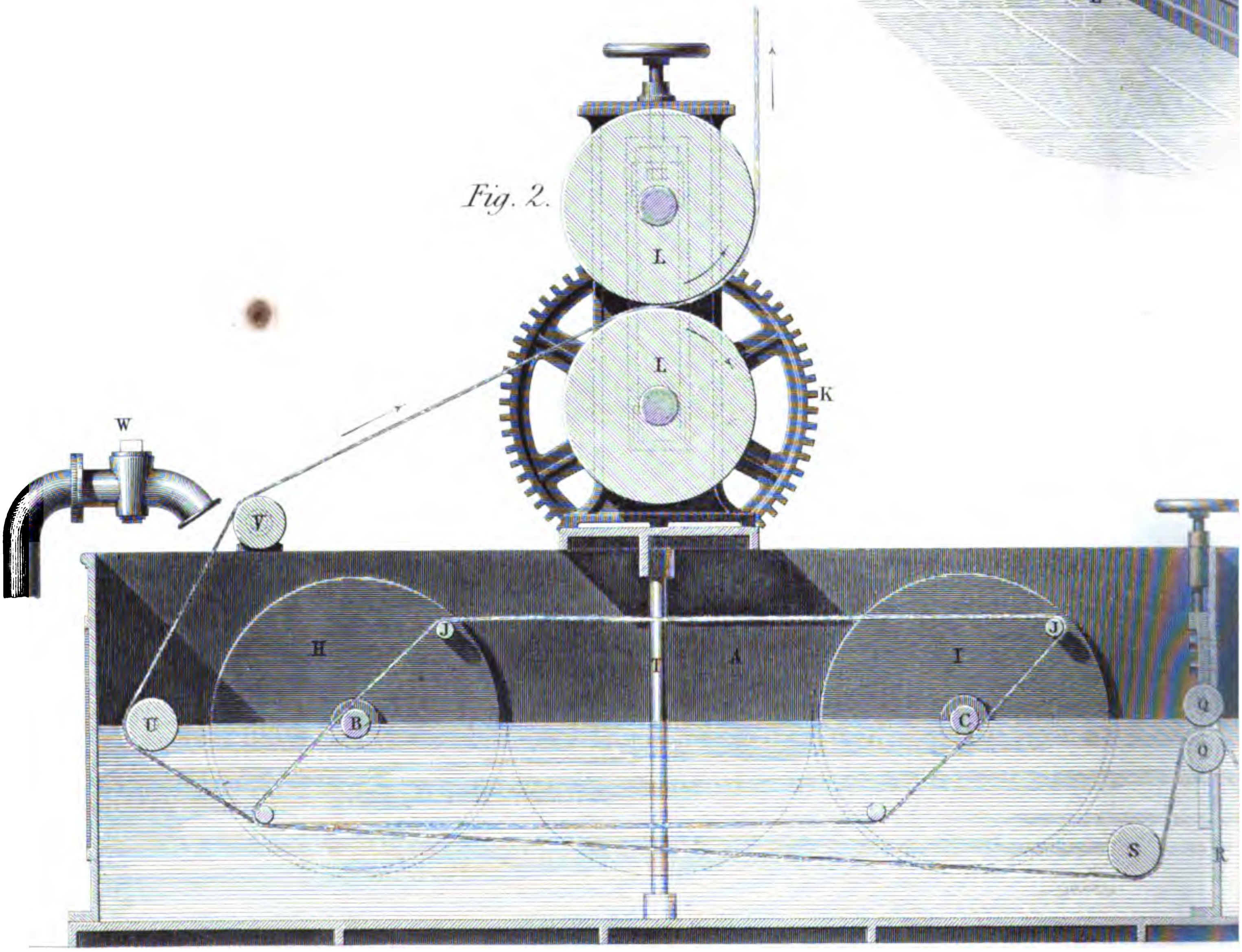


Fig. 2.

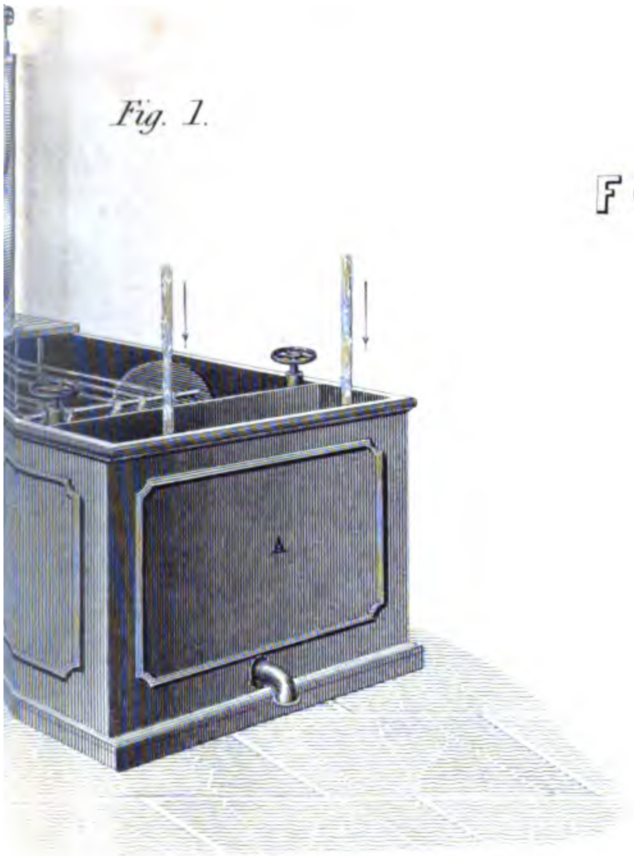


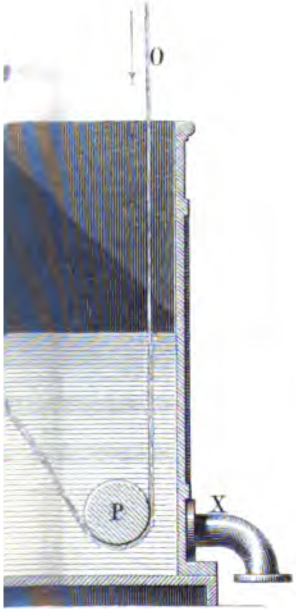
S

W. Johnson Offices for Patents

FOR TEXTILE FABRICS.

Fig. 1.





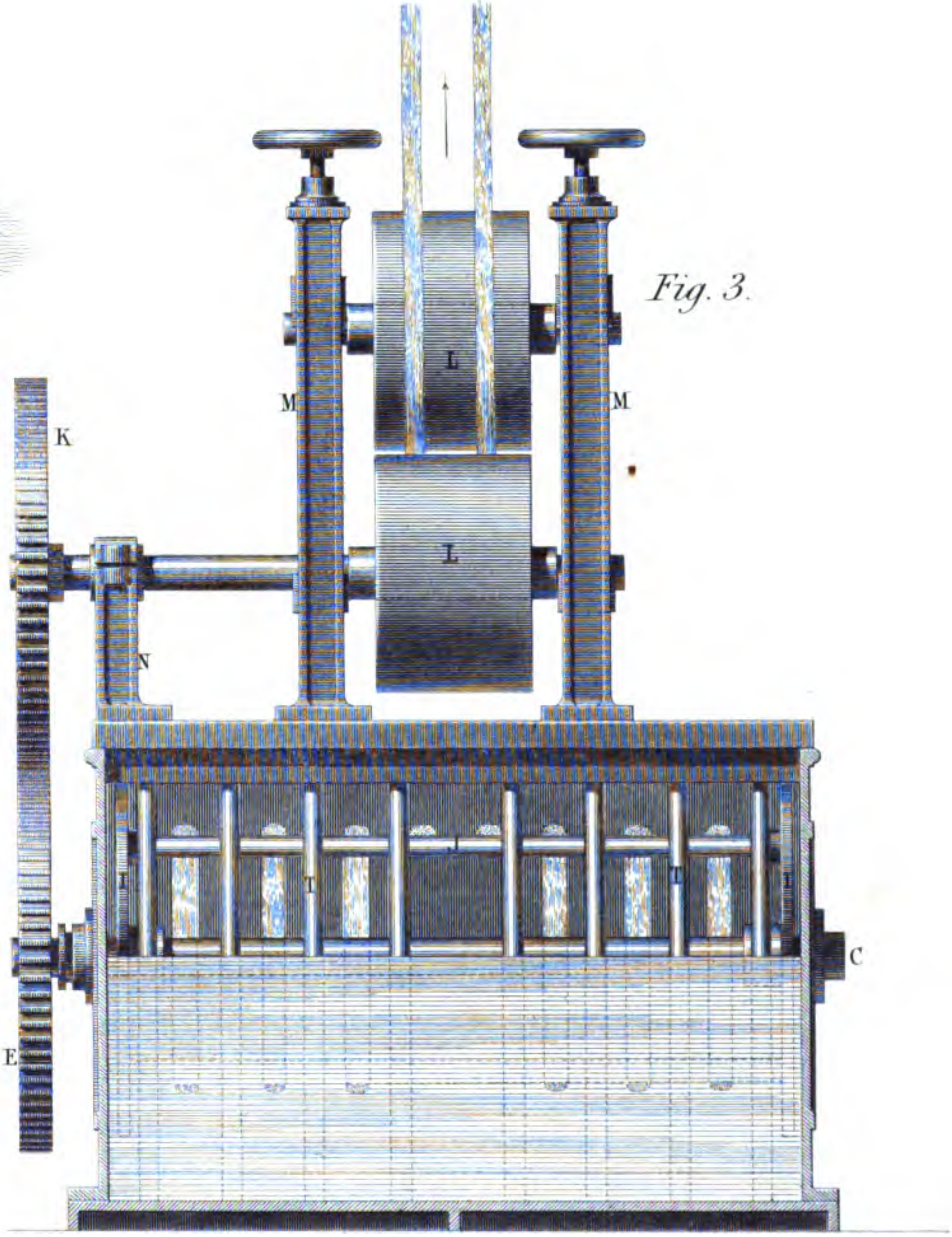


Fig. 3.

ALE

5 6 7 8 9 10 Feet

[illegible]

to be passed through with the lesser shifts of the shuttle-box, to make the shifting movement with such lesser shifts as slow as possible; whilst, with the single cam and catch-bar arrangement, the movement, on a comparatively small shift occurring, is as rapid as the latter portion of the movement, when a comparatively large shift takes place. In applying shifting shuttle-boxes to both sides of the loom-lathe, the movements of both shuttle-boxes may be derived from apparatus constructed on the hereinbefore described system, suitably modified in its minor details to suit the object in view. For example, if the loom is to be arranged for the weaving or manufacture of "corded" fabrics, that is to say, fabrics with occasional lines of coarser or thicker weft, this is accomplished in a very simple manner, by carrying a shaft across the loom to connect the shuttle-boxes on each side with the cam and pattern movement. Instead of setting the lever, F, on a stud-pin, as represented in Fig. 1, this lever is made fast upon a shaft set in bearings in the two ends of the loom-frame, and the opposite end of this shaft carries a single-armed lever, from the end of which a rod passes upwards to the shuttle-boxes on that side of the loom. Under such an arrangement, the cam or tappet action of the loom is contrived so that the shuttle-boxes shall shift at every pick of weft, so that the "cord" weft shuttle shall always be held and retained in its box after each pick, to whichever side it may be thrown.

In carrying out the arrangements for giving a variable taking-up of the cloth-beams in looms working with more than one shuttle, a motion is either taken from the shuttle drop-box, or from the pins of the pattern-barrel, which afford abundant means of taking any motion from them for working or acting upon the palls and catch of the take-up motion so as to give a variable feed in accordance with the nature of weft or stripe being woven in relation to the particular shuttle-box in use throughout the pattern.

This checking-loom certainly possesses many advantages, in so far, that patterns varying from 8 to 2000, or more shots, can be worked, requiring little time or trouble, and no extra expense in making the change from one pattern to another. Another advantage is, that the shuttles can be shifted to the extremes, or to any other positions found most desirable in arranging the pattern; whilst the loom is easily understood, simple to work, and not liable to go wrong.

WASHING MACHINE FOR PIECE GOODS.

EXAMPLE PLATE, G.

The three views on this plate illustrate a machine, first in perspective elevation, and then in plain geometrical section, presenting good studies for "effect," and contrasting the working drawing with the perspective picture. Fig. 1 is a perspective elevation of the washer complete, showing the whole of the gearing for actuating the moving details; Fig 2 is a longitudinal section on a larger scale; and Fig. 3 is a corresponding transverse section—that is, at right angles to Fig. 2. The main body of the machine consists of an open rectangular cistern, A, of cast-iron, which is kept about half full of the cleansing water. A couple of horizontal transverse shafts, B, C, are passed through this cistern, being carried in stuffing-box bearings in the two opposite side plates, and projecting through on one side to carry the spur-wheels, D, E. These shafts are made to revolve in the same direction by the revolution of the intermediate driving-shaft, F, carrying a third spur-wheel, G, in gear with the other two. The two shafts, B, C, have each a pair of end discs, H, I, fast upon them, to hold the diametrically opposed parallel rail-bars, J, which form the flat winces, or revolving frames, for acting upon the goods in the washing movement. These details, indeed, constitute the whole of the action.

The same central wheel, G, also drives an overhead wheel, K, of similar size, for actuating squeezing rollers, L. These rollers are of large diameter and of considerable breadth, and are set in bearings in the pair of vertical standards, M, carried on a cross bar on the top of the cistern, on which also is a low standard, N, for the bearing of the overhanging end of the bottom roller driving shaft. The bearings of this bottom roller are fixed; but those of the upper one are adjustable in the central vertical slots of their standards, by means of screws and hand-wheels at the top, to give any required pressure to the issuing goods.

In erecting the machine for work, the shafts and rails of the washing movement are set in one plane, and the water level is adjusted to the line of the shaft centres. The fabric to be cleansed is then fed in at one end, O, of the cistern, where two lengths are represented as being entered. Here it descends beneath a fixed guide-roller, P, and then passes between the pair of nipping rollers, Q, set in bearings on the side of a division plate, R, and adjustable by hand-wheels and screws. After leaving these rollers, the course of the goods is again downwards, beneath the fixed guide-roller, S, and thence between the first pair of vertical guide-bars, T. The direction is then round the under side of the flat wince at the opposite end of the cistern, the fabric being turned back over the bars, J. It then returns towards the front end of the cistern, and is similarly passed round the bars, J, of the disc, I, from the upper side—this return course being through the second pair or space of the division bars, T. The fabric finally returns through the third guide-space, and in contact with the wince bars, and is then passed up beneath the guide-roller, U, set at the water-level at the delivering end. From this point, it passes over the top of the external guide-roller, V, and is finally delivered through the squeezing rollers, L. As there are two lines of goods shown under treatment, it is obvious that both follow the same course.

SCREW CUTTING SLIDE LATHE.

EXAMPLE PLATE, H.

This carefully-drawn, and highly-finished example plate, H, of a screw cutting slide lathe, is given to illustrate some of the most recent improvements in this class of machine tools, and also to show the mode of treating such subjects as a finished drawing.

Fig. 1, is a complete front vertical elevation, and Fig. 2 a complete plan corresponding with Fig. 1.

The lathe-bed, A, supports the "dead-head," and one end of the sliding bed-piece, B, on which the saddle, C, is mounted. This bed-piece, B, which is shown removed about two feet away from the dead-head, can be adjusted so as to form any convenient gap, in accordance with the figure of the material to be operated upon, and when cutting screws, or turning, shafting, and such like, the bed-piece can be brought quite close up to the dead-head. The main screw, D, is carried in brackets bolted on to the front part of the bed, and is driven through a series of change wheels in accordance with the pitch of screw required to be cut. Between this main screw, D, and on the lathe bed-piece, B, a rack, E, is formed, extending the whole length of the bed-piece, so that a pinion carried by the saddle, O, can, when in gear with the rack, cause the back and forward traverse of the saddle. The shaft, F, which runs along the back of the lathe, carries two sliding screws, G, in gear with two worm-wheels, H. One of which is carried on the back end of the transverse screw, I, of the slide in the saddle, O, and when in gear, through the motion of the shaft, F, regulated by the series of change wheels, a regular transverse motion of the slide and tool holder, J, is the result. By so proportioning the speed of the main screw, D, with that of the transverse screw, I, of the slide, any angular or conical figure can be thus accurately turned by the self-acting duplex motion of the cutting tool holder, J. The principal weight of the saddle and the vertical force of the cut is sustained upon an incline formed on the front slide of the bed-piece, B, and has the effect of bringing the vertical parts of the slides of the saddle into close contact, whereby the traverse of the saddle, in a perfectly straight line, is thus ensured, whilst the best form of resistance to the thrust of the cutting tool is by this means presented. The steadiness of the saddle, and consequent accuracy of its motion, is one of the most difficult features in any self-acting lathe, and our example plate, M, shows very clearly the value of the important improvements recently made by Messrs Greenwood & Batley, Machinists, Leeds. The other parts of this lathe resemble many of those now in general use, so that we consider a lengthened description quite unnecessary at this advanced stage of the work. The front elevation, Fig. 1, is the first view to be drawn, and is begun by drawing a horizontal line for a base upon which the whole machine is to be erected. From this elevation the different parts must be squared down to the plan view, Fig. 2, immediately underneath, and with reference to what we have given whilst treating of other drawings in this work the practical draughtsman will have no hesitation how to proceed with such a subject as the one we have just described. Some of these lathes, represented by our example plate, M, have been supplied by Messrs Greenwood & Batley to the Royal Arsenal, Woolwich, and have given every satisfaction, and having been accurately drawn from the actual machine itself, we recommend this subject as being worthy of the attention of the draughtsman.

Our task is accomplished. The pages now before the student are given to him in the hope that they may be found to form—

> "A volume of detail, where all is orderly set down,"

and that they really redeem the promises held out in our prefatory remarks. We have endeavoured to bring before the reader all that can be of practical service to him in the attainment of that most useful art, mechanical drawing, commencing with illustrated descriptive details of the several instruments employed in drawing, to enable the student to understand more readily the purpose of their construction, as well as to enable him to select those instruments most suitable for his purpose, or to refer to the description of others not so generally met with. From this introductory essay, the student has been conducted from the simplest rudimentary instruction in the formation of geometrical figures, onwards through the elementary details of mechanical structures, to the delineation of the complete machine. The British measures of length now used in this work enables the practical student to enlarge or reduce our examples with ease, and transfer them to his drawing-board, as well as imparting to the work a more extended sphere of usefulness. We have also made considerable and important additions to the isometrical projections and architectural sections of our work, which we doubt not will be appreciated for their utility by the mechanical draughtsman, as well as proving of assistance to the student of architectural drawing. Our series of example plates are nearly all new, and have been extended. They have been drawn and engraved with the greatest care, and we may, without the slightest approach to self-laudation, say that the student will find in them a store of valuable matter for study. Other portions of the work have also been enlarged with the view of giving the student all the assistance he requires.

No book, however profoundly it may be designed, or however clearly it may be written, can make a draughtsman. To certain inherent qualifications, the ambitious student must add attentive assiduity and patient toil. The first elements will be useful; the second are absolutely essential. Let each of our readers recall the admirable words of Gibbon:—"Every man who rises above the common level has received two educations—the first from his teachers; the second, more personal and important, from himself." These are words which Bacon would have said must be "chewed and digested." Another able writer has most happily said—"The student must rest assured that, without industry, and that confidence which an ardent love of fame, and an enthusiastic desire for improvement, never fail to inspire, he will not become eminent; and he must remember that society forms a very different opinion of the man of sound judgment and perseverance, who seldom fails in his attempts, and the fanciful and vain, who, whilst they imagine themselves *more capable* to learn than others, pass their lives in indolent or trifling pursuits, without acquiring that knowledge of their profession which ensures accuracy and success in practice." These are the well-considered ideas of men who have combined both thought and action. Their pithy eloquence embodies whole chapters of matter worthy of sinking deeply into the student's memory, where they must arouse him from any dreamy and deceptive contemplations of comparative excellence; for the superficial thinker is but too apt to be thus led away, forgetting that such comparisons continually lessen the advantages on his side, as the acquisitions which he perhaps superciliously boasts are mouldering away.

If the work now before the reader fulfils but a small portion of the purposes for which we have designed it, we shall rest satisfied in having accomplished our mission of doing something towards the extension and diffusion of that particular branch of industrial education hitherto neglected to a great extent in this country, or at least rendered more difficult of attainment, owing to the want of a suitable manual of instruction. With these views, and to further such ends, we have written this work, and in such a spirit do we now commit it to the world of industrial readers. Perhaps we may be permitted to hope, that in years to come there may be many among those readers who, having by well-directed industry attained a position in their profession, will recall with feelings of grateful esteem the instruction and assistance they received in early life from the pages of the PRACTICAL DRAUGHTSMAN.

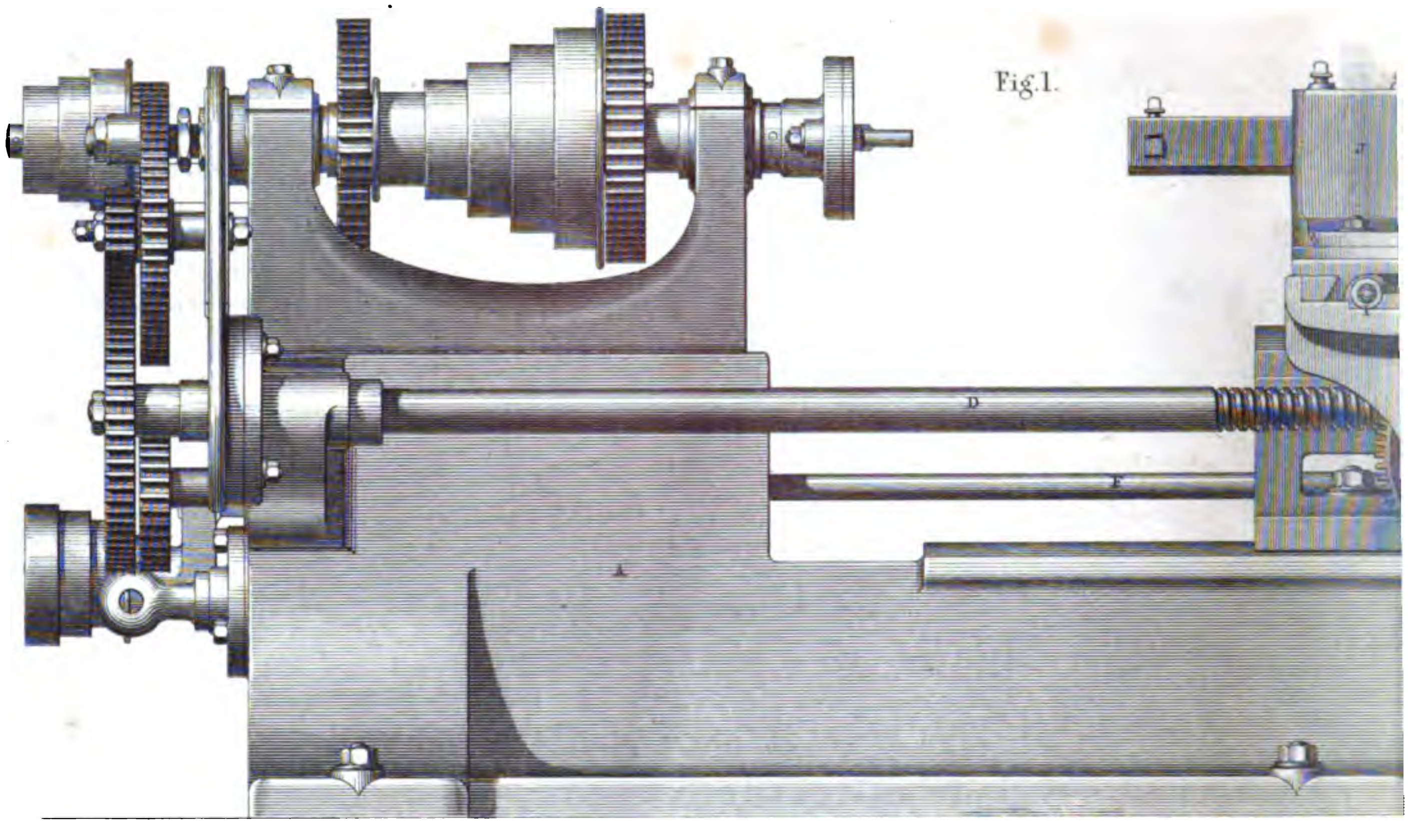


Fig. 1.

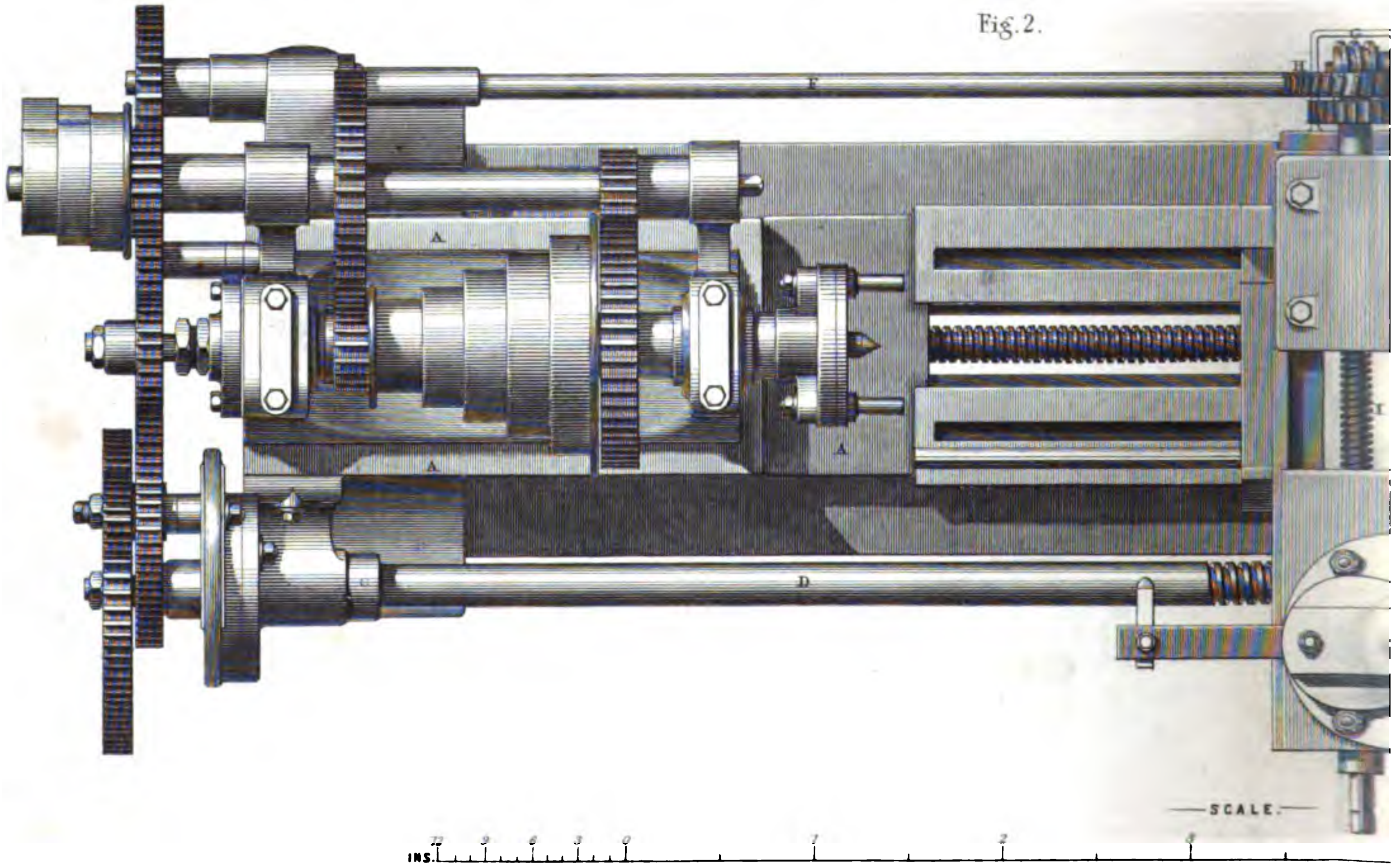
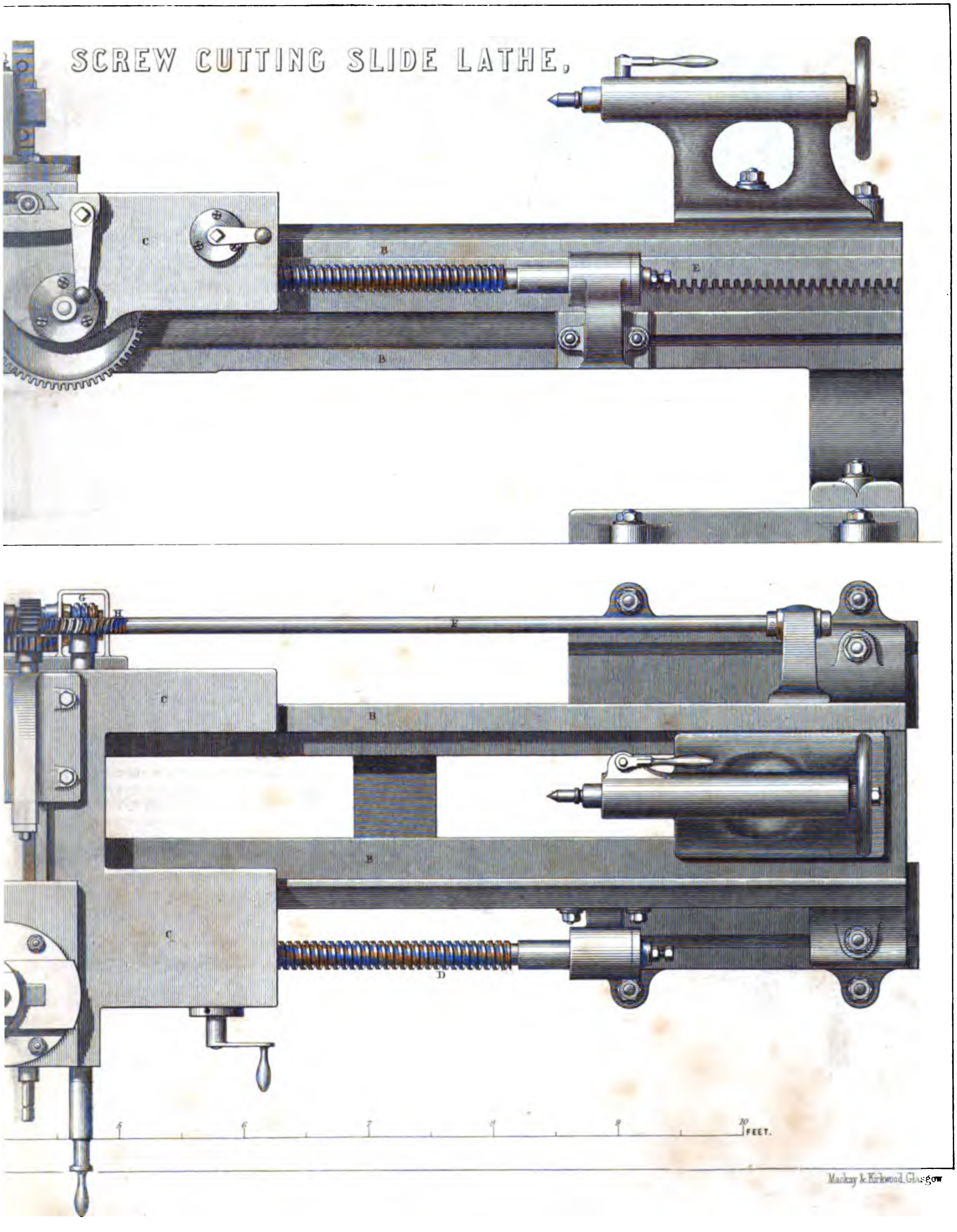


Fig. 2.

W. Johnson, Patent Offices
47 Lincoln's Inn Fields & Glasgow



SCREW CUTTING SLIDE LATHE,

Mackay & Kirkwood, Glasgow